Computational Models in Architecture: Understanding Multi- Dimensionality and Mapping

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Abstract. The aim of this paper is to discuss computational thinking and parametric design and thus “what model is” in the realm of mathematics in architecture. Here we investigate how multi-dimensionality and reference systems should be contemplated, along with what mapping is and how it is related with algorithms in computational design. Set theory and functional relations are re-visited and similarity/similitude concepts and the importance of non-dimensional parameters relating different reference systems (interpreted as domain and range relations) are pointed out. In this context, computational design approaches based on modern biomimetic studies in architecture which are beyond metaphors or analogies are considered as the prominent cases of the present inquiry.

Reference systems, dimensions and multi-dimensionality in the realm of computational modeling

A rapid change has taken place in the architectural design paradigm, the result of advances in information and computational technologies, as happened in several other disciplines. Enormous data handling capacities of computers, cutting-edge analysis techniques, sophisticated but user-friendly hardware and software allows us to address much more complex problems than ever before, making it possible to develop new solution techniques, providing massive data library and collaboration among disciplines. But on the other hand, all these available technologies and techniques require not only techno skills but also new mind sets, enabling algorithmic thinking and even a new level of cognition. This means that computational thinking and the resulting **computational models** are not only the substitutes of “phenomena, things, forces and more”, but also that new interfaces between different domains and the role of mathematics in this mind shift play a crucial role.

In any discipline, it is possible to find a definition explaining what a model is. These definitions are continuously evolving with the introduction of new technologies and interdisciplinary collaborations. Hence today, a model goes beyond the traditional description given in the *Oxford Dictionary*:

... a three-dimensional representation of a person or thing, typically on a smaller scale, (in sculpture) a figure made in clay or wax which is then reproduced in a more durable material, something used as an example, a simplified mathematical description of a system or process, used to assist calculations and predictions, an excellent example of a quality,

Furthermore it becomes a **new medium** of experience, exploration and learning revealing different level of complexities and dimensions.

Although models and modeling have always been part of disciplines and contribute to their progress, their role in any type of design – whether in art, engineering, medicine, architecture, etc. – is more explicit, starting from the very early examples to contemporary ones. In these disciplines, nature has always been the major source of inspiration, intuition and creation (fig. 1).

Fig. 1. Different levels of modeling of nature in art, design and engineering. From left to right: Mondrian trees (<http://paintings.name/piet-mondrian-biography.php>), Lotus effect (http://conservationreport.files.wordpress.com/2008/12/lotus-effect_biomimicry.jpg); Bionic car (<http://www.yankodesign.com/2009/06/03/ten-inspirational-and-creative-bionic-designs/>); Bionic Handling Assistant (http://www.festo.com/cms/nl-be_be/16306.htm)

But starting in the 1970s, the way disciplines learn from nature and the way the models are generated and used was redefined: first by Jencks and then by Benyus considering nature as a “model, measure and mentor” [Benyus 1997], then by the introduction of the fundamental principles of biomimicry and the advent of computational technologies. While the precision and the complexity of the models are increasing by the use of computational technologies offering massive amount of data and information, modeling is still a challenge in any discipline with the requirements of new mind sets [Arslan Selçuk, Gönenç Sorguç and Çakıcı 2011].

Fig. 2. Examples of nature as a “model, measure and mentor” in architecture [Pearson 2001]

In this context, the present study focused on computational models which require algorithmic thinking, comprehending the parameterization and thus the meaning of **reference systems** and **multi-dimensionality**. To understand what a computational model is in the context of this study, it is first necessary to explain how reference systems and multi-dimensionality are conceived, based on fundamentals of mathematics. For a mathematician, understanding dimension and dimensionality is straightforward issue which is not bounded by three-dimensional space at all. For example a point can be defined as n -tuples in n -dimensional space as

$$P(x_1, x_2, x_3, \dots x_n)$$

But for an architect or an engineer "a point" is mostly conceived by its 3D coordinate system as

