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Keywords: green space layout, urban square, fractal, hierarchical scale, design evaluation

The Design Evaluation of the Green Space Layout of Urban Squares Based on Fractal Theory

Presented at Nexus 2012: Relationships Between Architecture and Mathematics, Milan, 11-14 June 2012

Abstract. Many scholars have applied fractal geometry to analyze non-linear urban forms at the regional, city, and building levels. However, little attention has been given to the level of urban design. The aim of this paper is to test the validity of fractal analysis on the green space layout of urban squares. Four research cases – including one in USA, one in Argentina, and two in China – are analyzed using Bovill's box-counting method. We find that continuous hierarchical scales are essential to the fractal nature of the built environment, and the scales derived from the human-scale theories of open space are used to develop the box sizes. The authors demonstrate the advantage of using calculation charts to provide direct visual insights into the fractal distribution of the layout, through lacunas and shadowed boxes. The significance of new lacunas is presented and discussed in particular. The proposed method allows for quantitative measures, convenient evaluation, and potential feedback to improve the fractal quality of the green space design.

1 Introduction

Fractal geometry has been applied to understand non-linear forms and complex shapes since the late 1970s, when Mandelbrot [1977] declared and illustrated the fractal properties of nature. Subsequent works in broader disciplines, including geography, urban planning, landscape planning, and architecture, have demonstrated the internal link between fractal geometry and the built environment. Since the 1990s, a wide range of fractal analyses have been conducted at both macro and micro scales.

At the **regional level**, geographers have explored the spatial quality of cities and urban development [Batty and Longley 1994; Benguigui et al. 2000; Chen 2008; Frankhauser 1998], and have interpreted the formation of regional landscape [Thomas et al. 2008]. At the **city level**, researchers have studied the visual complexity of urban skylines [Oku 1990; Stamps 2002], aesthetic preferences for landscape silhouettes [Hagerhall et al. 2004], street plans of medieval cities [Hidekazu and Mizuno 1990], and the hierarchy and connectivity of urban networks [Salingaros 2003]. At the **building level**, scholars in architecture have compared the fractal dimensions of selected elevations and plans of modern [Bovill 1996; Ostwald and Vaughan 2008; Vaughan and Ostwald 2008] and traditional buildings [Bechhoefer and Bovill 1994], as well as architectural elements [Capo 2004]. However, little attention has been paid to the **district level**, or the level of urban design, except for Cooper [2003; 2005; 2008], who has provided fractal assessments of street-level edges, vistas, and skylines. This paper also adopts fractal

geometry at the district level, but focuses on the green space layout of urban squares, which is rarely the subject of fractal evaluation.

The spatial layout of urban squares is often investigated subjectively and discussed qualitatively, from the perspective of urban and landscape design. The percentage of green space, as a quantitative index, has been commonly used to measure an urban square. Although objective, this index is an oversimplification, neglecting the intricate qualities of the form, scale, and distribution of various green spaces. Therefore, more sophisticated measures of a green space layout are necessary for the purpose of evaluation.

The objectives of this paper are to develop a mathematical description of the green space layout at the district level, to test the validity of fractal analysis for measuring the spatial layout, and to provide assessment and feedback to improve the fractal quality of the green spaces of urban squares.

2 Methodology

Fractal dimension

Many natural objects, such as tree branches, river systems, mountain ranges, lightning bolts, and coastlines, are found to possess irregular shapes that could be best described by fractal geometry. By definition, a fractal is a rough or fragmented geometric shape that can be split into parts, each of which is (at least approximately) a reduced-size copy of the whole [Mandelbrot 1977]. At the regional and city level, researchers found that cities can be conceptualized as fractals at multiple interrelated scales. At the district level, we assume that the green space layout of an urban square should display fractal features to some extent, in order to provide a sense of naturalness to its users.

