Erratum

Erratum to: Finite Section Method for a Banach Algebra of Convolution Type Operators on $L^p(\mathbb{R})$ with Symbols Generated by PC and SO

Alexei Yu. Karlovich, Helena Mascarenhas and Pedro A. Santos

Erratum to: Integr. Equ. Oper. Theory (2010) 67:559–600 DOI 10.1007/s00020-010-1784-9

We correct Theorem 3.2 and Corollary 3.3 from [2]. This correction amounts to the observation that the proof of the main result in [2] contains a gap in Lemma 10.6 for $p \neq 2$. The results of [2] are true for $p = 2$.

1. Spectra of the Operators $P_{(\alpha,\beta)}$ and $Q_{(\alpha,\beta)}$

Let the notation be as in [2].

Theorem 3.1 in [2] was misstated (see [1, Chap. 9, Theorem 4.1]), it is valid only for $\alpha, \beta \in \mathbb{R}$. This led to the mistake in the calculation of the spectra of $P_{(\alpha,\beta)}$ and $Q_{(\alpha,\beta)}$ if either $\alpha = -\infty$ or $\beta = +\infty$ in Theorem 3.2 of [2]. This theorem must be replaced by the following.

Theorem 1.1. (a) *If $-\infty < \alpha < \beta < +\infty$, then*

$$\mathrm{sp}_{\mathcal{B}(L^p(\alpha,\beta))}(P_{(\alpha,\beta)}) = \mathrm{sp}_{\mathcal{B}(L^p(\alpha,\beta))}(Q_{(\alpha,\beta)}) = \mathfrak{L}_p.$$

(b) *If $-\infty < \beta < +\infty$, then*

$$\mathrm{sp}_{\mathcal{B}(L^p(-\infty,\beta))}(P_{(-\infty,\beta)}) = \mathfrak{A}_p(0, 1), \quad \mathrm{sp}_{\mathcal{B}(L^p(-\infty,\beta))}(Q_{(-\infty,\beta)}) = \mathfrak{A}_q(0, 1).$$

(c) If $-\infty < \alpha < +\infty$, then

$$\mathrm{sp}_{\mathcal{B}(L^p(\alpha, +\infty))}(P_{(\alpha, +\infty)}) = \mathfrak{A}_q(0, 1), \quad \mathrm{sp}_{\mathcal{B}(L^p(\alpha, +\infty))}(Q_{(\alpha, +\infty)}) = \mathfrak{A}_p(0, 1).$$

Proof. The proof of part (a) given in [2, Theorem 3.2] remains unchanged.

(b) It is clear that λ belongs to the spectrum of $P_{(-\infty, \beta)}$ (resp. $Q_{(-\infty, \beta)}$) if and only if $2\lambda - 1$ (resp. $1 - 2\lambda$) is in the spectrum of $S_{(-\infty, \beta)}$. By [3, Theorem IV(ii)],

$$\mathrm{sp}_{\mathcal{B}(L^p(-\infty, \beta))}(S_{(-\infty, \beta)}) = \mathfrak{A}_p(-1, 1).$$

Hence

$$\mathrm{sp}_{\mathcal{B}(L^p(-\infty, \beta))}(P_{(-\infty, \beta)}) = \{\lambda \in \mathbb{C} : 2\lambda - 1 \in \mathfrak{A}_p(-1, 1)\} = \mathfrak{A}_p(0, 1),$$

$$\mathrm{sp}_{\mathcal{B}(L^p(-\infty, \beta))}(Q_{(-\infty, \beta)}) = \{\lambda \in \mathbb{C} : 1 - 2\lambda \in \mathfrak{A}_p(-1, 1)\} = \mathfrak{A}_p(1, 0).$$

It remains to observe that $\mathfrak{A}_p(1, 0) = \mathfrak{A}_q(0, 1)$.

(c) The proof is analogous to that of part (b) because in view of [3, Theorem IV(ii)], $\mathrm{sp}_{\mathcal{B}(L^p(\alpha, +\infty))}(S_{(\alpha, +\infty)}) = \mathfrak{A}_q(-1, 1)$. $\square$

Since $\mathrm{sp}_{\mathcal{B}(L^p(-\infty, -1))}(Q_{(-\infty, -1)})$ and $\mathrm{sp}_{\mathcal{B}(L^p(1, +\infty))}(P_{(1, +\infty)})$ are contained in $\mathfrak{L}_p$, the result stated in Corollary 3.3(a),(b) of [2] remains valid and the proof is the same. However, Corollary 3.3(c) in [2] is wrong and must be replaced by the following.

