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Research

From Mediaeval Stonecutting to Projective Geometry

Abstract. We tend to think about technology as the application of abstract science to practical problems, but sometimes the inverse is true, as in the case of modern orthogonal projections, which originated empirically in mediaeval workshops and only after a long historical process gave birth to abstract projective geometry. However, this evolution is marked by strong transformations in the media of knowledge transmission, the social groups that control these forms of knowledge, and the very nature of this branch of knowledge. This article charts these transformations, and also serves as an introduction to the eight articles in this special issue of the the NNJ which examime particular issues raised by these historical processes, such as rib vaults by Juan de Álava, the use of ovals at the Escorial, the surbased vault at Arles town hall, staircases in the treatise of Juan de Portor y Castro, axonometric drawing in stonecutting treatises, Frézier's treatise on stereotomy as an antecedent to Monge's Descriptive Geometry, Monge's studies on developable ruled surfaces, and Jules Maillard de la Gournerie's criticisms on Monge's system.

During the twentieth century, a great number of books and articles dealt with the history of linear perspective. In contrast, scholarly works about the historical development of orthographic projections [La Gournerie 1855; Loria 1921; Taton 1954; Sakarovitch 1997] are rather scarce. Such a state of affairs probably reflects an extended misconception; for many architects, engineers and mathematicians, orthogonal projection is a trivial operation, resulting simply from taking away a coordinate from a Cartesian system. However, from the standpoint of the history of science, such a conception is anachronistic and inconsistent with historical evidence; in fact, orthogonal projection appeared as the result of a complex process that lasted centuries and predates Descartes by more than four hundred years.

In particular, the use of orthogonal projection in Classical Antiquity and the Early Middle Ages is, at best, quite limited. The best known Roman plan, the *Forma Urbis Romae* does not involve projection [Sakarovitch 1997: 27-29, 34-36]; it is a bare foundation plan, obtained as a horizontal section of a vast ensemble of buildings; of course, the depiction of objects lying on the same plane cannot be taken as a proof of the use of any kind of projection. As for vertical projections, such as the full-scale working drawing for the pediment of the Pantheon, laid out in the outer terrace of the Mausoleum of Augustus, include only elements that are placed at close vertical planes, such as the cornices and the inner face of the pediment [Haselberger 1983; 1994]; this happens also in miniatures of the Early Middle Age depicting besieged cities or Heavenly Jerusalem. That is, orthographic representations of Antiquity and the Early Middle Ages cannot be considered projections in the architectural sense, since they represent at most a quite shallow portion of space.

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When the draftsmen of Antiquity wanted to convey the impression of space, they resorted to other means of representation, in particular cavalier perspective and "transoblique" projections of the kind used much later by John Hedjuk [Eisenmann 1975]. We should be wary here of another frequent oversimplification. It is usual to remark that orthogonal projection involves no deformation, but this statement can be applied only to figures that are parallel to the projection plane. Actually, orthographic projections mask the figures that are orthogonal to projection planes, such as vertical walls in plans and side façades or flat rooftops in elevations, turning them into lines. Thus, if one wants to represent solids in space, one should resort to linear perspective, almost unknown in Antiquity, or a forerunner of axonometry.

Fig. 1. Ravenna, Mausoleum of Theodoric. Groin vault on the lower story, c. 520

In contrast, orthogonal projections of distant objects were used in the portfolio of Villard de Honnecourt [c. 1225; 2009] and a fair number of Gothic architectural drawings. Villard's plans for the cathedral of Meaux (fol. 15r) or the abbey at Vaucelles (fol. 17r) include both the section of the pillars and the horizontal projection of the vault ribs. Vertical projections of objects in distant planes are also present at Villard's well-known drawings for Reims cathedral (fol. 31v), including the exterior wall at aisle level and the high windows, both in the exterior and interior elevations. The emergence of this method of representation seems to be connected with such constructive practices as the use of the plumb line for the geometric control of Gothic vaults, attested much later by the manuscript of Rodrigo Gil de Hontañón [c. 1540, fols. 24v-25v]. Masons hung plumb lines from rib voussoirs to check that these pieces were lying over their theoretical positions, with the help of a full-size tracing drawn on planks laid on a scaffolding under the vault. The inclusion of different planes in elevations, deriving probably from the use of the square in stone dressing, does not seem to advance so quickly [Rabasa 2007]; such elevations as the ones for the cathedral of Strasbourg, the baptistery of Siena or the one in the "Reims palimpsest" are not much bolder that the tracings for Greek and Roman pediments. By contrast, such plans as the one for a pinnacle in Vienna involve a plethora of horizontal projections of the different stages of the pinnacle, superimposed on one another [Recht 1995, 38-43, 59-66; Branner 1958; Ackerman 1997; Sanabria 1984, 78-82].

