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Research

Living on the Moon: Topological Optimization of a 3D-Printed Lunar Shelter

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Abstract. Long-term permanence of human beings on the surface of the Moon poses several problems, due both to the health hazards against which it is necessary to take shelter, and to the economical sustainability of the mission. We briefly describe the mathematical and numerical tools needed to project a 3D-printed lunar shelter aimed at overcoming such problems, and we present and discuss the resulting optimized architectural design, provided by Foster + Partners.

Keywords: design analysis, design theory, automation, CAD, engineering, minimal surfaces, modeling, patterns, physics, polytopes, shapes, 3D printing, structures/structural engineering, tessellations/tilings, topology

1 Introduction

Ever since the Apollo missions in the late 1960s, the idea of having a permanent base on the Moon has been the focus of many research projects. Indeed, human exploration programmes of major space agencies of the world envisage colonization of the Moon as an intermediate step towards crewed trips to Mars, the asteroid belt and the moons of the outer planets.

Nevertheless, long-term permanence of human beings on the surface of the Moon poses several problems. On one hand, it is necessary to provide suitable protection against health hazards such as radiation, micrometeoroids and harsh thermal environment. On the other hand, the very high cost of transporting infrastructure or construction materials to the Moon hardly make these projects sustainable from the economic viewpoint.

Three aspects can often be found in most lunar habitation proposals, all aimed at overcoming the problems outlined above: *inflatables*, *cylindrical modules* and *shielding*.

Radiation protection is often achieved by protection through bulk material. In order to minimize the amount of material that has to be brought from Earth (thus reducing the mission's costs), research focuses on in-situ resources utilization. Local materials such as lunar soil, also known as *regolith*, can in fact be used to provide this cover (fig. 1). There are quite a few ways to apply this bulk regolith to a structure: piling of loose regolith, retention walls and regolith sandbags. Most of these concepts rely on an underlying rigid structure, such as a standard cylindrical module.

A strategy based on inflatables rather than solid shells has been often proposed for lunar habitation.

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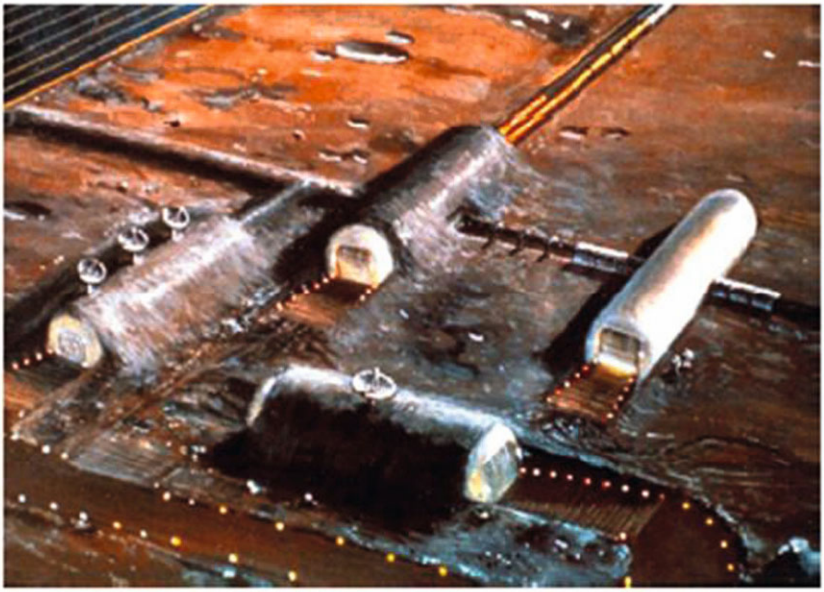


Fig. 1. Lunar Base with regolith used as a protection shield. Image courtesy of NASA

Indeed there are numerous examples of inflatable architecture on Earth. Inflatable structures have the advantage of being not limited by space constraints of transportation, and are potentially lighter in weight. They are usually semi-rigid. Many recent space habitation proposals, such as TransHab (fig. 2), have combined a core cylindrical module with an inflatable module around the core. This hybrid approach exploits advantages provided by both systems.

Fig. 2. Cut away axionometry of the TransHab module. Image courtesy of NASA

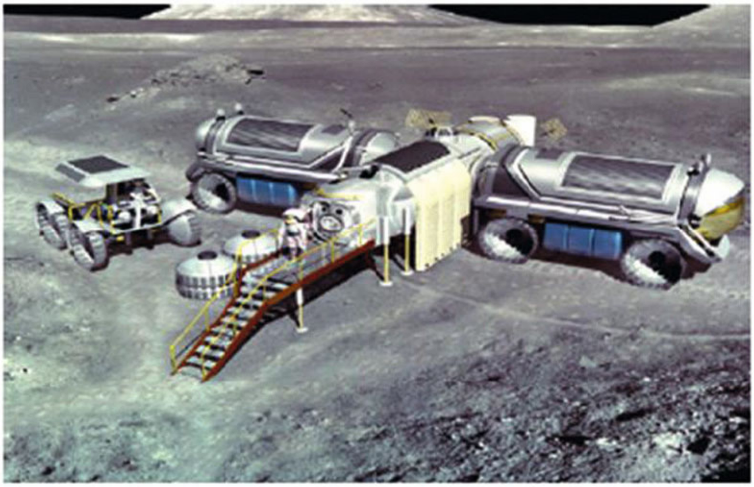


Fig. 3. Lunar outpost based on pressurized rovers, working in conjunction with fixed, shielded core modules. Image courtesy of NASA

