12th International Conference on Geometry and Applications

Varna (Bulgaria), September 1–5, 2015

The conference took place in the International Home of Scientists “F. Joliot-Curie”, St. Constantine and Helena, Bulgaria.

The Scientific Committee which led the conference consisted of W. Benz (*coChairman*), G. Georgiev, S. Grozdev, J. Davidov, R. Fritsch, H. Havlicek (*representing Journal of Geometry*), S. Ivanov, H. Karzel, A. Kreuzer (*representing Journal of Geometry*), H.–J. Kroll, I. Landjev, M. Manev, M. Marchi, O. Muskarov, S. Pianta, I. Sabitov, A. Salimov, U. Simon, G. Stanilov (*Chairman*), C. Udriste, H. Wefelscheid. The Organizing Committee consisted of G. Stanilov, S. Grozdev, M. Teofilova, S. Staneva.

Laudations in honour of Ralph-Hardo Schulz and Herbert Zeitler were given by Grozio Stanilov and Heinrich Wefelscheid, respectively. The following are abstracts of talks presented at the conference.

Loxodromes on Canal Surfaces

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A canal surface in Euclidean 3-space E^3 can be defined as envelope of a moving sphere whose trajectory of centers is a spine curve $m(u)$ with varying radius $r(u)$. Thus the parametrization of the canal surface is

$$C(u, v) = m(u) + r(u) \left(\sqrt{(1 - r'(u)^2)} n(u) \cos v \right. \\ \left. + \sqrt{(1 - r'(u)^2)} b(u) \sin v - t(u) r'(u) \right),$$

where u is arc-length parameter, $0 \leq v \leq 2\pi$, t , n and b are the unit tangent, principal normal and binormal vectors of $m(u)$, respectively. The first fundamental form in $\{C_u, C_v\}$ of the canal surface C is given by the following equation

$$ds^2 = ((1 - \kappa g \cos v - h')^2 + (g\tau + h\kappa \sin v)^2 + (g' - h\kappa \cos v)^2) du^2 + 2(g^2\tau + gh\kappa \sin v) dudv + g^2 dv^2,$$

where $g = g(u) = r(u)\sqrt{1 - r'(u)^2}$ and $h = h(u) = r(u)r'(u)$; $\kappa = \kappa(u)$ and $\tau = \tau(u)$ are the curvature and torsion of $m(u)$, respectively [1].

Thus the arc-length of any curve on the canal surface C between u_1 and u_2 is given by

$$s = \int_{u_1}^{u_2} \sqrt{F(u, v)} du,$$

$$F(u, v) = (1 - \kappa g \cos v - h')^2 + (g\tau + h\kappa \sin v)^2 + (g' - h\kappa \cos v)^2 + 2(g^2\tau + gh\kappa \sin v) \frac{dv}{du} + g^2 \left(\frac{dv}{du} \right)^2.$$

A curve on the canal surface $C(u, v)$ in E^3 which cuts all meridians ($v = \text{constant}$) or parallels ($u = \text{constant}$) at a constant angle is called as a loxodrome. The angle between the loxodrome and the meridian is given by the following equation

$$\cos \theta = \frac{G(u, v)}{\sqrt{H(u, v)}},$$

where

$$G(u, v) = ((1 - \kappa g \cos v - h')^2 + (g\tau + h\kappa \sin v)^2 + (g' - h\kappa \cos v)^2) du + (g^2\tau + gh\kappa \sin v) dv,$$

$$H(u, v) = ((1 - \kappa g \cos v - h')^2 + (g\tau + h\kappa \sin v)^2 + (g' - h\kappa \cos v)^2) du^2 + 2((1 - \kappa g \cos v - h')^2 + (g\tau + h\kappa \sin v)^2 + (g' - h\kappa \cos v)^2) \times (g^2\tau + gh\kappa \sin v) dudv + ((1 - \kappa g \cos v - h')^2 + (g\tau + h\kappa \sin v)^2 + (g' - h\kappa \cos v)^2) g^2 dv^2.$$

From this equation, the differential equation of the loxodrome which cuts all meridians at a constant angle θ is

$$\begin{aligned} & (\cos^2 \theta ((1 - \kappa g \cos v - h')^2 + (g\tau + h\kappa \sin v)^2 + (g' - h\kappa \cos v)^2) g^2 - \\ & - (g^2\tau + gh\kappa \sin v)^2) \left(\frac{dv}{du} \right)^2 - 2\sin^2 \theta ((1 - \kappa g \cos v - h')^2 + (g\tau + h\kappa \sin v)^2 + \\ & + (g' - h\kappa \cos v)^2) (g\tau + h\kappa \sin v) \frac{dv}{du} \\ & = \sin^2 \theta ((1 - \kappa g \cos v - h')^2 + (g\tau + h\kappa \sin v)^2 + (g' - h\kappa \cos v)^2)^2. \end{aligned}$$

In this talk, we also compute the differential equations and arc-lengths of loxodromes on canal surface which cut all parallels at a constant angle. Moreover we give examples of loxodromes on some canal surfaces such as cylinder, helical canal surface, cone and torus.

Mathematical Subject Classification (2010). 53A04, 53A05.

Keywords. Loxodrome, canal surface, Euclidean space.

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Curvature Properties of Riemannian Manifolds with Circulant Structures

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An interesting problem in the geometry of Riemannian manifolds with additional structures is the studying of sectional curvatures of some special two-planes in the tangent space of the manifolds, for example [1,2,3].

We introduce a four-dimensional differentiable manifold M with a Riemannian metric g , whose matrix in local coordinates is a circulant matrix with first row

$$(g_{1j}) = (A \ B \ C \ B), \quad (1)$$

where $A = A(p), B = B(p), C = C(p) \in FM, p \in M$. We assume that

$$A > C > B > 0. \quad (2)$$

Then the conditions for g to be a positive definite metric are satisfied [4].

Let q be an endomorphism of the tangent space $T_p M$. We suppose the local coordinates of q are given by the circulant matrix, whose first row is

$$(q_1^j) = (0 \ 1 \ 0 \ 0). \quad (3)$$

Then q satisfies

$$q^4 = \text{id}, \quad q^2 \neq \pm \text{id}.$$

The structure q of the manifold (M, g, q) is an isometry with respect to the metric g . Further, x, y, z, u will stand for arbitrary elements of the algebra on the smooth vector fields on M or vectors in the tangent space $T_p M$. A basis of type $\{x, qx, q^2x, q^3x\}$ of $T_p M$ is called a q -basis. We say that the vector x induces a q -basis of $T_p M$.

Let (M, g, q) be a manifold with the property $R(qx, qy, qz, qu) = R(x, y, z, u)$, where R is the curvature tensor of type $(0, 4)$. Let a vector x induce a q -basis of $T_p M$. Then

- (1) $\mu(x, qx) = \mu(qx, q^2x) = \mu(q^2x, q^3x) = \mu(q^3x, x), \mu(x, q^2x) = \mu(qx, q^3x)$, where μ is the sectional curvature of the q -sections;

- (2) the sectional curvature $\mu_1(\varphi)$ of the q -section $\varphi = \angle(x, qx)$ is expressed by

$$\mu_1(\varphi) = \frac{1}{1-\cos^2 \varphi} \left((1 - 4 \cos^2 \varphi) \mu_1\left(\frac{\pi}{2}\right) + \frac{3}{4} (\cos \varphi + 2 \cos^2 \varphi) \mu_1\left(\frac{\pi}{3}\right) + \frac{3}{4} (2 \cos^2 \varphi - \cos \varphi) \mu_1\left(\frac{2\pi}{3}\right) \right);$$

- (3) the sectional curvature $\mu_2(\theta)$ of the q -section $\theta = \angle(x, q^2x)$ has an analogous expression by θ .