$$P(x_1, x_2, x_3).$$

This perception is mostly related to the mind sets developed in the course of education, in learning/teaching the concept of reference systems and dimensions based on Cartesian coordinates which is adequate to define most geometries and spatial relations. However, when the essence of parameters employed in the definitions $x_1, x_2, \dots x_n$ are investigated, it can be seen that these are related with the information/data to be incorporated in the definition and thus in the **reference system**. In this regard, reference systems can be conceived of as the **vocabulary** of the models, determining the dimensions and precision of the model as well as relations between the **modeled** (i.e., *domain*) and the **model** (i.e., the *range*). This can be exemplified by considering what a pixel is. A pixel cannot be defined as a simple point on the screen but it is defined as

$$P(x, y, \text{number of bits, RGB, intensity, address in memory}),$$

which requires additional parameters describing all the features of it. Hence a pixel can be completely defined by at least five parameters. In other words, a pixel can be considered as belonging to a higher-dimensional reference system and accordingly multi-dimensional. Each parameter used in the definition of the pixel can also be regarded as the information to be incorporated belonging to the reference domain.

Fig. 3. Modeling process

The same argument can be illustrated in the process used to model the seashell *Turitella terebra* shown in fig. 3. In this modeling process, the parameters chosen to identify the *Turitella terebra*, which is the domain of the model, also constituted the reference system of the reconstruction, that is, the computational model. Several samples of *Turitella terebra* were analyzed with regard to their form and structural stability and the following parameters constituting their reference system defined: coiling axis, section; shape of the aperture or shape of the shell's tube cross section, horizontal displacement;

departure from the coiling axis of the section in the horizontal direction, vertical displacement; translation along the vertical direction of the coiling axis, growth; aperture expansion or the rate of increase of section size, outer shell, material. Hence, in the present context *Turitella terebra* can be considered as having seven dimensions.

In any modeling process, description of the appropriate parameters and thus the reference system in relation with the context is essential to construct a model. Besides, how these parameters are reflected (i.e., mapped to the model) should be questioned. The mapping (i.e., relations or **rules** in general from domain to range) can be considered as the basis of algorithms.

The search for a proper reference system and how these references are incorporated has always been a concern in architecture. The golden ratio, which relates width and height by a number, is a rule of mapping or a simple algorithm for the designer. The human body provided the scale for designing columns in Greek and Roman architecture (fig. 4). The Vitruvian man of the Renaissance, Le Corbusier's Modulor, and nature as tutor as seen in Zimbabwe towers are some other examples illustrating how the domain (nature) is related to the range (model) as shown in fig. 5.

Fig. 4. Vitruvius's illustrations on the proportional evolution of Doric and Ionian Columns from a man to a slender woman (from left to right) [Vitruvius 2001]

Fig. 5. From left to right: Vitruvian man of the Renaissance [Vitruvius 2001]; the Modulor [Le Corbusier 2000], Eastgate Centre Building in Zimbabwe inspired by the ventilation of termite towers (<http://www.asknature.org/product/373ec79cd6dba791bc00ed32203706a1#changeTab>)

Why computational modeling?

In questioning the motive behind modeling, the very first plausible answers are: **need to solve a problem, curiosity, inspiration, to learn, to compare, to re-experience.** Yet, computational models should not be perceived by their parameters and what they represent alone, but also with the process of modeling consisting of several mappings, that is, algorithms constructing the end model. Whatever the reason for the model, models in general resulting from observing a phenomenon/force/process/thing may serve to find a solution, or an answer, or may constitute the problem itself. In either case, the cycle that is followed in any modeling process has a similar pattern as tabulated in fig. 6.

Observation of actual phenomena, simplification regarding the need, precision, present knowledge, solution methods and present level of technology to obtain a working model; translating working model to mathematics; solution methods including mathematics and techniques developed by related disciplines resulting in computational model and results of the solution of computational model as an answer or a feedback for further inquiries. This computational cycle helps designers to discipline and systematize their thoughts. The major steps of the computational thinking process are explained in Table 1.