In any case, the full development of such methods took more than two centuries. Late mediaeval plans and elevations depart frequently from present-day rules of orthogonal projection; for example, it is fairly usual to represent roses in oblique planes as perfect circles, when they should be depicted as ellipses. During this process, and probably as a result of their efforts to overcome such difficulties, draftsmen or masons found that horizontal and vertical projections can be coordinated in some way [Sanabria 1984: 60-82]. An early fourteenth-century elevation of a bell-tower, probably copied from the Giotto project for the *campanile* at Florence cathedral, as a model for that of the Siena cathedral [Recht 1995: 63-67], includes a precisely constructed octagonal upper story, topped by a spire. Such a drawing cannot be prepared without starting from a plan or resorting to complex calculations regarding the geometry of the octagon, which were quite probably beyond the reach of masons and architects of the period. We must, therefore, surmise that masons or architects in fourteenth-century Tuscany mastered a method for constructing elevations starting from the plan.

Fig. 2. Laon, Cathedral. Sexpartite vault over the nave, c. 1180

However, the implementation of such procedures seems to have been neither quick nor easy. A century later, Mathes Roriczer's *Buchlein der fialen gerechtigkeit* [1486] explains how to obtain the elevation of a pinnacle starting from the plan; rather than using reference lines, as any student of Descriptive Geometry would have done, he draws an axis of the elevation, constructs orthogonals to this axis, and painstakingly transfers measures from the plan to these perpendicular lines in the elevation. Once again, such a procedure bears the mark of stonecutters' full-size tracings, inscribed on floors or walls or drawn on planks. In such tracings, it is not easy to construct parallels, while in contrast, orthogonals can be drawn with relative ease with a large set square [Martínez de Aranda c. 1600: 16; Calvo-López 2000b: II, 82]; this explains the use of the axis of the pinnacle.

The late Middle Ages brought about another classical graphic tool: revolution. Late Gothic German books [Bucher 1972; Recht 1995: 111] or Renaissance texts such as those by Hernán Ruiz [c. 1550], De l'Orme [1567] or Vandelvira [c. 1585] compute the spatial position of keystones in rib vaults through an idiosyncratic procedure. While ribs that are parallel to the vertical projection plane project as circular arcs, oblique ribs should project as ellipses. Masons of the period had no clear understanding of the ellipse; instead they used four-centre ovals for surbased arches, simplifying the tracing procedure, as we shall see below when we discuss Ana López Mozo's paper. Besides, an oval or an ellipse would have been useless for the dressing of the rib voussoir; by contrast, a true-scale representation of the axis of the ribs, obtained by revolution around a vertical axis, is quite useful. In particular, German masters carried this process to levels of high virtuosity in the *Prinzipalbogen* technique, which involves multiple vertical rotations of a series of ribs in order to unfold them over a single vertical plane [Müller 1990; Tomlow 2009].