Fractals can be quantified and measured by a parameter D , the fractal dimension. In Euclidean geometry, it is well known that line, plane, and volume have integer dimensions of 1, 2, 3 respectively. The fractal dimension, in contrast, may have a value of 0.7, 1.3 or 2.4. Imagine yourself touching a table surface with cracks and bumps; it feels more than a two-dimensional surface, yet it is not three-dimensional either. Its value of D is somewhere between 2 and 3, depending on how smooth or rough the texture is. In fact, D indicates “the extent to which a structure exceeds its base dimension to fill the next dimension” [Hagerhall et al. 2004: 248].

The box-counting method

Based on Mandelbrot’s proposal, Carl Bovill [1996] performed a version of “box-counting” method for calculating the fractal dimensions of the architecture of Frank Lloyd Wright and Le Corbusier. We can take a line drawing of a building elevation as an example. The elevation is first covered by a large grid with the side length of the grid box recorded as δ_1 . The number of grid boxes intersected by the line image is counted as $N(\delta_1)$. Next, a smaller grid is placed over the elevation. The side length of the smaller box is recorded as δ_2 , and the number of intersected boxes is counted as $N(\delta_2)$. The fractal dimension D can be calculated by the following equation:

$$D_{\delta_1-\delta_2} = \frac{\log N(\delta_2) - \log N(\delta_1)}{\log(1/\delta_2) - \log(1/\delta_1)}$$

By repeating this process over a sequence of grids of decreasing sizes, we can record $\delta_1, \delta_2, \delta_3, \dots, \delta_n$ and $N(\delta_1), N(\delta_2), N(\delta_3), \dots, N(\delta_n)$. If we plot $\log N(\delta_n)$ against $\log(1/\delta_n)$, the regression slope of the diagram is the approximate fractal dimension D of the elevation.

This present paper adopts Bovill's box-counting method, because it is clear, simple, and applicable for almost any type of line image. The authors want to test how a similar approach can be used as an appropriate tool to evaluate the green space layout of urban squares.

When applying the box-counting method, determining the most appropriate sizes of the grid is the primary task. The green space of urban squares, like natural objects, only reveals its fractal quality in a particular scale range. Hence, the box sizes need to be defined within this range. In addition, the scaling ratio of successive box sizes varies among researchers, although a ratio of 1/2 has often been employed by many people, including Bovill.

Spatial hierarchy

Approximate fractals in nature are generally self-similar, governed by a scaling and recursive principle over an extended, but finite, scale range. This leads us to the notion of spatial hierarchy, which is an important parameter to reveal the fractal properties of the green space of an urban square.

Green space can not only provide ecological and visual amenities to users, but also define and divide urban space to facilitate people's experience of the square. Since the human being is the ultimate subject to interact with the built environment, the spatial hierarchy of green space should be developed in accordance with the theories of human scales of open space.

E. T. Hall [1966] developed a series of measurable distances of interpersonal space (Table 1). These dimensions are widely used to study human behaviors in interior space. According to Ashihara's [1981] one-tenth theory, it is adequate for the scale of exterior space to be eight to ten times of that of interior space. Multiplying Hall's distances by 10, the corresponding outdoor scales of human interaction is obtained at a rough estimate (Table 2). It is amazing to see the close correlation of interpersonal distances between indoor and outdoor measures. Hall's four different zones also appear valid to describe various distances between people as they interact in an open space.

Research by other scholars also confirmed the distances calculated in Table 2. Lynch [1984] considered 12 m as an intimate scale for outdoor spaces because one can see a person's face in rich detail at about 12 m to 14 m. Alexander [1977] proved a distance of 21 m to 24 m for people to recognize a face. This distance supported the modular unit of 20 m to 25 m proposed by Ashihara [1981] in the design of exterior space. Gehl [1987] found that 70 m to 100 m is the maximum distance for watching outdoor events. In addition, Hall [1966] found that an indoor distance above 3.7 m is more suitable for one-way communication with larger audiences for public speech. This implies that in an open space, at a distance of over 37 m, an individual tends to watch people or events without being personally involved, since one-on-one interaction can no longer be maintained among people with anonymous identities.