Fig. 6. Simplified computation cycle for learning/experiencing by modeling

A problem to be solved, imagination, inspiration, observation, curiosity	
1	Skill and some background knowledge
Description	
2	Description of the reference system, identification of parameters
Outlining the model	
3	Mapping and algorithms
Construction of model	
4	Mathematics and any others
Testing the model	
5	Precision, information, learning

Table1. Graphic representation of the steps of modeling process

It can be claimed that the first three phases of computational modeling are strongly dependent on the skills and knowledge of the “modeller”. Real phenomena are usually very complex, and parameters and variables can be quite hard to identify and their numbers huge. Hence the identification of the reference systems is crucial. The level of precision expected in the model and the subject matter determines the description of the reference system (the parameters to be included) and dimensionality (parameters to be employed to generate the model). Moreover, each parameter included in the model describing its domain and range (usually the model itself) actually transforms the problem from ill-defined to well-defined. Therefore in modeling the very first step is the identification of parameters and the construction of a reference system and related dimensions, together with given or established rules; in a broad sense algorithms are mappings from m dimension to n dimension.

The development of algorithms or rules from one domain to another means that designer (modeller) determines the process, controls the process and thus plans/designs the process. The computational model obtained though this cycle can be linear or non-linear, stochastic or deterministic, static or dynamic, lumped or distributed, depending on the context and the algorithm developed within the modeling process. Computational thinking and the resulting models are not black boxes but with their clearly declared structures and procedures are transformed into white boxes for any user, as demonstrated in Table 2.

Table 2. Black Box Model

Table 3. Modeling Process Transforms Black Box to White Box

Tables 2 and 3 showing the major steps of computational modeling are analogous to functional relations in mathematics, which is based on set theory. This analogy is illustrated by fig. 7.

Fig. 7. Basic set theory

The last phase of the modeling can be considered as the new medium of exploration and experience where learning occurs on two levels: learning through the modeling process, and learning by testing the end product, that is, the end model.

Designing the “architectural design process”

Computational design and its technologies force the architectural design practice to evolve from designing not only the end product but also designing the process. The role of inspiration, intuition, cognition and creativity versus technology has been questioned, accompanied by discussions on “what creativity is”. We believe that algorithmic thinking and computational models, transforming design and the design process from a black box to a white box for which intuition, inspiration, tactile knowledge determine the model, enrich design skills and creativity and their pursuit for which “forces and phenomena, data and information” are to be included in the design implicitly or explicitly, by inspiring, by calculating or computing.

As discussed in the previous section, each endogenous or exogenous variable taken into consideration determines the reference system and the dimensionality of the problem/design. Therefore, it is the designer’s responsibility to determine the rules, that is, the algorithm to relate/map domain and range parameters in the design process. As the number of parameters included in the definition increases, the complexity of the design cycle and the resulting model increases, demanding more complex mapping/algorithms.

This requirement can be exemplified in performance-based design approaches in architecture. Shaping the building by the requirement(s)/constraints specified in the initial design phase (defining domain and the range) necessitates well defined relations/rules/algorithms which are not only limited by formal relations or formal similarities as shown in some contemporary examples in fig. 8.

Fig. 8. Simulation models constructed to design performative architecture
[Kolarevic and Malkawi 2005: 48, 141, 143, 211]

These broadened relations, which are beyond formal similarities observed in performative design and the determination of the mapping rules, can be contemplated by the similitude concept.

Mapping through similitude/ similarity

Similitude is a concept frequently employed in mathematics, physics and engineering to evaluate and test the qualities, precision and accuracy of the models proposed. The idea behind the similitude approach engineering is to guarantee the reliability of the behavioral qualities of the model regardless of its small scale or number of parameters incorporated. It is evident that the models are much simpler both in scale and in dimensionality; yet they should be reliable enough to represent the phenomena they are substituted for. Hence, the mapping from real to model through similitude analysis establishes the rules for how reference systems and the related parameters should be related. Although this definition is more appropriate for engineering models, architecture too, under the influence of the computational design paradigm, is becoming more and more transdisciplinary, resulting in more complex design processes and thus the end models. In this perspective, similitude offers the potential to develop algorithms allowing *reduced complexity and number of parameters without any loss of the precision or mimicking capacity of the model*. The fundamental technique used in similitude is the dimensional analysis, which expresses *the system with as few independent variables and as many dimensionless parameters as possible*. The complementary part of the dimensional analysis is the generation of non-dimensional parameters, which make it possible to compare and combine different reference systems and map the domain onto the range, that is, onto the model having different dimensions. It is important to note that in the mapping process by similitude, the exogenous and endogenous variables should be preserved for the precision and reliability of the model.