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Identities for the Vector-Rows of the Essential Matrix

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We consider two perspective projections, or equivalently two camera systems, determined by the pairs $(\overline{S}, \omega)$ and $(\overline{S'}, \omega')$. Assume that their normalized camera matrices $\mathbf{P}$ and $\mathbf{P}'$ are related to the first camera frame. Then, the first matrix is $\mathbf{P} = [\mathbf{I} | \mathbf{o}]$, where $\mathbf{I}$ is the 3×3 identity matrix and $\mathbf{o} = (0, 0, 0)^T$ is three-dimensional zero vector. The second matrix expresses the position of the second camera with respect to the first one, and therefore it can be written in the form $\mathbf{P}' = [\mathbf{R} | \mathbf{t}]$, where $\mathbf{R}$ is the 3×3 rotation matrix and $\mathbf{t} = \overrightarrow{\overline{S} \overline{S'}}$ is a three-dimensional translation vector. Let $(x_{\overline{S'}}, y_{\overline{S'}}, z_{\overline{S'}})$ be the Cartesian coordinates of the second projection center $\overline{S'}$ with respect to the

first camera frame. Then, the position vector $\overrightarrow{S S'} = (x_{\overline{S'}}, y_{\overline{S'}}, z_{\overline{S'}})^T$ of $\overline{S'}$ coincides with the translation vector $\mathbf{t}$ and the skew-symmetric matrix $\mathbf{S}$ in the equation $\mathbf{E} = \mathbf{S} \mathbf{R}$ is given by $\mathbf{S} = \mathbf{t}_\times = \begin{bmatrix} 0 & -z_{\overline{S'}} & y_{\overline{S'}} \\ z_{\overline{S'}} & 0 & -x_{\overline{S'}} \\ -y_{\overline{S'}} & x_{\overline{S'}} & 0 \end{bmatrix}$. The

rotation matrix $\mathbf{R} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$ in the same equation has vector-

rows $\mathbf{r}_i = (r_{i1}, r_{i2}, r_{i3})$, $i = 1, 2, 3$. Similarly, if $\mathbf{E} = \begin{bmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \end{bmatrix}$ is

the essential matrix determined by $(\mathbf{m}')^T \mathbf{E} \mathbf{m} = 0$, the vector-rows of $\mathbf{E}$ are denoted by $\mathbf{e}_i = (e_{i1}, e_{i2}, e_{i3})$, $i = 1, 2, 3$.

Using the matrix equation $\mathbf{E} = \mathbf{S} \mathbf{R}$, we obtain the basic relations between vector-rows of the essential matrix and the position vector of the second projection center. If

$$(\mathbf{e}_1 \cdot \mathbf{e}_2)(\mathbf{e}_1 \cdot \mathbf{e}_3)(\mathbf{e}_2 \cdot \mathbf{e}_3) < 0,$$

then

$$\begin{aligned} (\mathbf{e}_1 \cdot \mathbf{e}_1)(\mathbf{e}_1 \cdot \mathbf{e}_2)(\mathbf{e}_1 \cdot \mathbf{e}_3) &= -(\mathbf{e}_2 \cdot \mathbf{e}_3) \left\{ (\mathbf{e}_1 \cdot \mathbf{e}_2)^2 + (\mathbf{e}_1 \cdot \mathbf{e}_3)^2 \right\}, \\ (\mathbf{e}_1 \cdot \mathbf{e}_2)(\mathbf{e}_2 \cdot \mathbf{e}_2)(\mathbf{e}_2 \cdot \mathbf{e}_3) &= -(\mathbf{e}_1 \cdot \mathbf{e}_3) \left\{ (\mathbf{e}_1 \cdot \mathbf{e}_2)^2 + (\mathbf{e}_2 \cdot \mathbf{e}_3)^2 \right\}, \\ (\mathbf{e}_1 \cdot \mathbf{e}_3)(\mathbf{e}_2 \cdot \mathbf{e}_3)(\mathbf{e}_3 \cdot \mathbf{e}_3) &= -(\mathbf{e}_1 \cdot \mathbf{e}_2) \left\{ (\mathbf{e}_1 \cdot \mathbf{e}_3)^2 + (\mathbf{e}_2 \cdot \mathbf{e}_3)^2 \right\}. \end{aligned}$$

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Existence of Continuous Solutions of a Perturbed Linear Volterra Integral Equation

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In this paper we study the existence of continuous solutions on compact interval of a perturbed linear Volterra integral equations. The existence of such a solution is based on well-known Leray–Schauder principle for fixed point in

Banach space. A special interest is devoted to the study of uniqueness of continuous solution. A numerical approach is based on fixed point method and quadrature rules to approximate a solution for the perturbed linear Volterra integral equations. The convergence of the numerical scheme is proved. Some numerical examples are given to show applicability and accuracy of the numerical method and ensuring the theoretical results.

Keywords. Perturbed linear Volterra integral equation, Leray–Schauder principle, fixed point method, quadrature rules.

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Multitime Multiple Recurrences

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The discrete multitime multiple recurrences are common in analysis of algorithms, computational biology, information theory, queueing theory, filters theory, statistical physics etc. We discuss in detail the cases of linear recurrences with constant coefficients, highlighting in particular geometrical aspects and original theorems of existence and uniqueness of solutions.

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A Generalization of a Chesar Kosnita Theorem

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It is well-known the following Kosnita theorem for a non-rectangle triangle $\triangle ABC$ with circumcenter O : If the circum-centers of $\triangle BCO$, $\triangle CAO$ and $\triangle ABO$ are denoted by O_a , O_b and O_c , respectively, then the lines AO_a , BO_b and CO_c have a common point.

It turns out that this Kosnita theorem could be generalized replacing the circum-circle of $\triangle ABC$ by a circum-conic $\bar{k}(O)$ of the triangle, whose center

O is not on the sides BC , CA and AB . Let l_a be the conjugate line to OA with respect to $\bar{k}(O)$, which passes through the mid-point of the segment OA . In a similar way the lines l_b and l_c could be defined through the mid-points of the segments OB and OC . Let $l_b \cap l_c = O_a$, $l_c \cap l_a = O_b$ and $l_a \cap l_b = O_c$. The points O_a , O_b and O_c are centers of circum-conics of $\triangle BCO$, $\triangle CAO$ and $\triangle ABO$, respectively. In addition, these conics are homothetic to $\bar{k}(O)$. A generalization of the Kosnita theorem is the following: The lines AO_a , BO_b and CO_c have a common point.

Further, let l'_a be the conjugate line to O_bO_c with respect to $\bar{k}(O)$, which passes through the mid-point of the segment O_bO_c . In a similar way the lines l'_b and l'_c could be defined through the mid-points of the segments O_cO_a and O_aO_b . The lines l'_a , l'_b and l'_c have a common point O_1 which is the center of a circum-conic of $\triangle O_aO_bO_c$. This conic is homothetic to $\bar{k}(O)$. The point O_1 verifies the following property: If G is the gravity center of $\triangle ABC$, then O_1 , O and G are co-linear. In the particular case when $\bar{k}(O)$ is a circle, then the circum-center of $\triangle O_aO_bO_c$ is located on the Euler line of $\triangle ABC$. In this case in addition the Kosnita point is isogonally conjugated to the center of the Euler circle with respect to $\triangle ABC$.

On the Existence Pre-Semigeodesic Coordinates

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Let $A_n = (M, \nabla)$ be an n -dimensional manifold M with the affine connection ∇ , and let $U \subset M$ be a coordinate neighborhood at the point $x_0 \in U$. A couple (U, x) is a coordinate map on A_n .