When generated properly, the scales of spatial hierarchy can help establish the grid sizes of the box-counting approach. To accommodate the 1/2 scaling ratio of the box-counting method, the interpersonal distances have to be adjusted to their optimal approximation. A new set of dimensions is thus proposed as the final hierarchical scales applied in this study, which is 5 m, 10 m, 20 m, 40 m, and 80 m. Our underlying hypothesis is that to reveal the fractal nature of spatial arrangements, and to provide human scale and psychological comfort to users, the design of green spaces should

maintain the proposed hierarchical scales in a continuous manner. Of course, the largest scale varies depending on the size of the urban square.

Zone	People to interact	Range	Distance (m)	Hall's distance x 10 (m)	Visual perception	Proposed hierarchical scale (m)
Intimate distance	lovers, children, pets, close friends and family members	Close phase	0 ~ 0.15	0 ~ 1.5		
		Far phase	0.15~0.46	1.5~4.6		5
Personal distance	friends and associates	Close phase	0.46~0.76	4.6 ~7.6		
		Far phase	0.76 ~ 1.2	7.6 ~ 12	see a person's face in rich detail at 12 m to 14 m	10
Social distance	new acquaintances and strangers	Close phase	1.2 ~ 2.1	12 ~ 21	recognize a person's face at 21 m to 24 m	20
		Far phase	2.1 ~ 3.7	21 ~ 37	watch without being personally involved over 37 m	40
Public distance	larger audiences	Close phase	3.7 ~ 7.6	37 ~ 76	maximum distance for seeing events at 70 m to 100 m	80
		Far phase	>7.6	>76		

Table 1.
E.T. Hall's indoor scales of human interaction

Table 2.
Outdoor scales of human interaction

3 Data

Case selection

To search for applicable cases for fractal analysis, the concept ‘urban square’ needs to be clarified. In this paper, C. C. Marcus’s [1997] definition of the city square is borrowed and modified as follows.

- a. centrally located in the city center or district center, where major thoroughfares intersect;
- b. not attached to a particular building;
- c. encompassing one or more city blocks and bounded by streets on three or four sides;
- d. integrating both planting and hardscape elements, with the percentage of green space between 40% to 75%;¹
- e. attracting a variety of users and activities from a greater distance.

The city square, according to Marcus, is one of the largest types of urban plaza. The reason why we choose this category is to maximize the number of overlays of the descending grids in the box-counting approach. Besides, the dimensions of selected urban squares should be comparable to maintain the consistency of the measuring grids. This leads us to the exploration of the size of an urban square.

Ashihara [1981] found that the adequate size of the exterior space to be 0.8 hm² to 1.6 hm².² Gehl [1987] suggested a maximum dimension of 70 m to 100 m to see events, which implies that the width of a successful enclosed square should be less than 100 m. Sitte and Stewart [1945] proposed that the length of a square should not exceed twice of the width. Combining the above research findings, we conclude that the appropriate size of the urban square is about 0.8 hm² to 2 hm², which is added as another criterion for our sample screening.

After a careful search and comparison using Google Earth, four urban squares were chosen for presentation in this study (fig. 1). They are Union Square in the USA, May Square in Argentina, and Central Square and Seven-Star Square in China.

Fig. 1. The aerial photographs of selected urban squares (Source: Google Earth)
The location and size information of these squares is summarized in Table 3.³

Name	City	Country	Size (hm ²)
Union Square	San Francisco	USA	0.90
May Square	Buenos Aires	Argentina	1.35
Central Square	Qiqihaer	China	1.22
Seven-Star Square	Jincheng	China	1.32

Table 3. The location and size of selected urban squares

Image processing

When applying the box-counting method, an urban square has to be converted into a two-dimensional plan. A line drawing is thus produced by tracing the perimeter of the green space in AutoCAD, based on the aerial image and scale provided by Google Earth. On-site photos collected via Internet are also utilized to verify image details in the digitizing process.

