Book Review

Hydrodynamic Instabilities, by François Charru, Cambridge University Press, 2011; ISBN 978-0-521-14351-6

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Hydrodynamic (in)stability analysis is the process of examining the response of laminar flow to a disturbance of infinitesimal or finite amplitude. It has a rich history, going back to George Stokes, Lord Kelvin, Lord Rayleigh and Osborne Reynolds in the late nineteenth century.

The book is divided informally into two parts. The first part deals with the linear stability problems and consists of seven chapters. The second part involves four chapters and is dedicated to nonlinear systems and nonlinear instability issues.

Chapter 1 provides a brief introduction to the subject. The elementary notions from dynamical systems theory, like phase space, phase portrait, linear stability of a fixed point, together with bifurcations in finite-dimensional dynamical systems, are presented first. Model examples from hydrodynamics include the stability of a soap film, a bubble, a colloidal suspension, convection in a ring and the development of structures as in the case of double diffusion of heat and matter. The chapter closes with a concise explanation of the phenomenon of transient growth of infinitesimal perturbations. This effect is connected with the non-normality of the linearised operator in a considered stability problem.

Chapter 2 discusses *Instabilities of fluids at rest*. Some details of the general stability analysis through Jeans' formula as a simplified model of a gravitational instability of a cloud of interstellar gas are explained. Then the application of the presented technique to particular flow instability situations is

described and discussed. These include: the Rayleigh–Taylor interface instability, the Rayleigh–Plateau capillary instability, the Rayleigh–Bénard thermal instability and the Bénard–Marangoni thermocapillary instability. These phenomena form a natural basis, which allows us to introduce and understand such concepts as control parameter of an instability, symmetry-breaking bifurcations, and pattern formation.

Chapter 3 deals with *Stability of open flows: basic ideas*. A terse survey of necessary mathematical tools for the analysis of this class of flows is presented with examples based on the Ginzburg–Landau (GL) equation. Linear dynamics of a wave packet with the help of the stationary phase method is studied first. Subsequently, the precise definition of stability in the Lyapunov sense, together with the notion of asymptotic stability, is formulated. Also, linear stability and two types of instability, namely, convective and absolute, are presented and explained in terms of the Green function. The criteria for linear stability, and for instabilities both convective and absolute, are also established and illustrated. The chapter ends with discussion of Gaster's relation (that is, a transformation formula, used to convert the growth rate of temporal instability to an equivalent spatial growth rate).

Chapter 4 treats the *Inviscid instability of parallel flows*. After some qualitative remarks about Kelvin–Helmholtz instability, the linearised perturbation equations for infinitesimal disturbances when viscous effects are negligible, are derived. Using the Squire theorem, the Rayleigh equation is obtained and analysed. This equation is then applied to Couette flow and flows with uniform vorticity. The important Rayleigh theorem, which states the necessary

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condition for instability of a parallel flow, is formulated and proved also. Various aspects of the Kelvin–Helmholtz instability mechanism are the subject of subsequent section. The stability of Taylor–Couette flow between two coaxial cylinders, to which the following section is devoted, is itself the theme of considerable texts. So, only the essential physics of the problem is presented with the Rayleigh stability criterion and the Taylor critical criterion in terms of Taylor number.

Chapter 5, *Viscous instability of parallel flows*, logically continues the study introduced in Chapter 4. It begins with the celebrated experiment of Reynolds on the instability of Poiseuille flow in a tube then addresses the boundary layer flow on a flat surface. The next section presents the general mathematical tools for these problems: the suitable perturbation equations, the generalized Squire theorem, and the famous Orr–Sommerfeld equation. The destabilizing effect of viscosity near the wall related to the non-slip condition, among others, is also partially explained. The remaining sections in this chapter present detailed study for Poiseuille flow, both plane and in a pipe, and for boundary layer flows, with conclusions.

Chapter 6 concerns *Instabilities at low Reynolds number*. Such instabilities can arise in diverse fluidal situations. Two cases are considered: liquid films falling down an inclined plane subject to gravity, and sheared liquid films. The basic mathematical apparatus is presented including Orr–Sommerfeld equations with the appropriate boundary conditions on a deformable interface, solved by regular asymptotic series. Numerical and experimental studies are performed and discussed also. A brief exposition of the stability problem of sheared liquid film (in the case of viscous flow of two liquids) to long-wavelength disturbances, is fulfilled by dimensional analysis.

Avalanches, ripples, and dunes are the subject of chapter 7. The author is a well-known specialist in this field. First, the avalanche phenomenon is described and modeled both theoretically and experimentally. A simple criterion for instability is found. Some approaches to sediment transport modeling are then presented: hydraulic, and involving relaxation phenomena. The structures, which are visible on a granular bed sheared by a liquid flow, namely, ripples

and dunes are outlined briefly. The formation of subaqueous ripples in steady viscous flow is described, based on a model for the erosion and deposition of the moving grains. Thereafter, an analogous discussion of the origin of ripples in oscillating flow is given. The last section presents an analytical approach to the description of subaqueous dunes, based on the Saint–Venant equation. Unfortunately, this simple model has no possibility to properly reproduce the process of dune formation in terms of instability mechanisms.