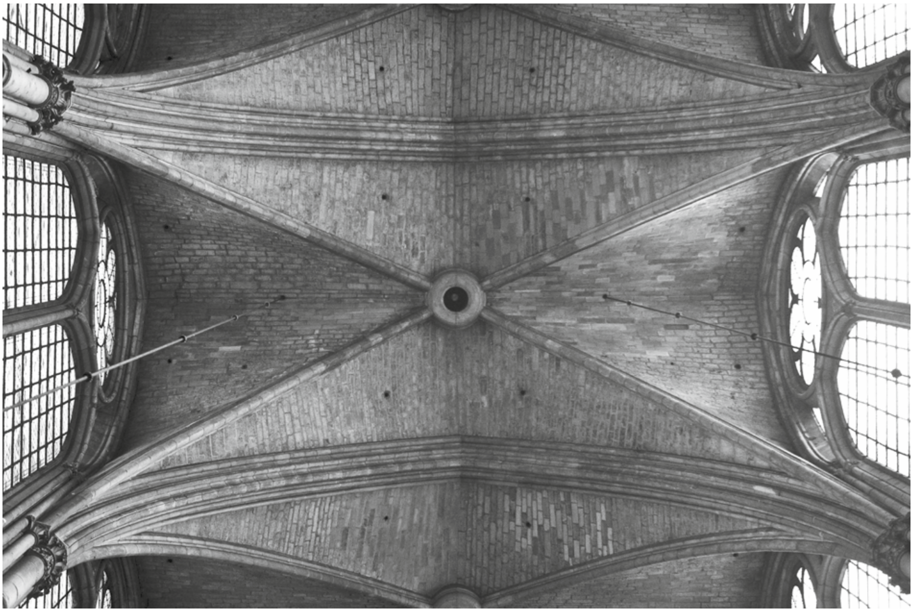

Fig. 3. Reims, Cathedral. Quadripartite vault over the nave, c. 1250

The introduction of the Renaissance in France and Spain posed new problems. Broadly speaking, Gothic construction involves a point and line paradigm, where ribs and their intersections at the keystones provide a supporting network for loosely defined severies. In contrast, Renaisssance vault design stems from surface and volume: the faces of the voussoir are portions of the intrados surface and the bed joints, and must meet tightly with abutting voussoirs. New methods of geometrical control were required to build such architectural members; since vaulting in ashlar masonry is quite unusual in the Italian Renaissance, neither French and Spanish masons nor Italianate artists, who were responsible for much architectural work in the period, had previous experience with this problem.

French and Spanish masons, building on the Gothic tradition in some aspects, but departing from it in a number of essential traits, quickly put together a new set of geometrical tools to tackle these problems. Reference lines connecting plan and elevation appear twice in Dürer [1525/c. 2008: fol 15v, 84v]; in both passages the author mentions masonry lore. Forty years later, in the treatise of Philibert De l'Orme [1567] and the manuscripts of Alonso de Vandelvira [c. 1585] and Ginés Martínez de Aranda [c. 1600], reference lines are used everywhere, providing a method to correlate different projections that is clearly more efficient than Roriczer's axes. This allowed Renaissance masons to carry out easily transpositions of the vertical projection plane and to use orthogonal projections to control the dressing of the voussoirs of the most complex masonry members, generating projecting planes or cylinders by means of the square.

However, such procedure, known as squaring, is not the most efficient dressing method; in fact, it involves waste of both labour and material, as Philibert De l'Orme [1567, 73v] remarked. Thus, Renaissance masons devised a number of alternative methods, sometimes involving rotations around horizontal lines or graphical computations of the angles between voussoir edges and, quite frequently, the use of templates of the voussoir faces. These templates could be rigid, obtained by means of triangulations or rabattements, or flexible, based on a sophisticated system of cone developments that was known as early as 1543 [Palacios 1987; Palacios 1990: 18-20; Rabasa 1996b; Potié 1996; Ruiz de la Rosa 2002] and was fully mastered by De l'Orme's time [1567: 113]. Later on, Girard Desargues [1640; see also Bosse 1643] advocated the use of a suitably sloped plane instead of a horizontal projection plane, although his method was violently contested by traditional masons, led by Jacques Curabelle [1644].