‘Green space’ generally refers to an area composed of ‘softscape’ elements, such as grass, shrubs, flower beds, and trees, in contrast to ‘hardscape’ elements, such as pavement, sculptures and small building structures. To avoid complications, we have defined additional rules for extracting the information about green spaces:

- a. The object of our research is green space, not individual softscape elements. Hence, attention is only given to the boundary between the softscape and hardscape on the ground. The closed outline of a grass area with all planting inside, is considered as a single green space entity.
- b. Limited by the resolution and accuracy of a source image, cartographic detail needs to be controlled. We decided to ignore any isolated green space if its longest dimension is less than 0.5 m. The same rule applies to isolated pavement areas within a green space. We believe that such a detail plays a minor role in the overall distribution of green space, and has no significant influence on our analysis.
- c. Our study only concerns two-dimensional features on the ground. Any softscape elements elevated above ground, such as tree canopy, are not included in the green space. However, a planting hole of a tree or shrubs on pavement could be considered as green space, if the hole size is larger than 0.5 m.
- d. Water elements are excluded from the green space.
- e. Light fixtures, outdoor furniture, flowerbeds enclosed by hard materials, garbage bins, and manhole covers located inside a green space are all ignored, regardless of their sizes. Ornamental flower pots for temporary display on pavement areas are also disregarded.

Calculation

Controlling boundary

Fractal characteristic is not the only, nor even the most important, criterion affecting the quality of green space. Landscape design, visual attraction, cultural and historical context could be more critical. Therefore, we are neither interested in the design evaluation of the selected urban square, nor concerned with making a comprehensive assessment on the green space quality. The research purpose is to find inspiring prototypes which best illustrate the fractal properties of green space. As a result, whether our study area covers the entire urban square or not is less important, as long as the area covers all the green spaces, revealing the interrelationship of softscape and hardscape surfaces.

Consequently, we assume that cropping all the sidewalks along the perimeter of the urban square to be acceptable. Due to the internal deficiency of the box-counting method, the fractal dimension D can be significantly affected by the white space surrounding the source image. To minimize the interference, the sidewalks of all selected

cases are cropped for the sake of consistency, even though some sidewalks might be considered as an integral part of the design scheme.

Determining grid size

According to the previous discussion on spatial hierarchy, the green space layout should display continuous hierarchical scales of 5 m, 10 m, 20 m, 40 m, and 80 m. In other words, these numbers could be assigned as the side lengths of the box-counting grids in our analysis. Since the dimensions of selected urban squares are between 80 m to 200 m, 40 m is chosen as the largest box-counting grid to generate meaningful data.

People might anticipate that the fractal dimension $D_{(box, 20-10m)}$ indicates the fractal nature of green spaces of 20 m to 10 m. However, this is not true. In fact, we found that $D_{(box, 20-10m)}$ partly indicates the fractal spaces of the scale above 20 m. This means that to understand the fractal spaces of 10 m to 5 m, it is necessary to calculate $D_{(box, 5-2.5m)}$. As a result, the smallest size of the box-counting grid is extended to 2.5 m, although this dimension is not included in Table 2. In summary, the box sizes employed in our analysis is 40 m, 20 m, 10 m, 5 m and 2.5 m.

Placing the grid

When carrying out the box-counting analysis, how the image is placed against the initial grid may lead to differences in D values. To lower the impact of this problem, and to reduce the distortion caused by the empty space surrounding the image, two rules are followed to overlay the initial grid. The primary rule is to cover the image with the minimum number of grid boxes, and the secondary rule is to center the image within the measuring grid if possible.

4 Results

Based on Bovill’s classic method of box-counting, the fractal dimension of each box range is calculated by the equation given earlier:

$$D_{\delta1-\delta2} = \frac{\log N(\delta2) - \log N(\delta1)}{\log(1/\delta2) - \log(1/\delta1)}$$

The results are summarized in Table 4 and the corresponding box-counting charts are presented in fig. 2.