This can be achieved by the use of the Buckingham Pi Theorem, which describes how every physically meaningful equation/relation involving n variables can be equivalently rewritten as an equation/relation of $n - m$ dimensionless parameters, where m is the number of fundamental dimensions used. This theorem also provides a method for computing these dimensionless parameters by the use of parameters describing the reference systems.

The relations used to derive scaling laws, that is, mapping from domain to range (model), determine the precision and reliability of the model. The idea of similitude offers great potential for architectural design. Architecture can adopt such a methodology especially in performance-based design approaches as well as biomimesis in design.

Re-experiencing the nature-architecture relation. Biomimesis: learning from nature

The idea of learning from nature is as old as humankind, contributing to the progress of several disciplines, innovative solutions, optimized designs and more. Biomimesis in architecture, which can be defined as mimicking nature, strongly manifests the need to develop methods to model nature in architecture. It is obvious that the model is meant to go far beyond questioning the forms, to understanding natural forces and phenomena. Researchers studying biomimicry conceive nature as a model, as a measure, and as a mentor. Hence modeling becomes more important than ever before, as mentioned earlier. In what follows, a biomimetic study carried out as part of the Ph.D. research

performed in Middle East Technical University by Semra Selçuk Arslan under the supervision of Asst. Prof. Dr. Arzu Gönenç Sorguç will be discussed.

Seashells have always been a source of inspiration in art and architecture. Today, studies about them are not limited to their forms alone, but also consider their material, form and structural integration. In the present example, the seashell *Turitella terebra*, which is shown in fig. 3, was studied to explore the relation between its form and its structure to furnish more insight for shell design in architecture.

Fig. 9. The relation between seashells and architecture in terms of form and structure
[Arslan Selçuk and Gönenç Sorguç 2010]

The study was designed in three stages: first, the experimental study to apprehend the structural behavior of selected and prepared shells under compression; second, the development of computational model for form generation based on the actual seashell and for structural simulation tests; third, the derivation of non-dimensional parameters learnt by studying the seashells for designing optimized shells in architecture.

In the experimental part of the study, more than 80 *Turitella terebra* shells were prepared and subjected to compression tests with the same boundary and loading conditions to determine their structural strength according to some dimensions (fig. 10). The form of the *Turitella terebra* was also examined extensively in this stage and a mathematical model generating a three-dimensional solid model of the shell for further studies was developed through a C++ compiler. The Finite Element Analysis (FEA) technique was employed to evaluate the precision and the efficiency of the model by comparing the experimental and the FEA results (fig. 11).

The comparison proved that the model of *Turitella terebra* is precise enough to further investigate the structural behavior of the shell. Moreover, the experiments and simulations showed that structural behavior of the seashells and their resistance to compressive forces can be expressed as a **function** of their **base-to-height ratio** which enables us to represent the basic dimension of these seashells. The characteristic base-height ratios with corresponding average loading before structural failure (i.e., fractures) are shown in Table 4.

Fig. 10. Preparation before physical compression tests and FEA

Fig. 11. Examples from test results

The mathematical model developed in the realm of the study was not only able to generate *Turitella terebra* with the utmost precision, but also many others based on the model, which may be called subsets or classes of the seashell. But more important than the form generation process, the model enabled us to determine their structural behavior. The parameter representing the relation between **height** to the **base area** of the shell enabled us to map the knowledge gathered though the study on seashell form and its structural behavior.

base/height ratio	0.35-0.38	0.40-0.48	0.50-0.59	0.60-0.69	0.70-0.76	0.79-0.86	0.90-1.00
load (N)	543	548	628	649	662	869	931

Table 4. Representing base-height ratio range to load

Then the experiment was extended by sectioning the shell and introducing two more parameters, namely, **whorl** and **weight** of the seashell, as shown in Tables 5 and 6.

weight (gr)	7.54-8.18	8.48-9.20	9.50-10.89	12.12-14.00	15.38-16.03	17.18-20.08
load (N)	380	450	820	670	750	820

Table 5. Representing weight to load relation

whorl number	1.5-2	2.5-3	3.5-4	4.5-5	5.5-6	7.5-8	9
load (N)	500	540	560	550	570	620	630

Table 6. Representing increasing whorl number to load

In the study, it was found that not only the base to height ratio, but also other parameters, such as weight-to-whorl number, base-to-whorl number or weight-to-surface area can be employed to represent the structural behavior of *Turitella terebra* under compression. This information has been exemplified in the design of tapered cone shells which are very common forms in the minarets of mosques. Initially, a tapered cone form with a 10 meter base diameter and 6 meter top diameter made of concrete was proposed and the height providing the maximum structural stability was explored. The base-to-height ratio observed in the seashell examples were employed as the “educated guess” to find the “best height” yielding the structural stability. It was found that the dimensions 5, 8 and 10 meters might be used in architecture, resulting in the structural stabilities shown in fig. 12.