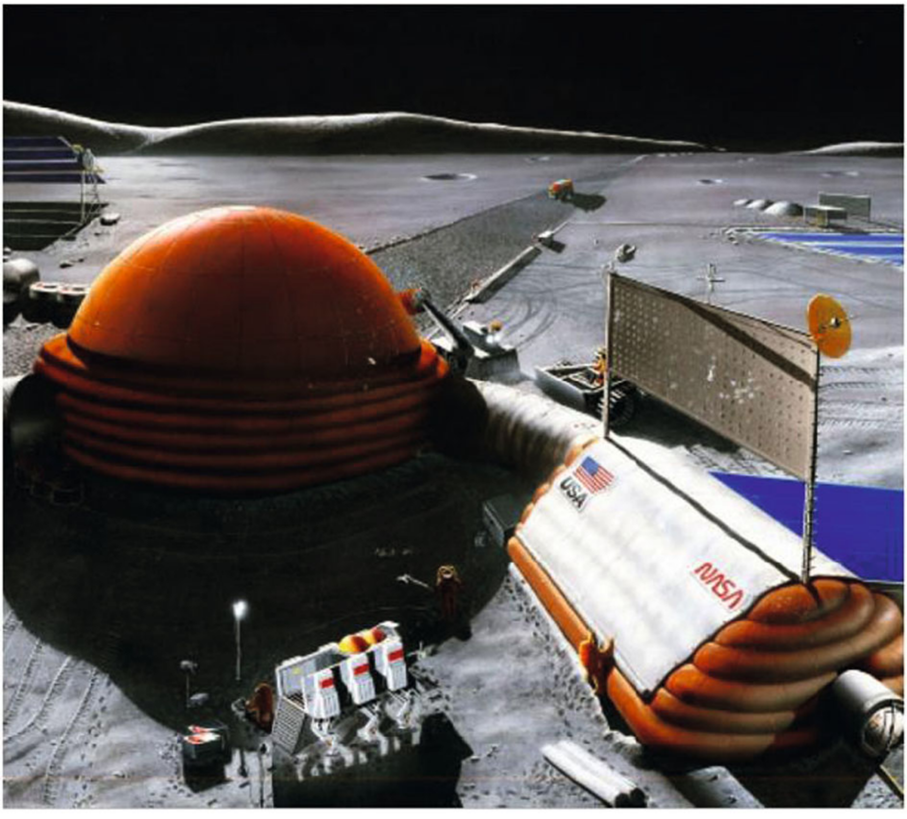


Fig. 4. Lacus Veris Inflatable habitat concept. Image courtesy of NASA

In 2009 the European Space Agency (ESA) awarded a General Study Programme contract to an industrial consortium formed by Alta S.p.A. (Italy, a company with a strong heritage in space technology development), Monolite Ltd. (UK, holding the patent for the D-Shape construction scale 3D printing technology), Foster + Partners (UK, one of world's major architecture practices) and the Perceptual Robotics Laboratory of the Scuola Superiore Sant'Anna (Italy, a leading laboratory in the field of robotics and automation). The objective of the study was to assess the concept of 3D printing technology as a potential way to build a habitat on the Moon using lunar regolith.

Here we will not present the results of that study, which is described elsewhere [Ceccanti et al. 2010], but rather we will provide a description of the conceptual design of the regolith-based shelter which originated from that study, as such a design represents an original application of topological optimization in architecture.

As in the cases described above, this design is also based on inflatable modules and regolith covers are used. More precisely, an inflatable module is used to provide the pressurized shell for the breathable environment of the habitat, while the regolith is used to create an external shell covering the inflatable, in order to shelter the habitat from radiation and micrometeoroids, with additional thermal insulation and thermal capacitance. Unlike the previous studies, however, the regolith is neither loose nor contained by external elements, but is partially consolidated to form a stone-like material; this is made possible by the application of the patented D-Shape 3D-printing technology which is briefly described in §1. The consolidated regolith provides containment of that part of the regolith which is left loose, and also provides rigidity and structural support to the shelter. This method makes it possible to reduce the amount of regolith that requires processing (harvesting or digging, transporting, lifting, seting in place), once the shielding requirements and the usuable internal volume are fixed.

The goal of the architectural design was therefore to select the shape of both the inflatable and the regolith cover in order to minimize the amount of required regolith. Furthermore, the ratio between consolidated regolith and loose regolith also required minimization, as the process (3D printing) requires a consolidating agent (the printing 'ink'), which has to be (at least to some extent) brought from Earth. Reducing the amount of 'ink' as far as possible may make the mission viable. To this end, the internal layout of the shelter wall, i.e. the shape of the solid cells whose wall is made of 'printed' regolith and whose core is made of loose regolith, was subject to topological optimization, in order to achieve a load-bearing, consolidated structure which was as light as possible. The shelter design and its features are described in § 3.

For the reader's convenience, in § 4 we list the main mathematical tools leading to the choice of the design of the habitat. Finally, conclusions are drawn in § 5, where possible future developments are also mentioned.

2 D-Shape 3D-printing technology

In recent years rapid prototyping (or "3D-printing") technologies are the objects of increasing attention in the architecture community for their promise to allow direct construction of buildings with virtually any shape. Some of these technologies have the capability of agglomerating inert materials like sand using a special 'ink'. This feature is especially attractive for the space community, in particular for the utilization of in-situ resources related to manned space exploration, as discussed in the introduction.

Direct manufacturing techniques derive from the rapid prototyping systems developed in Japan and USA during the late 1980s and the 1990s. Rapid prototyping is defined as an additive process that builds up three-dimensional objects which are going to become end-use parts, by the automated curing and/or deposition of successive layers of material. There is a wide range of direct manufacturing machines, commonly known as 3D printers, which use different additive processes and materials. More properly, the matter of direct 3D printing of buildings or building blocks falls into the general field of construction-scale rapid manufacturing.

The patented D-Shape technology is one among the few which is already capable of building a construction scale artefact through rapid manufacturing, as shown by the 2 m. tall "Radiolaria Pavilion" prototype shown in fig. 5. A larger, 6 meter tall version, was planned for construction and installation in a roundabout in Pontedera, a small town in the province of Pisa, Italy.

Fig. 5. The Radiolaria Pavilion prototype obtained with the D-Shape system.
Photo courtesy of Monolite UK Ltd.

The D-Shape printer is a sort of gigantic plotter (fig. 6). Seen from the outside, it appears as a big aluminium structure inside which the building is constructed. The structure is composed of a rigid square frame that lifts along four columns. Each corner of the frame is equipped with an electro-pneumatic climbing device, controlled by an encoder. A beam ("bridge") is mounted on the lifting frame and supports the moving printing head, which is the real core of the technology. CAD-CAM software drives the machinery during the building process. More precisely, the .STL file derived from the 3D CAD model is imported into the printer control system.