Fig. 4. Amiens. Cathedral. Tierceron vault over the crossing, c. 1250

Such powerful graphical instruments were indispensable in Renaissance and Baroque ashlar construction, since the geometrical challenges posed by the architecture of the period were quite complex. Arches opened in oblique or sloping walls or at the junction of two walls generate elliptical openings; lunette vaults and arches in round walls bring about cylinder intersections; windows opened in domes involve cylinder and sphere intersections; splayed arches replace cylinders with cones, adding further complexity; rear-arches and stairs involve ruled surfaces (see, for example, [Jousse 1642] and [Derand 1643] and the articles in this issue by Giuseppe Fallacara et al., Rocío Carvajal and Snezana Lawrence].

All this led Jean-Baptiste De La Rue [1728], to include at the end of his *Traité de la coupe des pierres* a chapter on cylinder and cone sections, considered in abstract, under the name of *Petit traité de stéréotomie*. This last word, used for the first time by Curabelle in his *querelle* with Desargues, literally means "cutting of solids" and at first stood for the purely geometrical aspects of stonecutting. Ten years later, Amedée-François Frézier transformed De La Rue's short final appendix into a solid scientific foundation for the craft of masonry construction. As Marta Salvatore explains in her contribution to this issue, Frézier devoted the first volume of his *Théorie et pratique de la coupe des pierres et des bois ... ou traité de stéréotomie* [1737-1739] to stereotomy, a new science which encompassed projections, developments, angular measures and surface intersections, leaving the application of these concepts to actual stonecutting or *tomotechnie* for the other two volumes.

Salvatore also remarks that Frézier's *Stéréotomie* acted as a prodrome, or forerunner, to Gaspard Monge's Descriptive Geometry, a discipline that was intended to serve at the same time as a representational language for engineers and as a method for solving geometrical problems both abstract and applied. Monge, a former Professor of the Theory of Stonecutting at the Engineering School of Mézières, built his Descriptive Geometry on several foundations, including artillery and topography; however, double orthogonal projection, mastered by Monge thanks to his stonecutting teaching, played a central role in this new science [Monge 1798; La Gournerie 1855; Taton 1954; Sakarovitch 1992; Sakarovitch 1994a; Sakarovitch 1995; Sakarovitch 1997: 189-282].

Monge's approach poses a number of problems. First, the expression *Géométrie Descriptive* took over the semantic field that had been given to *Stéréotomie* by De La Rue and Frézier; as a result, the word "stereotomy" invaded the area that had been granted to *tomotechnie* and the meaning of the word – but only the *word* – passed quickly from theoretical and geometrical to practical and mechanic. In contrast, the discipline became progressively more abstract, dealing with such problems as the tangency between two surfaces in the *arrière-voussure de Marseille*, a rear-arch that rests on a round and a surbased arch. Nineteenth-century treatises solve this problem neatly using Hachette's theorem; however, as Sakarovitch [1992; 1997: 307-309] and Rabasa [1996a] have pointed out, it is impossible to distinguish "correct" rear-arches executed using this theorem from the ones built by masons in earlier periods, who smoothed the junction between the two surfaces empirically.

The separation of nineteenth-century Descriptive Geometry from actual constructive practices was shown clearly by the neglect of such practical methods as transpositions of projection planes, revolutions and rabattements. Monge had used them here and there in his lessons at the École Normale, but did not mention them explicitly. Later on, Olivier [1843-1844: in particular 18-19] campaigned for the inclusion of such procedures in the canon of Descriptive Geometry, dubbing them "the fundamental problems" of Descriptive Geometry. However, as Enrique Rabasa points out in his article in this issue, La Gournerie [1860: vi-viii; 1874: 154] remarked that changes of projection plane were not as new as Olivier hinted; at the same time, he made clear that the substitution of a sloping plane for the horizontal projection plane is useless in stone construction and other practical applications, since such an operation tampers with the natural position of horizontal and vertical planes.

Another important point that shows Monge's abstract approach is the issue of spatial representation. Gradually, writers on stonecutting had complemented their operational diagrams in double or multiple orthographic projection with more intuitive cavalier or linear perspectives, as Miguel Alonso Rodríguez, Elena Pliego de Andrés and Alberto Sanjurjo Álvarez explain in their contribution to this issue. Monge placed a heavy burden on the shoulders of orthographic projection: it had to fulfill both roles, operative and representative; in fact, he went as far as tearing cavalier perspectives out from the pages of the copy of De La Rue's treatise used at the École Polytechnique, as Rabasa explains in his article. However, orthographic projection is not the best method to represent volume, as I said at the beginning of this paper. This led Monge and the professors at the École Polytechnique to complement orthogonal projections with shades and shadows, building on similar practices at the École de Génie de Mézieres; this in turn led to the study of such elaborate problems as the shadow of the triangular-section screw [Sakarovitch 1997: 85-94].

