Research Institute of Theory and
History of Architecture and
Town-planning
Russian Academy of the
Architecture and Construction
Sciences
21-a 7th Parkovaya St.
Moscow, 105264, RUSSIA
kozlov.dmitri@gmail.com

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Structures of Periodical Knots and Links as Geometric Models of Complex Surfaces for Designing

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Abstract. Practical modeling of spatial surfaces is more convenient by means of transformation of their flat developments made as topologically connected kinetic structures. Topologically, any surface in 3D space consists of three types of elements: planar facets, linear edges and point vertexes. Planar facets and linear edges can be identified respectively with the structural units of folding structures and kinematical nets. Here we consider a third possible type of flat transformable structures with vertexes as form-generative units, in which flat developments of surfaces are formed by arranged point sets given by contacting crossing points of the periodic knots and links made of elastic-flexible materials, so that their crossing points have real physical contacts. This new form-generative method can be applied to modeling of both oriented and non-oriented differentiable topological 2D manifolds. The method of form generation based upon the developing properties of periodic structures of knots and links may be applied to many practical fields, including art, design and architecture.

1 Models of surfaces

Practical modeling of spatial surfaces is more convenient by means of transformation of their flat developments made as topologically connected kinetic structures. According to topology, any graph on a surface consists of three types of elements: planar facets (F), linear edges (E) and point vertexes (V). There are two well-known types of transformable structures, based upon the structural units corresponding to the planar (F) and linear (E) elements of surface division. Those two types of surface models can be represented respectively by the folding structures with hinged flat faces (fig. 1a), and the kinematical nets with non-triangle meshes (fig. 1b).

Fig. 1. Transformable surface models: a) folding structure [Vranka 1990: 9];
b) kinematical net [Otto, Burkhardt and Hennicke 1974: 130]

In addition to the planar and the linear types of the flat developments of surfaces there is a third possible type of flat transformable structures with vertexes (V) as structural units. Approximations of surfaces by ordered sets of points is the common method in mathematics and computer graphics. In these cases a separate point is represented by a dot in the virtual space, and its numerical value relates to the three Cartesian coordinates.

The physical model of a point may be represented as the contact of two physical 3D bodies such as tangent solid spheres or tangent cylinders with non-parallel axes. A flat or spatial system of the contact points produces the vertex surface model, that is, the point approximation of real continual surfaces.

To work as kinetic structures such surface models must have the real physical contacts at their crossings that can slide along the cylindrical rods. It may be done when the contacting cylindrical rods are flexible enough to form the structure as a woven fabric and resilient enough to keep the contacting of the crossings by the forces of bending (fig. 2a-b). At the same time the neighboring ends of the cylindrical rods should be connected to each other to provide the structural continuity of the models and to synchronize the slide movement for all of the contacting points.

Fig. 2. Woven cylindrical rods as point model of surface (a), (b) and cylindrical rods joined into knotted structure. Image: author

The joining of the neighboring ends of the resilient cylindrical rods turns the woven structure into a knot or a linkage of knots (fig. 2c). The structures of all knots and links made of resilient cylindrical materials give models of the vertex (V) or point surfaces, but not all of them can serve as kinetic transformable flat developments of 3D surfaces. The sets of the contacting points of the resilient knots and links form flat point surfaces that under certain circumstances may be transformed into the spatial shapes.

2 *Knots as surface models*

Knots have been known since before the beginning of recorded history, though they were used mainly to attach different objects to each other, actually not as independent structures but as joints between other parts. In addition, knots and links have been known in design and decorative art from ancient times, especially in the form of “woven ornaments”. At the end of the nineteenth century, thanks to works by Lord Kelvin and Peter Guthrie Tait, who studied knots as models of physical phenomena [Knott 1911], the mathematical theory of knots as a part of algebraic topology began to arise. In recent decades some new and promising applications of the theory of knots have been found in molecular biology (knotting of DNA molecules [Liu et al. 1976]), physics (the quantum field theory [Witten 1989]) and statistical mechanics [Kauffman 1988].

