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# Weak automorphisms of dihedral groups

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Abstract. Let $\mathcal{A} = (A; F)$ be an algebra with $T$ the set of all its term operations. For any permutation $\tau$ of $A$, the induced mapping $f \to \tau \circ f \circ \tau^{-1}$ defines a permutation $\tau^\star$ of the set of all finitary operations on the set A. We say that $\tau$ is a *weak automorphism* of $\mathcal{A}$ if and only if $\tau^*(T) = T$. Of course any automorphism $\alpha$ of $\mathcal{A}$ is a weak automorphism, because $\alpha^*(t) = t$ for all $t \in T$. The set of all weak automorphisms of $\mathcal{A}$ forms a subgroup of the symmetric group on $A$. In this paper, we describe weak automorphisms of the dihedral groups $\mathcal{D}_n$ for $n \geq 3$. We show that the weak automorphism group of $\mathcal{D}_n$ is a semidirect product of the group of automorphisms of $\mathcal{D}_n$ and some group related to the group of invertible elements of the ring $\mathbb{Z}_n$.

## 1. Introduction

The notion of a weak automorphism of a general algebra was introduced by A. Goetz in [1]. The first algebras in which this notion was studied were Boolean and Post algebras [13] and algebras having a basis [7]. Then some results were obtained for finite fields and rings [2,3] and mono-unary algebras [8]. Recently in [11,12], the author has found a characterization of algebras whose weak automorphism groups are $k-$transitive for $k \geq 3$. In the case of groups, the image of the group operation $x \cdot y$ under the induced mapping $\tau^\star$ of a weak automorphism $\tau$ of a group $\mathcal{G} = (G; \cdot, ^{-1}, 1)$ is a new group operation. One can ask under what conditions the mapping $\tau_n$ for $n \in Z$, defined by $\tau_n(g) = g^n$ for $g \in G$, is a weak automorphism of $\mathcal{G}$. This question was studied in [9,14] for finite groups. The permutation $\tau_{-1}$ is a weak automorphism in any group. Clearly $\tau_{-1}^\star$ maps $x \cdot y$ onto $y \cdot x$. It is known that in abelian groups [4] and in the absolutely free groups [6], every weak automorphism is a superposition of an automorphism with $\tau_{-1}$ or simply an automorphism. There are some papers in which the authors are looking for group operations other than $x \cdot y$ or $y \cdot x$, which are not induced by a weak automorphism. In [5], nilpotent groups of class 2 were studied and in [10] such group operations were found in nilpotent groups of class 3 and 4. In this paper, we determine weak automorphisms of dihedral groups $\mathcal{D}_n$ for $n \geq 3$, and we give a complete description of the structure of the group of weak automorphisms of $\mathcal{D}_n$. It turns out that this group is a semidirect product of the automorphism group

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of $\mathcal{D}_n$ and some subgroup of the symmetric group on $D_n$ closely related to the group of invertible elements of the ring $\mathbb{Z}_n$.

## 2. Some properties of weak automorphisms

Let $\mathcal{A} = (A; F)$ be a finitary algebra with $T^{(n)}$ the set of all $n-$ary term operations. A permutation $\tau$ of the set $A$ is said to be a *weak automorphism* of $\mathcal{A}$ if for all $n = 0, 1, 2, \ldots$, the induced mapping $\tau^\star : A^{A^n} \to A^{A^n}$ defined by

$$(\tau^\star f)(a_1, a_2, \ldots, a_n) = \tau(f(\tau^{-1}(a_1), \tau^{-1}(a_2), \ldots, \tau^{-1}(a_n))$$

for $a_1, a_2, \ldots, a_n \in A$, transforms each set $T^{(n)}$ onto itself. This means that for every term $t \in T^n$ for $n = 0, 1, \ldots$, there exist two terms $t_1, t_2 \in T^n$ such that $\tau^\star(t) = t_1$ and $\tau^\star(t_2) = t$ or, what is the same, the equalities

$$\tau(t(a_1, a_2, \ldots, a_n)) = t_1(\tau(a_1), \tau(a_2), \ldots, \tau(a_n)),$$
$$\tau(t_2(a_1, a_2, \ldots, a_n)) = t(\tau(a_1), \tau(a_2), \ldots, \tau(a_n))$$

hold for all $a_1, a_2, \ldots, a_n \in A$. The elements of the set $WAUT(\mathcal{A})$ of all weak automorphisms of an algebra $\mathcal{A}$ form a subgroup of the symmetric group $(S_A; \circ, ^{-1}, Id)$ of the set $A$, which contains the automorphism group $AUT(\mathcal{A})$ as a normal subgroup.

