Remarks on “Floating Bodies in Neutral Equilibrium”

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Abstract. This note offers historical comments on mathematical life in the city of Lwów, and points out that the paper cited in the title contains a solution of Problem 19 in “The Scottish Book”, prepared just prior to World War II in the “Scottish Café” located in that city.

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During the interval between the two World Wars, the city of Lwów in Poland was a major mathematics center, and hosted feverish activity by persons such as Banach, Schauder, Steinhaus, Mazur, Orlicz, Kac, and Ulam. An active seminar met regularly in a coffeehouse called “The Scottish Café”; the participants proposed problems to each other, and those judged of continuing interest were inscribed by hand into a notebook that was guarded by the headwaiter in a secret location. The proposers would offer prizes such as a bottle of wine to those who could solve the problems (presumably such prizes were shared). The seminar continued to thrive during the Russian occupation, with contributions from Russians such as P. Alexandroff, Liusternik, and Sobolev, but following the later German invasion the university was closed, seminar participants went elsewhere (some to extermination camps), and the seminar died. Remarkably, the notebook was somehow preserved and apparently brought to the city of Wrocław by Banach’s son. There it came into the possession of Hugo Steinhaus, who survived the war and sent a typewritten copy to Stanislaw Ulam (then in the USA). Ulam translated and arranged for limited distribution of the notes, under the title “The Scottish Book”. The notes later appeared as a formal publication [1]. I am indebted to Sergei Tabachnikov for referring me to that volume, and to Mohammed Ghomi for referring me to Professor Tabachnikov.

Problem 19 of this collection (proposed by Ulam) reads:

Is a solid of uniform density that will float in water in every position a sphere?

In turn this question was anticipated in a still earlier work [4]. Both references clearly anticipate the central question addressed in the recent papers [2, 3].

No response appears in the text of [1]. The problem as formulated is open to some interpretation. Responses containing counterexamples that appeared in [4] and in ensuing literature [5, 6, 7] were based on hydrostatic theory going back to Archimedes that took no account of surface tension. Independently in 2007 Mattie Sloss and I came on to the problem from a different point of view taking a partial account of surface tension, and published in [3] an affirmative answer to the question in that context, for smooth convex three-dimensional bodies. Prior to [3] I had already obtained in [2] a negative answer for the two-dimensional case of an infinite cylinder floating horizontally in the water. For the particular case $\gamma = \pi/2$, I constructed explicitly in [2] sections distinct from circles that lead to floating in any orientation, as particular “curves of constant width”. My attempt to do so for other choices of γ had only limited success, see the discussion in [2]. The source for my difficulty was recently clarified for me in an unexpected way using the mathematical theory of billiards, see [8].

As an aside, it may be noted that Lwów has since become the city of Lviv (Львів) in the Ukraine.

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