The process consists in printing sections of a 3D form on successive layers: the system operates by depositng and levelling a thin substrate of sand (pre-mixed with a certain amount of metallic oxide which will react later with the inorganic binder), then the printing head sprays the inorganic binder (the 'ink') only on selected areas (that is, the local section of the final solid), then another substrate layer is deposed and so on, until the final height of the printed item is reached. This system allows the designer to conceive quite complex architectural structures, such as the Radiolaria Pavilion prototype shown above, the only theoretical limits being physics (parts cannot float in the air without support) and the mechanical properties of the consolidated material.

Fig. 6. The D-Shape printer. Photo courtesy of Monolite UK Ltd.

During the typical process on Earth an outer shell is also created to hold the unbound material in place; this shell (as well as remaining unbound sand) is removed when the process is complete. In contrast, for the Moon process, the structure will be created by adding layers whose printing area decreases progressively with height, exploiting the angle of repose of regolith instead of needing a containment box. Moreover, the single big 3D printer will be replaced by a couple of small robots (rovers) with a 3D printing head and a regolith-gatherer tail (fig. 7).

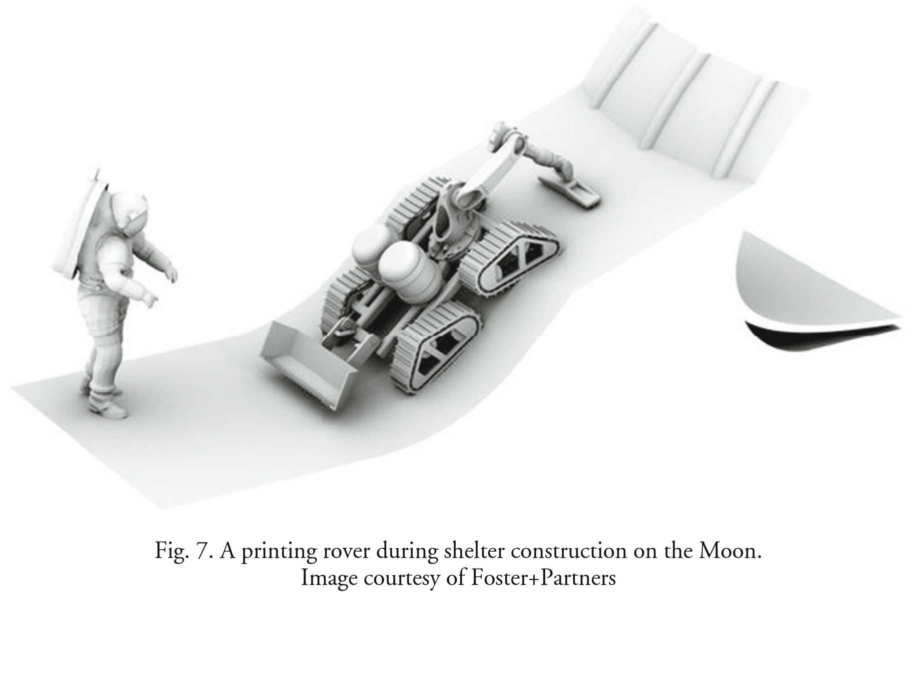


Fig. 7. A printing rover during shelter construction on the Moon.
Image courtesy of Foster+Partners

Fig. 8 shows a possible configuration and manufacturing sequence of an habitation module on the Moon, whose shape will be described in detail in § 3.

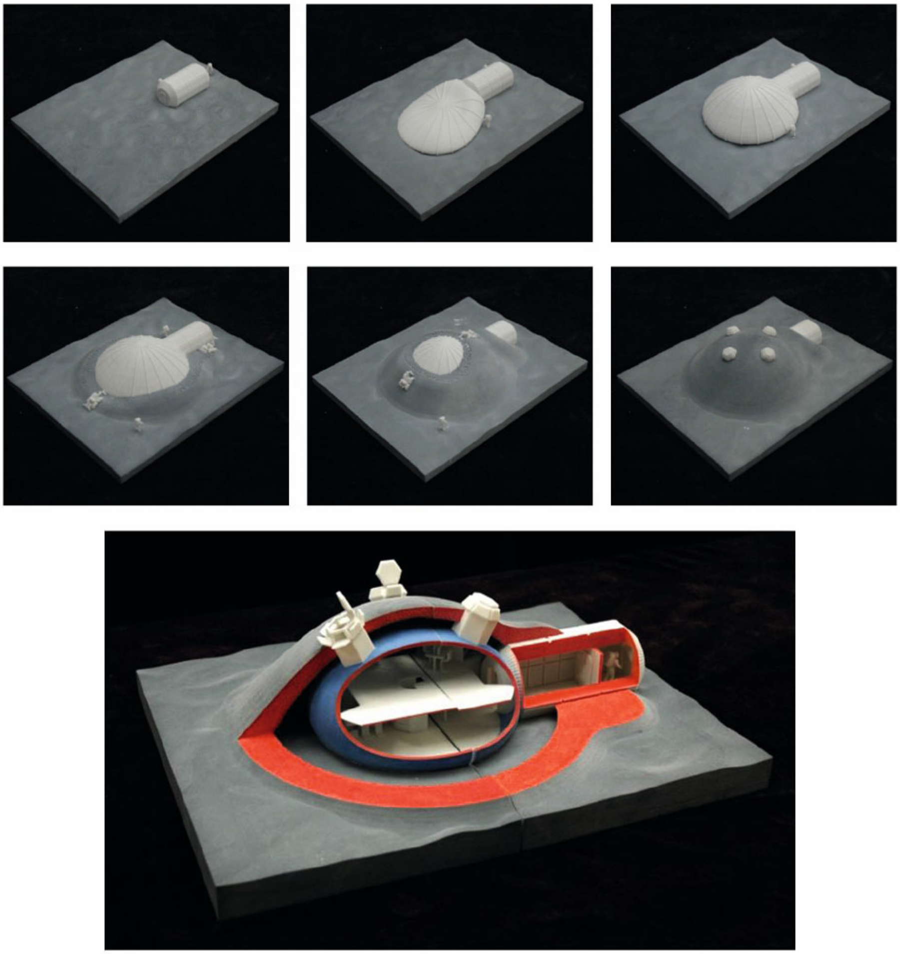


Fig. 8. Habitat module assembly sequence. Photo courtesy of Foster+Partners

Further details on the D-Shape technology can be found on the D-Shape website, http://d-shape.com.