Well known semigeodesic coordinate systems on surfaces and (pseudo-) Riemannian manifolds in [1, p. 43] is generalised by the following way:

Definition 1. Coordinates in A_n are called *pre-semigeodesic coordinates* if one system of coordinate lines are geodesics and their natural parameter is just the first coordinate.

It is known the curve $\ell : x^h = x^h(t)$ is a *geodesic* if under it exists a parallel tangent vector. Geodesic ℓ is characterized by the following equation $\nabla_{\lambda(t)} \lambda(t) = \rho(t) \cdot \lambda(t)$ where $\lambda(t) = dx^h(t)/dt$, $\rho(t)$ is a function depend on the parameter t . The parameter t is *canonical* if $\rho(t) \equiv 0$. This parameter we denote by τ .

Theorem 1. The conditions $\Gamma_{11}^h(x) = 0$, $h = 0, 1, \dots, n$ are satisfied in (U, x) if and only if the parametrized curves $\ell : I \rightarrow U$, $\ell(s) = (s, a_2, \dots, a_n)$, $s \in I$, $a_i \in \mathbb{R}$, $i = 2, \dots, n$, are canonically parametrized geodesics of $\nabla|_U$ (I is some interval, a_k are suitable constants chosen so that $\ell(I) \subset U$, Γ_{ij}^h are components of connection ∇).

Theorem 2. *The conditions $\Gamma_{11}^h = 0$, $h = 1, \dots, n$, are satisfied in (U, x) if and only if the coordinate system (U, x) is pre-semigeodesic.*

We thought that the existence of this chart is trivial. This problem is obviously more difficult, than we supposed. This observed by Z. Dušek and O. Kowalski [3] who precisely proved the existence of pre-geodesic charts in the case when components of affine connection are real analytic functions.

We proved following

Theorem 3. *For any affine connection determined by $\Gamma_{ij}^h(x) \in C^r$, $r \geq 1$, there exists a local transformation of coordinates determined by $x' = f(x) \in C^{r+1}$ such that the connection in new coordinates satisfy $\Gamma_{11}^h = 0$ for $h = 1, \dots, n$.*

The existence of this chart is not excluded in the case if components are only continuous.

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Manifolds Admitting a Structure of Four Dimensional Algebra of Special Type

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The purpose of this note is to describe some properties of manifolds endowed with an almost tangent structure T , $T^2 = 0$ and an almost complex structure J , $J^2 = -E$, $E = id$.

We consider a linear connection ∇ on N , which is compatible with the algebraic structure, i.e. $\nabla J = 0$, $\nabla T = 0$. The existence of ideals in the corresponding algebra implies the existence of autoparallel submanifolds of N .

Keywords. Four dimensional associative algebra, affine connected manifold, algebra of fiber-preserving operators.

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Solutions to Strominger System with Non-Constant Dilaton in Dimensions 7, 6 and 5

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The bosonic fields of the ten-dimensional supergravity of the form $R^{1,9-d} \times M^d$ are the spacetime metric g , the NS three-form field strength H , the dilaton ϕ , the gauge connection A with curvature F^A and are non-trivial only on M^d , $d \leq 7$. One considers the metric connections with skew-symmetric torsion $\nabla^\pm = \nabla^g \pm \frac{1}{2}H$, where ∇^g is the Levi-Civita connection of g .

A heterotic geometry preserves supersymmetry iff there exists at least one Majorana-Weyl spinor ϵ such that the Killing-spinor equations hold:

$$\nabla^+ \epsilon = 0; \quad \left(d\phi - \frac{1}{2}H\right) \cdot \epsilon = 0; \quad F^A \cdot \epsilon = 0, \quad (1)$$

where $\cdot$ means Clifford action of forms on spinors.

The Green-Schwarz anomaly cancellation mechanism requires that the three-form Bianchi identity receives an α' correction of the form

$$dH = \frac{\alpha'}{4} (p_1(\nabla^-) - p_1(F^A)) = \frac{\alpha'}{4} (Tr(R^- \wedge R^-) - Tr(F^A \wedge F^A)). \quad (2)$$

The system consisting of (1) and (2) is called the *Strominger system*.

We construct in [1, 2] a complete solutions to the Strominger system with non trivial field strength H , non-trivial instanton F^A and non-constant dilaton ϕ in dimensions 7, 6 and 5 based on the quaternionic Heisenberg group and its contractions.

For a negative α' the dilaton depends on one variable and it is determined as a real slice of the Weierstrass' elliptic function.

For a positive α' the dilaton depends on four variables and it is determined by the fundamental solution of the Laplacian in $\mathbb{R}^4$.

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Loops Related to Reflection Geometries

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In the book “Theory of K-Loops” by Hubert Kiechle we find the following construction theorem for left-loops and loops: Let $(G, \cdot)$ be a group, 1 the neutral element, U a subgroup of G and let $L \subseteq G$ be a *transversal* of the pair (G, U) , i.e.

(T1) $\forall x \in G \exists_1 \bar{x} \in L$ with $\bar{x} \cdot U = x \cdot U$.

(T2) $1 \in L$.

For $t, s \in L$ let $t \oplus s := \overline{t \cdot s}$. Then $(L, \oplus)$ is a left loop (called *derived left-loop*), i.e. $\forall l \in L : 1 \oplus l = l \oplus 1 = l$ and if $\forall s, t \in L, \exists_1 x \in L$ with $s \oplus x = t$. $(L, \oplus)$ is a *loop* (i.e. also the equation $x \oplus s = t$ has a unique solution) if and only if the set L satisfies the condition (T3) - stronger as (T1) - (then L is called *L-transversal*)

(T3) $\forall x, y \in G : |L \cap x \cdot U \cdot y| = 1$.

Under the notion “reflection geometry” one can find all absolute planes. To any reflection geometry we can associate a so called kinematic space or group space $(\Gamma, \mathfrak{F}, \mathfrak{G})$ and to each subgroup $F \in \mathfrak{F}$ we find in form of a plane of $(\Gamma, \mathfrak{G})$ a transversal L of (Γ, F) . With this method we can associate to each reflection geometry in a natural way loops $(L, \oplus)$, which shall be studied in this talk in particular in the case of a hyperbolic plane.

Acute Triangles in the Context of the Illumination Problem

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Let V_n denote the set of all vertices of the unit cube $[0, 1]^n$ in the Euclidean space $\mathbb{R}^n$. We deduce by recursion on n a precise formula for the total number r_n of those right triangles whose vertices belong to V_n . The formula is as follows: $r_n = 2^n ((3^n + 1)/2 - 2^n)$.

Actually, r_n coincides with the total number of those right triangles whose vertices belong to the set of vertices of an n -dimensional right rectangular parallelepiped P in $\mathbb{R}^n$ (see [3]). By applying the above formula, we give a

direct proof of the fact that there exists a set $X \subset \mathbb{R}^n$, such that all three-element subsets of X form acute triangles and the cardinality of X is of an exponential growth with respect to n (cf. [1] where the same result is proved by using a probabilistic argument). The set X is then associated with a certain version of the well-known illumination problem for convex bodies in $\mathbb{R}^n$ (see, e.g., [2]). Namely, it is shown that there exists a compact convex body K in $\mathbb{R}^n$ such that:

- (a) the total number of singular points of the boundary of K is equal to $\text{card}X$;
- (b) for illumination of the whole boundary of K , at least $\text{card}X$ many rays in $\mathbb{R}^n$ are necessary.