While nineteen-century stereotomy languished in such trivial issues, Poncelet, a pupil of Monge at the École Polytechnique, was imprisoned in Saratov, in the wake of the Napoleonic Wars. Deprived of books, paper and pencil, he conceived Projective Geometry [1822], an abstract branch of mathematics dealing with the properties of figures that are left unchanged by projections. Thus, while closing its historical cycle, stonecutting lore furnished vital inputs for the creation of an important branch of mathematics; a common thread ties together mediaeval masons, Renaissance architects, enlightened engineers and nineteenth-century mathematicians, leading to the formation of an essential part of present-day science.

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However, this evolution is punctuated with strong mutations. First, throughout the historical span that goes from the Late Middle Ages to the nineteenth century, this body of knowledge was snatched from the hands of one social group by another on several occasions. At the onset of the Renaissance in France and Spain, the control of the subject passed from masons, the heirs of the mediaeval tradition, to "professional architects", in Wilkinson's [1977] terminology. Of course, archetypal, "Albertian" Renaissance architects were not particularly interested in this field, since they focused on conception, rather than execution, and there was no strong tradition of vault construction in ashlar masonry in Italy. However, a new kind of architect emerged during the sixteenth century, in particular in France and Spain: a professional who had learnt architectural theory from Italian treatises, but who had also mastered constructive skills through hands-on practice, along the lines of the mediaeval tradition. Such figures as Philibert De l'Orme, put forward by Wilkinson as the archetype of this new kind of architect, or Hernán Ruiz and Alonso de Vandelvira in Spain, fit exactly into this model.

In any case, such a transition was not peaceful. Philibert De l'Orme remarks that the architect should master construction techniques in order to command the artisans, not the other way round [De l'Orme 1567: fol. 2r, 81r].

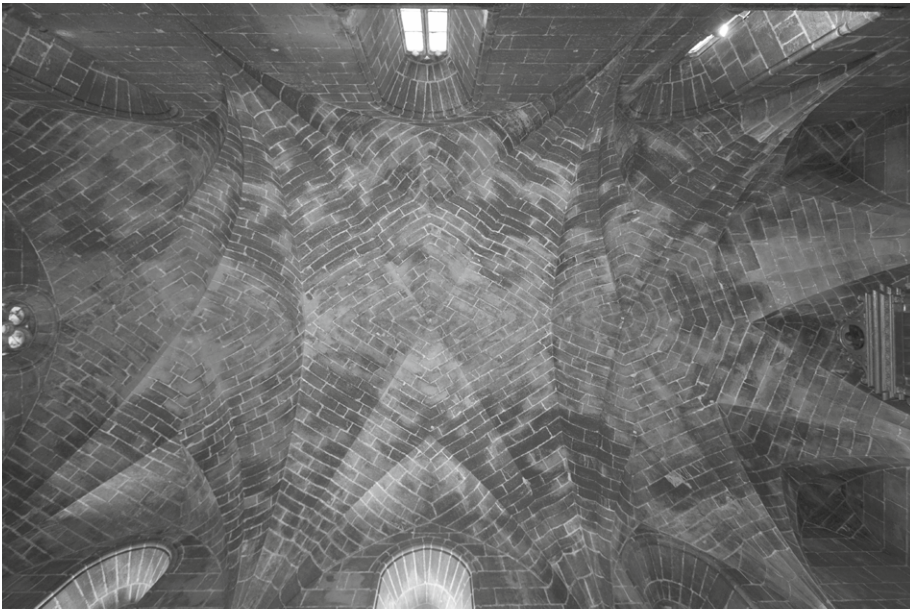

Fig. 5. Valencia, Blackfriars convent. Ribless tierceron vault over the Kings Chapel.
Françesc Baldomar, c. 1450

