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Dimensioning of Ancient Buildings for Spectacles Through *Stereometrica* and *De mensuris* by Heron of Alexandria

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Abstract Vitruvius explains in detail how to design Latin and Greek theatres through a geometrical approach that does not involve the problem concerning cavea's capacity which is the starting requirement of every building for spectacles. Two treatises, *Stereometrica* and *De mensuris*, written during the first century A.D. by Heron of Alexandria, explain through numerical examples some relevant formulas aimed at calculating the capacity of buildings like theatres, as well as circuses and amphitheatres. Thanks to new and accurate surveys of some buildings for spectacles, we will show some applications of these formulas and how the dimensioning problem was converted by Heron into a particular case of squaring the circle.

Keywords Building type (theatre, amphitheatre) · Design analysis · Design theory · Geographical area (Italy, Spain, Algeria, Jordan) · Historical era (classical age) · Historical treatise · Algorithms · Arithmetic progressions · Circles · Geometry · Geometric analysis · Grids · History of mathematics (Heron of Alexandria) · Measurement/measuring systems · Modules · Proportional analysis · Shapes (circular ring) · Squaring the circle (quadrature of the circle) · Symmetry

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Introduction

Heron of Alexandria is widely known for his inventions and writings on many different topics related to mathematics, geometry, mechanics, etc. All the surviving manuals and treatises from this author have been collected in five volumes entitled *Heronis Alexandrini, Opera quae supersunt omnia* published by Taubner in 1976 and in particular the fifth volume, edited by Heiberg (1914), contains *Stereometrica* and *De mensuris*, whose formulas may be very helpful for investigating ancient architectures for spectacles. This paper is focused on some relevant aspects of the design of those buildings types that developed over the centuries from the original Greek typologies into the mature architecture of the Roman Empire: circus (the Roman version of the Greek hippodrome), theatre, amphitheatre stadium and odeion. Architects of antiquity who had to design buildings belonging to those typologies had to solve similar design problems, although each type had its own specificity. The common constructive, functional and cultural aspects of their designs reflect the results achieved through the application of mathematical formulas aimed at solving specific requirements: in particular Heron's formulas are focused on the number of spectators that can be accommodated within specific shapes such as semi-circular rings. Other parts of amphitheaters and circuses are also calculated by means of this set of simple formulas, such as the perimeter of the arena or the number of spectators at a given height of the *cavea*. The application of these formulas to new and highly accurate surveys made by means of 3D laser scanner allow us to add some new keys for understanding the ancient techniques of design by clarifying the rules guiding the dimensioning of tiers of seats. The analyses carried out on the *ichnographia* of ancient buildings for spectacles shed light on some additional research topics, such as the possible use of standard areas (*clima, actus, iugerum,* etc.) for pre-dimensioning the *cavea*. Thanks to these results it is possible to provide an alternative geometrical–mathematical approach to understanding the different constructive phases of several important Roman theatres.

Ancient Treatises and Manuals

The study of ancient architecture is made possible by means of different intellectual, cultural, material and technological tools. Among these, treatises and manuals assume great relevance in the investigation of architectural design, as do other written materials: epigraphs, inscriptions, technical documents such as drawings and scale models that include brief texts, annotations, etc. Two of the main references for scholars and researchers on architecture and construction in antiquity are Vitruvius's *De architectura* (2002, 2009), Varro's *Disciplinarum libri IX*, and the set of writings belonging to the *Corpus Agrimensorum Romanorum*. This last work includes all the surviving ancient texts concerning rules, guidelines and technical topics that deal with territory and its measurement, and in some cases they cross the boundary into some interesting notions on architectural design. The *Corpus* was probably intended as a didactic tool for the management of territory: from this standpoint it can be regarded more as a handbook than as an actual treatise, although it includes some geometrical rules and formulas for the measurement of lengths and areas of different figures. To this same category, but with more general scope, belong a great part of Heron's writings. The Alexandrian mathematician was particularly famous during antiquity (and also nowadays) for his skill in the fields of applied mathematics, geometry and mechanics. His abilities made him one of the most popular science communicators of the ancient world. To be honest, the kind of mathematics contained in Heron's writings is quite distant from the most famous works by Apollonius of Perga (*Conics*) or Euclid (*Elements*), both soundly based on a formal and theoretical approach we can hardly discern in the *Corpus*. Heron is in fact representative of a different kind of knowledge: more practical and aimed at providing numerical examples instead of theoretical demonstrations. As already noted by Boyer two ways for studying geometrical figures seem to in fact coexist: "There evidently were two levels in the study of configurations … one of which, eminently rational, might be known as geometry and the other, crassly practical, might better be described as geodesy" (Boyer 1991: 172).

In the first 14 years of the twentieth century the publishing house Teubner collected in five volumes of the series *Bibliotheca Scriptorum Graecorum et Romanorum Teubneriana* the "Heronis Alexandrini opera quae supersunt Omnia" containing all the texts from the Alexandia's scientist that have arrived to our age. Within this wide and heterogeneous set of writings on mathematical applications, two sections focus on architectural construction*: De mensuris* and *Stereometrica* (Heiberg 1914). The majority of these two works was written by Heron, although the authorship of some parts is disputed because of many changes probably introduced by later editors, in particular the Byzantines.

Technical and Theoretical Issues

It is possible to assert that the information provided by the whole set of texts on technical and theoretical issues referred mainly to ancient architecture: on one hand we can in fact deduce formulas concerning calculations, while on the other it is possible to shed light on some conceptual tools for the interpretation of architectural design. However, trying to be as rigorous as possible, other relevant factors come to light, extending the research to the field of standard units of measurement, in particular those used by *Gromatici Veteres*, Varro, and Columella.

All the writings in the *Corpus Agrimensorum Romanorum* begin with a list of standard areas. Depending on many factors (the author, his origin, the time in which he writes) is possible to note the presence of overlapping terms, changes, disappeared obsolete words, etc.

Other authors not belonging to the *Gromatici Veteres*, such as Varro and Columella, also included within the framework of their writings some topics concerning standard areas for land measurement. The measurement of lengths, perimeters, areas by means of geometry and mathematical formulas also formed part of the know-how of architects who worked together with surveying experts for the measurement of buildings and not just for the division of terrain or for the construction of roads, as clearly expressed by Columella:

> "Quod ego non agricolae sed mensoris officium esse dicebam, cum praesertim ne architecti quidem, quibus necesse est mensurarum nosse rationem, dignentur consummatorum aedificiorum, quae ipsi disposuerunt, modum conprehendere, sed aliud existiment professioni suae convenire, aliud eorum, qui iam extructa metiuntur et imposito caluclo perfecti operis rationem computant"; …(Carena 1977: 336–337).[1]

This brief quote forms part of the introduction to the fifth book of the treatise on the Art of Agriculture and is placed before one of the more complete lists of standard measures, *areae mensura*, that can be found (also in comparison to *Gromatici Veteres*). The more relevant aspect is that he refers to the professional collaboration between architects and surveyors in order to calculate the cost of a new construction or to assess the dimensions of built works of architecture. On the active role of surveyors within the general context of the architectural profession, in particular when it was needed high accuracy for tracing the plan (*ichnographia*), we can also quote Balbus, perhaps one of the best known among the *mensores aedificiorum*. At the beginning of the second century A.D., he writes:

> "Ex pluribus circulis forma sine angulo, ut harenae ex quattuor circulis…[2] (quoted in Masci 2004: 102)

It is an explicit reference to the oval shape of the amphitheatre's arena, geometrically constructed by means of a compass with two different radiuses, in order to achieve a continuous curve, as clearly explained by Mario Docci and Riccardo Migliari in their works on the Colosseum (Docci 1999: 23–31; Migliari 1999: 33–50).

In this paper we will underline the existing relationships between Heron's formulas for the designing of building for spectacles and the standard areas used for subdividing, measuring and planning territories and urban areas during Roman Empire (summarized in Table 1) (Fig. 1).

This set of writings, as mentioned earlier, are characterized by a different purpose in comparison to *De Architectura*, since Vitruvius wanted to elevate architecture to the rank of a "liberal art", by staying away from too technical aspects in favor of a more "theoretical journey" inside architectural principles. In any case, before starting with the analysis of typologies for spectacles using Heron's formulas and standard areas from *Gromatici veteres*, it is important to give the correct name to some of the steps of the design process, as explained by Vitruvius in the first book of his treatise. Those terms provide the theoretical framework in which every formula and concrete measure is converted into architecture.

[1] "… and I said it was a task for a land surveyor, not a farmer, and I brought the example of architects, which certainly have to know the science of measurement, but they consider measurement of their designed buildings a not appropriate task, once finished, since they consider their purpose as professionals another, another task is the one of those who measure finished constructions and calculate their cost;….".

[2] From a set of circles is possible to draw a shape without corners, like the arena of the amphitheatre, which is made of four circles.

Table 1 Standard areas from *Gromatici Veteres* (GV), Varro (V) and Columella (C)

Pes constratus (GV)	Passus (GV)	Scripulum (V), (C)	Actus minimus (GV)	Clima (C)	Versus (V)	Actus quadratus (GV)	Iugerum (GV)	Porca (C)	Heredium (V)	Stadium (GV)	Centuria (GV)
1 × 1 pedes	1 × 5 passi	10 × 10 pedes	120 × 4 pedes	60 × 60 pedes	10 × 10 pedes	120 × 120 pedes	240 × 120 pedes	180 × 30 pedes	240 × 240 pedes	125 passi	200 iugeri
1 PC	5 PC	100 PC	480 PC	3,600 PC	100 PC	14,400 PC	28,800 PC	5,400 PC	57,600 PC	625 PC	5,760,000 PC