From the mathematical point of view, a knot is a one-dimensional curve placed in ordinary three-dimensional space so that it begins and ends at the same point and does not intersect itself [Neuwirth 1979]. A piece of knotted threadlike material, such as

string, with its ends joined together is the common physical model of an abstract mathematical knot. The material and geometrical properties of the object are of no importance. They must be just long and flexible enough to provide the possibility of tying the knot and joining the ends together.

Fig. 3. Classification of prime knots up to nine crossings after [Burde and Zieschang 1985].
Drawing: author

Plane projections of spatial knotted curves are known as knot diagrams. The diagrams may be treated as planar graphs or flat curves and analyzed from the point of view of symmetry. Nevertheless, the forms of the diagrams in the classifications are not equal to the real forms of the knots because practically any knot may be depicted as several diagrams of different shapes classified in knot theory in accordance with the number of their crossing points and topological complexity (fig. 3). It is not an easy task to identify knots for the purpose of surfaces modeling by simply studying the classifications such as the one shown in fig. 3.

The knots of resilient material turn from arbitrary one-dimensional curves into flat woven structures with gentle rather than sharp bends and contacting points at the crossings. The making of a knot from a resilient material, like steel wire or fishing-line, turns the physical properties of the material into the generation factor for the spatial form of the knot. In this case it is important to let the ends of the resilient knot rotate in space and release the excess twist forces. The elastic energy of the knotted resilient filament

forces it to assume the shape defined by the minimum value of the energy. The loops of the closed knotted filament become smooth and near to equal size, and the number of their inflexions tends to be the minimal one. The central line of the knotted filament tends to coincide with the plane surface and its crossings come into contact. As a result, the knot turns into the smooth and energetically balanced pattern.

The results of my own experiments with knots of resilient filaments have demonstrated that the knot representations with the greatest value of the axial symmetry correspond to the most energetically stable knot structures. For example, the trefoil knot of the resilient materials with D_2 symmetry tends to turn into the trefoil with D_3 symmetry, and the figure eight knot with D_1 symmetry turns into the same knot with D_2 symmetry (fig. 4). The number of crossings in the both cases is the same.

Fig. 4. Trefoil (a) and figure eight knot (b) made of soft (left) and resilient (right) materials.
Image: author

Further exploration of the classified prime knots up to nine crossings (fig. 3) makes it possible to find several other examples of smooth projections of both alternating and non-alternating knots with the same number of crossings. The alternating knots of this type have the general name “Turk’s-Heads” [Ashley 1944] and can be classified in accordance with the numbers of their bights (petals) and leads (coils). Turk’s-Heads serve not only as the binding knots but also as decorative ones. The beauty of the cyclic knots derives from their structural stability and smooth symmetrical shapes.

3 Properties of cyclic knots

In modern prime knot classifications, many of the cyclic Turk’s-Head-like knots are depicted as non-cyclic diagrams. The alternating knots 4_1 , 8_{12} and 9_{40} can be transformed into the shapes with circular periodicity (fig. 5a-h). However, in the knot table by K. Reidemeister [1932] the knot 9_{40} , known as the Chinese Button Knot, was depicted in the Turk’s-Head shape. The shapes of Turk’s-Heads diagrams are similar to the shapes of torus knots like knot 8_{19} (fig. 5b). The torus knots with one orbit of crossings (knots 3_1 , 5_1 , 7_1 , etc.) coincide with Turk’s-Head knots. Some of the non-alternating and non-torus knots 8_{20} , 9_{46} , and 9_{47} can also be diagrammed as Turk’s-Head-like shapes (fig. 5c, f, g).

All of the cyclic knots of this type may be named “torus Turk’s-Heads,” in contrast with another type of knots with circular periodicity known as “chain Turk’s-Heads” [Ashley 1944]. The simplest chain Turk’s-Head knot, knot 9_{41} , has nine crossings (fig. 5e). The petals of chain Turk’s-Heads do not include the central region of their diagrams. The thickness of the flat woven patterns of Chain Turk’s-Heads remain practically constant and this characteristic makes them more stable than torus Turk’s-Heads.

Turk’s-Head knots are often used to braid cylindrical, toroidal or sometimes spherical surfaces. The braiding knots are made of soft materials and consist of many leads and bights with dozens of contacting crossings. This principle of regular weaving is good for duplicating an existing 3D surface, but not sufficient to make a model of the surface as a self-supporting independent structure.