3 Design and topological optimization of the regolith shelter for the lunar habitat

In order to decouple functions and ease technical feasibility, the lunar habitat is made of different layers. The inner layer is a pressurised inflatable, providing breathable air at an almost atmospheric pressure. In order for it to contain two levels, the inflatable non-rigid structure could have a height of 5 m, and its overall dimension could be in the region of 10 x 5 metres in section (fig. 9). Moreover, its overall shape should have a continuous curvature, as in that way it will be able to withstand the internal pressure more effectively.

It is quite obvious that such a fragile structure has very limited rigidity and needs to be protected. Therefore, the successive layer of the habitat is a kind of a shield, whose catenary structure spans the internal pressurised volume, in a way that ensures that mostly compression forces act on the structure. A protection offset of 0.3 metres is left between the inflatable structure and the catenary shield, so that the two layers come nowhere directly into contact. Sitting above this offset boundary the catenary is 'draped' so that it maintains the closest fit to the offset boundary below (fig. 10).

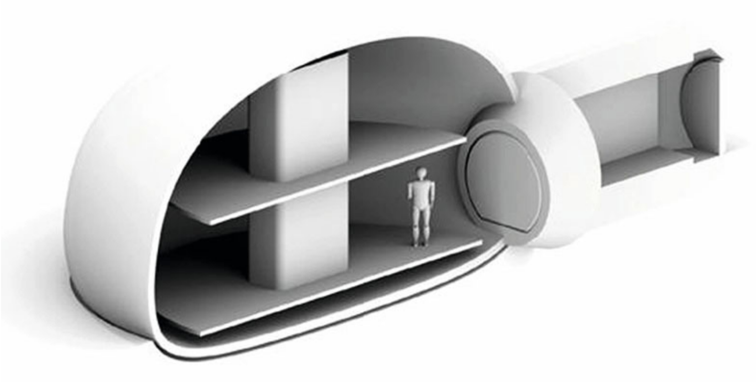


Fig. 9. Cutaway of an inflatable non-rigid structure with two levels.
Image courtesy of Foster+Partners

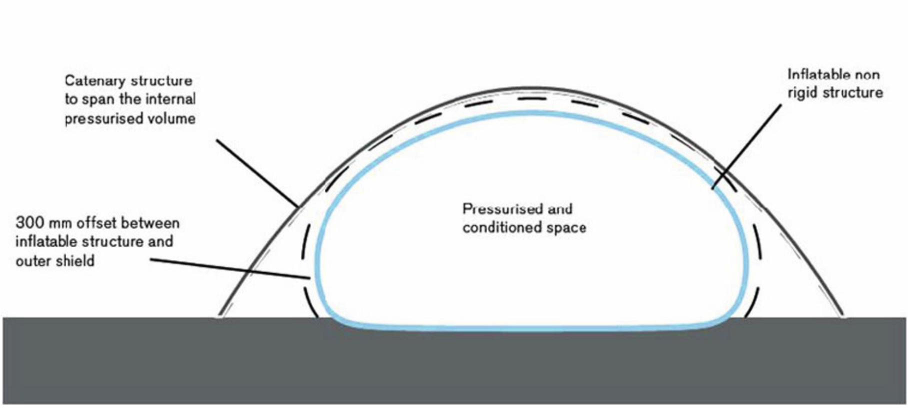
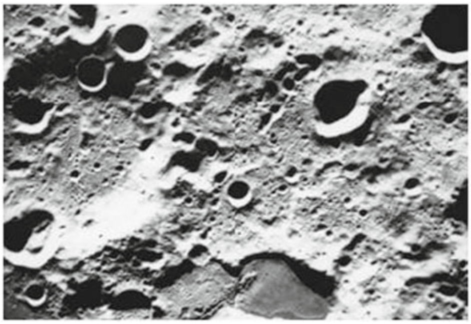


Fig. 10. Catenary profile of the inner part of the sheltering wall. Image courtesy of Foster+Partners



As there is almost no atmosphere on the Moon, meteorites impact on its surface at speeds close to 18 km/s (to put this into perspective, a bullet leaves a rifle at about 2 km/s), giving the Moon's surface its characteristic riddled geometry.

Fig. 11. Surface geometry of the Moon is mainly shaped by the impact of meteorites. Image courtesy of NASA

Although large meteorites are rather rare, a sufficient protection layer against meteorite impacts is necessary. According to the requirements developed in the frame of the study, this protection is achieved by offsetting the catenary structure by 800 mm, with the offset being radial since meteorites can impact the surface under any angle (fig. 12).

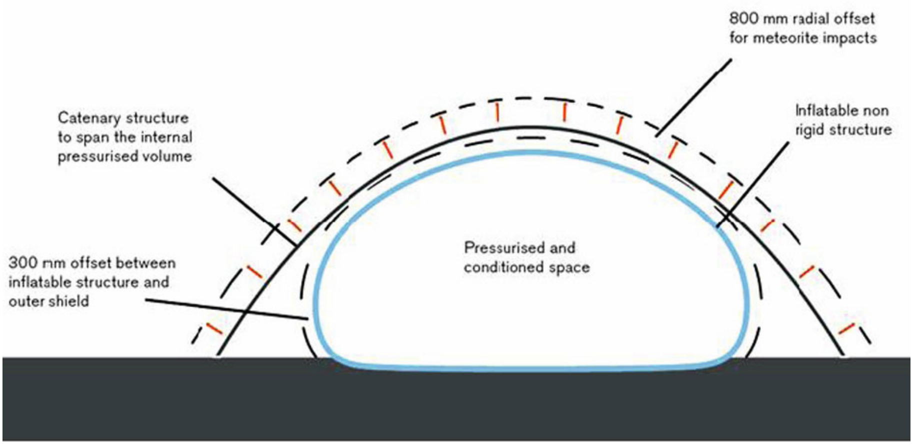


Fig. 12. Radial offset for protection from micrometeorites. Image courtesy of Foster+Partners

Due to the virtual non-existence of atmosphere and magnetic field on the Moon, space radiation on its surface is far higher than on Earth. More precisely, radiation reaching the Moon's surface is associated with three different sources: solar wind, solar flares and galactic cosmic rays.