In the case of a group $\mathcal{G} = (G; \cdot, ^{-1}, 1)$, the set $T^{(n)}$ consists of all words $w$ in $n$ variables of the form

$$w(x_1, x_2, \ldots, x_n) = x_{i_1}^{n_1} x_{i_2}^{n_2} \cdots x_{i_k}^{n_k},$$

where $1 \leq i_1, i_2, \ldots, i_k \leq n$ and $n_1, n_2, \ldots, n_k \in \mathbb{Z}$.

As a simple consequence of the definition of a weak automorphism, we get

**Proposition 2.1.** *Let $\mathcal{G} = (G; \cdot, ^{-1}, 1)$ be a group, and let $\tau$ be a permutation of the set $G$. Then $\tau$ is a weak automorphism of $\mathcal{G}$ if and only if*

(1) $\tau(1) = 1$,
(2) $\tau(g^{-1}) = (\tau(g))^{-1}, g \in G$,
(3) *there exist two words $u(x, y)$ and $v(x, y)$ such that equalities*

$$\tau(g \cdot h) = u(\tau(g), \tau(h)) \quad \textit{and} \quad \tau(v(g, h)) = \tau(g) \cdot \tau(h)$$

*hold for all $g, h \in G$.*

*Proof.* Routine. □

**Theorem 2.2.** *Let $\mathcal{G} = (G; \cdot, ^{-1}, 1)$ be a group. Suppose that $\tau$ is a weak automorphism of $\mathcal{G}$ such that $\tau^\star(x \cdot y) = w(x, y) = x \circ y$ for some word $w(x, y)$; that is, the equality*

$$\tau(g \cdot h) = \tau(g) \circ \tau(h) \tag{2.1}$$

*holds for all $g, h \in G$. The following equalities hold for all $g, h, k \in G$ and all integers $n$:*

(1) $\tau(1) = 1$,

(2) $1 \circ g = g \circ 1 = g$,
(3) $(g \circ h) \circ k = g \circ (h \circ k)$,
(4) $g^{-1} \circ g = g \circ g^{-1} = 1$,
(5) $\tau(g^n) = (\tau(g))^n$.

*Therefore* $(G; \circ, ^{-1}, 1)$ *is a group and* $\tau$ *is an isomorphism between the groups* $\mathcal{G}$ *and* $(G; \circ, ^{-1}, 1)$.

*Proof.* We have $\tau(1) = 1$, because $T^{(0)} = \{1\}$. This together with (2.1) gives

$$1 \circ \tau(g) = \tau(1) \circ \tau(g) = \tau(1 \cdot g) = \tau(g) = \tau(g \cdot 1) = \tau(g) \circ \tau(1) = \tau(g) \circ 1,$$

which proves (2). We have also

$$\begin{aligned}(g \circ h) \circ k &= \tau\big((\tau^{-1}(g) \cdot \tau^{-1}(h)) \cdot \tau^{-1}(k)\big)\\ &= \tau\big(\tau^{-1}(g) \cdot (\tau^{-1}(h) \cdot \tau^{-1}(k))\big) = g \circ (h \circ k).\end{aligned}$$

Further, $\tau(g) \circ \tau(g^{-1}) = \tau(g \cdot g^{-1}) = 1 = \tau(g^{-1} \cdot g) = \tau(g^{-1}) \circ \tau(g)$, which means that $\tau(g^{-1})$ is the inverse of the element $\tau(g)$ with respect to the operation $\circ$ and (4) follows. Now by easy induction on positive $n$ together with (4), we get (5), as required. □