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On Volume Type Functionals in Euclidean Geometry

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The notion of elementary volume for sufficiently simple geometric figures is one of the most important in geometry of Euclidean space. Discussions of this concept occupy a substantial place in all standard university lecture courses of Euclidean geometry. There are many text-books, manuals and monographs devoted to the subject (see, for example, [1] H. Hadwiger, *Vorlesungen über Inhalt, Oberfläche und Isoperimetrie*, Springer-Verlag, Berlin, 1957; [2] V. G. Boltyanski, *Hilbert's Third Problem*, M., 1977 (in Russian); [3] A. V. Pogorelov, *Geometry*, M. 1984 (in Russian)). In [1] and [2] the notion of elementary volume is introduced and investigated by using the method of mathematical induction, which is somewhat preferable from the teaching viewpoint. Another approach based on general concepts of measure theory allows to avoid induction method. In the present report some volume type functionals in Euclidean space are considered. Close connections with basic notions of the theory of invariant measures and with equidecomposability theory are demonstrated.

Newton, Halley and Tchebychev Iterative Methods as Methods for Simultaneous Determination of Multiple Polynomial Zeros

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There is no doubt that the most popular iterative methods in the literature are Newton, Halley and Tchebychev methods. These methods has an elegant geometrical interpretation for real functions. Namely, every iteration x_{n+1} is a zero of a tangent curve to the graph of the function f at the point $(x_n, f(x_n))$. It is well known that the different iterative methods correspond to the different tangent curves. The tangent curves of Newton, Halley and Chebyshev methods are line, hyperbola and parabola, respectively.

On the other hand there are a lot of iterative methods for simultaneous finding of polynomial zeros. Recently, Proinov and Ivanov [4] proved convergence theorems for Halley's method, considered as an iterative method for simultaneous computation of all simple zeros of a polynomial f over an arbitrary normed field.

In this paper we establish local convergence theorems with error estimates for Newton, Halley and Chebyshev methods as methods for simultaneous determination of multiple polynomial zeros. Note that the obtained results for Newton and Chebyshev methods are new even in the case of simple zeros. The proof of the main results are based on convergence theorems due to Proinov [2], Proinov and Ivanov [3] and Ivanov [1]. The numerical examples which demonstrate the convergence properties of the proposed methods are also provided.

Mathematical Subject Classification (2010). 65H04, 12Y05.

Keywords. Newton's method, Halley's method, Chebyshev's method, simultaneous methods, polynomial zeros, multiple zeros, local convergence, error estimates.

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Near-MDS Codes Revisited

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Near-MDS have been introduced in 1995 in [2]. They are defined by weakening some restrictions in the definition of the MDS codes. Let us denote by $m'(k, q)$ the maximum possible length for which there exists a $[n, k]_q$ near-MDS code. In this talk, we present some old and new results on the exact value of $m'(k, q)$ for various integers k and prime powers q .

The following theorem summarizes some straightforward observations about $m'(k, q)$.

Theorem 1. *Let k be a positive integer and let q be a prime power. Then*

- (i) $m'(2, q) = 2q + 2$;
- (ii) $m'(k, q) \leq m'(k - \alpha, q) + \alpha$, for every α with $0 \leq \alpha \leq k$;
- (iii) $m'(k, q) = k + 1$ for $k > 2q$;
- (iv) $m'(2q, q) = 2q + 2$;
- (v) $m'(2q - 1, q) = 2q + 1$.
- (vi) $m'(k, q) \leq 2q + k - 2$;

The following theorem gives an improvement over Theorem 1(vi) for sufficiently large dimensions.

Theorem 2. *There exist no $[2q + k - 2, k]_q$ near-MDS codes for $k \geq q + 4$ and $q \geq 9$.*

Due to extensive computational work by Bartoli, Marcugini, Milani and Pambianco [1, 3, 4, 5], the exact values of $m'(k, q)$ were determined for all fields of order $q \leq 9$, as well as lower and upper bounds on the size of the longest near-MDS code for some larger fields. Even more results were obtained for the case of dimension three, which corresponds to the problem of the maximal size of an $(n, 3)$ -arc in $\text{PG}(2, q)$. These results are summarized in the table below.

q/k	2	3	4	5	7	8	9	11	13
2	6	8	10	12	16	18	20	24	28
3	7	9	9	11	15	15	17	21	23
4	8	10	10	12	14	16	16	20–21	21–24
5		11	11	11	13	15	16	18–22	21–25
6		12	12	12	13	14	16	18–23	21–36
7			9	11	14	15	17	18–24	21–27
8			10	12	13	16	18	18–25	21–28
9				11	13	14	19	19–26	21–29
10				12	14	15	20	20–27	21–30
11					14	15	16	18–28	21–31
12					15	16	16	18–29	21–32
13					15	15	16	18–30	21–33
14					16	16	17	18–31	21–34
15						17	17	18–32	21–35
16						18	18	18–33	21–36

The following two conjectures for near-MDS codes can be viewed as analogues of the famous Main Conjecture for MDS codes.

Weak Main Conjecture for NMDS codes. For all positive integers k and all prime powers q it holds $m'(k, q) \leq 2(q + 1)$.

Strong Main Conjecture for NMDS codes. There exists a universal constant c (not depending on q) such that $m'(k, q) \leq N_1(q) + c$, where $N(q)$ denotes the maximum number of $\mathbb{F}_q$ rational points on an elliptic curve defined over $\mathbb{F}_q$.

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Contactly Conformal Transformations of 3-Dimensional Almost Contact B-Metric Manifolds

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The present work deals with differential geometry of almost contact B-metric manifolds. These manifolds are the odd-dimensional extension of almost complex manifolds with Norden metric and a neutral-signature analogue of almost contact manifolds with compatible metric. The differential geometry of the studied manifolds is developed as a whole but in the case of the lowest dimension some special characteristics are valid.

Let $(M, \varphi, \xi, \eta, g)$ be an almost contact B-metric manifold, i.e. M is a $(2n+1)$ -dimensional differentiable manifold, φ is an endomorphism on the tangent bundle, ξ is a Reeb vector field, η is its dual contact 1-form and g is a pseudo-Riemannian metric (called a B-metric) of signature $(n+1, n)$ such that: $\varphi\xi = 0$, $\varphi^2 = -\text{Id} + \eta \otimes \xi$, $\eta \circ \varphi = 0$, $\eta(\xi) = 1$, $g(\varphi x, \varphi y) = -g(x, y) + \eta(x)\eta(y)$, where Id is the identity and x, y are arbitrary smooth vector fields on M .

The almost contact endomorphism φ is an almost complex structure on the contact distribution $\ker(\eta)$ and φ acts as an anti-isometry with respect to the restriction of the B-metric g on $\ker(\eta)$. In distinction from the Hermitian case, the basic B-metric generates an associated B-metric $\tilde{g}$. In this way, any hypersurface of an almost contact B-metric manifold can be considered as an almost complex Norden manifold and viceversa.

There are studied contactly conformal transformations of almost contact B-metric structures. These transformations act on the considered manifolds from the so-called main classes. For these manifolds, the covariant derivatives of the structure tensors are expressed explicitly by the metrics.

Our main goal is a construction and a characterization of almost contact B-metric manifolds of dimension 3 obtained by contactly conformal transformations.

Objects of special interest are 3-dimensional almost contact B-metric manifolds derived as extensions of almost complex Norden manifolds. Since the latter manifolds are 2-dimensional, they are space-forms and contactly conformal equivalent to Kähler-Norden manifolds.

There are used two different methods for construction of the B-metric on the almost contact manifold, which is a warped product of a real line and a 2-dimensional space-form. The former method is constructing by the cone and

the latter one—by the S^1 -solvable extension, which is a doubly warped product of the pair of Norden metrics (the given one and its associated).