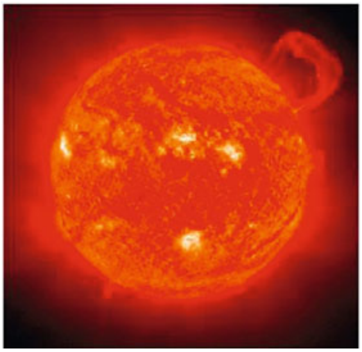


Fig. 13. The sun pictured during a large solar flare. Image courtesy of NASA

Solar radiation, in particular during solar flares, is actually the main design driver (fig. 13). For the purpose of this study, it was calculated that a regolith cover of 1500 mm in the direction of the sun rays is sufficient to have a reasonable safety factor. Moreover, since the proposed location for the lunar habitat is near the Moon's south pole, any solar radiation will come in at a very low, almost horizontal angle. Hence the catenary curve has to be horizontally offset by 1500 mm to protect the inner layer against solar radiation (fig. 14).

The two offsets, for meteorite and radiation protection, in three dimensions create two intersecting surfaces. Hence, to create one overall skin, a best fit catenary curve is draped over the two curves. In addition to the offsets for meteorite and radiation, a third offset from the catenary gives the shell thickness, which is calculated to support the necessary addition regolith above plus additional loads. The resultant structure has a variable thickness over its cross section: it has a greater thickness at the rim where it meets the horizontal ground plane, and it is thinner at the zenith.

When using a 3D-printer on the Earth, the machine that prints is always larger than the printed part. In contrast, to be able to print on the Moon, a much more bottom up approach has to be taken. Namely, in our case, two small rovers build the shelter one layer on top of another, climbing on the last finished level to spread and process the next one. When granulates are dropped on top of each other, they will naturally form a pile with a constant angle of repose, which in our case is about 40°. Hence, in order to minimize the amount of regolith that needs to be displaced, the design of the structure is built up in such a way that it falls geometrically within that angle of repose.

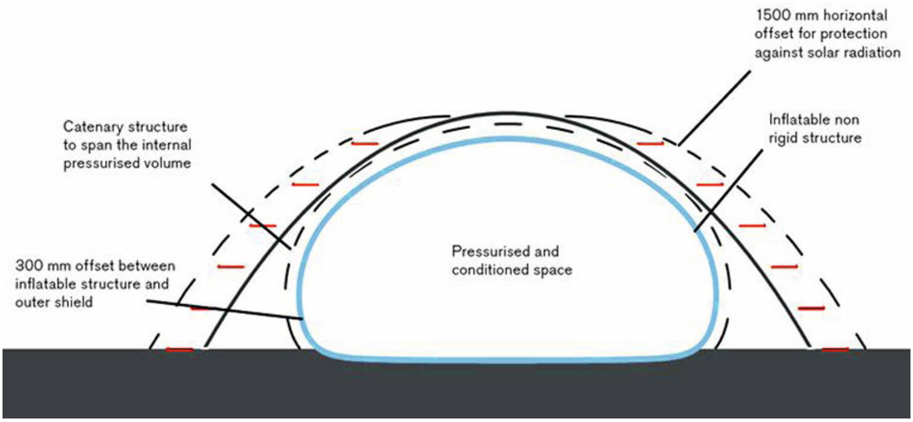


Fig. 14. Horizontal offset for protection from radiation. Image courtesy of Foster+Partners

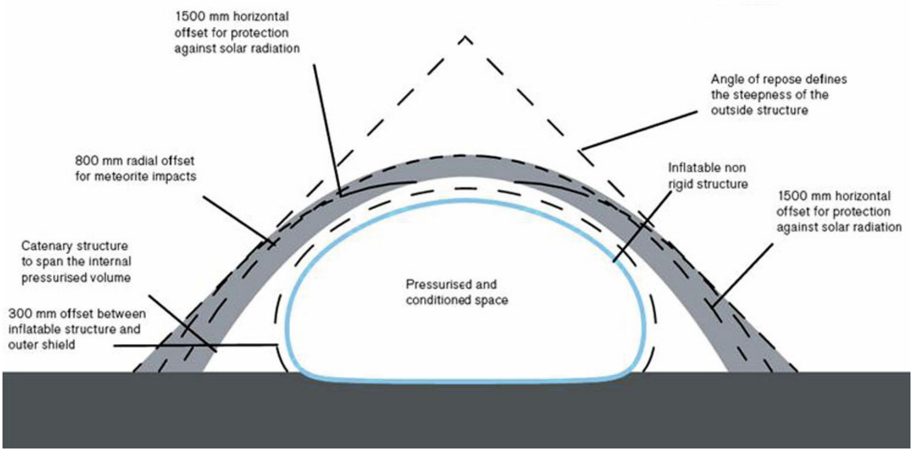


Fig. 15. Effect on external shape of angle of repose. Image courtesy of Foster+Partners

Once we have understood the optimal external profile of the protective layer, we still need to optimize its internal structure, in order to minimize the amount of binder used in the structure while maintaining its overall structural rigidity. More precisely: all the considerations discussed above prescribe the optimal thickness and shape of the protective shield, as well as the amount of regolith needed to construct it. Nevertheless, we still need to optimize the internal elements of the wall structure, with the aim of minimizing the ratio between consolidated material and rough regolith. In fact, since certain components for the printer ink must be transported to the Moon, in order to minimize the costs we need to create a structure that uses a minimum amount of binder per volume of regolith. In other words, we don't need to completely solidify the regolith cover, and we need to develop a strategy to see where the regolith needs to be bonded and where it can stay in a loose state.

In order to determine the optimal structure, a series of trials were performed, inspired both by the observation of architectural terrestrial analogues, and by the analysis of natural structures (fig. 16).

