Preface

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This special issue is a result of two interdisciplinary workshops on *Signal and Image Processing, Integral Transforms, Cognition and Robotics Using Geometric Algebra*, organized by Eduardo Bayro-Corrochano, Elio Conte, David Hestenes; respectively by Eckhard Hitzer and Roxana Bujack, as part of the 10th International Conference on Clifford Algebras and their Applications in Mathematical Physics (ICCA) in August 2014 at the University of Tartu in Tartu, Estonia.

The first workshop ranging from n -dimensional signal processing to understanding consciousness, dealt with applications of Clifford and geometric algebras that have surprisingly spread into many fields of applied mathematics, computer science and engineering. Such applications include problems in modeling and detection of high order geometric primitives, estimation and tracking of spinors, diverse formulations in applications of Clifford, Fourier and wavelet transforms, and major progress in geometric computing for neural computing, quantum computing, robotics and control.

The thematically closely related second workshop was concerned with quaternion and Clifford Fourier and wavelet transforms as refinement and generalization of the one-dimensional complex Fourier and wavelet theory, which play an increasing role in different areas of mathematics, physics, computer science, engineering, and beyond. Its objective was to discuss the state of the art in mathematics and application of these hypercomplex Fourier and wavelet transforms and to present ideas for new developments.

In the following we give a brief overview of the content of the various contributions collected in this special issue.

The contribution *Four Forms Make a Universe* by Bernd Schmeikal approaches the mind-matter interface regarding a commutative subspace as oriented logic core within space-time algebra, that allows for arbitrary real and complex truth classifiers. From the existence of algebraic features like polarity between two appearances, identification and rearrangement fundamental to human cognition and construction, one can understand that the intelligence of space-time is prior to cognition, as it contains within its representation the basic self-reference necessary for the intelligible de-convolution of space-time.

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Following this, in the paper *The Cosmological Constant from the Extended Theory of Gravitation in Clifford Spaces* by Carlos Castro, the author explores the novel physical consequences of the Extended Theory of Gravity in C-spaces (Clifford spaces). One of the most salient physical feature of the extended gravitational theory in C-spaces is that one can generate an effective stress energy tensor mimicking the effects of *dark matter*/energy understood entirely from a purely Clifford algebraic and geometric perspective.

Next Thomas Batard and Timothy Raeymaekers study *Some properties of the spinor Fourier transform*, applied to functions taking values in the entire Clifford algebra. They determine two bases under which this Fourier transform is diagonalizable and introduce a proper application relevant convolution theorem.

After this David Eelbode and Eckhard Hitzler formulate *Operator Exponentials for the Clifford Fourier Transform on Multivector Fields*. Applying Clifford analysis, they study the Clifford Fourier transforms (with multivector square roots of -1) of multivector signal functions. They derive an operator representation expressed as the Hamilton operator of a harmonic oscillator.

Moreover, in *The Quaternion Domain Fourier Transform and its Properties*, Eckhard Hitzler defines and analyzes in detail a quaternion Fourier transform for quaternion valued signals over quaternion domains. This quaternion domain Fourier transform transforms quaternion valued signals (for example electromagnetic scalar vector potentials, color data, space-time data, etc.) defined over a quaternion domain (space-time or other four-dimensional domains) from a quaternion position space to a quaternion frequency space.

Finally, in *Multidimensional Quaternionic Gabor Transforms*, Lakshmanan Akila and Rajakumar Roopkumar treat quaternionic functions in $L^p(\mathbb{R}^d, \mathbb{H})$, $p = 1, 2$. They define a right sided quaternionic Fourier transform (QFT) with a single kernel factor. They use a Mustard type of convolution (the QFT of the convolution then gives the product of the QFTs of the factor functions) to define two types of Gabor window transforms and study their properties. They also add a discussion of a Gabor transform mapping Boehman spaces to Boehman spaces, in the context of convolution quotients of continuous functions.

The next group of contributions concentrates on geometric and visual data processing, hyperbolic homothetic motion and the control of a snake robot. In *A conformal geometric algebra based clustering method and its applications*, Minh Tuan Pham and Kanta Tachibana introduce new geometric data clustering methods based on the multivector description of data in conformal geometric algebra $Cl(4, 1)$. Their experiments show that the detection ability of their new method is superior to conventional data clustering methods.

Next in *Geometric Data Manipulation with Clifford Algebras and Möbius Transforms*, Mijail Guillemard, Armin Iske and Udo Zoelzer use the Möbius transform defined in Clifford algebra for the geometric study of point clouds in arbitrary dimensions. They introduce an application to signal classification in a dimensionality reduction framework. The characteristics of this method are shown with a computational experiment.

Furthermore, in *Geometric Entities Voting Schemes in the Conformal Geometric Algebra Framework*, authors Gehovaa Lopez-Gonzalez, Gerardo Altamirano-Gomez and Eduardo Bayro-Corrochano resort to randomized Hough transforms and tensor voting schemes applied to three-dimensional visual space objects, described in terms of conformal geometric algebra $Cl(4, 1)$ k -blades. They demonstrate their results with a number of real image experiments.

Following that, in *A New Expression for Higher Order Accelerations and Poles under the One Parameter Planar Hyperbolic Homothetic Motions*, Serdal Şahin and Salim Yüce study higher order velocities, accelerations and poles for one parameter hyperbolic homothetic motion. They further demonstrate the use of the rotation angle as motion parameter.

Moreover, in *Control of 3-link robotic snake based on Conformal geometric algebra*, authors Jaroslav Hrdina, Aleš Návrát, Petr Vašík treat the local controllability of a three link snake robot in conformal geometric algebra $Cl(4, 1)$. They state the non-holonomic kinematic equations in geometric control theory and propose an efficient control law. The functionality is shown in a virtual model implementation, and the robot's dynamics is discussed. Finally, in *Geometric Perception of Pose and Tracking*, authors G. Osuna-González, O. Carbajal-Espinosa, A. Loukianov and E. Bayro-Corrochano use reflectional symmetry in conformal geometric algebra $Cl(4, 1)$ for a novel way of three-dimensional pose-estimation for object tracking and manipulation. Real time implementation results of object pose estimation with a stereo vision system are presented as well.

We gratefully acknowledge the excellent cooperation with the chair of the International Advisory Board of the ICCA conference series Wolfgang Sprössig, the local conference chair Viktor Abramov and his dedicated team, as well as the wonderful work of our session co-organizers, and the fine collaboration with the AACA editorial team, especially the editors in chief Waldyr Rodrigues Jr. (outgoing) and Rafał Abłamowicz (incoming), with Thomas Hempfling of Birkhäuser/Springer and with the expert technical editor Saranya Karunakaran. Furthermore, we thank all authors for their precious academic contributions to the two interdisciplinary workshops at ICCA 2014, and finally the many expert reviewers for their essential and time-intensive contributions to selecting, improving and correcting the contributions contained in this special issue (names in alphabetical order): M. Akar, M. Bahri, S. Bernstein, M. Berthier, R. G. Calvet, O. Conradt, H. De Bie, L. Dorst, T. Ell, L. Fuchs, E. Guendelman, D. Hestenes, D. Hildenbrand, J. Hrdina, N. Le Bihan, W.-Q. Lim, S. Mann, T. Nitta, E. Ozdamar, Z. Oziewicz, M. T. Pham, J. Ryan, G. S. Staples, W. Sprössig, K. Tachibana, S. Vacaru, and J. Zamora-Esquivel.

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