	Union Square San Francisco, USA	May Square Buenos Aires, Argentina	Central Square Qiqihaer, China	Seven-Star Square, Jincheng, China
$D_{(box, 40-20m)}$	2.000	2.000	1.750	1.874
$D_{(box, 20-100m)}$	1.737	1.766	1.477	1.295
$D_{(box, 10-5m)}$	1.446	1.360	1.196	1.122
$D_{(box, 5-2.5m)}$	1.394	1.113	1.109	1.060

Table 4. The fractal dimensions of selected urban squares

Fig. 2. The box-counting charts of selected urban squares

Terms

A few terms which are critical for our further discussion on the results should be explained. In fact, these terms are directly influenced by the technical features and limitations of the box-counting method.

Space

Since the box-counting method is performed on two-dimensional line drawings, the figure and ground relationship of softscape and hardscape is undifferentiated. Hence, the *fractal dimension of green space* is not a very accurate expression. The fractal dimension actually reflects the interplay of the green space and hard surface area, with both components equally emphasized. In the fractal analysis of an urban square, when we talk about the concept of *space*, it refers to either the solid green space consisting of softscape elements, or the void space enclosed by several solid green spaces, which is a paved area.

A space of a particular scale

When we speak of a space of a particular size, we mean that within the enclosed boundary of a solid space or a semi-closed area of a void space, the biggest square box that can be accommodated is a square of that size. For example, if we speak of a space of 20 m, it means that the biggest square box accommodated is 20 x 20 m. Similarly, if the biggest square box accommodated is 35 x 35 m, we call it a space of 35 m.

Lacuna

Although fractal dimension is determined by the number of boxes counted, the boxes not counted, which reveal crucial information about the organization of space, have a much greater significance in our study. The uncategorized or white boxes in each calculation chart could be classified into four categories depending on their positions (fig. 3).

Fig. 3. The classification of uncategorized boxes in the box-counting

- Old lacuna: the white box shared the same location with that of the preceding chart;
- Grown lacuna: the newly emerged white box located inside the boundary of the urban square, and adjacent or connected to the old lacuna;
- New lacuna: the newly emerged white box located inside the boundary of the urban square, and separated from the old lacuna;⁴
- Border lacuna: the newly emerged white box located outside the boundary of the urban square.

From the formula of the fractal dimension, we know that the value of D is not affected by old lacunas (a), but by newly emerged white boxes of b, c, and d, which influence the number of counted boxes.

Spatial implications of lacunas

To explore the relationship between lacuna and spatial characteristic, let us compare the calculation charts of a hypothetic model (fig. 4). It has a total of six spaces of descending sizes. There are two spaces with the scale of 80 m to 40 m, two of 40 m to 20 m, and two of 20 m to 10 m. In T40, the space of 64 m generates a new lacuna, but the space of 48 m does not. This is because of the particular way the grid is laid out. Apparently, a space over 40 m should generate at least one lacuna in a 20 x 20 m grid no matter how it is placed. This is why a new lacuna appears in the space of 48 m in T20. Meanwhile, among the two spaces between 40 m to 20 m, the space of 36 m creates a new lacuna, and the space of 24 m does not. In T10, two new lacunas emerge in the space of 24 m and 18 m respectively, but the space of 12 m remains grey.

Fig. 4. The box-counting charts of a hypothetic model

The model leads to the generalization that at the box range of $2\delta_1$ to δ_1 (e.g., 40 m to 20 m), a new lacuna is caused by either a space of $4\delta_1$ to $2\delta_1$ (e.g., 80 m to 40 m), or a space of $2\delta_1$ to δ_1 (e.g., 40 m to 20 m). However, the probability of the latter decreases as the scale of space descends toward δ_1 (e.g., 20 m).⁵

We find family genes a convenient metaphor to explain the spatial implications of lacunas. Considering the chart of T40, T20 and T10 as the *parent*, *child* and *grandchild* layers respectively, and the spaces of 80 m to 40 m, 40 m to 20 m, and 20 m to 10 m as *parental genes*, *genes of child*, and *genes of grandchild* respectively, we can make the following statements.