Fig. 5. a-d, f, g) Torus-type; e, h) chain-type Turk's-Heads. Image: author

According to my experiments with cyclic knots made from resilient materials, the most important feature affecting their ability to generate and maintain form, is the quantity of the contacting crossings and the order of their distribution. The non-alternating torus Turk's-Head knots cannot keep their flat woven patterns stable, because they have fewer of the real contacting crossings than the alternating ones. At the same time, the non-alternating knot 9_{47} with the symmetrically arranged crossings (fig. 5g) maintains the form much better than the knot 9_{46} with the randomly arranged crossings (fig. 5f). The simple torus and chain Turk's-Head-like prime knots 9_{40} or 9_{41} (fig. 5d, e) may be transformed from plane into spatial positions, but they don't have enough contacting crossings to either fix the spatial form, or keep their flat form stable.

The force of friction in the contact crossings of the cyclic knots must be sufficient to maintain connection, but not so strong as to force them to slide along each other. The balance of these two demands is important when flat cyclic knots are turned into kinetic transformable structures. At the same time, the quantity of contact crossings must be large enough to form a fragment of the point surface with uniformly distributed friction forces. The periodicity allows development of the simplest cyclic knots into more complicated ones by means of the sequential repeating of their shaping principles.

4 Development of cyclic knots

To develop a cyclic knot with a small number of crossings, it is necessary to identify the basic shaping principles of its structure in order to repeat them sequentially. For example, each of the successive cyclic knots in the sequence starting with the trefoil, is derived by adding one bight and one lead to the previous one (fig. 6). The quantity of bights and leads grows according to the linear sequences $a_n = n + 2$ (3, 4, 5, 6, 7, 8 ...) and $a_n = n + 1$ (2, 3, 4, 5, 6, 7 ...) respectively, but the corresponding quantity of the crossings grows as a quadratic sequence $a_n = n(n + 2)$ (3, 8, 15, 24, 35, 48 ...). Similar

methods of knot development are typical for decorative knots and knotted ornaments [Merne 1987].

Fig. 6. Sequence of cyclic knots with common shaping principle. Image: author

All of the torus Turk’s-Head knots and links can be classified according to the numbers of the leads – the turns of coils (p) and the bights petal loops (q) – in the system of orthogonal coordinates. The numbers p and q may be equal to any natural number: if they are coprime then the structure is a knot (fig. 7). If p and q are not coprime, the structure is a link of equivalent knots, and number of linked knots is equal to the greatest common divisor of p and q .

$p \backslash q$	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

Fig. 7. Classification of torus-like Turk’s-Head knots. Image: author

The total quantity of crossings depends of the number of bights and leads. The quantity of the physically contacting crossings depends of the number of waves on elastic

knotted and closed filaments. The number of the waves corresponds to the bridge number of knots. A bridge is a linear fragment of a knot that starts from under one crossing, goes over other linear fragments of the knot, and ends by passing under another crossing. For alternating knots the bridge number is equal to the number of crossings, though it is not necessarily its minimal bridge number. In the case of the knots of resilient material the waves or bridges play the leading role in strengthening the structure of the knots: they bend and stretch the fragments of knotted filaments between the neighboring contacting over-crossings or under-crossings. As a result, the knots come to work as structures with tension in the bridges and compression in the crossings. Such knot structures are synergetic in Buckminster Fuller's meaning of the word as "behavior of integral, aggregate, whole systems unpredicted by behaviors of any of their components or subassemblies of their components taken separately from the whole" [Fuller 1982: 3].

5 Transformation of developed cyclic knots

Cyclic knots and links with the numbers of bights and leads large enough to form a stable flat structure with a sufficient quantity of contacting crossings have the unique property of being transformable from flat into spatial positions. To transform a structure of the developed cyclic knot it is necessary to reduce the size of its perimeter while keeping the size of the center fixed. The distributed forces applied to the peripheral points of the flat structure (fig. 8a) cause the increase in elastic energy in the knotted resilient filaments and the structure takes the form of a spherical segment (fig. 8b).