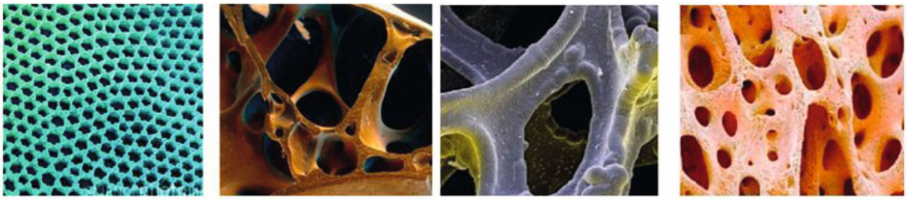


Fig. 16. Natural microstructures

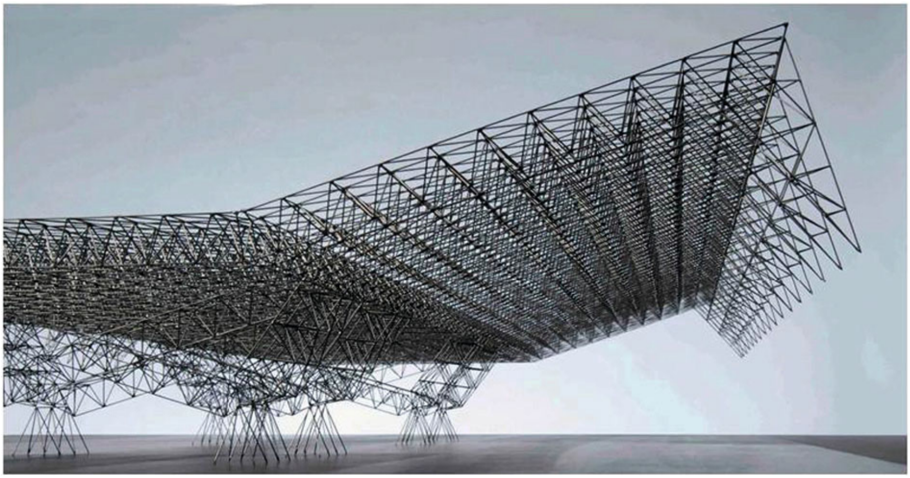


Fig. 17. Physical model of large spaceframe structure. Image courtesy of Foster+Partners

The design team explored many possibilities, including some prospected by the software program named solidThinking Inspire (www.solidthinking.com), and some others simulated by a digital hanging chain model. Tessellated structures were then considered, together with closed and open foam structures. At the end of this trial process, a "pseudo-foam" structure that is a close approximation of a foam using a 3D Voronoi tessellation was chosen.

To verify the current capability of the D-Shape printer, a demonstration model, representing a solid sector of the resulting optimized structure, was manufactured via 3D printing using simulated lunar regolith as the base material. A representation of the demonstration piece is shown in fig. 18.

In order for the internal structure to be visible, the "bubbles" in the demonstration piece are empty, that is, only the consolidated part of the structure is shown. Instead, in the real shell the bubbles will be completely filled by loose regolith. The block, made of simulated regolith, was displayed from March to November 2011 at the "Stazione Futuro" exhibition in Turin.

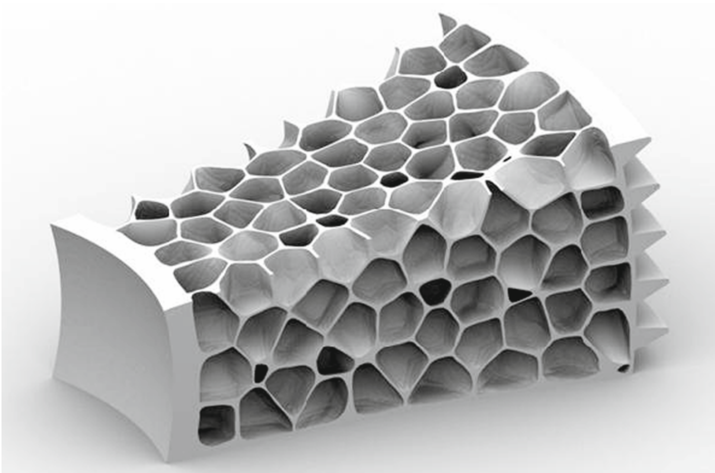


Fig. 18. The model of the pseudo-foam structure. Image courtesy of Foster+Partners

3 Mathematical tools

3.1 The catenary arch

The catenary is the curve that an idealised hanging chain or cable assumes when supported at its ends and acted on only by its own weight (fig. 19).

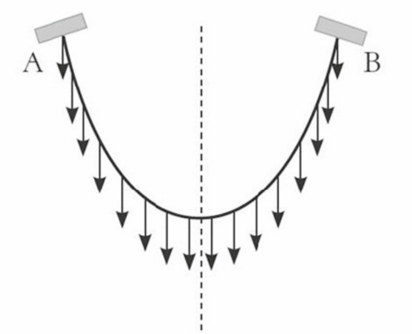


Fig. 19. The catenary. Drawing: authors

Such a curve is subject only to tension forces. Therefore, once we reverse a catenary curve with respect to the horizontal axis, we get an arch, called a catenary arch, in which only compression forces may occur due to the weight of the arch itself. Consequently, along a catenary arch the load is evenly distributed and therefore, unlike other types of arch, the catenary arch is able to stand without buttresses or other supporting elements. Moreover, it is an optimal structure in the sense that, for a given material of construction, it is the structure of minimal weight which is able to withstand a fixed attached load.

There are a number of examples in architecture of hanging chains models used to calculate the optimum form of structures. Between 1889 and 1908, for instance, Antoni Gaudí designed the Colònia Güell church using preceisely this method. Following the form of the catenary allowed Gaudí to develop a very lightweight brick masonry structure.

Fig. 20. Reconstruction of the original Gaudí's hanging model, Casa Milá, Barcelona. Photo: authors

Nowadays, we are able to design with new digital tools that simulate hanging chain models, allowing direct manipulation of geometry with live physics feedback. Moreover, the use of the 3D-printer technology solves another practical problem: the construction of Roman (round) or Gothic (lancet) arches is particularly easy due to the fact that they require identical voussoirs (except possibly for the central ones, corresponding to the keystone). In contrast, the making of a catenary arch is more complex, as it requires voussoirs which are all different from one another. Such a problem is completely overcome using a 3D-printer such as D-Shape, which is in fact able to construct any possible shape, no matter how fancy it is.