- Old lacunas located inside the boundary of an urban square indicate the existence of family genes, as illustrated by the old lacunas inside the space of 64 m in T20 and T10.
- Grown lacunas further depict the family genes, as shown in the space of 48 m and 36 m in T10.
- New lacunas reflect the genes acquired from either parent, or child, or both, as indicated by spaces of 48 m (parent) and 36 m (child) in T20.
- Border lacunas locate outside the image, and do not contribute to the understanding of the spatial layout. These lacunas create noise and distortion of D , as displayed by six white boxes at the right border of T20.

Among different categories of lacunas, we find that the category 'new lacuna' is the most critical indicator of the fractal nature of an image. Fig. 5 shows the probability of a space appearing as new lacuna in the corresponding calculation chart.

Fig. 5. The probability of a space appearing as new lacuna in a calculation chart by scale of space

Taking the chart of 10 x 10 m grid for example, all spaces over 20 m generate lacunas, but only some spaces of 10 m to 20 m (about 45.5% statistically) generate new lacunas in this chart, the rest (about 54.5%) generate new lacunas in the chart of 5 x 5 m grid. To be precise, a new lacuna in the chart of 10 x 10 m grid indicates the existence of a space of 10 m to 40 m. Considering the probabilities, we can narrow down the scale of space as a rough estimate. In general, new lacunas in the chart of 20 x 20 m grid reveal the existence of spaces of 30 m to 60 m, and new lacunas in the chart of 10 x 10 m grid reveal that of 15 m to 30 m, and so on, as indicated by four rectangles in Fig. 5. Although exceptions may occur, this conclusion is basically correct.

We can conclude from Fig. 5 that the family genes of all generations are fully indicated by lacunas in the child layer. However, its own genes are either concealed or partially manifested by lacunas in this layer, but are fully planted in its offspring layers. Since lacuna reflects fractal dimension, the philosophy behind the metaphor of family genes is that each fractal segment is an integral part of a continuous whole, and cannot be fully understood if isolated from its context. Additionally, the principle illustrates the recursion and iteration of fractal geometry, and discloses the hidden logic of the box-counting method in measuring fractal dimensions.

Continuous hierarchical scales

The importance of new lacuna is substantiated by comparing the charts of Central Square with a Sierpinski Carpet (fig. 6). The similarity between the two is that new lacunas always emerge as the box size descends at each step. The difference is that the pattern of the Sierpinski Carpet is simple and fixed with infinite iteration,⁶ and Central Square is more irregular and flexible with a finite scale range.

The recurring of new lacunas at each step indicates that the layout is designed with continuous hierarchical scales. For the case of Central Square, it verifies the existence of spaces of 60 m to 30 m, 30 m to 15 m, and 15 m to 7.5 m respectively. According to Tables 1 and 2, these scale ranges generally correspond to the open space of public scale, social scale, and personal scale. This prototype of continuous hierarchical scales is exactly what we are looking for, which best illustrates the fractal nature of green space layout.

Fig. 6. left) A Sierpinski Carpet; right) the box-counting charts of Central Square

In the Sierpinski Carpet, the number of new lacunas emerges at an exponential rate as the box size decreases. In reality, the spatial layout of a green space can hardly match this rate. It seems that a linear rate is more practical for designers. We recommend an inverse proportion between the number of new lacunas and the box size as the minimum requirement. For our cases, if the number of new lacunas in a current layer is equal to or more than twice the number of the preceding layer, we consider the current layer as having adequate new lacunas. In order to find out if sufficient spaces at each hierarchical scale are provided in selected cases, new lacunas are counted respectively (Table 5).