Fig. 8. Stages of transformation of surface model given by cyclic knot. Photos: author

If the forcing is continued, the structure further transforms (fig. 8c) and successively takes the forms of a hemisphere (fig. 8d), a truncated sphere (fig. 8e, f) and finally a sphere, that is, the position when the size of the peripheral circumference equals the central one. The structures of chain-type cyclic knots have a fixed radius to their ring-shaped areas. In contrast to them, the structures of torus type cyclic knots are not stable and need supplemental fixings, such as rings for their central polygonal facets.

I have named the developed transformable cyclic knots created out of resilient materials “*NODUS*” structures (*nodus*, meaning “a knot” in Latin) [Kozlov 1991a]. The spatial transformation of the *NODUS* structures consists in the sliding of the contact crossings along the resilient filaments and in the twisting of the filaments about their central axis. This is why the cross-sections of the filaments must be round. The waves on the filaments move and change their lengths to adapt to the current disposition of the contact crossings. As a result, the geometry of the vertex surfaces specified by the *NODUS* structures changes integrally.

From the geometry of the neutral (parabolic) Gaussian curvature, *NODUS* structures can be transformed into positive (elliptic) or the negative (hyperbolic) geometry. These three types of the surface curvatures completely exhaust all of the possible internal geometries of two-dimensional manifolds [Hilbert and Cohn-Vossen 1999]. In contrast to solid models of surfaces, the point surfaces of *NODUS* structures can change their Gaussian curvatures without breaks or folds (fig. 9).

Fig. 9. Flattening of *NODUS* structure. Photos: author

I believe that *NODUS* structures may be treated as the third type of flat transformable kinetic structures based upon the vertexes (V) that complements the two well-known types of the structures based upon the planar (F) and linear (E) elements of the surface division. Although the vertexes do not exist separately, and they are given by the contacting points of knotted resilient filaments, they are nevertheless the only elements of the structures that belong to the surfaces in the strict sense of the word as two-dimensional objects.

6 Combinations of *NODUS* modules

The method of form finding described here may be extended to a practically unlimited variety of surfaces. The torus-type and the chain-type *NODUS* structures can serve as the basic modules in different combinations and modifications. Two chain type *NODUS* structures joined at their perimeters form an elliptic surface (fig. 10a), and joined at their centers form a hyperbolic one (fig. 10b). These structures can be transformed one into another due to the phenomenon of torus rotation. A torus can rotate around its central circle axis, interchanging the outer and the inner parts of the surface. The *NODUS* structure, as the outer (elliptic) or the inner (hyperbolic) parts of the torus, can also rotate around the circle axis of an imaginary torus. The structure can turn inside out and take the forms of the outer and the inner parts of the torus surface.

The complete surface of a torus is the linkage of two *NODUS* modules of the elliptic and hyperbolic curvatures (fig. 10c). Another method of making a torus is to join two torus-type *NODUS* structures by both their central and peripheral circumferences. The structure thus derived consists of two (or any other even number) linked mirror torus knots (fig. 11a). The volume increases by compression of the inner hyperbolic part of the structure (fig. 11b).

Fig. 10. *NODUS* structures of elliptic (a), hyperbolic (b) and combined (c) curvature.
Photos: author

Fig. 11. Torus surface as a linkage of two torus knots. Photos: author

Fig. 12. Pretzel surface as a linkage of two pretzel knots. Photos: author

Fig. 13. Combinations of *NODUS* modules with one (a and b) and two self-crossings (c).
Photos: author

The pretzel surface with two holes is a combination of two torus structures on a plane (fig. 12) and result from a linkage of two (or any other even number) pretzel knots.

The method of multi-layered combinations of the *NODUS* modules makes it possible to create many other forms, such as surfaces with different numbers of self-crossings (fig. 13).

The *NODUS* structures shown in figs. 10-13 were made of spring steel wire about 1 mm in diameter. The sizes of the structures are not more than 30 cm, which permits transforming them by hand. The transformation of larger structures, however, demands a regular distribution of compressed force on the peripheral points.

7 Large-sized *NODUS* Structures

To transform large-sized *NODUS* structures, I designed and built a special mechanism. It transmits the force of the rotating horizontal spindle to the vertical jackscrew, which is connected with eighteen rolls placed on the circle 1 m in diameter. Rotation of the wheel handle transforms the flat *NODUS* structure into a spherical one and undoes it, as well. Although this mechanism was initially used only as an instrument for research, it has also been displayed in several art exhibitions as a piece of kinetic art (fig. 14).