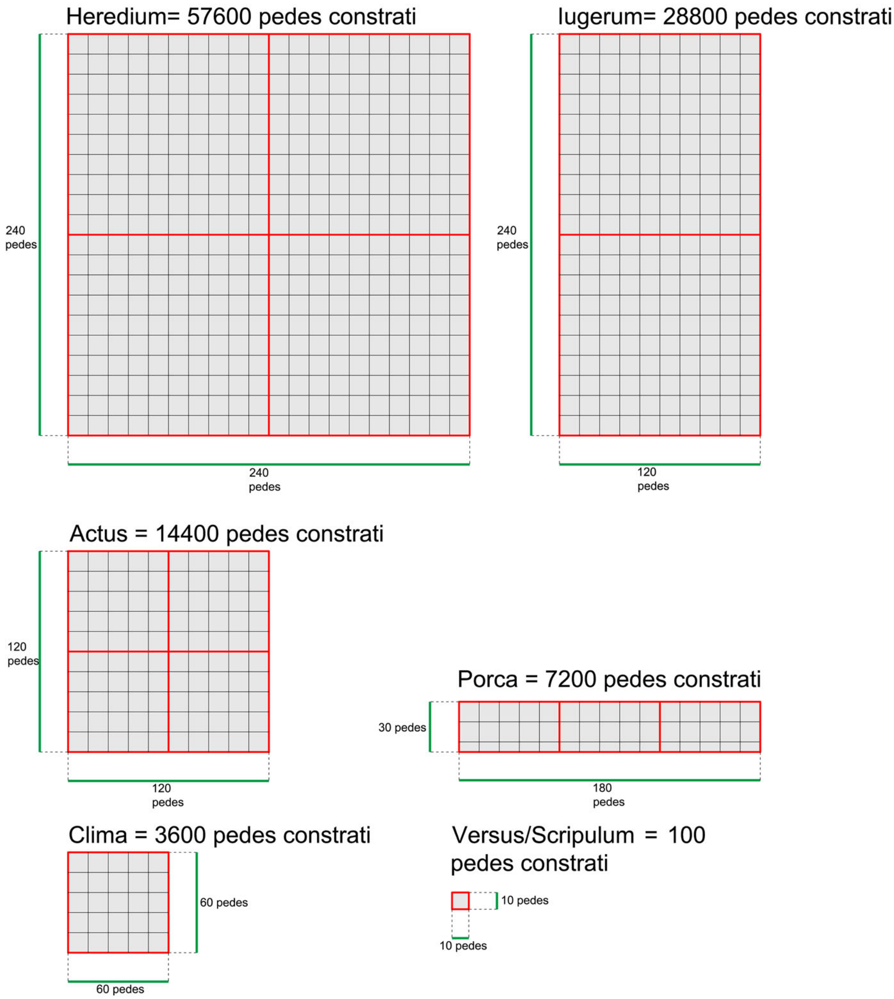

Fig. 1 Comparison between different Roman units of measurement of areas. Image: authors

In the first book of his treatise Vitruvius lists a set of words, in great part derived from similar terms used by Greek architects, whose meanings range from ideological/theoretical aspects, to operative/technical rules, but also imply archaic standards for architectural drawing, aesthetic, social habits, and economy, etc.: *ordinatio*, *dispositio*, *eurythmia*, *symmetria*, *decor*, and *distributio*. Not all of these have the same relevance and in some cases they present overlapping meanings, but amongst them the importance of *symmetria* should be commented as primary: it means the use of a three-dimensional modular grid aimed at giving common proportions to the whole building and its parts. The *eurythmia* is the achievement of a harmonic composition of all the elements characterizing the parts of a building: those proportions imply the use of a modular grid that will facilitate the composition process and the calculations needed for the distribution of all the areas and lengths leading to the final construction. *Dispositio* is divided into three parts: *ichnographia, orthographia* and *scaenographia* that are forms of representation and verification during the design process (Bartoli 1997).

These last three terms should not be confused with our current technical drawings, but they were undeniably the "ancestors" of the current orthographic and perspective views. The *ichnographia* was a kind of plan of the building and contained the result of the calculations made by the architect in order to distribute areas and lengths proportionally (*eurythmia*) by means of the modular structure (*symmetria*). It is thus reasonable that the modular structure adopted by the architect (like a grid, or a modular element) could be a main feature in this form of representation. The *orthographia* represented instead how the *ichnographia* had to be "extruded" and it contained relevant information about the main façades of the building. The last form of representation could have dealt in some degree with the current concept of perspective, as the term *scaenographia* seems to suggest, but this concept should be filtered by means of the ancient theories on vision and optics (Euclid, Heron of Alexandria). The other terms—*ordinatio*, *decor*, and *distributio*—played an important role in the general architectural principles expressed by Vitruvius, but they deal with aspects of the architectural design that may somehow be included in the set of terms and meanings already explained. The main concepts that are going to be used in the framework of this present paper for analysing and interpreting Heron's formulas are *symmetria* (the modular structure), *eurythmia* (the system of proportions between parts) and *ichnographia* (the way plans were represented).

The Theatre and Other Typologies for Spectacles

Ancient buildings for spectacles developed over the centuries from original Greek typologies to mature examples of Roman Empire architecture. As mentioned, the main types are the circus, theatre, amphitheatre, stadium and *odeion* (Fig. 2).

The different building types share several aspects, ranging from cultural to technological and constructive ones. For this reason, ancient architects had to solve similar problems when approaching the design of these buildings, even if each type had its own specificity. For example, problems related to the capacity are deeply connected to ergonomics of the seats, to the correct visual perception of the stage or of the circus track, as well as many other features (which incidentally also belong to contemporary buildings with the same functions). Nowadays these common aspects, due to their practical nature, are managed by technical regulations; there was presumably something homologous in the ancient world, even if quite different. Vitruvius writes unsystematically on topics such as rules and guidelines for a correct design, but due to the theoretical nature of his treatise, he prefers to explain historical and conceptual development of the architecture with the addition of some geometric constructions aimed at establishing a common way to obtain some specific shape. Among those "graphic algorithms" the only one dedicated to buildings for spectacles is the one referring to the theatre's *ichnographia*. Vitruvius

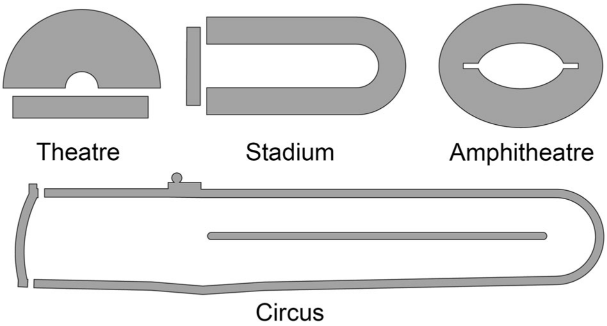

Fig. 2 Different typologies of buildings for spectacle. Image: authors

is apparently very specific when providing this very well-known graphic construction based on circles and triangles: for this reason the theatre became one of the most famous and studied typology from the ancient world. Other authors also wrote about the theatre in poems and historical writings (Publius Maro Virgilius, Livius, Pliny the Elder, Tacitus, just to mention the best known), handing down to us plenty of information about the history, the development and the social significance of these buildings, and attesting to their importance in the social life of ancient populations. On the whole, these factors meant that theatres were living works of architecture, constantly used and redesigned for the most diverse reasons, pushing to the limit the flexibility of the original Greek prototype, as underlined by Pierre Gros (2001: 325–331). This process of evolution involved not just the theatre, but also the other ancient buildings for spectacles that shared many common features with those original and homologous Greek typologies. Different factors caused morphological and dimensional variations on the architecture; in many cases these are related to the presence of former buildings whose structures could be reused, but they also dealt with local propensity towards different kinds of spectacles, local traditions and, last but not least, laws and preferences of the Roman ruling class.

Theatres are found all over the Mediterranean Basin because during the Roman Imperial age this typology was chosen as a representative urban sign of the process of Romanization. Other typologies developed from Greek prototypes (for example, those related to sport as the stadium) did not actually achieve the same popularity and can be regarded as more localized phenomena (Greece, Asia Minor, and Microasia). Amphitheatres are widely spread inside western provinces, but it is quite difficult to find them at east of the Greek peninsula, because those countries never considered *munera* (gladiatorial games) and *venationes* (hunting) spectacles essential in comparison to tragedy, comedy and athletic games (the Roman term used for them, *certamina graeca*, is indicative). Chariot races were a kind of entertainment that had more fortune in eastern provinces, and in many cases circuses served several different purposes thanks to provisional arrangements, in many cases made of wood. The increased number of functions, as well as the cultural and urban differences that arose during the Imperial Age, reshaped this set of public buildings in many directions (Gros 2001: 304), but one in particular can be considered the distinctive feature of Roman constructions: the conversion of Greek "open" compositions (opened to the town, to the environment) into closed volumes with clear stereometries. This tendency is attested to by many examples in Italy and in the provinces from the late Republican Age, but due to its significance the model provided by the theatre erected by Pompey in 55 B.C. in Rome can be elected as the main witness of Latin prototype (Moracchiello and Fontana 2009: 154). The Roman tendency to "close" the building, isolating it from the external environment, can also be noticed in restorations or *ex-novo* constructions of other typologies such as stadiums; a classic example can be found in the city of Nikopolis founded by Augustus in the 30 B.C. to celebrate the victory of Actium. The stadium of the town was built close to the theatre and was devoted to athletic competitions, but it has the peculiar shape of a "stretched" amphitheater, more than a Greek stadium because the rows of seats are laid all around the rectangular track (*dromos*) without interruptions forming semicircles at the two smallest ends (*sphendonai*): the construction is thus completely "sealed" (Fantini 2008: 114–116). Only a few stadiums present the same feature of that of Nikopolis, which probably served as a sort of prototype for those built in Caria (Turkey), Nysa and Afrodisias: another example can also be found at Laodicea ad Lycum in ancient Phrygia. Furthermore, a marble maquette found during the 1970s at Hadrian's Villa shows some similarities that suggest the hypothesis that an amphitheater with that shape could also have been planned for Tivoli. Other examples are the amphitheatres at Scythopolis (Beit She'n, Israel), Lucera (Italy), Virunum (Klagenfurt, Austria), Flavia Solva (Steiermark, Austria), and Caesarea (Cherchel, Algeria).

Studies carried out on these buildings suggest their use both for *munera* and *venationes*, but the oblong form of the arena would also have easily allowed the performance of hunting spectacles. Stadiums and theatres have systematically undergone many changes, such as enlargements of theatres' *cavea* or subdivisions of the *dromos* as in the case of the stadium of Aphrodisias, or the circus of Cesarea Maritima (Adembri and Fantini 2013) Finally, by their very nature buildings for spectacles were multipurpose, and thus they had to be capable of being easily renovated, reshaped and especially enlarged by means of provisional or permanent structures. This evolutive trend has actually made quite difficult the work of scholars, who, starting from Renaissance, have tried to make graphic constructions "fit" with the surveys they made (Salvatore 2007: 64). This aspect still determines many of the ambiguities, as underlined by Salvador Lara Ortega in his in-depth investigation of the theatre of Sagunto (1992: 151–179). The Spanish architect states that the final configuration achieved by Roman theatres cannot be explained merely as the product of a single design solution, but rather as the development of a sequence of successive steps aimed at managing changes, adaptations, and extensions of theatrical structures carried out with the purpose of providing more comfortable and efficient buildings. Many authors offer interpretations of ancient theatres based on a severe observance of the Vitruvian pattern, even for those buildings that during antiquity undergone heavy restorations or enlargements; in those cases, Lara Ortega considers that trying to make fit the original graphic scheme by Vitruvius is a mistake, since it produces no more than confusion and worthless graphic complexity. Ortega demonstrated, in the case of Sagunto's theatre, in what way Roman architects never ceased to follow the first construction scheme. They simply used the geometric model more freely at the moment of re-designing a theatre, because they considered more advantageous to reuse existing structures of both cavea and walls of the scenic building, rather than demolish and rebuilt new structures just for complying with a theoretical graphic pattern. For instance, when a bigger *cavea* was needed, they would just re-apply the Vitruvian pattern, from the same center of the original theatre, but with a longer radius; they would also move and increase the dimension of the scenic building.