We determine the form of contactly conformal transformations which convert the constructed manifolds into almost contact B-metric manifolds belonging to the main classes. Finally, we deliver several examples of the studied manifolds using the commented methods.

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Invariant Tensors Under the Twin Interchange of Norden Metrics on Almost Complex Manifolds

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Hermitian metrics on almost complex manifolds are well known. Then the almost complex structure J acts as an isometry with respect to the (Riemannian or pseudo-Riemannian) metric. The associated $(0,2)$ -tensor of the Hermitian metric is a 2-form. Other case is when the almost complex structure acts as an anti-isometry regarding a pseudo-Riemannian metric. Such a metric is called a Norden metric. The associated $(0,2)$ -tensor of any Norden metric is also a Norden metric. So, in this case we dispose with a pair of mutually associated Norden metrics, known also as twin Norden metrics. These *almost Norden manifolds* are studied in the latter three decades, in the beginning under the names generalized B-manifolds, almost complex manifolds with Norden metric and almost complex manifolds with B-metric.

An interesting problem on almost Norden manifolds is the presence of tensors with some geometric interpretation which are invariant under the so-called twin interchange. This is the swap of the counterparts of the pair of Norden metrics g , $\tilde{g}$ and their Levi-Civita connections ∇ , $\tilde{\nabla}$. Similar results for the considered manifolds in the basic classes $\mathcal{W}_1$ and $\mathcal{W}_3$ are obtained in other papers. The aim of the present work is the solving of the problem in the general case.

The presentation is organised as follows. Section 1 contains some preliminaries on the considered type of manifolds. In Section 2 we report the main results on the topic about the invariant objects and their vanishing. For instance, anti-invariants under the twin interchange are the potential Φ of $\tilde{\nabla}$ with respect to ∇ , the associated Nijenhuis tensor of J ; whereas invariants under the twin interchange are all classes of almost Norden manifolds, the average connection D of ∇ and $\tilde{\nabla}$, the associated 1-forms of Φ , the Nijenhuis tensor of J , the

curvature tensor K of D , the average tensor P of the curvature tensors for ∇ and $\tilde{\nabla}$, any linear combination of the tensors P and K . In Section 3 we consider an example of the studied manifolds of dimension 4 by means of a construction of an appropriate algebra depending on 4 real parameters. Then we compute the basic components of the invariant objects which are found in the previous section.

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Non-Umbilic Hypersurfaces in Hyper-Kaehler Manifolds

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It is known that any totally umbilic hypersurface in a hyper-Kaehler manifold carries a natural induced 3-Sasakian structure. The converse is also true: Any 3-Sasakian manifold can be embedded into a hyper-Kaehler manifold, namely into its hyper-Kaehler cone. The 3-Sasakian structures are a special type of quaternionic contact (qc) manifolds. In the present talk we consider qc manifolds embedded in a non-umbilic way into a hyper-Kaehler space. We show that, if compact, any such qc manifold is necessarily a quotient of the 3-Sasakian sphere.

Totally Umbilical and Screen Totally Umbilical Radical Transversal Lightlike Hypersurfaces of Kähler-Norden Manifolds

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For any semi-Riemannian manifold there exist lightlike submanifolds. The geometries of non-degenerate and lightlike submanifolds are different because in lightlike case the tangent bundle and the normal bundle have a non empty intersection.

A $2n$ -dimensional almost complex manifold with Norden metric $(\overline{M}, \overline{J}, \overline{g}, \tilde{\overline{g}})$ is a semi-Riemannian manifold equipped with an almost complex structure $\overline{J}$ which is an anti-isometry with respect to the metric $\overline{g}$. Due to this there exist two Norden metrics (or B-metrics) $\overline{g}$ and $\tilde{\overline{g}}$ on $\overline{M}$ which are necessarily indefinite of signature (n, n) .

In this talk we present totally umbilical and screen totally umbilical radical transversal lightlike hypersurfaces of Kähler-Norden manifolds of constant totally real sectional curvatures $\overline{\nu}$ and $\tilde{\overline{\nu}}$. Since there exist two Norden metrics, we

consider two types of such hypersurfaces—with respect to the induced metrics g and $\tilde{g}$ by $\bar{g}$ and $\bar{\tilde{g}}$, respectively. We find classes of Kähler-Norden manifolds which do not contain some of the examined hypersurfaces. We study curvature properties of semi-symmetric type of totally umbilical radical transversal lightlike hypersurfaces of Kähler-Norden manifolds of constant $\bar{\nu}$ and $\bar{\tilde{\nu}}$. An example of a totally umbilical radical transversal lightlike hypersurface, which is locally symmetric, semi-symmetric, Ricci semi-symmetric and almost Einstein is constructed. Examples of totally umbilical and screen totally umbilical radical transversal lightlike hypersurfaces of Lie groups belonging to a 4-parametric family of Lie groups as 4-dimensional Kähler-Norden manifolds are obtained.

On Darboux Helices and W-Direction Curves in Lie Groups

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In this paper, we define Darboux helices in three dimensional Lie groups with a bi-invariant metric and obtain a characterization of Darboux helices. Moreover, we give some relations between slant helices and Darboux helices. Also, we show that curves of constant precession are Darboux helices in a three dimensional Lie group. And then, we show the position vectors of Darboux helices in a three dimensional Lie group. Finally, we define W-direction curves in a three dimensional Lie group G and investigate the relationship between Darboux helices and W-direction curves.

Keywords. Darboux vector, Darboux helices, constant precession, position vector, W-direction curves, Lie groups.

Mathematical Subject Classification (2010). Primary 14H45; Secondary 22E15.

On Almost Contact Metric Structures and G_2 Structures

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Each manifold with G_2 structure has an almost contact metric structure induced by the G_2 structure. In this work, we investigate the deformation of a G_2

structure by a vector field and study the new almost contact metric structure obtained by the deformation of the G_2 structure. In addition, we give some examples.

On the Product of Two Almost Contact Metric Manifolds

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In this study, we investigate the almost Hermitian structures on the product of two almost contact metric manifolds. First, we show that if two factors are semi-cosymplectic (quasi-K-cosymplectic), then the product is semi Kähler (quasi-Kähler). Then, we observe that if two factors are $\mathcal{G}_1$ -Sasakian with an extra property, then the product manifold is an almost Hermitian manifold of class $\mathcal{G}_1$. In addition, we prove that if two factors are $\mathcal{G}_2$ -Sasakian, the product manifold is in the class $\mathcal{G}_2$ of almost Hermitian manifolds.

Keywords. Almost contact metric manifold, almost Hermitian manifold, product manifold.

Mathematical Subject Classification (2010). 53D10, 57R15.

The Quaternionic Contact Heat Equation and Some Its Applications

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For a quaternionic contact (QC) manifold of dimension $4n+3$ we consider the QC heat equation

$$\frac{\partial}{\partial t} u = -\Delta u, \quad (1)$$

where $u(x, t) : M \times [0, +\infty) \rightarrow \mathbb{R}$ is a smooth function and $\Delta : \mathcal{F}(M) \rightarrow \mathcal{F}(M)$ is the sub-Laplacian on M . The *energy functional* of (1) is defined by

$$\mathcal{F}(\varphi) = \int_M |\nabla \varphi|^2 e^{-\varphi} \text{Vol}_\eta, \quad \varphi \stackrel{\text{def}}{=} -\ln u. \quad (2)$$

The main technical result of this talk is a *Perelman's entropy formula* for the QC heat equation (1) on a compact QC manifold. We obtain as a main application of this formula the monotonicity of (2) along the QC heat equation under some a-priori conditions. This result is in a close relationship with the

Lichnerowicz type results for the first eigenvalue of the sub-Laplacian established in [1] and [2] for the bigger dimensional and the seven-dimensional cases, respectively, because of the similarity of the a-priori conditions.