Union Square and May Square do not have any new lacuna at the box range of 40 m to 20 m. Strictly speaking, this indicates the lack of space with the scale over 40 m in both squares. Additionally, it seems that the number of spaces of 15 m to 7.5 m in May Square is out of proportion, and should have been decreased by half. For both squares in China, the spaces of 7.5 m to 3.75 m are extremely insufficient. Meanwhile, Seven-Star Square does not have enough spaces of 15 m to 7.5 m either. At the last column of Table 5, the authors provide an example of desired counts at each hierarchical scale for general reference.

Box range (m)	Union Square San Francisco, USA	May Square Buenos Aires, Argentina	Central Square Qiqihaer, China	Seven-Star Square Jincheng, China	Spatial scales indicated by new lacunas (m)	An example of desired counts
40-20	0 ↓	0 ↓	1	1	60-30	1
20-10	3	5	5	6	30-15	5
10-5	6	18 ↑	11	5 ↓	15-7.5	10
5-2.5	21	18	5 ↓	2 ↓	7.5-3.75	20

Table 5. New lacuna counts at each box range of selected cases (↑ oversupply ↓ insufficient)

It is interesting to see different design orientations between the two Chinese and the North and South American cases. The former generally focus on the spaces of public and social scales, but the spaces of personal scale are neglected. In contrast, the latter pay much more attention to providing sufficient spaces of personal scale, and the public scale seems secondary in design considerations. Although this difference might be caused by cultural and historical conditions or the influence of modernism, the sample size is too small here to make further generalization. What we can learn from these cases is that spaces of all hierarchical scales are equally important in the design layout of an urban square.

We notice that there is a monument or a column at the center of both Union Square and May Square. Such a structure forms a strong focal point and visual interest in the design layout. In fact, the designers did provide a large space of public scale right in the middle of each square. Unfortunately, the space is divided into fragments by the structure. In May Square for example, if the monument component of May Pyramid were absent from the plan, the fractal dimensions would have been improved. The authors have no intention of suggesting the removal of this historical landmark. However, the erection of a monument at the geometric center of an urban square is driven by the aesthetics of Baroque, which showcases the power of human control. It is not surprising to see its conflict with the aesthetics of fractal, which modestly imitates nature. This finding challenges our narrow-minded focus in designing an urban square, which emphasizes the solid structure over void space. While not speaking from the perspective of fractal, Sitte and Stewart [1945] also proposed that statues should always be placed on the edges instead of the center of a square. He quoted Vitruvius that the center should be reserved for gladiators, whose modern counterparts are people and events.

Fractal dimensions

In Table 4, the $D_{(\text{box}, 40-20\text{m})}$ values of Union Square and May Square are equal to 2.0. The direct cause of this extreme result can be observed from the charts of A20 and B20 in Fig. 2. Obviously, new lacunas are absent from the charts. The truth is that both squares have a considerable number of spaces with the scale under 20 m. When the layout of the square is covered by the grid of either 40 m or 20 m, all boxes intersect the image lines, and all are counted. Therefore, the entire image appears as homogeneous plane which corresponds to dimensional value of 2.0 in Euclidean geometry.

Seven-Star Square has a $D_{(\text{box}, 5-2.5\text{m})}$ close to 1.0, which corresponds to a line. Comparing the charts of D5 and D2.5 in Fig. 2, we notice that the pattern of the counted boxes in D2.5 quickly conforms itself to the image outlines. This indicates its spatial feature of line dominates over the feature of plane.

At the box range of 5 m to 2.5 m, Union Square has a fractal dimension of 1.394. This value is much higher than the other cases. Examining the chart of A2.5 in Fig. 2, we can see two shadowed clusters at the bottom of the square, which are counted boxes. If we claim that lacunas in a child layer indicate all the family genes of its higher layers, then the shadowed boxes encompass all the offspring's genes of its lower layers. Meanwhile, the child's own genes are also concealed partially in the shadowed boxes. In the chart of A2.5, the shadowed clusters reveal the existence of small spaces grouped together; most of them with the scale smaller than 3.75 m. In summary, a calculation chart with considerable areas of shadowed boxes generally results in a relatively high fractal dimension.