Design of *Cavea* and Amphitheatre's *Podium*

Although chapters 3–9 of Book V book of Vitruvius's *De Architectura* provide a great deal of information about theatre design, it still seems to be missing a final word concerning the theatrical building in terms of an organic complex formed by three mutually-linked parts: *orchestra*, *cavea* and *scaena frons*. Casting a glance beyond Vitruvius's graphic pattern made of triangles inscribed into a lower circle,[3] some interesting suggestions may come from Heron's writings dating back to the first century A.D. In *De mensuris* 24, entitled "Measurement of theatres", the Alexandrine mathematician writes (Fig. 3):

> "We can measure a theatre in this way: if the bigger perimeter of a theatre is equal to 100 feet and the smaller is 40 feet, we can know how many people get into it. Calculate in the following way: the bigger perimeter plus the smaller perimeter is equal to $100 + 40 = 140$ feet, $1/2 \times 140 = 70$ feet. Count the rows (steps) of the theatre and we find out they are 100; $100 \times 70 = 7000$ foot; this is the number of people that fit, 7000"[4] (Heiberg 1914: 180–181).

In *Stereometrica* 40–43 (Heiberg 1914: 45–49) Heron also provides interesting examples related to the computation of the theatre's capacity; the title of the chapter is also indicative: "*different way for calculating basins*". The first example, at paragraph 42, is quite similar to *De mensuris* 24 (Fig. 4):

> "A theatre whose external circumference is equal to 420 feet and the internal is 180 feet is provided with 280 rows of seats; if we want to determine the capacity of spectators it should be done like this: the external circumference plus the internal circumference is equal to $420 + 180 = 600$ feet; $600/2 = 300$; the number 300 multiplied by the number of rows, that is 280, leads to $300 \times 280 = 84000$ spectators; because each foot corresponds to a person. In the case we had on the whole 600 feet, we divide them in two, in order to obtain the average: $1/2 \times 600 = 300$. If we have 50 rows: $50 \times 300 = 15000$ feet. This is the number of persons that fit because the space for a person is equal to the width of a foot" (Heiberg 1914: 47–49).

Paragraph 43 illustrates other two examples about capacity of theatres (Fig. 5):

[3] The *perimetros imi*, as properly underlined by Marta Salvatore (2007).

[4] Translated by authors.

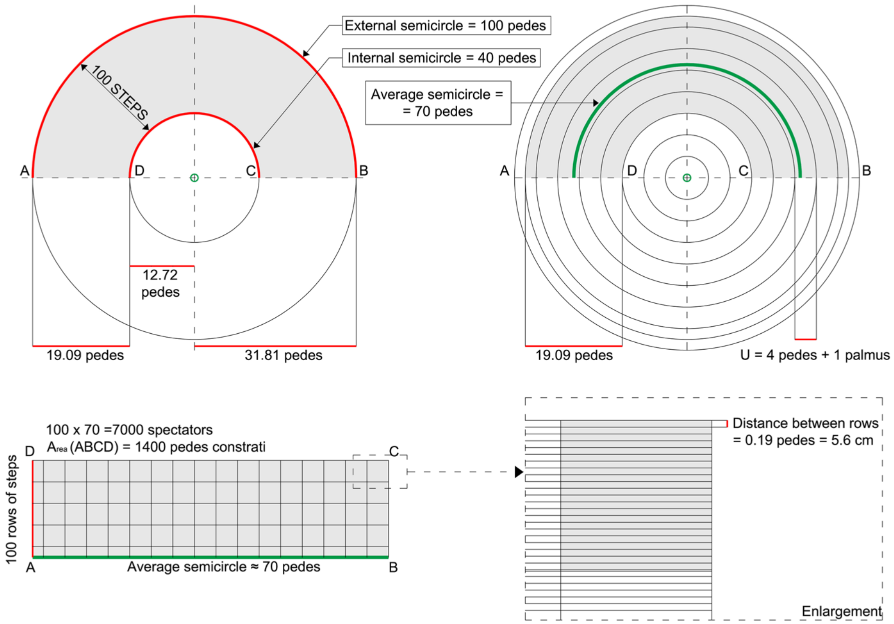

Fig. 3 Graphic interpretation of Heron's formula from *De mensuris*, 24. Image: authors

> "In another theatre, with 250 steps, the first row contains 40 people, the last row of seats 120 people for calculate the total number of people must do the following: make the sum of the first step and the last, which is 160 persons and divide by two: 160/2 = 80; 80 × 250 = 20000 people, that is the capacity of the theatre" (Heiberg 1914: 49).

There is also an example of an arithmetic progression, as underlined by Heiberg (1914: 50–51) aimed at calculating the number of spectators at a certain height of the *cavea* (Fig. 6):

> "If you want to know how many people contains the last step of a theatre in which we know that starting from the first step we have a steady increase equal to 5 spectators, the first step contains 40 people and the theatre is made up of a total of 250 steps, then do the following: 250 steps minus 1 is equal to 249 steps, 249 × 5 = 1245 + 40 of the first step. 1285 is the number of people that contains the last step" (Heiberg 1914: 49–51).

A general analysis of these relevant formulas for the design of *cavea* reveals that Heron basically illustrates in both books, by means of numerical examples, the same methodology for squaring the circle. The dimensions are however totally out of reality (in particular the depth of the seats as they range from 5.6 cm to few millimeters). But Heron's purpose is not to provide "standards"; on the contrary, it is to supply examples of dimensioning the *cavea* in relation to its capacity in terms of spectators. This same remark concerns the width of the seat (*locus*), which he

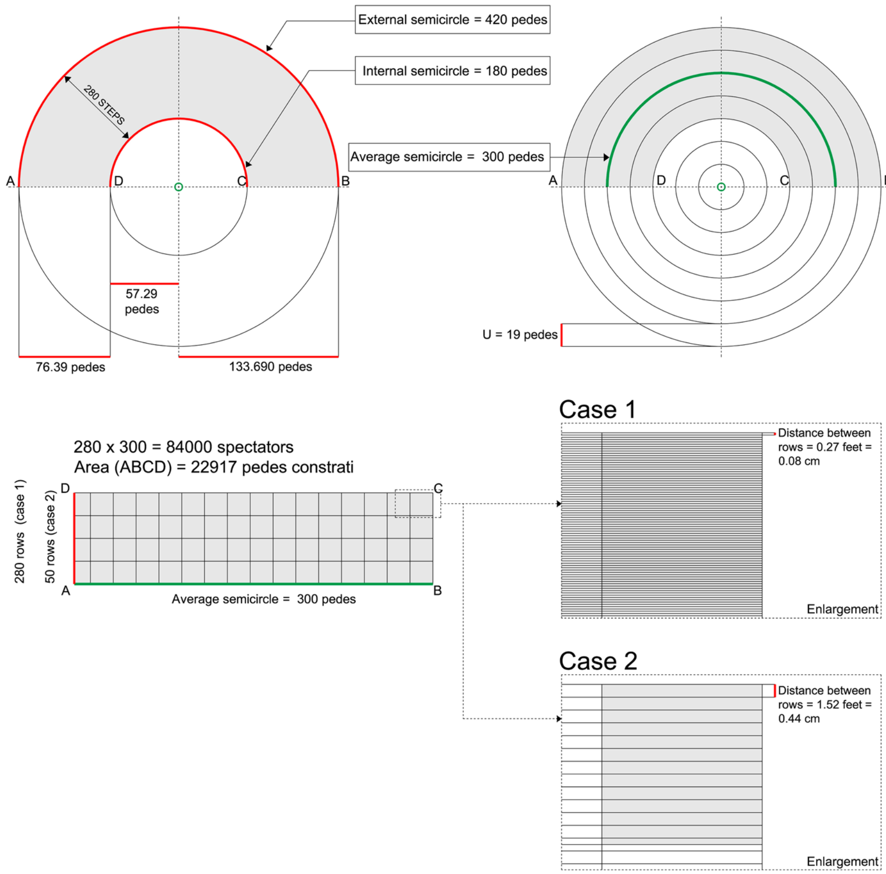

Fig. 4 Graphic interpretation of Heron's formula from *Stereometrica*, 42. Image: authors

gives as equal to one *pes*[5] (29.57 cm), as this measure seems to be used inside these formulas just to simplify the computation.

Like Vitruvius, Heron does not use the word *orchestra* for the identification of the main circle (actually the geometric entity from which the graphic algorithm begins), simply referring instead to the two half-circles describing the *cavea*'s surface. He stresses the importance of the procedure leading to the subdivision of the *analemma* (the containment walls of the *cavea* facing the *scaenae frons*). Basically the semi-annular ring is converted into a rectangular surface, with one side equal to the *analemma*'s length and the other equal to the average of the two half-circles. The graphic representation of the examples reveals for the *cavea* a common module that starts from the *perimetros imi* (or the orchestra boundary, with the exception of the *proedria*, the larger seats reserved for senators and other important people, obtained inside the orchestra) and reaches the upper *praecintio*. This

[5] On the theme of the units of length in use during Roman Empire, and in particular on the percentage variations of the *pes monetalis*, see Rosada (2010: 125–125) and Duncan-Jones (1980: 127–133).

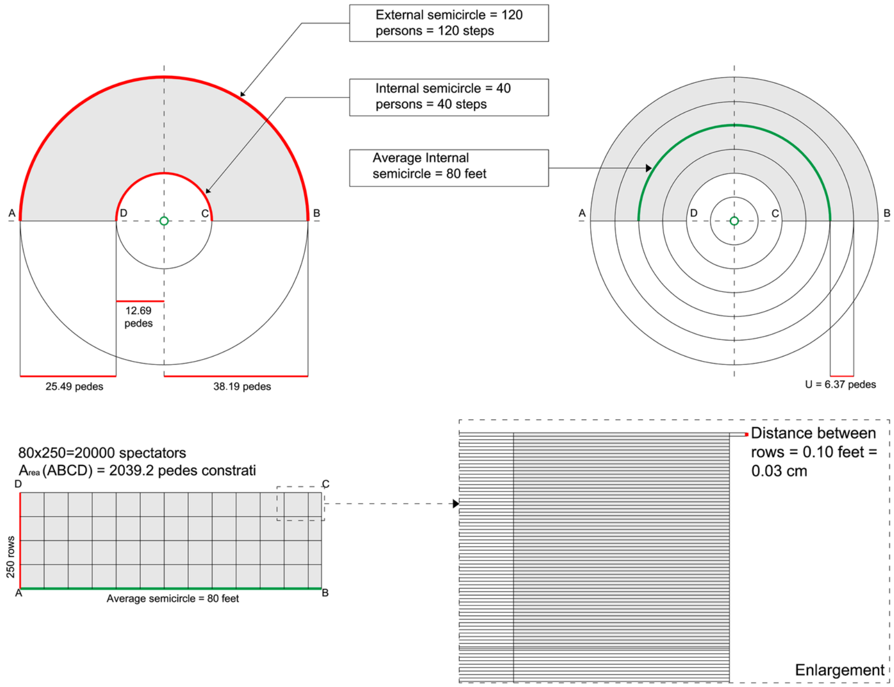

Fig. 5 Graphic interpretation of Heron's formula from *Stereometrica*, 43. Image: authors