Fig. 14. Mechanism for transformation of large *NODUS* structures. Photos: author

The material of the large-sized *NODUS* structures must be resilient and lightweight at the same time. Large structures of steel wire are too heavy to carry their own weight. It is better to use some resilient materials like plastics or fibro-plastics. To test the possibility of assembling and erecting a large *NODUS* structure, I undertook an experiment with the structure of a torus-type knot that was 6 m in diameter (fig. 15). The material involved was PVC tubes, 4 m long and with a 16 mm cross section. The assembling method was the consecutive butting of the tubes beginning with the central fixing ring (fig. 15a).

To control the correctness of the knot pattern, I used a small-scale diagram drawn on letter-sized paper. I did not draw any diagram on the ground. Then I went along the knotted line and fixed every finished loop with the central ring (fig. 15b). At the same time, I wove the tubes to make the contact crossings and the waves (fig. 15c). When the flat *NODUS* structure was completed (fig. 15d), I lifted the central ring and the structure assumed the shape of a deformed spherical segment (fig. 15e). As the final step, I connected the second fixing ring to the peripheral points of the structure, and it became springier and took the shape of a truncated ellipsoid (fig. 15f). The contact crossings kept the structure stable by the friction forces, without any supplemental fixing devices. I believe that the structure of more elastic materials, such as fibro-plastics or resilient metal alloys would behave much better and take a springier shape.

Fig. 15. Experiment with large-sized *NODUS* structure of PVC tubes. Photos: author

Fig. 16. Bamboo dome based on *NODUS* structures. Sketches by Yona Friedman [Kozlov 1991]

Another interesting variant would be to try to assemble and erect a dome-shaped structure of natural resilient materials like bamboo. In 1990 I took part in a UNESCO project headed by Yona Friedman and dedicated to simple roof-building techniques with

the use of inexpensive local materials available to people who have little education and almost no resources [Kozlov 1991b]. The designs were primarily prepared for the lowest income people in India. As a result of the work the album with the collection of Friedman's hand-drawings was published (fig. 16). My part of this album was about building a light-weight transformable bamboo dome. The sequence of the operations for assembling and erecting of the dome was practically the same as for the dome out of PVC tubes (fig. 17).

Fig.17. Internal (a) and external (b) views of large-sized *NODUS* structure. Photos: author

The large-sized *NODUS* structure looked beautiful in natural surroundings. The pattern of the interwoven curve line grid offered a different aesthetic impression from the internal (fig. 17a) and the external (fig. 17b) point of view. This experiment proved that cyclic knots of resilient materials offer a new method of form finding that may find application in many practical fields including art, design and architecture.

8 Knotted Stained Glass Folly

One of the latest architectural applications of *NODUS* structures is the design of the Knotted Stained Glass Folly in the Mathematical Park project sponsored by ARPAM [Bruter n.d.]. The goal of the project is to build visualizations of mathematical concepts and facts and beautiful mathematical shapes and objects. This project focuses on mathematical objects of large size, or “follies”. “Folly” is an architectural term from late eighteenth-century Western Europe used to indicate structures that were built in outdoors settings such as parks and gardens, and includes unusual sculptures, buildings, even sometimes miniature museums. Today ARPAM is working closely with the European Society for Mathematics and Art (ESMA) to promote the concept with state, local, public and private agencies in order to find a physical location to host the first Mathematical Park and sources of funding to sponsor and support this project.

The idea of the Knotted Stained Glass Folly is based on a singular representation of the Boy surface and belongs to Claude P. Bruter, now the president of ESMA. French mathematician François Apéry described it in mathematical terms and produced several wire models (fig. 18a) of the Boy surface based on the equations he established [Bruter 2009]. This representation has a vertical axis of symmetry. When the object is cut by a plane perpendicular to this axis, it becomes a kind of flower or a symmetrical trefoil knot projection. This “flower” is the basis of a fiber-space; the fibers are ellipses that all meet in a singular point on the vertical axis. Two such ellipses, being relatively close one to the other, form the boundary of a singular Möbius band with the singular point as a singularity (fig. 18b).