Fig. 12. Demonstration of the working “base to height ratio” relation with loading on a tapered cone (FEA results)

The stability of the shells increases when the base-to-height ratio increases and approaches 2 as observed in the *Turitella terebra* example. In this study the base-to-height ratio was conceived as an “educated guess” to design shell forms which require the integration of material, form and structure, and can be extended to further explorations. The study has been extended to the existing masonry mosques and the same ratio has been tabulated in Table 7. It is seen that the base diameter-to-height ratios were close to 2 in the Hagia Sophia, which was built in 537 A.D. Similarly, in the mosques built by Sinan, which are still standing without any structural failure, this ratio approaches 2 as well.

Building	Construction year	Diameter/height ratio
Hagia Sophia	537	2.02
Fatih Mosque	1470	2.01
Sultan Beyazid Mosque	1486	1.31
Şehzade Mosque	1548	1.63
Süleymaniye Mosque	1557	1.64
Rüstem Paşa Mosque	1561	1.84
Mihrimah Sultan Mosque	1565	1.54
Lala Mustafa Paşa Mosque	1565	2.03
Selimiye Mosque	1574	1.97
Sokullu Mehmet Paşa Mosque	1577	1.62
Azapkapı Mosque	1578	1.74

Table 7. Diameter-to-height ratios of some important dome structures (dimensions achieved from the drawing files available at <http://archnet.org>)

The model can be employed to examine other potentials of seashells as well, but within the context of the present example, it is aimed at pointing out the systematic of computational design, the model and its context and the mapping from one domain to another, as shown in fig. 13. The choice of seashell has actually no importance and could be anything and the same computational design methodology would be applicable.

Fig. 13. Proposed methodology for learning from nature, a case on seashells: biomimicry in architecture

Conclusions

It has been accepted that architecture and design are undergoing change, and that informatics and computer technologies have transformed the design concept from a result to an interactive process. On the other hand, the computational technologies and continuously expanding data result in the paradoxical relation between the potentials of dealing with more complex problems and the requirement of new mind sets to deal with the massive amount of data and complexity. Computational thinking enabling this required mind shift also demand complete understanding of reference systems, dimensions and thus parameterization together with mapping as the fundamentals of algorithms. Many disciplines have already been developing their own ways of “computational thinking”, which is evolving in accordance with the developing technologies. The question of “what about in architecture” deserves more attention. In this regard, the role of mathematics in the computational thinking and in the accompanied models has become more crucial in all disciplines, and so too in architectural design, as we have shown here. Revisiting the fundamental concepts of set theory, functions and relations, and interpreting those concepts in the context of computational thinking is essential to deal with complex systems and generating models elaborating the way architects learn, think and design.

Biomimetic studies provide the best representations of the necessity of computational thinking and thus modeling. The potentials that biomimesis offers to researchers in science and design in terms of “inspiring/learning/adapting and/or implementing ideas from nature” impose this way of thinking and a system designed to transfer the answers/knowledge it encompasses. Hence the example chosen to elucidate the present inquiry was a biomimetic study. It emphasises that computational modeling is important in the reflection of biomimesis on architecture, and that this multi-dimensional and multi-disciplinary concept could be transferred to another discourse – architecture – accurately and efficiently through a computational process.

We believe that assembling quantitative data with computational models and using these models to investigate quantitative methods for transferring them to architectural design is an accurate exemplification of the present discussion. What distinguishes this study from other, similar ones is its emphasis on the necessity that inspiration from nature not be restricted to formal aspects, and that the modeling process is important in architectural design, with an understanding of reference systems, dimensions and parameterization yielding new design approaches and technologies with new levels of precision.

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