Corollary 1.2. *Let χ_+ and χ_- be the characteristic functions of $(0, +\infty)$ and $(-\infty, 0)$, respectively, and $\mathcal{B} := \mathcal{B}(L^p(\mathbb{R}))$. Then*

$$\mathrm{sp}_{\mathcal{B}}(\chi_+ P_{\mathbb{R}} \chi_+ I + \chi_- Q_{\mathbb{R}} \chi_- I) = \mathfrak{A}_q(0, 1).$$

Proof. Let $\lambda \in \mathbb{C}$. The operator $\chi_+ P_{\mathbb{R}} \chi_+ I + \chi_- Q_{\mathbb{R}} \chi_- I - \lambda I$ is represented with respect to the direct sum $L^p(\mathbb{R}) = L^p(-\infty, 0) \dot{+} L^p(0, +\infty)$ by the matrix

$$\begin{bmatrix} Q_{(-\infty, 0)} - \lambda I & 0 \\ 0 & P_{(0, +\infty)} - \lambda I \end{bmatrix}.$$

Hence, by Theorem 1.1(b)–(c),

$$\begin{aligned} & \mathrm{sp}_{\mathcal{B}}(\chi_+ P_{\mathbb{R}} \chi_+ I + \chi_- Q_{\mathbb{R}} \chi_- I) \\ &= \mathrm{sp}_{\mathcal{B}(L^p(-\infty, 0))}(Q_{(-\infty, 0)}) \cup \mathrm{sp}_{\mathcal{B}(L^p(0, +\infty))}(P_{(0, +\infty)}) \\ &= \mathfrak{A}_q(0, 1) \cup \mathfrak{A}_q(0, 1) = \mathfrak{A}_q(0, 1), \end{aligned}$$

which finishes the proof. $\square$

2. Gap in the Proof of Lemma 10.6 in [2]

Since the proof of [2, Lemma 10.6] uses the wrong [2, Corollary 3.3(c)], it contains a gap. Replacing [2, Corollary 3.3(c)] by Corollary 1.2 in the proof of [2, Lemma 10.6], we arrive at the following.

Lemma 2.1. *Let $\eta \in M_\infty(SO)$. Then*

$$\mathfrak{A}_q(0, 1) \subset \text{sp}_{\mathcal{L}_{\infty, \eta}^{\mathcal{J}}}(\Phi_{\infty, \eta}^{\mathcal{J}}(\mathbf{P}\mathbf{X}_-\mathbf{W}_-\mathbf{X}_-\mathbf{P} + \mathbf{Q}\mathbf{X}_-\mathbf{W}_+\mathbf{X}_-\mathbf{Q})) \subset \mathfrak{L}_p,$$

$$\mathfrak{A}_q(0, 1) \subset \text{sp}_{\mathcal{L}_{\infty, \eta}^{\mathcal{J}}}(\Phi_{\infty, \eta}^{\mathcal{J}}(\mathbf{P}\mathbf{X}_+\mathbf{W}_+\mathbf{X}_+\mathbf{P} + \mathbf{Q}\mathbf{X}_+\mathbf{W}_-\mathbf{X}_+\mathbf{Q})) \subset \mathfrak{L}_p.$$

We believe that both spectra coincide with $\mathfrak{L}_p$, however we were so far unable to prove this.

If $p = 2$ then both $\mathfrak{A}_2(0, 1)$ and $\mathfrak{L}_2$ collapse to the segment $[0, 1]$. Hence for $p = 2$ the previous result reads as follows.

Lemma 2.2. *Let $p = 2$ and $\eta \in M_\infty(SO)$. Then*

$$\text{sp}_{\mathcal{L}_{\infty, \eta}^{\mathcal{J}}}(\Phi_{\infty, \eta}^{\mathcal{J}}(\mathbf{P}\mathbf{X}_-\mathbf{W}_-\mathbf{X}_-\mathbf{P} + \mathbf{Q}\mathbf{X}_-\mathbf{W}_+\mathbf{X}_-\mathbf{Q})) = [0, 1],$$

$$\text{sp}_{\mathcal{L}_{\infty, \eta}^{\mathcal{J}}}(\Phi_{\infty, \eta}^{\mathcal{J}}(\mathbf{P}\mathbf{X}_+\mathbf{W}_+\mathbf{X}_+\mathbf{P} + \mathbf{Q}\mathbf{X}_+\mathbf{W}_-\mathbf{X}_+\mathbf{Q})) = [0, 1].$$

Thus, the main result of [2] remains true (and new) at least for $p = 2$.

References

- [1] Gohberg, I., Krupnik, N.: One-Dimensional Linear Singular Integral Equations, vols. 1 and 2. Operator Theory: Advances and Applications, vol. 53–54. Birkhäuser, Basel (1992)
- [2] Karlovich, A.Yu., Mascarenhas, H., Santos, P.A.: Finite section method for a Banach algebra of convolution type operators on $L^p(\mathbb{R})$ with symbols generated by PC and SO . Integr. Equ. Oper. Theory **67**, 559–600 (2010)
- [3] Widom, H.: Singular integral equations on L^p . Trans. Am. Math. Soc. **97**, 131–160 (1960)

Alexei Yu. Karlovich (✉)

Departamento de Matemática
Faculdade de Ciências e Tecnologia
Universidade Nova de Lisboa
Quinta da Torre
2829-516 Caparica
Portugal
e-mail: oyk@fct.unl.pt

Helena Mascarenhas and Pedro A. Santos
Departamento de Matemática
Instituto Superior Técnico
Universidade Técnica de Lisboa
Av. Rovisco Pais
1049-001 Lisboa
Portugal
e-mail: hmasc@math.ist.utl.pt
pasantos@math.ist.utl.pt