Fig. 18. Wire model of Boy surface by F. Apéry (a), its general view (b) and design of the folly (c). Images: [Bruter 2009: figs. 5-7]

Bruter describes the structure of the folly as some well-chosen ellipses of the Boy surface, materialized in chrome steel. The main part of the folly consists of two spaces: one which is visible from the outside, and the other one which is only visible from the interior of the folly. A few Möbius bands are designed as stained glasses. By looking up and down, one can see them completely from inside the folly. A series of vertical arches in the same material as the frame will cover the entrance of the two symmetric rooms. A preliminary visualization of the folly, without any decoration, gives a general idea of its structure (fig. 18c). Convenient rods transversal to the elliptic frame will define panels and make it possible to cover the folly's exterior with glass and mirrors. Some of these panels will be stained glass windows showing either tiling or knots and braids with symmetries [Bruter 2009].

Though in the initial variant the folly was designed as a system of separated ellipses, from my point of view it is possible to take a *NODUS* structure of appropriate form to build the Boy surface as a single knotted woven structure with self-crossings. The solution of this task leads to another direction of the *NODUS* form-finding method that is consisted in the changing topological representations of the basic modules. The ring-shaped area of cyclic knots with two concentric circle edges may be turned into the knotted one with two concentric closed knotted curves. As a result, a knotted surface with self-crossings will emerge (fig. 19a). In addition, the two concentric circle edges of the ring-shaped area may be joined together to form a single double-covered circle that can serve as an edge of the Möbius band surface with self crossing (fig. 19b, c). I believe that these two models which represent separated parts of the Boy surface furnish the necessary key for designing the Boy surface as an integral *NODUS* structure.

Fig. 19. Knotted surface with three self-crossings (a, left) and one-side surface with self-crossing (b, c). Photos: author

9 Conclusion

The shape of architectural objects in general can be treated as envelopes: 2D surfaces embedded into 3D space. In the fifteenth century Italian architect and theorist Leon Battista Alberti claimed that the architecture consists in the outlines and the

structure (*lineamenta et structura* in the original Latin text of *De re aedificatoria*). The visible shape (outlines) exists only because it is determined by a structure that is not directly perceived. Modern mathematical theories of form also distinguish the shape as an exterior surface and the form itself as an internal structure [Lord and Wilson 1984]. The topological approach to the problem of physical modeling of surfaces gives us the new tools for describing, modeling and working practically with objects which, until recent times, remained completely in the field of abstract mathematics.

Modeling of any 3D surface follows the common logic of designing and manufacturing. Practically it is more convenient to work with two-dimensional representations in the form of sketches, perspective drawings or computer graphic display images. In the same way it is more convenient to create the surfaces of three-dimensional objects in two-dimensional space as their developments, evolvments, patterns, etc. and then transform them into spatial position. The practical possibilities of physical modeling of spatial surfaces are determined in general by the structure and the material chosen for the plane developments.

Well-known facet and linear surface models do not exhaust all of the possible plane approximations, because apart from the two-dimensional facets and the one-dimensional edges there are also the zero-dimensional vertexes, so there must be a third type of plane surface models, namely vertex or point surfaces. The physical model of a point is the contacting of two round bodies, and cyclic knots and links made from resilient materials are the most natural way to organize a great number of point contacts into topologically connected structures. It is likely that knots in the form of *NODUS* structures will provide us with the new method of surfaces modeling that can be applied to different fields of design, including architecture, engineering and art.

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About the author

Dmitri Kozlov is an architect and research scientist. He received his Ph.D. in Technical Aesthetic and Design from the Russian Academy of Architecture and Construction Sciences, Research Institute of Theory and History of Architecture and Town-planning, Moscow, Russia. In the late 1980s and 1990s he worked for the Laboratory of Architectural Bionics headed by Yuri Lebedev and took part in international collaborative design and research projects and architectural exhibitions. He is an international lecturer on applications of topology and knot theory to form-finding in architectural design and the inventor of three-dimensional knotted string structures, which he calls *NODUS* structures. He is a member of ESMA and is the ARPAM project coordinator in Russia.