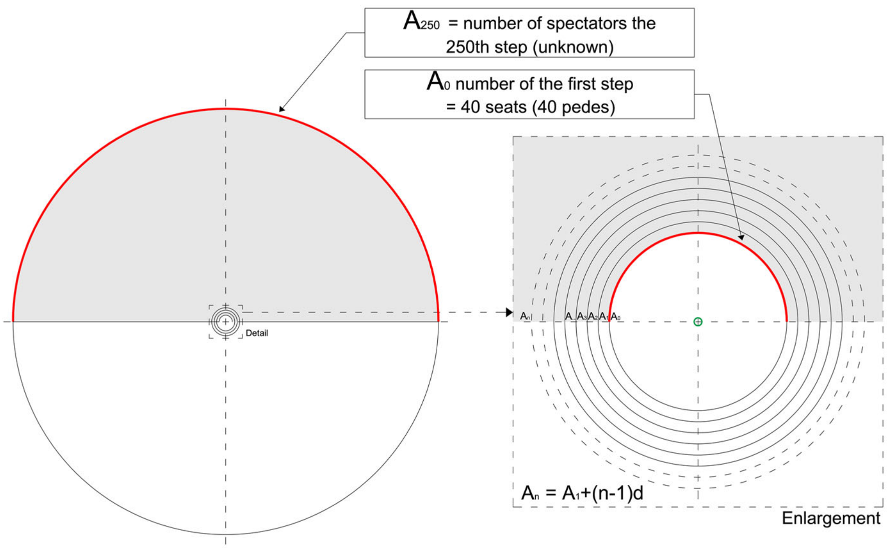

Fig. 6 Graphic interpretation of Heron's formula from *Stereometrica*, 43. Image: authors

modular design actually complies with the idea of *symmetria* and in fact Vitruvius provides indications for the modular subdivision of the orchestra when explaining the graphic algorithm for dimensioning the *scaenae frons*:

> "Orchestra inter gradus imos quod diametron habuerit, eius sexta pars sumatur, et in cornibus utrimque aditus eius mensurae <constituantur, ad> perpendiculum interiores sedes praecidantur[6]; ..." (Vitruvius 2002: 300–301).

Even more interesting is the following paragraph devoted to the calculation of one of the most relevant part of the amphitheatre, the *podium* (Heiberg 1914: 50–51). As correctly pointed out by Pierre Gros (2001: 366), the shape described by this formula does not refer to the "classic" oval stadiums and amphitheatres, but instead to that much rarer type made of straight sides and semi-circular ends mentioned previously (Fig. 7):

> "If you want to find the perimeter of an amphitheater whose length is 240 feet and width of 60 feet, you have to do this: $240 \times 240 = 57600$, $60 \times 60 = 3600$, length multiplied by width is equal to 14400 and then: $57600 + 14400 \quad +3600 = 75600$, then extract the square root of $75600 = 275$. $2 \times 275 = 550$ is the perimeter" (Heiberg 1914: 51–53).

The formula by Heron can thus be rewritten using the contemporary formalism as following:

$$P = 2 \surd\ (D2 + d2 + Dd).$$

Case Studies

De Mensuris and *Stereometrica* provide relevant information, not contained in *De Architectura*, that allow us to investigate the *theatre* typology from a design standpoint, especially suggesting some crucial questions: did ancient architects use standard quantities for the preliminary design of the *cavea*? Did they follow the original modular structure, the Vitruvian *symmetria*, when they had to modify and enlarge theatres?

It is quite difficult to answer to these questions because the majority of buildings have undergone profound transformations and, further, because the action of time and man, by means of additions or inappropriate interventions, has produced alterations that erased evidence that was crucial for correctly addressing the problem. In this complicated framework, however, consistent help has come from the ancient maquette found in Hadrian's Villa (Fig. 8).

Its purpose was probably to provide technical information for the construction: it is not exactly comparable to a contemporary maquette (ancient architects used different codes and techniques for modeling) as the representation offers a simplified view of the building to come; in fact it is a kind of shallow-relief or *stiacciato* aimed to store relevant information concerning the *ichnographia*.

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[6] "A sixth of the diameter of the orchestra between the lowest seats should be divided off; at the corners on either side (of the orchestra) where the entrances are, the innermost seats must be cut perpendicularly to a height equalling that sxith... (Vitruvius 2009: 139–141).

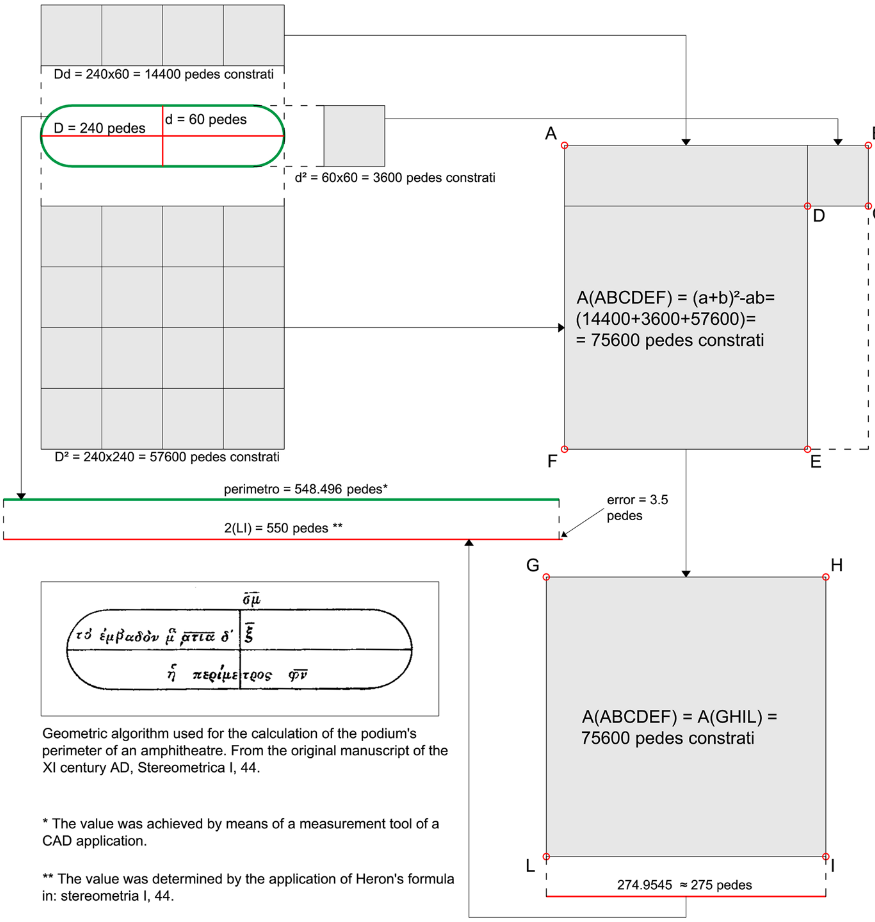

Fig. 7 Graphic interpretation of Heron's formula from *Stereometrica*, 44. Image: authors

Recent studies (Fantini 2008: 114–116) suggest that the whole maquette was carved from a marble slab approximately forming a double square measuring 89.78 by 172.22 cm (3.03 by 5.99 *pedes*); this rectangle represents the total area assigned for the building, probably a *iugerum* (Fig. 9). In this case the shorter edge of the slab, perfectly equal to three feet, would correspond to 120 *pedes* (35.4 m) at full scale.

Furthermore, assuming 1:40 as the scale of the *ichnographia*, the whole model seems to have been designed using a modular grid equal to 3/5 of a *palmus* (*symmetria*), which in the actual building would have been exactly 6 *pedes*. Tiers of seats are not represented as real rows: they represent instead the mentioned module of 6 pedes. Thus, if the 1:40 scale assumed is correct, then an *eurythmia* character also emerges from the areas of the tiers of seats: the two linear rows and the curved one have exactly the same area, which would be equal to 3,600 *pedes constrati* in

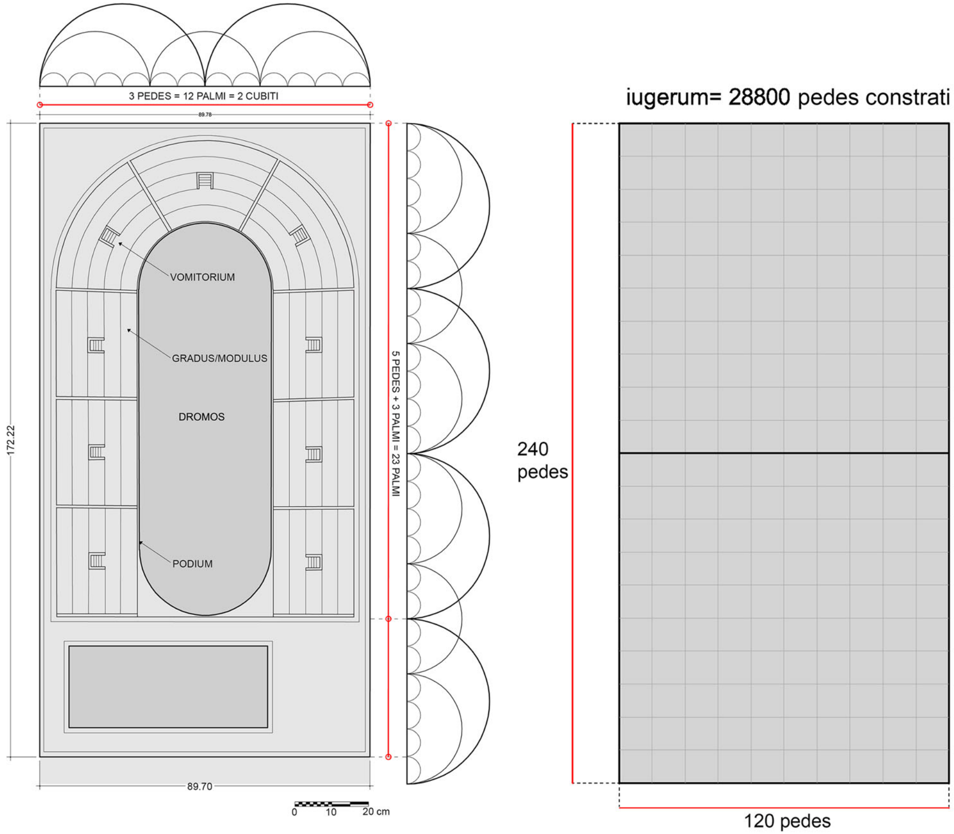

Fig. 8 Plan of the marble maquette from Hadrian's Villa. Comparison with a *iugerum* and modular interpretation using Roman measurement units. Image: authors