## 3. Notations and some properties of dihedral groups

Let $\mathcal{G} = (G; \cdot, ^{-1}, 1)$ be a group. Suppose that $\mathcal{H}$ is an arbitrary subgroup and $\mathcal{K}$ is a normal subgroup of $\mathcal{G}$. If each element $g$ from $G$ has the unique presentation $g = h \cdot k$ where $h \in H$ and $k \in K$, then the group $\mathcal{G}$ is said to be a *semidirect product* of $\mathcal{K}$ and $\mathcal{H}$. Denote $Z_n = \{0, 1, 2, \dots, n-1\}$, and let $Z_n^\times$ stand for the set of all invertible elements of the ring $\mathbb{Z}_n = (Z_n; +, \cdot, -, 0, 1)$. For $n \geq 3$, the dihedral group $\mathcal{D}_n$ is a semidirect product of the group $(\{1, -1\}; \cdot, ^{-1}, 1)$ and $(Z_n; +, -, 0)$, where $\varepsilon \in \{-1, 1\}$ acts on $Z_n$ as the natural automorphism, $i \to \varepsilon \cdot i$ for $i \in Z_n$, of the group $\mathcal{Z}_n^+ = (Z_n; +, -, 0)$. Thus any element of the group $\mathcal{D}_n$ can be presented as a pair $(\varepsilon, i)$, where $\varepsilon \in \{1, -1\}$ for $i \in Z_n$. The operations in $\mathcal{D}_n$ are given by the formulas

$$(\varepsilon, i) \cdot (\eta, j) = (\varepsilon \cdot \eta, \eta \cdot i + j), \quad (\varepsilon, i)^{-1} = (\varepsilon, -\varepsilon i), \quad 1 = (1, 0).$$

We use the standard notations:

$$x^{-1}yx = y^x, \quad [y, x] = y^{-1}x^{-1}yx, \quad x^{\alpha+\beta y+z} = x^\alpha (x^\beta)^y x^z, \quad x^{\beta y} = (x^y)^\beta,$$

for arbitrary group elements $x, y, z$ and all integers $\alpha, \beta$. The following equalities are identities in any group:

$$[y, x]^{-1} = [x, y], \quad [xy, z] = [x, z]^y [y, z], \quad [x, yz] = [x, z][x, y]^z. \tag{3.1}$$

Let $exp(\mathcal{G})$ denote the exponent of $\mathcal{G}$; that is, the smallest positive integer $t$ such that $g^t = 1$ for all $g \in G$.

**Proposition 3.1.** (i) *For all elements* $x = (\varepsilon, i)$, $y = (\eta, j)$ *from* $D_n$ *we have*

$$[y, x] = (1, (1-\eta)i + (\varepsilon - 1)j),$$
$$[y, x]^x = (1, (1-\eta)\varepsilon i + (1-\varepsilon)j),$$
$$[y, x]^y = (1, (\eta - 1)i + (\varepsilon - 1)\eta j),$$
$$[y, x]^{xy} = (1, (\eta - 1)\varepsilon i + (1-\varepsilon)\eta j),$$

*and therefore the equation*

$$[y, x]^{xy} = [y, x]^{-1-x-y} \tag{3.2}$$

*is an identity in* $\mathcal{D}_n$.

(ii) *The equalities*

$$[[y, x], [u, v]] = 1, \qquad [x^2, y^2] = 1, \qquad [x^2, [y, z]] = 1, \qquad U^{(1-x)(1+x)} = 1,$$

*where* $U$ *is any product of commutators, are identities in* $\mathcal{D}_n$.

*Proof.* This follows immediately from the definition of the group $\mathcal{D}_n$. □

**Proposition 3.2.** *Any word* $w(x, y)$ *of two variables* $x, y$ *in the group* $\mathcal{D}_n$ *can be written as*

$$w(x, y) = x^s y^t [y, x]^{A+Bx+Cy}, \tag{3.3}$$

*where* $s, t \in Z_n$ *and* $A, B, C \in Z_e$ *for* $e = exp(\mathcal{D}'_n)$.