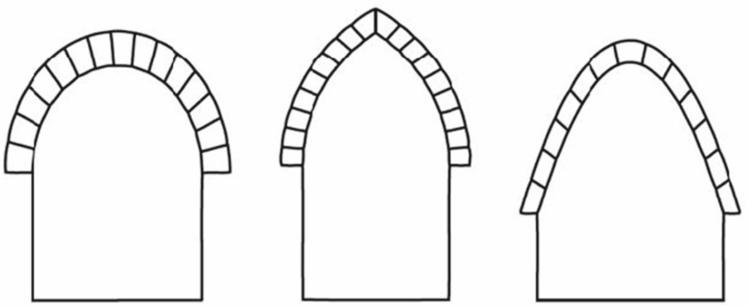


Fig. 21. A Roman arch, a Gothic arch and a catenary arch, showing the shapes of their voussoirs. Drawing: authors

3.2 Soap films, bubbles and foams

As we know since our early childhood, when we blow air through a ring dipped in soapy water, we get a soap bubble whose shape is a perfect sphere. This is due to the fact that a soap film always assumes the shape that minimizes its area while enclosing a fixed volume. Similarly, if we blow through a straw in a solution of soapy water, we get a mass of bubbles (a foam), clinging to each other in a way that is not at all random. More precisely, from the mathematical point of view, a foam is an infinite cluster of bubbles, built from cells separated by films that meet in precise singularities (fig. 22), encoded by the following Plateau's laws:

1. soap films are made of entire smooth surfaces, and the average curvature of a portion of a soap film is everywhere constant at any point on the same piece of soap film;
2. intersecting surfaces give rise only to two possible configurations: either three surfaces meet along an edge (called a Plateau border), or six surfaces (hence four Plateau borders) meet in a vertex;
3. the dihedral angles of three surfaces meeting along a Plateau border all measure 120°, while those formed by the four Plateau borders meeting in a vertex all measure (approximately) 109°.

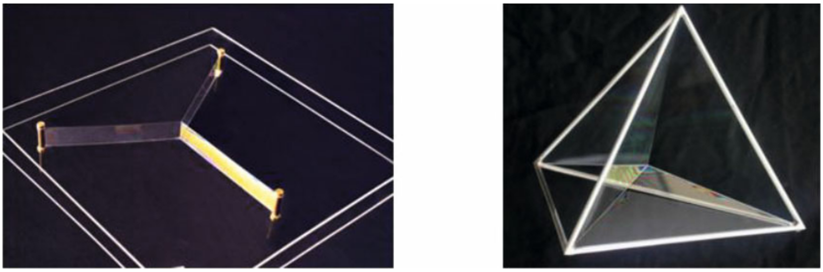


Fig. 22. Soap films: left) three surfaces meeting in an edge; right) six surfaces meeting in a vertex. Photos: courtesy of Italo Tamanini, *Matematica trasparente: superfici minime e bolle di sapone* *http://bolle.science.unitn.it/*

While obeying the above rules, the bubbles in a foam may have very irregular shapes, and they arrange themselves randomly. Speaking about "foams" in our context, we obviously do not mean soapy water foams, but regolith built shapes inspired by foams. In a broader sense, a foam is indeed a two-phase system in which typically a high volume of gas is enclosed in a liquid or solid in the form of a large number of relatively small cells. In our case, loose regolith is enclosed in solid bonded regolith. In our trials we compared two different kinds of foam, closed and open, differentiated by the fact that the walls of the bubbles of the open kind contain pores that are connected to each other and form an interconnected network. In other words, in a open cell foam there are no chambers within the structure and particles can flow through the foam. It is obvious, in particular, that open foams cannot be realized with soapy water.

Closed foams are currently used in car bodies, to reduce material weight whilst not compromising material strength, and have proven to be extremely good at absorbing impacts. This last feature is obviously very attractive, when considering absorbing impact from meteorites. However, the advantage of open foam in the context of a lunar construction is that, having less adhesive material, an open foam may be more lightweight than a closed one. Nevertheless, open foam structures are less strong from the structural point of view than closed foam structures, as they lack cell faces which act as plates reducing bending forces at nodes.

3.3 Tessellations, Voronoi diagrams and Delaunay triangulations

A tessellation of a plane or space is the complete covering of such plane or space with a non-overlapping shape, that is, is a pattern where the same shape is repeated as to cover the whole plane or space without any gaps or overlaps. A tessellation is regular if the shape that repeats is a regular polytope. In particular, since the regular polygons in a tessellation of the Euclidean plane must cover the plane at each vertex, the polygon's interior angle measure must be an exact divisor of 360°, which happens only for the triangle, square, and hexagon. Hence, only those three regular polygons can tessellate the Euclidean plane.

A generalization of this concept is given by the definition of the Voronoi tessellation (also called Voronoi diagram). A Voronoi diagram of the plane is a decomposition of the plane in "regions", determined by the distances of its points from the elements of a finite subset of points. In other words: let S be a finite set of points in the plane. For every point p in S we define $V(P)$ to be the set of points of the plane which are closer to p than to any other point in S. We get this way a finite number of regions (one for each point in S), whose union covers the whole plane, without overlapping or gaps. The resulting decomposition is called Voronoi diagram of the plane, relative to the set S, and it is clearly a generalization of a tessellation, in the sense that in a Voronoi diagram the shapes which cover the plane do not need to be equal. In a similar fashion, we may define a Voronoi diagram for any metric space.

A closely related concept is that of Delaunay triangulation: given a set P of points in the plane, the Delaunay triangulation of the plane associated to P is a triangulation $D(P)$ such that no point in P is inside the circumscribed circle of any triangle in $D(P)$. Such a definition may be extended to three or higher dimensions by considering circumscribed spheres or hyperspheres.

Voronoi diagrams and Delaunay triangulations are strictly related, since the Delaunay triangulation of a discrete point set **P** in general position corresponds to the dual graph of the Voronoi diagram for **P**.