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On Para-Sasakian η -Ricci Solitons

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In this paper, we study η -Ricci solitons on para-Sasakian manifolds. We prove that on a para-Sasakian manifold $M(\phi, \xi, \eta, g)$, the symmetric (0,2) tensor field $\alpha = \mathcal{L}_\xi g + 2S + 2\mu\eta \otimes \eta$ is parallel with respect to the Levi-Civita connection, as a result (g, ξ, λ, μ) is an η -Ricci soliton on M . In addition, the non-existence of certain kinds of para-Sasakian manifolds with η -Ricci solitons is shown. Finally, we discuss 3-dimensional and conformally flat para-Sasakian manifolds with η -Ricci solitons.

Mathematical Subject Classification (2010). 53C21, 53C44, 53C25.

Keywords. Para-Sasakian manifolds, Ricci solitons.

Binomial Configurations Containing Complete Graphs

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We propose to base a classification of partial Steiner triple systems (PSTS) on determining the structure of their complete subgraphs contained in them. There are two main problems that this theory starts with.

Firstly: what are the minimal parameters of a PSTS necessary to contain a K_n -graph? and does there exist a PSTS with these parameters which contains/does not contain a K_n -graph. It turns out that a minimal (with respect to its size) PSTS that contains K_n is a so called binomial configuration i.e. it is a (v_r, b_3) -configuration such that $v = \binom{n+1}{2}$, $r = n - 1$, and $b = \binom{n+1}{3}$ for an integer n . As the best known example of a binomial configuration we can quote any

generalized Desargues configuration (combinatorial Grassmannian) (see [3] or [2]). Then the answer to the ‘dual’ question: what is the maximal size of a complete graph that a binomial $\left(\binom{n}{2}_{n-2}\binom{n}{3}_3\right)$ -configuration may contain easily follows: this size is $n-1$. However, there are $\left(\binom{n}{2}_{n-2}\binom{n}{3}_3\right)$ -configurations that do not contain any K_{n-1} -graph. For example (for $n=5$) we can quote the known result: there are 10_3 -configurations without K_4 , see [1]. Moreover, a binomial $\left(\binom{n+1}{2}_{n-1}\binom{n+1}{3}_3\right)$ -configuration contains a complete K_n graph iff it contains a binomial $\left(\binom{n}{2}_{n-2}\binom{n}{3}_3\right)$ -configuration.

The second problem is: what is the possible number of K_n -graphs contained in a $\left(\binom{n+1}{2}_{n-1}\binom{n+1}{3}_3\right)$ -configuration $\mathfrak{M}$ and what is the structure they yield.

Theorem. *The number of K_n -subgraphs of a binomial configuration as above belongs to $M = \{0, 1, \dots, n-1, n+1\}$. For every $m \in M$ there is a binomial configuration which contains m subgraphs K_n .*

Binomial configurations with the maximal number of their complete subgraphs of corresponding size are exactly the generalized Desargues configurations. The structure of the intersection points of K_n -subgraphs of a $\left(\binom{n+1}{2}_{n-1}\binom{n+1}{3}_3\right)$ -configuration is isomorphic to a generalized Desargues configuration.

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Hyperplanes in Binomial Partial Steiner Triple Systems

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A *hyperplane* in a partial linear space $\mathfrak{M}$ is a proper subspace of $\mathfrak{M}$ which crosses every line of $\mathfrak{M}$. A (finite) partial Steiner triple system (a PSTS, in short) is a (finite) partial linear space with all the lines of size three.

For a set X we write $\wp(X)$ for the set of all subsets of X and $\wp_2(X)$ for the 2-subsets of X . If A, B are disjoint we set $\mathcal{H}(A|B) = \wp_2(A) \cup \wp_2(B)$. For any two subsets W, Y of a fixed X we define $W \pitchfork Y = (W \cap Y) \cup ((X \setminus W) \cap (X \setminus Y)) = X \setminus (W + Y)$.

Fact 1. *If W_1, W_2 are hyperplanes of a finite PSTS $\mathfrak{M} = \langle X, \mathcal{L} \rangle$ then $W_1 \cap W_2$ is a hyperplane of $\mathfrak{M}$. The family of the 3-sets of hyperplanes of the form $\{W_1, W_2, W_1 \cap W_2 : W_1, W_2 \text{ are distinct hyperplanes of } \mathfrak{M}\}$ defines the structure of the projective space $PG(n, 2)$ for some integer $n = -1, 0, 1, \dots$*

There appears a natural question what are the hyperplanes in some particular PSTS's known in the literature. Here we concentrate upon some binomial PSTS's (BSTS, in short) i.e. on $\left(\binom{n}{2}_{n-2} \binom{n}{3}_3\right)$ -configurations. So, we completely determine all the hyperplanes in so called generalized Desargues configurations $\mathbf{G}_2(n)$ (cf. [2] for a definition of this class), in combinatorial Veronese spaces $\mathbf{V}_k(3)$ (cf. [1]) and in quasi Grassmannians $\mathfrak{R}_n$ (cf. [3]). It turns out that when such a BSTS is suitably defined on the set $\wp_2(X)$ for an X with $|X| = n$ then each its hyperplane has form $\mathcal{H}(A|B)$ for a decomposition $X = A \cup B$.

Finally, for a BSTS $\mathfrak{M} = \langle \wp_2(X), \mathcal{L} \rangle$ 'spanned' (see [4]) by a complete graph we characterize which sets $\mathcal{H}(A|B)$, $A \cup B = X$ are hyperplanes of $\mathfrak{M}$.

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On the Structure of $(t \bmod q)$ -Arcs in Finite Projective Geometries

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Let q be a prime power and let $t < q$ be an integer. An arc $\mathcal{F}$ in $PG(r, q)$ is called a $(t \bmod q)$ -arc if $\mathcal{F}(P) \leq t$ for every point P from $PG(r, q)$ and $\mathcal{F}(S) \equiv t \pmod{q}$ for every subspace S of positive dimension.

The following theorems provide constructions for $(t \bmod q)$ -arcs.

Theorem 1. *Let $\mathcal{F}_1$ be a $(t_1 \bmod q)$ - and $\mathcal{F}_2$ be a $(t_2 \bmod q)$ -arc in $\text{PG}(r, q)$. If $t_1 + t_2 < q$ the arc $\mathcal{F}_1 + \mathcal{F}_2$ is a $(t \bmod q)$ -arc with $t = t_1 + t_2$. In particular, the sum of t not necessarily different hyperplanes is a $(t \bmod q)$ -arc.*

Theorem 2. *Consider the geometry $\Gamma = \text{PG}(r, q)$ and let S and T be subspaces in Γ with $S \cap T = \emptyset$, $\dim S + \dim T = r - 1$. Let $\mathcal{F}_0$ be a $(t \bmod q)$ -arc in T . Define an arc $\mathcal{F}$ in Γ as follows: for each point $Q \notin S$, $\mathcal{F}(Q) = \mathcal{F}(R)$, where $R = \langle S, Q \rangle \cap T$; for every $Q \in S$, $\mathcal{F}(Q) = t$. The arc $\mathcal{F}$ is a $(t \bmod q)$ -arc.*

A $(t \bmod q)$ -arc obtained by the construction described in Theorem 2 is called a lifted arc.