*Proof.* Since the group $\mathcal{D}_n$ is of finite exponent, we can assume that $w(x, y) = z_1 \cdot z_2 \cdots z_k$, where $z_1, z_2, \ldots, z_k \in \{x, y\}$. Using (3.1), we can transpose all $x$'s which follow after $y$'s. After this process, we obtain a word of the form $w' = x^t u$, where $u$ is a product of $y$'s and powers $[y, x]^{p(x)}$ of commutators for some polynomials $p(x)$. Now we remove all $y$'s on the second place after $x^t$. According to (ii) of Proposition 3.1, all commutators and all squares of elements commute with each other, and therefore we can rewrite all products of commutators as $[y, x]^{q(x,y)}$ for some polynomial $q(x, y)$, which by (3.2) is of the required form. □

## 4. Weak automorphisms of $\mathcal{D}_n$

We begin with the following

**Theorem 4.1.** *Let us define*

$$\tau_{A,a,b}(1, i) = \begin{cases} (1, ai) & \text{if } \varepsilon = 1, i \in Z_n, \\ (-1, b + (1-2A)ai) & \text{if } \varepsilon = -1, i \in Z_n, \end{cases}$$

*where* $a \in Z_n^{\times}$, $b \in Z_n$, *and* $A$ *is an element of* $Z_n$ *such that* $1 - 2A \in Z_n^{\times}$. *Then* $\tau_{A,a,b} = \tau$ *is a weak automorphism of the group* $\mathcal{D}_n$ *such that*

$$\tau(gh) = \tau(g)\tau(h)[\tau(h), \tau(g)]^{A+B(\tau(g)+\tau(h))}, \tag{4.1}$$

$$\tau\big(gh[h, g]^{A_1+B_1(g+h)}\big) = \tau(g)\tau(h) \tag{4.2}$$

*hold for every* $g, h \in D_n$, *where*

$$B = (1-2A)^{-1}(A - A^2) = -B_1, \;\; A_1 = -A(1-2A)^{-1}. \tag{4.3}$$

*Proof.* First of all observe that $\tau_{A,a,b} = \tau$ is a permutation of the set $D_n$. According to Proposition 2.1, we need only to check the equalities (4.1) and (4.2).

Let $g = (\varepsilon, i)$ and $h = (\eta, j)$ for $\varepsilon, \eta \in \{-1, 1\}$ and $i, j \in Z_n$. In view of Proposition 3.1, we have

$$\begin{aligned} [y,x]^{A+Bg+By} &= [y,x]^A([y,x]^g)^B([y,x]^y)^B \\ &= (1, i(1-\eta)(A + B(\varepsilon - 1)) + j(\varepsilon-1)(A+B(\eta-1)) \end{aligned} \tag{4.4}$$

Using this, we can compare the left $L(g,h)$ and the right hand side $R(g,h)$ of the equation (4.1). We have

$$\begin{aligned} L((1,i),(1,j)) &= (1, a(i+j)) = R((1,i),(1,j)), \\ L((1,i),(-1,j)) &= (-1, b + a(j-i)(1-2A)) = R((1,i),(-1,j)), \\ L((-1,i),(1,j) &= (-1, b + a(i+j)(1-2A)) = R((-1,i),(1,j)), \\ L((-1,i),(-1,j)) &= (1, a(j-i)), \\ R((-1,i),(-1,j)) &= (1, a(j-i)(1 - 4(A - A^2 + 2AB - B)). \end{aligned}$$

Thus for $B$ defined in (4.3), the equation $L(g,h) = R(g,h)$ holds for all $g, h \in D_n$.

