Erratum

Erratum to: On the pairs of multiplicative functions satisfying some relations

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In the original article we asserted (Theorem 1):

Theorem 1. *If f and g are complex valued multiplicative functions and*

$$g(2n+1) - Af(n) \rightarrow 0 \quad (n \rightarrow \infty) \quad (1.1)$$

with some constant $A \neq 0$, then either $f(n) \rightarrow 0$ ($n \rightarrow \infty$), or $A = f(2)$, $f(n) = n^s$ and $f(n) = g(n)$ for odd integers n .

This assertion is incorrect, as the following example shows:

Let $F(n) = (-1)^{n-1}$ ($n = 1, 2, \dots$), $G(2n+1) = \chi_4(2n+1)$, where χ_4 is the nonprincipal Dirichlet character (mod 4). Then $F(n)$ and $G(n)$ are multiplicative functions ($G(n)$ can be arbitrary on the powers of 2), furthermore

$$G(2n+1) = -F(n) \quad (n \in \mathbb{N}). \quad (1.2)$$

The failure comes from an incorrect assertion made in the proof of Lemma 8.

Let $H(n) = \frac{f(n)}{g(n)}$. Following the argument of the proof of Lemma 8, we obtain that $H(3) = H(Q)$ if $Q \equiv 3, 11 \pmod{12}$, thus $H(3) = H(3(4n+1)) = H(3)H(4n+1)$ if $(3, 4n+1) = 1$. For $Q = 11$ we have $H(3) = H(11)$. Since $\nu, \mu \equiv 3 \pmod{4}$, $(\nu, \mu) = 1$ implies that $H(\nu\mu) = H(\nu)H(\mu) = 1$, if $(\nu\mu, 3) = 1$, it can occur only if $H(n) = \text{constant}$ if n runs

over the integers $n \equiv 3 \pmod{4}$, $(n, 3) = 1$. Since $3^{1+2t} \equiv 3 \pmod{12}$, choosing $Q = 3^{1+2t}$, $H(3) = H(3^{1+2t})$. Let $Q = 11 \cdot 3^{2t}$. Then $Q \equiv 3 \pmod{12}$, and so $H(3) = H(11)H(3^{2t})$, consequently $H(3^{2t}) = 1$ ($t = 1, 2, \dots$).

Let $H(n) = \xi$ if $n \equiv 3 \pmod{4}$. There are two possibilities:

(A): $\xi = 1$

(B): $\xi = -1$.

In case (A) follow the further argumentation of Lemma 8, consequently the theorem formulated in [1] is true.

In Case (B) define

$$g_1(n) = \frac{g(n)}{G(n)} \text{ for odd } n, \quad f_1(n) = \frac{f(n)}{F(n)}.$$

From (1.1) we obtain that

$$g_1(2n+1) + Af_1(n) \rightarrow 0 \quad (n \rightarrow \infty)$$

and for $H_1(n) = \frac{g_1(n)}{f_1(n)}$ we obtain that $H_1(n) = 1$ for every odd n .

Consequently the following assertion is true.

Theorem. *Let f, g be complex valued multiplicative functions satisfying (1.1) with some constant $A \neq 0$. Then the following possibilities can occur:*

- (1) $f(n) \rightarrow 0$ ($n \rightarrow \infty$) and $g(m) \rightarrow 0$ ($m \rightarrow \infty$, $(m, 2) = 1$)
- (2) $f(n) = n^s$, $g(m) = m^s$ ($n, m \in \mathbb{N}$, $(m, 2) = 1$, $A = f(2)$)
- (3) $\frac{f(n)}{g(n)} = \chi_4(n)$ for odd n .

Let $g_1(2n+1) = \frac{g(2n+1)}{G(2n+1)}$, $f_1(n) = \frac{f(n)}{F(n)}$. Then $f_1(n) = n^s$, $g_1(m) = m^s$ ($n, m \in \mathbb{N}$, $(m, 2) = 1$).

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