This is not empty rhetoric; at the *Salle de Bal* at Fontainebleau, he put in his proper place Gilles Le Breton, a mason who had previously had his way against Serlio. Anyhow, De l'Orme was seen as an intruder by both the masons of his period and by such humanists as Ronsard, who mocked him with a pungent epigram [Potié 1996; Pellegrino 1996]; a few decades later, Ginés Martínez de Aranda [c. 1600: iv] complained that "even if craftsmen swear they are knowledgeable in their trade, they are not granted the authority they deserve according to the labour of their studies". During the seventeenth century the field was taken over by the literate, in particular by clerics. With the exception of Mathurin Jousse [1642], a blacksmith, the main treatises of the period were written by the Augustine Fray Laurencio de San Nicolás [1639], the Jesuits François Derand [1643] and Claude-François Milliet de Challes [1674], the Cistercian Juan Caramuel y Lobkowitz [1678], the Theatine Guarino Guarini (published 1737, but written no later than 1683), the Oratorian Tomás Vicente Tosca [1707-1715], or by such a wealthy bourgeois and prominent geometer as Girard Desargues [1640].

After Tosca, the last significant example of clerical involvement in stereotomy, the torch was passed to military engineers, through the hands of a somewhat obscure transition figure, Jean-Baptiste De La Rue [1728]. Very little is known about him; Pérouse de Montclos [1982: 100] makes him an architect, taking into account that he submitted his treatise and a number of papers on quantity surveying to the Academie Royale d'Architecture; however, he also built bridges and designed machines for placing foundation poles. If he was really an architect, his *Petite traité de stéreotomie* and his links with the Academie suggest he was groomed in the scientific architectural tradition of François Blondel and Claude Perrault, both of whom were interested in stereotomy [see Swanson 2003; Gerbino 2002; Gerbino 2005]. Ten years later, Frézier, a military engineer in charge of the Brittany fortifications, took his cue with *La pratique de la coupe des pierres ... ou traité de stéréotomie* [1737-1739]; further on, Gaspard Monge made stereotomy a main subject of the École Polytechnique, as the main application of his *Descriptive Geometry* [1798].

From this moment on, the most significant examples of the huge pile of nineteen-century treatises on stereotomy were written by professors of the École Polytechnique [Sakarovitch 1994a]. It has been remarked that, after the Restoration the school of Monge turned into the school of Laplace, Poisson and Cauchy [Olivier 1852: xi-xii, xv-xviii; Sakarovitch 1997: 321-322]. In any case, this development was anticipated at the end of the clerics' period: both Milliet-Deschalles and Tosca include *lapidum sectione* and *cortes de cantería* as sections in their mathematical treatises, although it is well established that Tosca's treatise, at least, was widely used for the instruction of military engineers in Spain [Capel 1998; Calvo-López 2007].

Fig. 6. Peterborough, Cathedral. Fan vault over the retrochoir. Attr. to John Wastell, c. 1500

Along with these shifts in the identity of the social groups that control this field, the media of knowledge transmission changed accordingly. Carl Barnes [2001] has remarked that Villard's product is not an "Album", a blank book gradually filled with drawings, but rather a haphazard accumulation of sheets from Villard and other masters. As late as the eighteenth century, this approach is frequent in Spanish manuscripts, such as the one ascribed to Juan de Portor y Castro [1708], including sheets from different authors and subjects. Such a conception is still present in the first printed works in the field; it is still not clear if there are two or three booklets from Roriczer [1486; c. 1490], since they are frequently bound together. Later on, these manuals take the form of the Renaissance treatise; however, sixteenth- and early seventeenth-century books still adopt the didactical methods of the "how-to book", giving instructions for each particular piece without furnishing the proofs of the geometrical procedures involved. Such proofs gradually appear from the seventeenth century on, in particular in Frézier [1737-1739]; thus, the treatise evolves into the scientific monograph. Just before the extinction of the species in the nineteenth century, the last of these mutations took place; a heated debate about skew bridges was carried out in books, but also in journal articles [C.L.O. 1837; Spencer 1839; Barlow 1841; La Gournerie 1851; La Gournerie 1853; La Gournerie 1872; see also Becchi 2002].