Similarly, let $L(g,h)$ and $R(g,h)$ denote the left and the right hand side of the equation (4.2), respectively. We have

$$\begin{aligned} L((1,i),(1,j)) &= (1, (i+j)a) = R((1,i),(1,j)), \\ L((1,i),(-1,j)) &= (-1, b + ia(1-2A)(2A_1 - 1) + ja(1-2A), \\ R((1,i),(-1,j)) &= (-1, b - ia + ja(1-2A)), \\ L((-1,i),(1,j)) &= (-1, b + ia(1-2A) + ja(1-2A)(1-2A_1), \\ R((-1,i),(1,j)) &= (-1, b + ia(1-2A) + aj), \\ L((-1,i),(-1,j)) &= (1, (j-i)a(1 - 2A_1 + 4B_1)), \\ R((-1,i),(-1,j)) &= (1, (j-i)a(1-2A)). \end{aligned}$$

This means that for $A_1$ and $B_1$ which satisfy (4.3), the equality $L(g,h) = R(g,h)$ holds for arbitrary choice of $\varepsilon, \eta$ and $i, j$. Therefore statement (4.2) is true, and consequently, $\tau_{A,a,b}$ is a weak automorphism of the group $\mathcal{D}_n$. □

**Theorem 4.2.** *Each weak automorphism of the group* $\mathcal{D}_n$ *is of the form* $\tau_{A,a,b}$ *for some triple of elements* $A$, $a$, *and* $b$ *from* $Z_n$ *such that* $1 - 2A, a \in Z_n^{\times}$.

*Proof.* Let $\sigma$ be a weak automorphism of $\mathcal{D}_n$ such that $\sigma^*(x \cdot y) = u(x,y)$ for some term $u$. By Proposition 3.2, we can assume $u(x,y) = xy[y,x]^{A+Bx+Cy}$ for some $A$, $B$, and $C$ from $Z_n$. Therefore the equality

$$\sigma(gh) = \sigma(g)\sigma(h)[\sigma(h), \sigma(g)]^{A+B\sigma(g)+C\sigma(h)}$$

holds for all $g, h \in D_n$. Since all elements of order $n$ belong to the set $X = \{(1, i) : i \in Z_n\}$, the element $\sigma(1, 1)$ has to belong to $X$. By Theorem 2.2, we have $\sigma(1, 1) = (1, a)$ for some $a \in Z_n \setminus \{0\}$. By (4.4) and induction on $i$, we obtain $\sigma(1, i) = \sigma(1, ai)$ for all $i \in Z_n$, and therefore $a \in Z_n^\times$. Suppose that $\sigma(-1, 0) = (-1, b)$ for some $b \in Z_n$. Using the equality $(-1, j) = (-1, 0)\cdot(1, j)$ and (4.4), we have

$$\begin{aligned}\sigma(-1, j) &= \sigma((-1, 0)\cdot(1, j)) = u((-1, b), (1, aj)) \\ &= (-1, b + ja(1 - 2(A - B + C))),\end{aligned}$$

for arbitrary $j \in Z_n$. Observe that $1 - 2(A - B + C)$ must be invertible in $\mathbb{Z}_n$, because otherwise $\sigma$ would not be a permutation of $D_n$. By putting $A' = A - B + C$, we get $\sigma(\varepsilon, i) = \tau_{A',a,b}(\varepsilon, i)$ for $\varepsilon \in \{-1, 1\}$ and all $i \in Z_n$, and the proof is complete. □

**Lemma 4.3.** *We have*

$$\begin{aligned}\tau^{-1}_{A,\,a,\,b} &= \tau_{\dot{-}A,\ a^{-1},\ -(1-2A)^{-1}a^{-1}b}, \\ \tau_{A,\,a,\,b} \circ \tau_{A',a',b'} &= \tau_{A\dot{+}A',\ aa',\ (1-2A)a'b+b'}, \\ \tau^{-1}_{A,\,a,\,b} \circ \tau_{0,\,a',\,b'} \circ \tau_{A,\,a,\,b} &= \tau_{0,\ a',\ (1-2A)ab'},\end{aligned}$$

*where* $\dot{-}A = -A(1 - 2A)^{-1}$ *and* $A \dot{+} A' = A + A' - 2AA'$.

*Proof.* This follows immediately from the definition of $\tau_{A,a,b}$. □

## 5. The group $WAUT(\mathcal{D}_n)$

Now we are going to describe the structure of the group $WAUT(\mathcal{D}_n)$. To do this we need an auxiliary result.