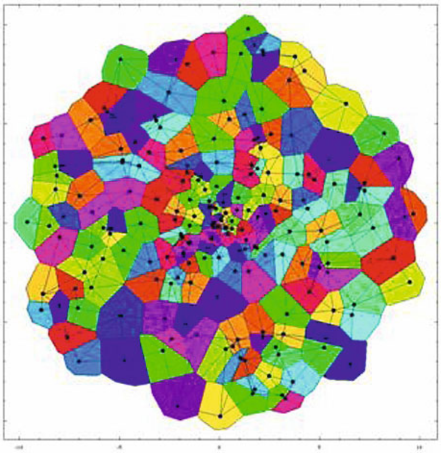


Fig. 23. The Voronoi diagram of a bounded plane region

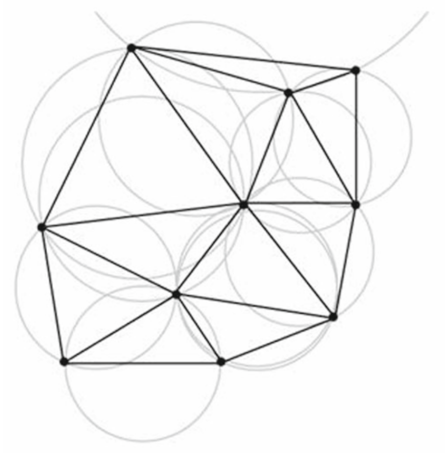


Fig. 24. A Delaunay triangulation in the plane with circumcircles shown. Drawing: authors

In our problem, we are concerned with internal tessellation of the three-dimentional shell whose shape was described in § 3. Two steps of the trial process are shown below in fig. 25.

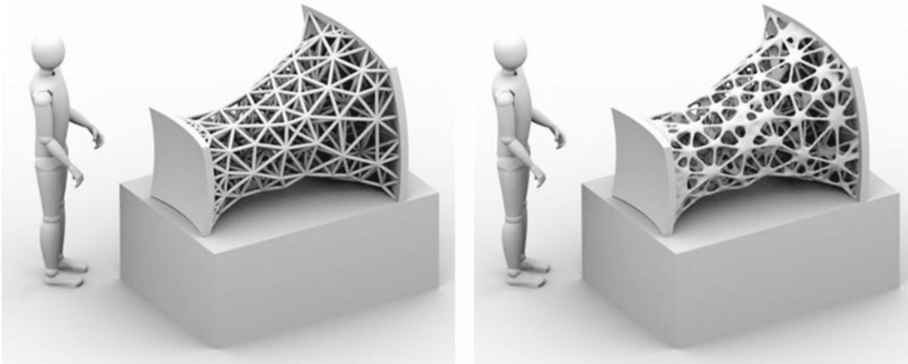


Fig. 25. Two examples from among possible tessellations of the lunar shelter prototype: left) pure triangulation; right) triangulation with reinforcing gussets. Image courtesy of Foster+Partners

3.4 Pseudo-foam

At the end of the trial, the selected layout for the internal structure of the habitat shelter was the so-called pseudo-foam that is shown by the model in fig. 18. The pseudo-foam is a close approximation of a foam created using a three-dimensional Voronoi tessellation. To become a real foam, the geometry would require one further optimization step using a foam evolving package such as Ken Brakke's Surface Evolver (www.susqu.edu/brakke/evolver/evolver.html), to achieve a configuration which is compliant with Plateau's laws (see §3.2 above). In order to generate the pseudo-foam, the bounding volume is specified, as is the density of cells to inhabit this volume. This gives both the foam density and average size of each cell. The cell centres are generated pseudo-randomly to fill the volume with the desired number of cells. These are then self-organised within the volume over a number of iterations.

4 Conclusions

Manned exploration of the solar system is an extremely demanding task, especially in terms of the amount of resources required. Exploiting in-situ resources, such as planetary soil, in the best way may make these missions viable in the future. The study "3D Printed Building Blocks Using Lunar Soil", funded by the European Space Agency, made it possible to assess the feasibility of manufacturing a shelter for a lunar habitat using lunar regolith and a novel direct manufacturing (3D printing) technique. The study required a combination of expertise from quite different sectors, including the space industry, architecture, rapid prototyping and automation. In particular, the study provided an opportunity to experiment with advanced tools in architecture, such as topological optimization of structures, in an uncommon and challenging environment. The role of mathematics, as emphasized in the paper, has been a crucial one, as mathematical tools were required for the computations leading to the engineering requirements, for the preliminary definition of the geometrical layout of the shelter, and for the topological optimization of its internal structure.

Future work will involve further development of the 3D printing technology focused on performance in a vacuum environment with regolith, testing of the mechanical properties of the materials, structural optimization with respect to actual material strength and achievable performance in terms of wall thickness reduction, further optimization of foam structure exploring the effect of average cell size, of cell size range and dispersion, of cell shape anisotropy and aspect ratio, etc. More precise definition of the construction sequence will also play a role in adapting or refining the design. Finally, the use of this kind of technology could eventually be extended to applications on Mars, since the composition of the base material plays a minor role in the working principles of the D-Shape process.

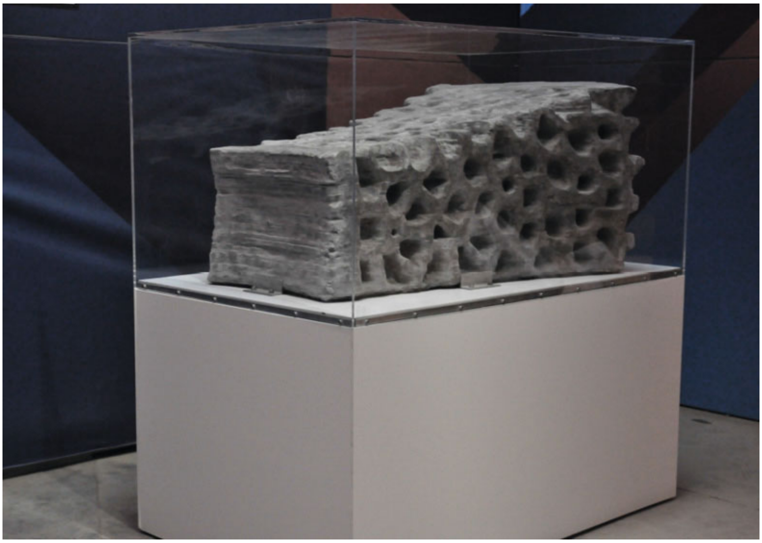


Fig. 26. The 3D-printed structure made of simulated regolith on display at the exhibition entitled "Stazione Futuro", Turin, March-November 2011. Photo: authors

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