The very nature of this branch of knowledge was affected by these social and formal shifts: starting as a purely empirical craft, it had taken on the striking form of an *experimental* geometry during the early seventeenth century. Shelby [1972] and Sanabria [1984] have stressed the fact that mediaeval masons' geometry is quite different from the practical geometry of Hugh of Saint Victor [c. 1125]; accordingly, they classify stonecutters' lore as "constructive geometry". Later on, Fray Laurencio de San Nicolás made an striking assertion: "by means of plaster models you will know that practical knowledge is in concordance with speculative knowledge, as I have experimented with my hands before writing" [1639: 70]. Thus, in Fray Laurencio's view, constructive geometry, in parallel with the learned geometry of the period, includes two branches, practical and speculative; in particular, the practical or experimental aspect of this branch of knowledge is carried out with the help of reduced-scale models. In fact, when Alonso de Vandelvira [c. 1585: 22r] is explaining a particularly difficult issue, the concavity or convexity of the templates for an arch opened in a round wall, he is afraid that the reader will be befuddled, and suggests that he prepare a model to verify his assertions empirically.

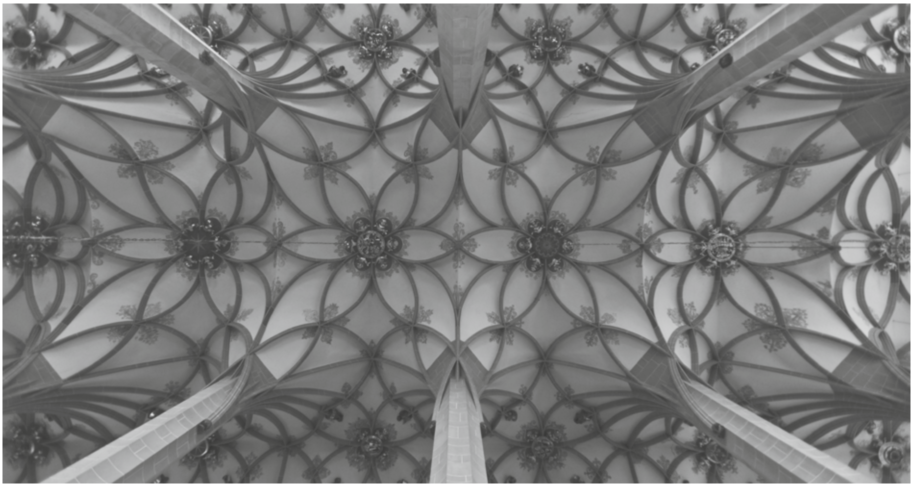

Fig. 7. Annaberg, Church of St. Anne. Net vault over the nave, c. 1500

However, a significant break took place at this moment. The clash between Desargues and Curabelle led to the proposal of a singular kind of duel: two groups of masons, each under the direction of one of the contenders, were to build an oblique arch according to the systems of their leaders; a substantial reward of two hundred *pistoles* was to be given to the winning team. However, Desargues pointed out that the correctness of an arch should not be judged by masons, but rather by geometricians; in other words, for Desargues, abstract geometry lies on a superior plane than empirical validation by means of practical stonecutting. In the end, the duel did not take place, since Curabelle obviously took the opposite stance [Desargues 1648; Sakarovitch 1994b]. This episode signals the final blow to empirical geometry; in our days, experimentation is mandatory in physics and mathematics, but is strictly forbidden in geometry. However, in the field of stereotomy, the resistance to abstract rationalism lasted for a long period; as Rabasa points out in his paper for this issue, La Gournerie [1874] mocked Olivier [1843-1844], saying that if he had read Frézier [1737-1739] he would have known that the transposition of the horizontal projection plane, employed by Desargues [1640; see also Sakarovitch 2009b], is useless in stereotomy and had been rejected by practitioners.