**Lemma 5.1.** *The elements of the set*

$$Z_n^{\dot{+}} = \{A \in Z_n : (1 - 2A) \in Z_n^\times\}$$

*form a group* $\mathcal{Z}_n^{\dot{+}} = (Z_n^{\dot{+}}; \dot{+}, \dot{-}, 0)$ *with respect to the operations* $\dot{+}$ *and* $\dot{-}$ *given by the following formulas*

$$A \dot{+} B = A + B - 2AB, \quad \dot{-}A = -A(1 - 2A)^{-1}.$$

*Proof.* We check

$$(A \dot{+} B) \dot{+} C = A + B + C - 2AB - 2AC - 2BC + 4ABC = A \dot{+} (B \dot{+} C),$$

and also

$$A \dot{+} (\dot{-}A) = A + (-A(1 - 2A)^{-1}) - 2A(-A(1 - 2A)^{-1}) = 0.$$

Let us observe that if $1 - 2A = u \in Z_n^\times$ and $1 - 2B = u' \in Z_n^\times$, then $1 - 2(A + B - 2AB) = uu'$. Thus $A, B \in Z_n^{\dot{+}}$ implies $A \dot{+} B \in Z_n^{\dot{+}}$. In view of the following equalities,

$$1 - 2(-A(1 - 2A)^{-1}) = (1 - 2A)(1 - 2A)^{-1} + 2A(1 - 2A)^{-1} = (1 - 2A)^{-1},$$

we infer that if $A \in Z_n^{\dot{+}}$, then $\dot{-}A \in Z_n^{\dot{+}}$, and consequently $\mathcal{Z}_n$ is a group. □

Observe that if $n = 2k$, then $1 - 2k \in Z_{2k}^{\dot{+}}$; consequently, the set $\{0, k\}$ is a subgroup of the group $\mathcal{Z}_{2k}^{\dot{+}}$. Let us define

$$\mathcal{T}_n = \begin{cases} \mathcal{Z}_n^{\dot{+}} & \text{if } n \text{ is odd,} \\ \mathcal{Z}_{2k}^{\dot{+}}/\{0,k\} & \text{if } n = 2k. \end{cases}$$

Let us put $\mathcal{T}_n = (T_n; \dot{+}, \dot{-}, 0)$. Now we are able to prove our main result.

**Theorem 5.2.** *For $n \geq 3$, the weak automorphism group $WAUT(\mathcal{D}_n)$ of the dihedral group $\mathcal{D}_n$ is a semidirect product of the automorphism group $AUT(\mathcal{D}_n)$ of $\mathcal{D}_n$ and the group $\mathcal{W}_n = \{\tau_{A,1,0} : A \in T_n\}$. A weak automorphism $\tau_{A,a,b}$ is an automorphism of $\mathcal{D}_n$ if and only if $A = 0$.*

*Proof.* In Theorem 4.2 we have established that

$$WAUT(\mathcal{D}_n) = \{\tau_{A,a,b} : A \in Z_n^{\dot{+}}, a \in Z_n^{\times}, b \in Z_n\}.$$

The weak automorphism $\tau_{A,a,b}$ is the identity permutation if and only if $ai \equiv 0 \pmod{n}$ for all $i \in Z_n$ and $b + a(1-2A)j \equiv j \pmod{n}$ for all $j \in Z_n$. This is equivalent to the statement $2A \equiv a - 1 \equiv b \equiv 0 \pmod{n}$. Therefore the mapping $\varphi$ defined by $\varphi(\tau_{A,a,b}) = A$ is a surjection of $WAUT(\mathcal{D}_n)$ onto the group $\mathcal{W}_n$. Lemma 4.3 shows that $\varphi$ is a homomorphism. From Theorem 4.1, we infer that $\tau_{A,b,a}$ is an automorphism of $\mathcal{D}_n$ if and only if $A = 0$. Since $AUT(\mathcal{D}_n)$ is a normal subgroup of $WAUT(\mathcal{D}_n)$, the proof is complete. □

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