In the plane case, non-trivial $(t \bmod q)$ arcs (i.e. arcs that cannot be obtained by Theorems 1 and 2) can be constructed as duals of certain blocking sets.

Theorem 3. *A $(t \bmod q)$ -arc in $\text{PG}(2, q)$ of size $mq + t$ exists if and only if there exists a $((m - t)q + m, m - t)$ -blocking set in $\text{PG}(2, q)$ with line multiplicities from the set $\{m - t, m - t + 1, \dots, m\}$.*

In this talk, we focus on the characterization of $(t \bmod q)$ -arcs for small values of t .

A $(1 \bmod q)$ -arc is either a hyperplane or the complete space. The plane $(2 \bmod q)$ -arcs have been characterized by Maruta [2]. This makes possible the characterization of $(2 \bmod q)$ -arcs in higher dimensions. For $(3 \bmod q)$ -arcs the situation is more complicated. We prove that if a $(3 \bmod q)$ -arc is not very large it is either the sum of hyperplanes or a lifted arc. We prove that in the particular case of $q = 5$, $r = 3$, every $(3 \bmod 5)$ -arc $\mathcal{F}$ is a lifted arc provided $|\mathcal{F}| < 158$.

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RSA vs. ECC—A Non-expert View

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In recent years, the RSA public key cryptosystem has faced much criticism, not only because of installation errors (over 12000 keys could be broken) but

also because substantial increases in the recommended key length have been required. In contrast, the parameters used in Elliptic Curve Cryptography (ECC) are of moderate length. There is just one drawback, the standard (Weierstrass) curves used until recently are suspected to have backdoors for intelligence agencies. Thus, some other curves, among them the so-called Bernstein curve (of Daniel J. Bernstein), are recommended and are said to be simpler to implement.

Vanishing Theorems for Projective and Harmonic Mappings

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Let $f : (M, g) \rightarrow (\bar{M}, \bar{g})$ be a mapping between Riemannian manifolds and assume that (M, g) is compact. The Bochner integral formula was obtained in [1]:

$$\int_M (Q(f) + \|Df_*\|^2 + \|\text{trace}_g Df_*\|^2) dVol_g$$

where $Q(f)$ depends on the curvature tensor $R(\bar{g})$ of $(\bar{M}, \bar{g})$ and the Ricci tensor $Ric(g)$ of (M, g) and D is the connection in $f^*T\bar{M} \otimes T^*M$ induced from the Levi-Civita connections $\nabla(g)$ and $\nabla(\bar{g})$. In [1] based on the equation, we have proved some vanishing theorems on projective map of constant rank ρ , umbilical and harmonic maps.

In the present work we shall show that $Q(f) \geq 0$ if $Ric(g) \geq f^*R(\bar{g}) \geq 0$ and $\text{sec}(\bar{g}) \geq 0$. Then based on these inequalities and the equation, we shall prove the theorem.

Theorem. *Let $(\bar{M}, \bar{g})$ be a Riemannian manifold with the sectional curvature $\text{sec}(\bar{g}) \geq 0$ and (M, g) be a compact Riemannian manifold with the Ricci tensor $Ric(g) \geq f^*R(\bar{g}) \geq 0$. If $f : (M, g) \rightarrow (\bar{M}, \bar{g})$ is any harmonic map (projective map of constant rank ρ), then f is totally geodesic ($\rho \leq 1$).*

This theorem is an analogue of the celebrated vanishing theorem which states that any harmonic map $f : (M, g) \rightarrow (\bar{M}, \bar{g})$ is totally geodesic if (M, g) is a compact manifold with the non-negative Ricci tensor and the section curvature of $(\bar{M}, \bar{g})$ is non-positive (see [2]). Moreover, from this theorem we obtain the corollary which generalizes the result in [3].

Corollary. *Let (M, g) be a compact Riemannian manifold with the sectional curvature $\text{sec}(g) \geq 0$ and the Ricci tensor $Ric(g) \leq \varepsilon g$ for some $\varepsilon \leq 1$. If $\bar{g}$ is another Riemannian metric on M with $Ric(\bar{g}) = g$, then $\nabla(g) = \nabla(\bar{g})$.*

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Linear Connections and Transformations on a Class Complex Manifolds with Norden Metric

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Almost complex manifolds with Norden metric (B-metric) are smooth even-dimensional manifolds equipped with an almost complex structure J and a compatible metric of neutral signature g which satisfy the condition $g(Jx, Jy) = -g(x, y)$. A classification of these manifolds with respect to the properties of ∇J , where ∇ is the Levi-Civita connection of g , is introduced by G. Ganchev and A. Borisov. This classification consists of three basic classes $\mathcal{W}_i$ ($i = 1, 2, 3$) two of which are integrable, i. e. with a vanishing Nijenhuis tensor (complex manifolds with Norden metric). One of the integrable classes is said to be a main class because its characteristic condition is an explicit expression of ∇J by means of the other structural tensors. This class is denoted by $\mathcal{W}_1$.

Object of interest in this work are linear connections on $\mathcal{W}_1$ -manifolds which possess special properties, like preserving the structural tensors under covariant differentiation: complex connections, metric connections, natural connections. Curvature properties of such linear connections are studied. Transformations of the considered connections are investigated, and invariant tensors under such transformations are obtained.

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Topology and Differential Structure on Descartes Folium

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The Descartes Folium carries families of group laws, defined entirely in terms of algebraic operations over the basic field. To highlight these structures and their

properties, we introduced and studied new topology and differential structure on Descartes Folium.

Geometry of Laplacian Eigenvalues and Eigenvectors on a Parallelepiped

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We analyse the dependence of eigenvalues and eigenvectors with respect to the edges (parameters) of the parallelepiped, finding new results connected to differential geometry. Then we interchange the parameters with the variables, in order to find qualitative properties related by the corresponding diffeomorphism.

Geometric Topology of Lagrange Multipliers

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The geometric topology of Lagrange multipliers is our news for optimization theory. In this context there are involved the interpretation of Lagrange multiplier as the rate of change of the optimal value with respect to changes in the constraint and PDEs reflecting this fact.

Geometry of Reliability Polynomials

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Looking for geometric modeling of reliability polynomials, we attend three important ideas: (i) find equivalent reliability polynomials via diffeomorphisms, (ii) cover a reliability hypersurface by probability straight lines and (iii) cover a reliability hypersurface by exponential decay curves. As models we shall use the reliability polynomials attached to some electric systems used inside aircrafts.

Properties of Voronoi Diagrams

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Voronoi diagram is a partitioning of a plane based on the positioning of a given set of points and which is the closest point in the set to each point in the plane. Each region is called a Voronoi cell.

This research was made to examine interesting properties of the Voronoi diagram and its connections to other constructions like Delaunay triangulation. The 3-dimensional case is also considered. There are different ways of constructing the diagram as intersecting halfplanes or using a union of parabolas. It can be generalized for multidimensional space where it has an order (it is one in the 2-dimensional case). It has a connection to convex hulls. If the Voronoi diagram is in n -dimensional space it is equivalent to certain convex polyhedral surfaces in $(n + 1)$ -dimensional space.

The Voronoi diagram has many applications in mathematics, informatics, artificial intelligence, biology, astrophysics and other sciences.

Traffic Noise During the Night: Addition, Mean and Evaluation

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It seems to be easy to measure the sound of traffic noise, because it exists a “sound level meter”. But if we want to decide

quiet or loud—sleeping or awakening

then we have many definitional problems, specially when applied to rail or air traffic sound:

The “awakening probability” during the passing of an airplane or a train depends on:

- time of night
- pass-by duration
- (average of) sound level during pass.