In Seven-Star Square, a sudden drop occurs between the value of $D_{(\text{box}, 40-20\text{m})}$ and $D_{(\text{box}, 20-10\text{m})}$ from 1.874 to 1.295. Comparing the charts of D20 and D10 in Fig. 2, we can see a large number of border lacunas appear in D10. Actually, the drop of D is half contributed by the border lacunas, which has nothing to do with the fractal distribution of green spaces. The similar problem also plagues Central Square. We find that border lacunas are related to the size and shape of the square, which is a known deficiency in the box-counting method. Consequently, the D values of two Chinese cases are not so dependable due to data distortion.

5 Conclusion

In conventional box-counting analysis, box sizes and calculation charts are utilized merely as mathematical tools. However, this study has discovered their lively meaning in measuring the green space layout of urban squares. First, we find that continuous hierarchical scales determine the fractal properties of the organization of green spaces, and the scales correspond to box sizes. Second, the calculation charts can provide direct visual insights into the fractal distribution of the layout, through lacunas and shadowed boxes, especially new lacunas. Our approach is suitable for describing the fractal interplay of the softscape and hardscape areas, in regard to their location, number, and size. The proposed method allows for quantitative measures, convenient evaluation, and potential feedback to improve the fractal quality of the green space design.

Because the box sizes are developed based on human scales of spatial hierarchy, the manipulation of the size and proportion of the initial image and grid is restricted. Hence, some fractal dimensions may be altered by white space around the source image, and data validity may be questioned. For this reason, we suggest relying on the visual observation

and counting of new lacunas at each box range, instead of D values, to avoid the interference of border lacunas.

The proposed method is new and primitive. Its assumptions and conclusions should be tested and refined with more case studies of different contexts and dimensions. Exploring new sets of hierarchical scales is also expected in future research. In-depth evaluation can be conducted by analyzing the actual size of the space generating new lacuna and its design implications. Furthermore, applying a similar approach to examine other urban design objects with a hierarchical nature can also be an interesting expansion.

Acknowledgements

This work is part of the research project #50978038 funded by National Natural Science Foundation of China (NSFC).

Notes

1. Wu et al. [2007] performed a study on the green space ratio of 58 city squares in 13 Chinese cities. Wu's original data indicate that all city squares have the green space ratio between 40% to 75% except one below 40% and five above 75%. We believe that 40% to 75% is a balanced range to make a place feel midway between a plaza and a park, and use this proportion as a criterion to search for our cases.
2. Applying his one-tenth theory, Ashihara [1981] found that the appropriate size of a large exterior space is somewhere between 57.6 x 144 m (0.8 hm²) and 90 x 180 m (1.6 hm²), which is consistent with Sitte and Stewart's [1945] finding that the typical size of the great squares of old cities is 58 x 142 m (465 x 190 ft).
3. When calculating the size in Table 3, all sidewalks along the perimeter of each urban square are excluded to reduce data distortion in the box-counting approach. Detailed reason is provided in the section "Data – Calculation – Controlling boundary" below.
4. On certain occasions, when newly emerged lacunas are barely connected to the old lacunas in a bottleneck fashion, we have the discretion to decide whether they are new lacunas or not. The decision is based on the design layout of the space generating these lacunas. If the space is relatively enclosed and has its own integrity, these lacunas are considered as new lacunas, instead of grown lacunas.
5. At the box range of $2\delta_1$ to δ_1 , the probability of a space with a scale of A ($2\delta_1 \geq A \geq \delta_1$) to generate a new lacuna is $4x((A-\delta_1)/A)^2$.
6. In the Sierpinski carpet, an initial square is divided into nine subsquares by a 3 x 3 grid, with the center subsquare being removed. Then, the same procedure applies recursively to the remaining eight subsquares to infinity. Fig. 6 illustrates the first two iterations of the Sierpinski carpet.

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