Erratum

Erratum to: “Skew-commuting and commuting mappings in rings” [Aequationes Math. 64 (2002), 136–144]

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The proof of Theorem 2.5 in *Skew-commuting and commuting mappings in rings*, Aequationes Math. 64 (2002), p. 139 is incorrect. The correct version is as follows.

Theorem 2.5. *Let $n \geq 2$, let R be a $n!$ -torsion-free ring with left identity e , and let H be an additive subgroup of R containing e . Let $G : R \times R \rightarrow R$ be a symmetric bi-additive mapping and g the trace of G . If g is n -skew-commuting on H , then we have $g(H) = \{0\}$.*

Proof. Suppose that

$$\langle x^n, g(x) \rangle = x^n g(x) + g(x) x^n = 0 \quad \text{for all } x \in H. \quad (2.6)$$

Using a similar approach to that in the proof of Theorem 2.1, we have $g(e) = 0$.

Let t be any positive integer. Replacing x by $x + te$ in (2.6) and using $g(x + te) = g(x) + t^2 g(e) + 2tG(x, e)$ for all $x \in H$, we obtain

$$tp_1(x, e) + t^2 p_2(x, e) + \cdots + t^{n+1} p_{n+1}(x, e) = 0 \quad \text{for all } x \in H,$$

where $p_k(x, e)$ is the sum of terms involving x and e such that $p_k(x, te) = t^k p_k(x, e)$, $k = 1, 2, \dots, n + 1$.

Replacing t by $1, 2, \dots, n + 1$ in turn, and expressing the resulting system of $n + 1$ homogeneous equations with the variables $p_1(x, e), p_2(x, e), \dots, p_{n+1}(x, e)$, we see that the coefficient matrix of the system is a Vandermonde matrix

$$\begin{pmatrix} 1 & 1 & \cdots & 1 \\ 2 & 2^2 & \cdots & 2^{n+1} \\ \vdots & \vdots & \cdots & \vdots \\ n+1 & (n+1)^2 & \cdots & (n+1)^{n+1} \end{pmatrix}$$

Since the determinant of the matrix is equal to a product of positive integers, each of which is less than $n + 1$, and since R is $(n + 1)!$ -torsion free, it follows

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immediately that for each $k = 1, 2, \dots, n+1$,

$$p_k(x, e) = 0 \quad \text{for all } x \in H.$$

In particular, we have, for all $x \in H$,

$$p_{n+1}(x, e) = 2\langle e^n, G(x, e) \rangle = 0 \quad (2.7)$$

and

$$\begin{aligned} p_n(x, e) &= \langle e^n, g(x) \rangle + 2\langle xe^{n-1}, G(x, e) \rangle \\ &\quad + 2\langle exe^{n-2}, G(x, e) \rangle + 2\langle e^2xe^{n-3}, G(x, e) \rangle \\ &\quad + \dots + 2\langle e^{n-2}xe, G(x, e) \rangle + 2\langle e^{n-1}x, G(x, e) \rangle = 0. \end{aligned} \quad (2.8)$$

By (2.7), we obtain that, for all $x \in H$,

$$2\{G(x, e) + G(x, e)e\} = 0 = G(x, e) + G(x, e)e; \quad (2.9)$$

and right-multiplying by e and using (2.9), we get $G(x, e) = 0$ for all $x \in H$. Hence this forces (2.8) to

$$\langle e^n, g(x) \rangle = g(x) + g(x)e = 0 \quad (2.10)$$

for all $x \in H$. Multiplying by e on the right and utilizing (2.10), we conclude that $g(x) = 0$ for all $x \in H$. This completes the proof.

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