The four final chapters cover the topics of nonlinear dynamics and bifurcation.

Chapter 8, *Nonlinear dynamics of systems with few degrees of freedom*, concentrates on describing various types of nonlinear oscillators governed by ordinary differential equations. It starts with a discussion of a classical problem of a particle in a double-well potential. This is used for the explanation of the idea of saturation of an instability. A concise discussion is given to the van der Pol oscillator and the Duffing equation. Amplitude saturation in the van der Pol equation is studied, whereas the frequency correction is analysed in the Duffing oscillator. The difficult and very interesting problems connected with the forcing of nonlinear oscillators are briefly addressed also. At last, the spatially periodic solutions of the Kuramoto–Sivashinsky equation are analysed, showing that, in the case of small Fourier modes, its behaviour can be described by the Landau-like amplitude equation near the instability threshold. It should be noted that the Landau amplitude equation serves as a prototype of a system with weakly nonlinear properties, which can be interpreted as a solvability condition of a differential (nonlinear) equation.

An elementary introduction to *Nonlinear dispersive waves* is presented in chapter 9. The deep water nonlinear Stokes wave is analysed briefly together with its instability (Benjamin–Feir (BF) instability). The general nature of BF instability is illustrated by means of the Klein–Gordon (KG) equation and the nonlinear Schrödinger (NLS) equation. Instability mechanism due to direct interactions among Fourier modes is described and explained using the KG equation. An alternative approach admits to an interpretation of BF instability in terms of the

modulation instability based on the NLS. The chapter finishes with the brief derivation of the NLS equation for the KG wave and some terse comments on the resonance related to a quadratic nonlinearity.

Chapter 10 is devoted to the *Nonlinear dynamics of dissipative systems*. A weakly nonlinear formalism is sketched which leads to the generic envelope equation (GL equation). For a model equation which describes the Rayleigh–Bénard convection, the suitable GL equation is derived. The stability analysis of stationary solutions of the GL equation is conducted and the Eckhaus instability criterion is obtained and interpreted. In this context, some experimental facts regarding the Rayleigh–Bénard convection and the Couette–Taylor flow are presented and discussed. Then, the analogous analysis is executed in the case of travelling waves near the instability threshold. The GL equation with complex coefficients is derived, the stability of the nonlinear travelling wave is examined, and the Benjamin–Feir–Eckhaus instability criterion is formulated. A role of the Tollmien–Schlichting waves in the process of laminar turbulent transition is described also. Translational and Galilean invariance of the model evolution equation expressed in the form of a conservation law, which is finally analysed, leads to a nontrivial coupling of two nearly neutral phase modes. An experimental evidence of this fact is illustrated by studies of the secondary instability of waves at the interface between two superposed sheared fluids.

The last chapter, chapter 11, is concerned with *Dynamical systems and bifurcations*. It provides the reader with minimal basis in dynamical systems theory and the necessary understanding of the terminology and methods used in the modern bifurcation theory. The ideas of generalized flow, trajectories, invariant sets (attractors), manifolds, Poincaré sections, etc., are introduced and explained. The Lorenz system is mentioned to illustrate the existence of very complex invariant sets having fractal structure (“strange attractors”). Linear stability analysis for the simplest non-trivial invariant sets, fixed points, is discussed. After explaining the notion of topological equivalence, the local hyperbolic theory is sketched with the concepts of stable and

unstable manifolds. These ideas are then used to define the center manifold, normal forms, structurally stable systems and genericity. The necessary conditions for structural stability using the notion of codimension are formulated. Thereafter, a bifurcation notion of a parameter-dependent system on a manifold is defined and two generic codimension 1 bifurcations, tangent (fold) and Andronov–Hopf, are briefly studied. The chapter ends with the spectacular example of a bifurcation of codimension 2 connected with the stability of a film flowing down an incline plane.

The book closes with an appendix containing derivation of the Saint–Venant equations.

Each chapter contains numerous exercises. Some of the problems and calculations fill gaps in the text. I strongly encourage readers to do the exercises because these subjects help the students understand better the essence of the described phenomena.

I am satisfied that I found in the book 11 short biographies of some of the most important persons in the study of hydrodynamic instabilities: Bagnold, Chandrasekhar, Helmholtz, Kapitza, Kelvin, Landau, Poincaré, Rayleigh, Reynolds, Stokes, and Taylor. I am disappointed by the lack of inclusion of Eberhard Hopf, a founding father, among others, of bifurcation theory! (See e.g. Denker M., *Jahresber. Deutsch. Math.-Verein.* **92** (1990), no. 2, 47–57; Icha A., *Nieuw. Arch. Wisk.* (4). **12** (1994), no. 1–2, 67–84).

Prof. Charu’s book is written in a precise and very readable style. There are many useful remarks, comments and figures throughout. It can be warmly recommended to anybody who is interested in hydrodynamics and fluid-like dynamical systems, and who has a basic knowledge of fluid mechanics, dynamical systems theory, and applied mathematics. Undoubtedly, this book will help a new generation of hydrodynamicists to stimulate the activity in this important area.

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