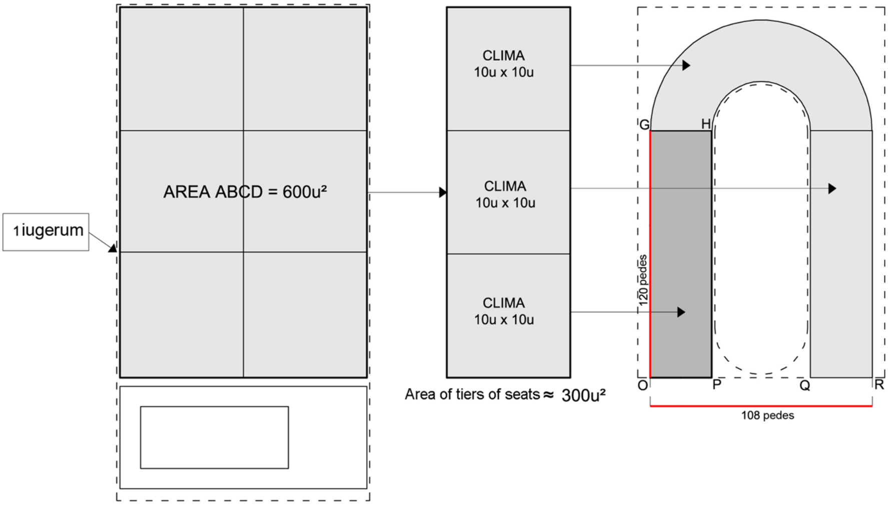

Fig. 9 Iugerum's sub modules (*climata*) in relation with the architectural design of the bleachers. Image: authors

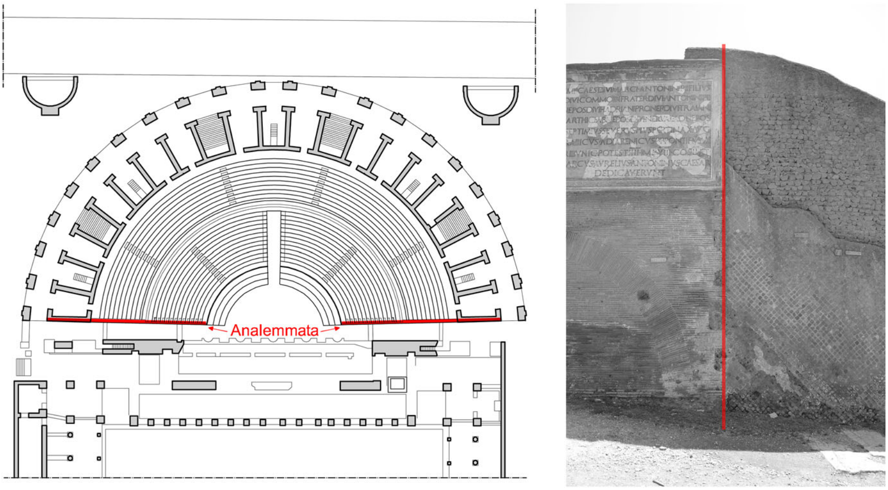

Fig. 10 Plan of Ostia's theatre with *analemmata* walls underlined (*left*), *analemmata* with a line pointing out the addition dating back to the second century A.D. (*right*). Image: authors

the actual construction; this extent corresponds to one *clima*, a standard unit of measure that is the fourth part of an *actus quadratus* (120 × 120 *pedes constrati*).

The quality and amount of information provided by this maquette are relevant to the purpose of this paper, but confronting theoretic matters, represented by this artefact (being a design and not a real building), is not enough for a true understanding of the incidence of Heron's formulas into facts. For this reason we switch now to real buildings, starting with two examples in which all constructive phases are well known and documented: the theatres of Sagunto and Ostia. In both cases their *caveae* have been concentrically extended, as can be seen by the traces of the process of enlargement that are still visible in the walls of the *analemmata*: in the Ostia theatre, the lateral walls of the *cavea*, leading to the *orchestra*, present a first constructive phase dating back to the first century B.C. (Fig. 10). The constructive technique belonging to this first phase is in fact an *opus reticolatum*, while the main enlargement that followed (end of the second century A.D.) is made in *opus latericium* (Calza and Beccati 2005: 25–26). Even if heavily restored during the 1930s, those traces are still evident.

Sagunto's theatre has also undergone significant restorations: a massive consolidation, filling of voids, strengthening of walls and vaults in the western *aditus* area in 1930, later followed by many other interventions up to the 1970s. The last one, carried out by Giorgio Grassi and Manuel Portaceli in 1990, aimed at functional renovation, can be defined as a typological restoration characterized by contemporary materials and constructive techniques. The intervention has unfortunately deleted many traces of the ancient building that could have facilitated the understanding of the constructive phases, but thanks the works by the archaeologist Emilia Hernández Hervás (1989) and by Salvador Lara Ortega (1992) is still possible to know exactly the sequence of all ancient extensions of the *cavea* and the scenic building (Fig. 11).

Fig. 11 The *cavea* of the Sagunto theatre after the typological restoration by Giorgio Grassi and Manuel Portaceli. The intervention has deleted many relevant traces of the original building, but at the same time the architects decided to follow the result of an in-depth archaeological investigation of constructive phases of the original building. Image: authors

In the case of Ostia a physical constraint, the presence of the *decumanus maximus*, prevented any enlargement of theatre beyond that road. On the contrary, in the case of Sagunto Roman architects could enlarge the *cavea* towards the natural slope of the mountain.

Heron's formulas, applied to the interpretation of the *ichnographia* of Ostia's theatre, sheds light on some relevant aspects of the *cavea*'s design: in the very first phase, it seems to have been bounded within a double square of 110 by 220 *pedes*; this area suggests a module, coming from the orchestra, equal to 10 *pedes,* in clear compliance with Vitruvius's geometrical rules (such as the division of the orchestra's diameter into six parts), and the general *symmetria* of the *cavea* also obeys the same modular structure. The intermediate semi-circle, calculated using the formulas from *Stereometrica* and *De mensuris* and approximated to the nearest integer formulas, is equal to: (346 + 94)/2 = 220 *pedes*. It is evident that the ancient architects wanted to achieve an average semi-circle strictly related to the general dimensioning of the *cavea*'s external boundary (a rectangle of 110 × 220 *pedes*). Unfortunately the exact number of rows is not so easily calculated due to the very bad state of conservation of the bleachers (before their reconstruction). In any case, twenty-four rows were rebuilt, leaving space between them for *precintiones*, as the remains suggested. In compliance with the representation used for the tires of seats in the *maquette* previously illustrated, we can presume that architects used some approximation to calculate the *cavea*'s capacity because they included within Heron's formula steps and annular paths (*gradationes* and *praecintiones*), subtracting from the bleachers the space required for passages, accesses, etc. only in a second phase of the designing process.

According to this hypothesis, the average depth of 0.77 cm (equal to 21/2 pedes with difference minor of 4 %) of the original seats was probably used by the ancient architect to calculate the number of spectators in the following way: 80/2.5 = 32 rows. The capacity of the first constructive phase of the *cavea* could so have been: 220 × 32 = 7,040 seats. A *pes* is equivalent to 29.57 cm and is identical to the measure of a locus as Heron wrote in his manuals. It is reasonable to believe that this equivalence was simply an expedient to facilitate the calculation; for this same reason Antonio Colini (1998) supported by Italo Gismondi, while investigating the capacity of the *Stadium Domitiani*, also supposed the real width of seats to be 0.444 m (1 + 1/2 *pedes*). In this framework, dividing the original 7,040 *loca* of Ostia's theatre by 1.5 allows us to estimate a capacity of about 4,700 spectators (or, to be precise, 4,693).

Moreover, beyond the numbers we can clearly perceive a specific design requirement: 7,040 is in fact quite a precise, neat quantity (very close to the round number 7,000), probably in close relationship to the required number of spectators. Referring instead to one of the questions at the beginning of this section, we started to search for "standard areas" inside the *ichnographia* of the *cavea*, since Heron's formula is a particular case of squaring the circle. Multiplying the *analemmata* by the average semi-circle the result is quite interesting: 80 × 220 = 17,600 *pedes constrati* (Fig. 12).

This area does not correspond exactly to a standard unit of surface adopted by Romans, but based on the observations made on the maquette from Hadrian's Villa,

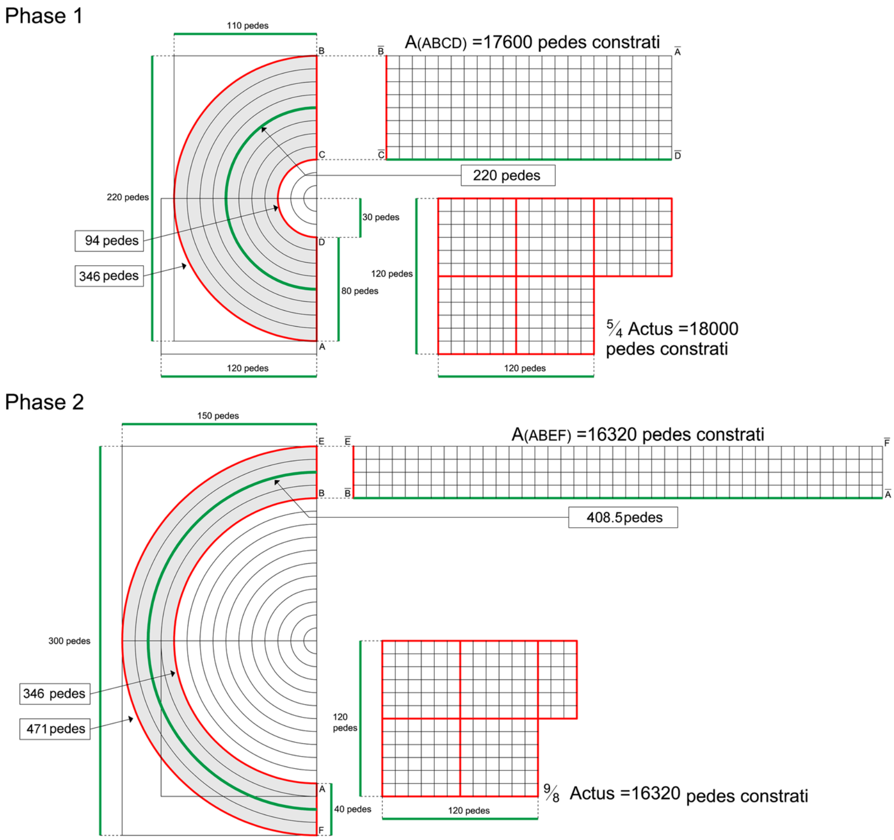

Fig. 12 Application of Heron's formula for theatre's capacity to different constructive phases of the Ostia theatre. Image: authors

we can presume that the *clima* (the sub-module of the *actus quadratus*) might have been used in the planning of the building's *ichnographia*: the *clima* is in fact equal to 3,600 *pedes constrati*, and so the area of the *cavea* can be approximated to 5 *climata* (18,000 *pedes constrati*, a deviation of 2.3 %).