But today it is usual to let the time unconsidered—that means: the awakening probability is the same in event 1 and event 2: depends on:

event 1: 22:00–23:00: 40 passings and 05:00–06:00: 40 passings,
event 2: every hour between 22:00 and 06:00: 10 passings.

Both events have the same night-pass-by level, but the awakening-probability is not the same. I will give a geometric calculation and description of the awakening probability, which depends on the (sleeping-)time and the sound levels.

Laudatio for Prof. Dr. Ralph-Hardo Schulz

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Ralph-Hardo Schulz studied at the University of Mainz and at Westfield College (London) where he was supported by a grant from “Studienstiftung des Deutschen Volkes”. In Mainz, he received his Diploma in Mathematics, passed the first teacher exam (“Staatsexamen”) in Mathematics and Physics and did his doctors degree under the supervision of Heinz Lüneburg. His diploma thesis “On block designs with transitive dilatation group” and his dissertation “On translation planes with collineation groups permuting the points of the line at infinity doubly transitively” were both published in “Mathematische Zeitschrift” (still in German).

After his second teacher exam at a Grammar school in his home town Bad Kreuznach, he was offered an assistant position by Peter Dembowski and a position as “Akademischer Rat” at University of Tübingen. Here, he gave lectures on Linear Algebra, Analysis, Projective Geometry, Topology, Probability Theory, Introduction into Statistics and Introduction into the Didactic of Mathematics. At the same time, he lectured on Mathematics at the teacher training college in Reutlingen.

In 1974, he accepted a position as “Professor for Mathematics and Mathematics Education” at “Freie Universität Berlin”. After the integration of parts of the teacher training college (PH) Berlin into Freie Universität he concentrated on teaching Mathematics, but coached several hundred candidates for their first teacher exam. He gave some lectures at Technical University Berlin, and during winter 1991/1992 he had an interim professorship at the University of Erlangen. Since 2010, he is officially retired but is still involved in teaching and research.

Ralph-Hardo Schulz has supervised 4 doctoral students; one of whose is Sabine Giese whom many of you know. He has written 3 books, one on Coding theory (in two editions), two on Mathematics revision courses, and has edited a further book (on “Mathematical Aspects of Applied Computer Science”). Of his 68 publications, 31 are on Mathematics, the others deal with Didactics of Mathematics and Computer Science. Over 70 of his reviews have been published in “Zentralblatt der Mathematik” where he was editor of section 94 (“Information and Communication, Circuits”) for several years.

His research profile can be divided into three main areas: Design Theory, Coding Theory and Didactics of Mathematics in general and in Coding and Cryptography. In Design Theory, the focus was directed at characterizing and constructing finite affine planes, block designs and divisible designs admitting 'nice' groups as auto-morphism groups either operating transitively in some sense or of special type. He showed e.g. that a finite group with a non trivial set of subgroups of equal size such that every element not equal to 1 lies in exactly λ subgroups is a p -group of exponent p , implying that the parameters v and k of a block design with transitive translation group are powers of the same prime number—a generalization of the situation in finite translation planes. An other example: A non-trivial divisible translation design (with blocks of equal size) admitting a Frobenius group (besides p -groups and Hughes-Thompson groups the only possibilities for point-regular translation groups) is a transversal design and, if the kernel is abelian, of known type. Together with coauthors (Antonino Spera, Cincia Cerroni, Armin Herzer, Hans Havlicek and Sabine Giese), he investigated divisible designs constructed from translation planes, from Laguerre geometries, divisible designs admitting an orthogonal group, a unitary group, a Suzuki group or a full dual translation group.

The second focus of his research was on error detecting codes. He investigated check digit systems which detect all single errors, all neighbour transpositions and many other double errors by check equations using groups with an anti-symmetric mapping T (i.e. with $xT(y) \neq yT(x)$ for all $x \neq y$). Introducing the concept of equivalence of check digit systems, there was a possibility, to define and determine, together with Sabine Giese and Shepanur Ugan, types of anti-symmetric mappings, especially for the quaternion group, for some small dihedral and small dicyclic groups.

One of his main didactic projects was to write understandably on coding theory and cryptography for a wider public. Already 1987, he edited a "Mathematik Unterricht" (MU)-booklet "Codieren und Chiffrieren" with Albrecht Beutelspacher. With Helmut Witten, Irmgard Letzner and Bernhard Esslinger, he published (in the journal "LogIn" for Computer Science teachers) 10 articles on "RSA and Co. in School", and articles on "Time experiments" for prime factorization, on "The quadratic sieve" and on "Elliptic curve cryptography".

Currently, he has been assigned by the Berlin senate for education to lecture in advanced training for teachers who, parallel to their work at school, seek accreditation in Mathematics as a further subject. He has also taken part in developing a new curriculum for the next course in teacher training which will start in September 2015.

Herbert Zeitler

July 26, 1923–August 24, 2012

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Over many years Herbert Zeitler and his wife Hermine came to our Geometry Conferences here in Varna. Their presence always stimulated these meetings. And when they could not come anymore we missed them very much.

As the only son of a porcelain painter he was born in Weiden/Bavaria, where he attended schools until after his matura in 1942 he was drafted to the army. Consequently he was sent to the east front to Riga. He had the good luck that his duty only consisted in transmitting wireless messages. On 8 May 1945 he came into a prisoner-of-war camp. This was not a nice time, but here he had also good luck. Because of health problems he had to go to the military hospital where a German medical doctor, with a wink, diagnosed: suspicion on typhus. from where the supervising Russian woman doctor panically exclaimed: *domoi, domoi – bistre, bistre*. Thus he was at home as early as October 1945.

Then in February 1946 Herbert began studying mathematics and physics Munich where he completed his studies to become a teacher at a gymnasium. Also during his study Herbert and Hermine married in 1946 the year in which also their only son Horst (who later became a medical doctor in Selb) was born.

In 1950 Herbert became a teacher at the very same school which he attended as pupil. He remained at this school until 1968 when he became headmaster of a gymnasium in the little town of Tirschenreuth, only some 35 km away from his hometown Weiden. During his time as a teacher Herbert published quite a lot of little articles on elementary mathematics relevant to school mathematics. As headmaster he now had more time for doing mathematical and didactical research. In 1977 Herbert completed his PhD thesis on (K,L) -planes with Bruno Bosbach (University Kassel) as his thesis advisor. Walter Benz (U Hamburg) was the second referee.

In 1978 he was invited by the president of the University of Bayreuth to apply for a full professorship for Didactics of Mathematics, and indeed he was elected to this position. Here he displayed a fruitful activity as researcher and lecturer during the 13 years of his duty until the completion of his 68th birthday in 1991.

Zeitler's research dealt with mathematics, pure and applied and on the other side with didactics of teaching mathematics.

In didactics he emphasized *concrete* didactics in contrary to *abstract* one. Didactics should improve the teaching and learning of mathematics.

“Abstract didactics is nonsense.”

Herbert’s interests in mathematics mainly focus on geometry in a wide sense:

- non-euclidean geometry, especially hyperbolic geometry;
- circle geometry (Möbius and Minkowski);
- finite geometries;
- Steiner triple systems, quasigroups;
- chaos theory, fractals, Julia and Mandelbrot sets.

The feature of his research consists in:

“Man muss es tun!”

(One has to do it!). He was not a man of creating great theories but in thoroughly calculating various kinds of examples. In this manner Herbert was a Non-Bourbakist.

At last we also should commemorate his wife Hermine who always provided for him an calm atmosphere for doing mathematics, She took care in managing all aspects of the daily life such that Herbert could concentrate on his research. She also had passed away shortly after him.

Thus, with their death also a part of our life has gone.

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