Applying Heron's algorithms to the second constructive phase we find $(471 + 346)/2 = 408.5$ *pedes*; with the thickness of the second phase *analemmata*, equal to 4 modules of 10 *pedes* and the depth of every row of the *ima* and *media cavea* equal to 2.5 *pedes*, the approximate number of rows is given by $40/2.5 = 16$. The number of spectators of the outer ring, the *summa cavea*, is given by $4{,}085 \times 16 = 6{,}536$.

The architect responsible for the extension of the *cavea*'s surface thus had to solve a big problem: how is it possible to double the surface of the first theatre (corresponding to 7,040 seats in accordance with Heron's formula) if only a narrow, limited space is available due to the presence of the *decumanus maximus*? Having no ample space to "manoeuvre" he was forced to design an addition equal to 4.5

climata: the surface of outer semi-circle would in this case be equal to 16,320 *pedes constrati*, and in fact the real surface of this part of the building is the very near that value: 16,336 (a deviation of less than one tenth of 1 %).

The theatre of Sagunto presents many similarities to that of Ostia: a double square of 84 × 168 *pedes* inscribed the very core of the *cavea*, its first development step (Fig. 13a). Appling Heron's Formula we find (264 + 75)/2 = 169.5 *pedes*. Multiplying this result by 60 *pedes,* the depth of the first *analemmata,* we get 10,170 *pedes constrati*, a number quite close again to 3 *climata* (10,800 *pedes constrati* − 6 % deviation) or, with a 2 % *spread,* to 100 *scripuli* (Traglia 1992: 620–621) or versus (a unit mentioned by Varro used in Campania of 10 × 10 *pedes constrati*).

Assuming the standard depth of a seat equal to 2 *pedes* (in accordance with Vitruvius), because the *cavea* underwent many interventions as described by Salvador Lara Ortega (1991: 266–270), we can presume that the first *cavea* had a capacity of about 5,000 spectators, ultimately not so far from that of Ostia.

Applying the same methodology to the intermediate phase (Fig. 13b) and to the third (Fig. 13c), the numerical results are quite interesting. The *ichnographia* of the semi-circular ring (corresponding to the second phase) can be estimated within values ranging between 9,330 and 10,050 *pedes constrati* (including the surface corresponding to the two *tribunalia*): in this case, more than in Ostia, it is evident that the requirements—the need for a bigger theatre for the increased population of Sagunto during the first century A.D.—, led the architect to double the first surface using the same modular structure (12 *pedes*). The last documented phase (around the third century A.D.) ranges between 6,957 and 7,389 *pedes constrati*; this last quantity, which includes two small surfaces of the bleachers connecting *cavea* to the *scaenae*, is quite close (±3 % deviation) to 7,200 *pedes constrati* that is to say to half an *actus* or two *climata*.

The presence of *climata* in the *ichnographia* of those buildings may also be a coincidence, or the product of inaccurate identification of constructive phases. In any case, a first conclusion can be reached: every enlargement of the *cavea* was most probably preceded by a survey of some of the significant dimensions of the building, such the *cavea*'s circumference and the external circumference of the bleachers, followed by the planning of the new addition in compliance with the original *symmetria* and using simple numerical relations between the first and the following phases.

The amphitheatre at Cesarea of Mauritania is most likely the first one with *double sphendonai*. It belonged to the African province of Mauretania and was built by Juba II, one of the most loyal client kings that served Rome and a personal friend of Augustus. The survey on the dimensioning system adopted for the construction of Roman public buildings of Caesarea confirms the usage of *actus quadratus* in each one of the three main developments phases of the town (Chennaoui et al. 2013: 15–18).

A key reading for the dimensioning of this building comes from its *ichnographia*; applying the same criteria that emerged from the results of the investigation carried out on the *maquette* and the theatres, it possible to find some relationship between linear and curved sectors of bleachers. In fact the first constructive phase of the amphitheatre's seats is related to *clima*: a common module inside the *ichnographia*

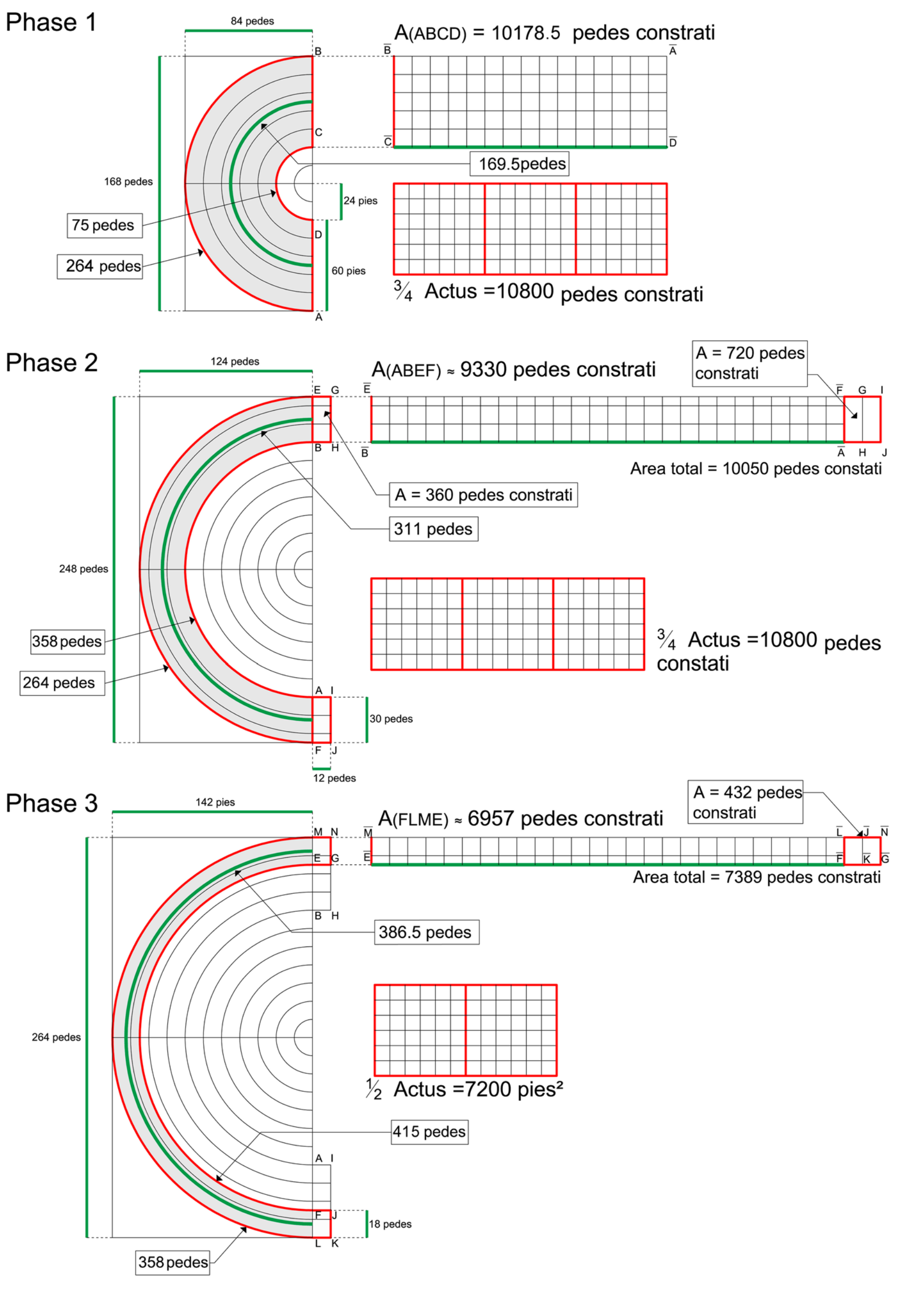

Fig. 13 Application of Heron's formula for theatre's capacity during different constructive phases of the Sagunto theatre. **a** first phase (*top*), **b** intermediate phase (*middle*), **c** third phase (*bottom*). Image: authors

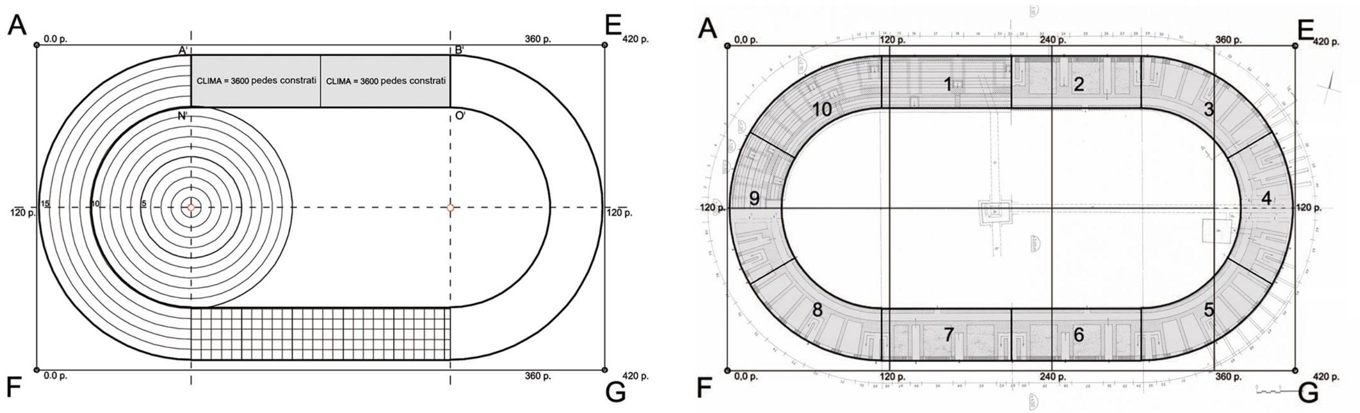

Fig. 14 The amphitheatre of Cesarea (today Cherchell, Algeria). In this building as well the presence of the Roman measurement unit called *clima* is evident: the area of the *sphendone* is equal to 3 *climata*, while the linear bleacher is equal to 2 *climata*. Image: authors

Fig. 15 The theatre of Mérida during the laser scanner survey carried out on November 2009. Photo: Filippo Fantini

is repeated all over the building starting from the two centres of the *sphendonai*. This module is equal to 5 *cubiti* (1 *cubitus* = 1.5 *pedes*). Each linear bleacher is equal to 2 *climata*; the semi-circular ones can also be divided by the same standard area equal to 3,600 *pedes constrati*; as a result they were equal to 3 *climata* (Fig. 14).

As part of the ongoing ATHENA Project,[7] a number of new survey campaigns on relevant theatres of the Mediterranean Basin have been carried out using 3D capturing techniques (Bianchini 2013). The highly reliable data coming from these activities helps researchers to arrive at a deeper knowledge of those ancient and complex monuments. In particular, 3D point cloud models provide highly detailed data compared to common survey techniques (Bianchini and Docci 2005), especially in case of difficult logistic conditions (Fig. 15).

One example is especially emblematic, the theatre of Petra (Fig. 16). Located 600 m from the centre of the city, it was constructed exploiting a natural slope, while the *scaenae frons* and *aditus maximi* were hewn out and partially built (Segal 1995: 91–92). The *Ima* and *media cavea* present a better state of conservation and a more accurate construction, while the *summa cavea* has a polygonal shape probably determined by the intrinsic difficulties of stone carving. The module, equal to 14 *pedes*, is obtained by dividing the *orchestra* into six parts. According to the Heron's formula we get an intermediate semi-circle of (484 + 132)/2 = 308 *pedes*

[7] Ancient Theatres Enhancement for New Actualities—Euromed Heritage IV Programme, http://www.athenaproject.eu.

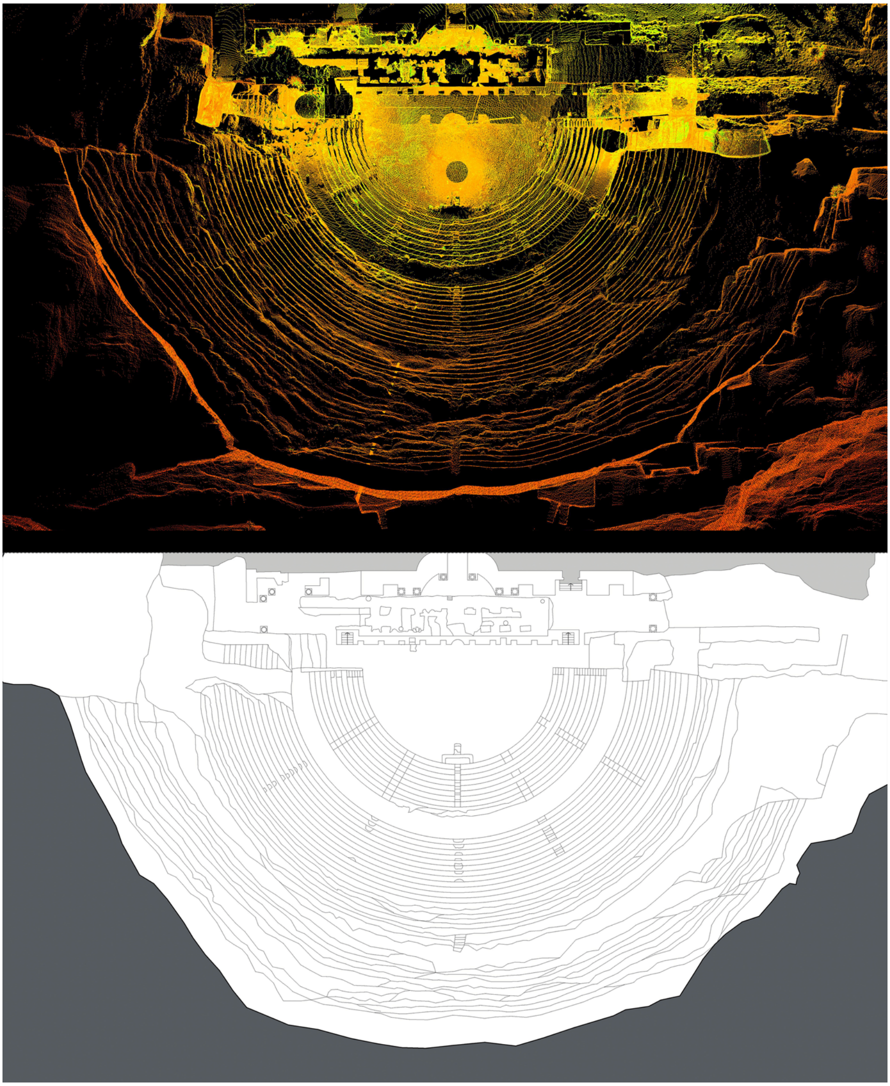

Fig. 16 Petra Theatre: point cloud from laser scanner survey and 2D drawing. Image: authors

(Fig. 17). The *analemmata* is formed by 8 modules of 14 *pedes* (112 *pedes*). The *analemmata* is formed by 8 modules of 14 *pedes* (112 *pedes*).

The average depth of the rows is 2.3 *pedes*, therefore the architect adopted this measure in order to get the number of rows according to Heron's formula: $112/2.3 = 49$ theoretical rows. In conclusion the number of spectators is so calculated: $308 \times 49 = 15{,}092$ *loca*. Also in this case a number quite close to what it seems a specific design requirement, 15,000 seats, obtained using the "rules" of *De mensuris* and *Stereometrica*. Assuming the hypothesis of Colini and Gismondi (actual width equal to 1.5 *pedes*), the number *ad abundantiam* may have been

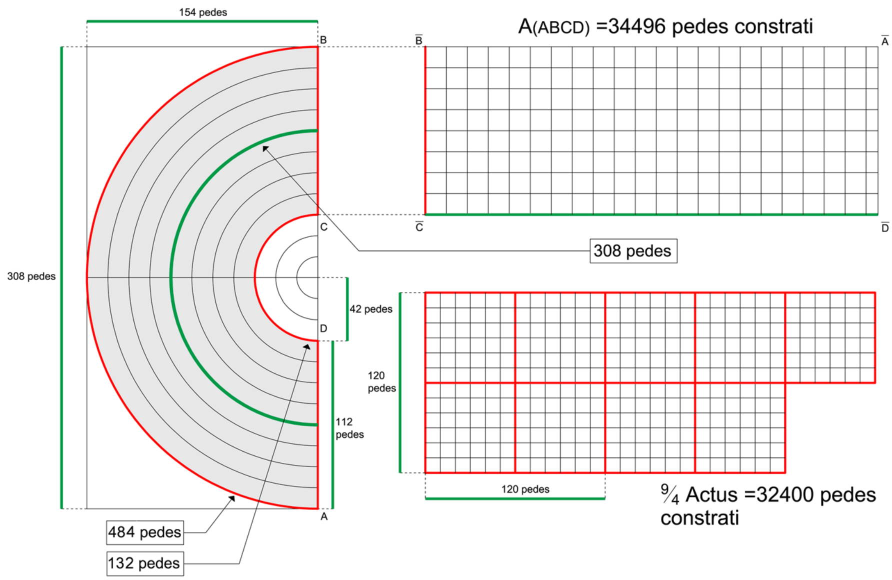

Fig. 17 Heron's formula applied to Petra theatre. Image: authors

calculated in order to receive 10,000 spectators. In this case no particular relationships to standard areas appear.

The case study of Jerash South theatre (Fig. 18) gives additional opportunities to test the hypothesis about the usage of standard measures for the dimensioning of the *cavea*: 11 *pedes* is the module, obtained as in the former cases, dividing the orchestra diameter into six parts. The *analemmata* is formed by 8.5 modules of 11 *pedes* (93.5 *pedes* in total). The average depth of the rows is 2 *pedes*, so the theoretical number of rows is 47. The intermediate semi-circumference: (397 + 104)/2 = 251 *pedes* (Fig. 19). Using Heron's formula we get 251 × 47 = 11,797 *loca*. As in Sagunto, in Jerash there are small rectilinear bleachers as long as a single module of 11 *pedes*; their contribution to the *cavea*'s global area is 47 × 22 = 1,034 *loca*. By adding 11,797 to 1,034 the complete number of spectators is achieved: 12,831 *loca*, about 8,500 real seats. But even more interesting is the global area of the tiers of seats if compared to standard areas: 25,482 *pedes constrati* correspond in fact to 7 *climata* (25,200 *pedes constrati*) or 2.5 versus or *scripuli* (25,000 *pedes constrati*) with deviations respectively of 1 and 2 %).

The last case study is the theatre of Mérida, where the module obtained from the *cavea* is equal to 15 *pedes*, the average depth of a row is 0.73 m, or 2.5 *pedes*. The *analemmata* is formed by 6.75 modules of 15 *pedes*, equal to 101 *pedes,* or 40 theoretical rows (Figs. 20, 21). The intermediate semi-circle is (459 + 141)/2 = 300 *pedes*. Applying Heron's formula, the total number of *loca* is (300 × 40) = 12,000. Using Colini's length for the *locus*, the result is 8,000 seats. The area used for the *cavea* is about 30,300 *pedes constrati*, corresponding to 8.5 *climata* (30,600 *pedes constrati*) or 3 versus or *scripuli* (30,000 *pedes constrati*) with deviation of 1 % in both cases.

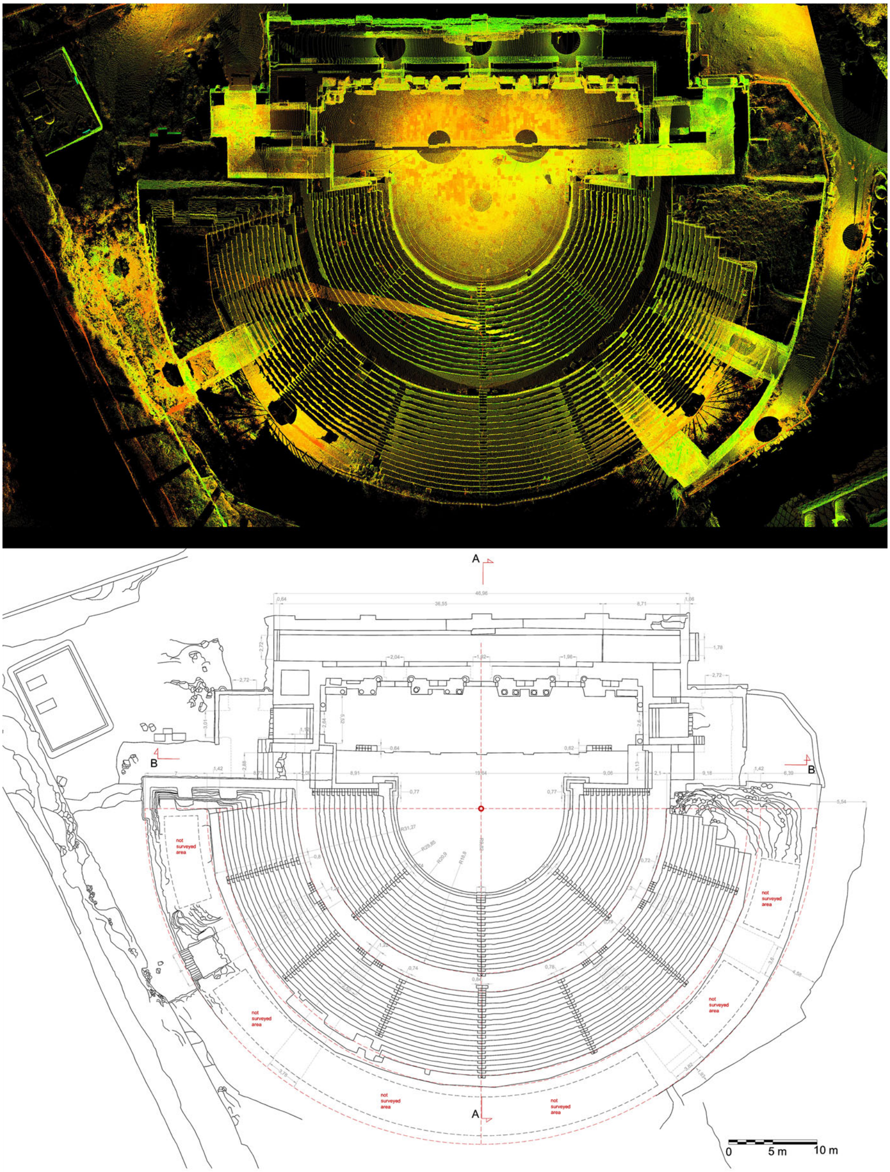

Fig. 18 Jerash South Theatre: point cloud from laser scanner survey and 2D drawing. Images: authors

Conclusions

Numerical tests seem to underline a constant usage of Heron's formulas in almost every building studied, as witnessed by the "round numbers" of almost every average semi-circle. Every one of those buildings conserves within their own masonry walls traces of interventions, enlargements and other modifications, in

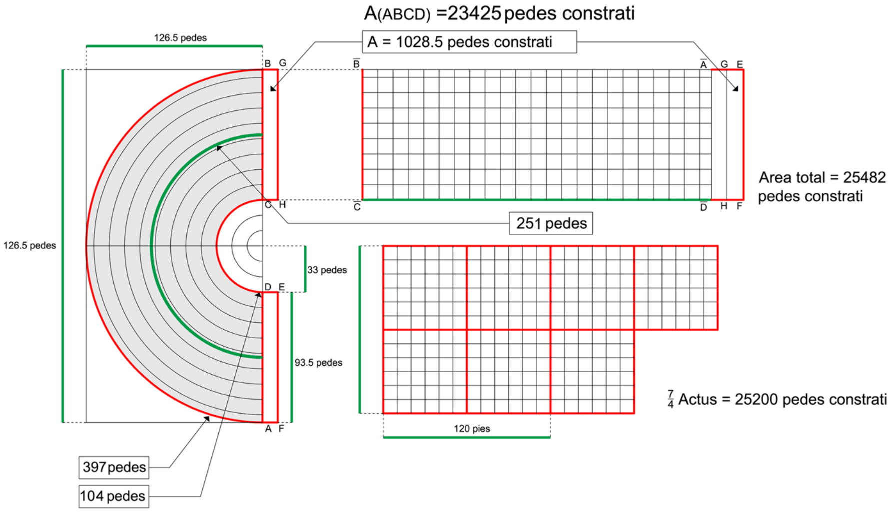

Fig. 19 Heron's formula applied to Jerash South theatre. Image: authors

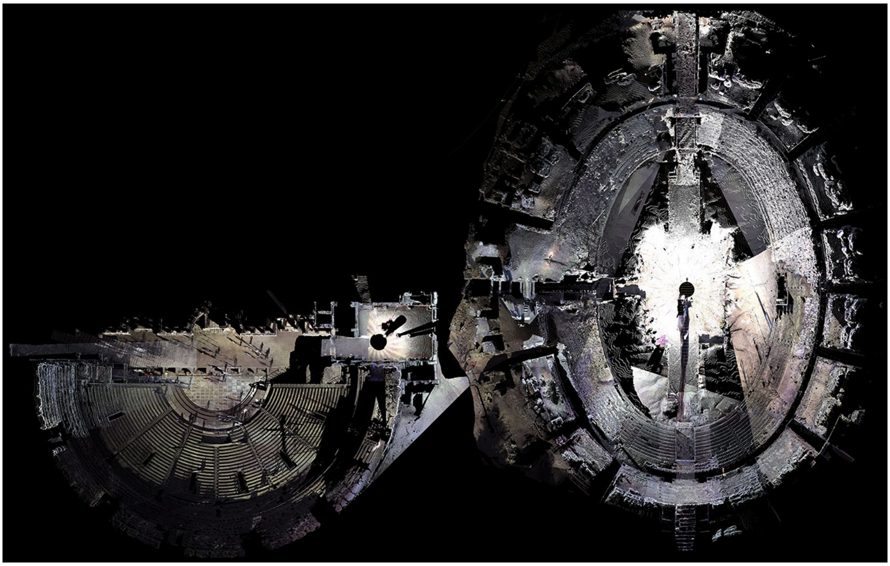

Fig. 20 Point cloud model of Mérida's theatre and amphitheatre. Image: authors

particular the heavy restorations carried out during the modern age. In any case, the presence of a common module in their *ichnographiae* is still easy to detect using Vitruvius's subdivision of the *perimetros imi*, which corresponds to a lower circle that does not include the *proedria*. The study of the historical development of theatrical buildings also appears to confirm respect for another concept expressed inside his treatise, *eurythmia*: the same common module persists in all documented

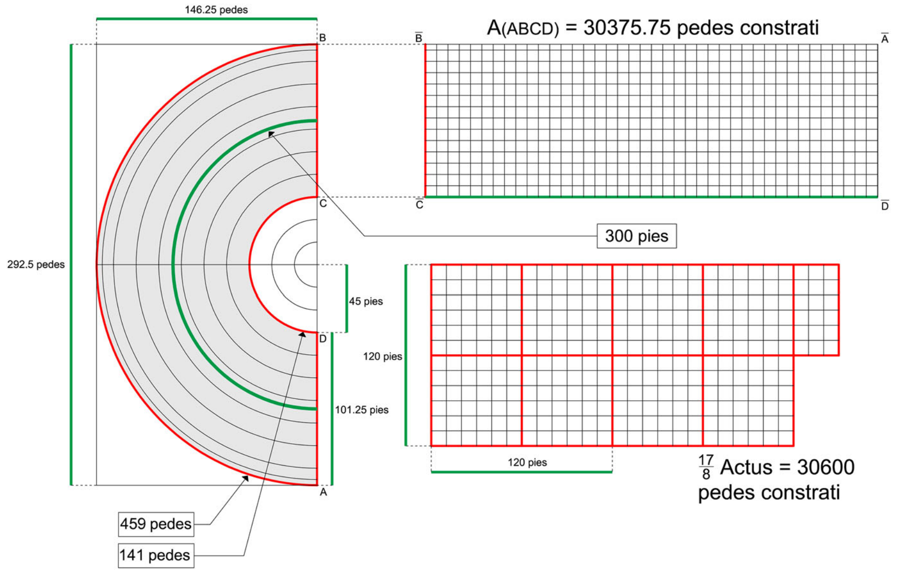

Fig. 21 Heron's formula applied to Mérida theatre. Image: authors

constructive phases of the set of analyzed theatres, as does a clear proportionality among the outer rings added to the first *cavea*. In the cases where it was not possible to respect this proportionality by means of an exact duplication of areas, as in the case of Ostia, it is reasonable to conclude that ancient architects, according to Heron's formulas, reduced the depth of the rows in order to accommodate the same required number of spectators. In the case of very deteriorated theatres, where the *summa cavea* has completely collapsed, this last hypothesis may aid virtual and real reconstructions. In some cases the usage of standard areas such as *clima* seems evident for the general dimensioning of bleachers global surface, as in the *maquette* from Hadrian's Villa in Tivoli, or the building for spectacles in Cesarea. Other examples seem instead to suggest that ancient designers preferred to use round numbers for the areas occupied by the *cavea* (Mérida, 30,000 *pedes constrati*; Jerash, 25,000 *pedes constrati*; Petra, 35,500 *pedes constrati*). Dimensioning criteria cannot be investigated with the goal of achieving exact measures, numbers, requirements, and so on, since ancient buildings for spectacles had been deeply modified in antiquity as well as in recent years. Nevertheless, Heron's *Stereometrica* and *De mensuris* provide useful hints and interpretative keys for detecting both constructive stages and requirements underlying the architectural design. This paper has been written with the purpose of disseminating some of the first results obtained applying this formula, which should be seen as a fundamental step of the cause-effect process originated by ancient architects for solving the practical problem of determining a theatre's capacity. The calculation of the *cavea*'s shape is in fact followed by the very well-known and studied graphic construction by Vitruvius, which, on closer inspection, aims at establishing a dimensional correspondence between the *orchestra* and the *scaenae frons*. Thanks to Heron's treatises, our comprehension of the process of theatre design can be understood in a more complete and exhaustive way.

Future Research

In this paper we tried to outline a new point of view for scholars of ancient theatres that stresses the fundamental role of Heron of Alexandria, not just as an *ante litteram* scientific disseminator, but also as provider of technical solutions for architectural design. As stated by Helge Svenshon (2009) the importance for the history of architecture of Heron's manuals has been completely overlooked and probably awaits a comprehensive reconsideration. The new possibilities disclosed by 3D capturing systems [such as laser scanners and photogrammetric applications based on Structure from Motion (SfM) algorithms] can actually lead to more reliable proportional and metric analyses, which could turn out to be crucial in this process.

A broader application and test of Heron's formulas to other buildings for spectacles (surveyed or in any case investigated in the past) could also concur to increase the reference base for assessing the existence of an actual design protocol. One example could be the amphitheater, the oval shape of which, generally obtained by using polycentric curves, might undergo a different kind of analysis where the "thickness" of every sector of bleachers characterized by different radii may lead to new interpretative keys. Another aspect deals with the possibility of developing an automatic software solution (parametric-generative approach) aimed at achieving a schematic 3D representation of a theatre starting from a small number of measurements, in particular those concerning *analemmata* and *perimetros imi*. Although solutions like this last one may lead to not perfectly accurate outcomes, they can nevertheless provide researchers from other fields of investigation, such as archaeology, with a kind of "rough preview" to be considered as the basis for further investigations.

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Filippo Fantini graduated with honors from the Polytechnic of Milan with a degree in museography in 2003. Starting from 2004 he collaborates regularly with the Department of Architecture—Design, History, Project of the University of Florence with which he participated in numerous surveying campaigns aimed at the documentation of sites from the particular historic and artistic interest such as: Villa Adriana (Tivoli), the Villa of Tiberius at Sperlonga (Latina), the walls of Massa Marittima (Grosseto), the Han Yang Ling Museum in Xian (Shaanxi, China) and the palace of Herod the Great at Masada (Israel). Professor of computer aided design at the Faculty of Industrial Design at the University of Florence in 2005. He won a PhD in the scientific field ICAR 17 in December 2005. From July 2009 to June 2011 had the scholarship "Santiago Grisolia" at the Instituto de Restauracion del Patrimonio (IRP) of the Polytechnic University of Valencia where he worked with a group of experts at a project funded by the European Commission, called "ATHENA" focused on enhancement and sustainable use of some of the most famous ancient theaters in the Mediterranean (Merida, Syracuse, Jarash, Caesarea, Carthage). From September 2013 he is research professor at the Università degli Studi di Bolonga, Campus of Ravenna. Main topics of research activity: Classical Archeology, Ancient Greek and Roman Theatre Classical Architectural Decoration, Villa Adriana, 3D Modelling, 3D Laser scanning, Digital Photogrammetry, Reverse Modeling.

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