Thus, the long thread that leads from mediaeval stonecutting to projective geometry poses a fair number of open problems for interdisciplinary research, including, for example, orthogonal projections in Gothic architectural drawings and their limitations; the transformations of orthogonal projection at the transition between Gothic and Renaissance; the multiple exchanges between masons, architects, engineers and cartographers in the Early Modern period; the distinctive traits of French treatises as opposed to Spanish manuscripts; the confrontation between empiricism and rationalism in Early Modern stonecutting, exemplified by the Desargues-Curabelle duel; the transition from stonecutting, artillery and other sciences to Descriptive Geometry during the Enlightenment and the French Revolution; and the historical roots of Projective Geometry in stereotomy and other practical disciplines.

In consequence, a Call for Papers was made for a monographic session at the eighth Nexus Conference on Relationships between Architecture and Mathematics, held in Porto, Portugal, 13-15 June 2010, asking for studies on the formal, social and epistemological shifts in stonecutting and stereotomy, in orthogonal projections and architectural drawing, and in descriptive and projective geometry. Of course, such a wide array of topics cannot be covered by a single conference session or even an entire issue of a journal; still, four papers were read in Porto, while other four papers have been selected for inclusion in this issue, focusing on a number of key problems in these subjects.

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To start with, in "Geometric Tools in Juan de Álava's Stonecutting Workshop" José Carlos Palacios and Rafael Martín deal with ribbed vaults in the school of Juan de Álava, a Spanish mason of the early sixteenth century, clearly influenced by late German Gothic, noted for his impeccable continuity between pillars and ribs. After briefly outlining Juan de Álava's career, the author puts forward a clear explanation of the origin and evolution of rib vaults. Groin vaults were frequently used in Roman architecture. They involve two horizontal, orthogonal cylinders of equal radii; the intersection of both cylinders results in two ellipses, the groins that give its name to this vault type. During the Imperial period, such vaults were often built in concrete and therefore did not require ribs. In the High Middle Ages, when commerce in the Mediterranean was risky and pozzolana was in short supply, such vaults were sometimes built in ashlar masonry, solving the union of the cylinders by means of L-shaped voussoirs that cross the intersection line and belong to both cylinders. However, such a solution, which was used, for example, in the lower story of the sixth-century Mausoleum of Theodoric in Ravenna (fig. 1), requires careful control of the dressing process and was not widely used in the Middle Ages; in fact, it was eschewed in the upper story of the Mausoleum for the well-known monolithic vault.

In the Romanesque period the groin vault was still used, in particular in church aisles. However, when building in rubble and mortar, it is not easy to obtain a neat profile at the groins, while the use of L-shaped voussoirs in ashlar masonry was forgotten, as we have seen. Thus, Romanesque builders began using ribs placed at the groins, to mask the junction between both cylinders, either in rubble or hewn masonry. At the same time, masons understood that, since ribs must be placed before the body of the vault, the ensemble could be built dispensing with the heavy formwork and centering required to build a ribless groin vault; in fact, a rib vault can be constructed using substantial centering for the ribs, and only a slight centering for the sections between ribs, known as severies, dispensing at the same time with real formwork.

Fig. 8. Santiago de Compostela, Cathedral. Horizontal ridge rib vault over the cloister galleries. Juan de Álava, 1521-1527

In any case, the construction of the elliptical ribs required to follow the groin profile was quite difficult, since the curvature of an elliptical arc changes constantly. In constructive terms, this requires the use of a different template for each voussoir of the rib. Early Gothic masons radically simplified this problem by raising the profile of the vault and using round diagonal arches at the groins. This required raising the transverse arches as well; however, instead of using raised elliptical arches, builders resorted to the pointed arch for wall and transverse arches. In addition to solving this problem, pointed arches allow for a remarkable simplification and rationalisation of the building process, resembling twentieth-century prefabrication, since all voussoirs of a pointed arch are identical, except the keystone, which can be materialised by a pair of obliquely cut voussoirs. Pushing this idea to the limit, pointed arches of different spans, and even diagonal round arches, can be built using only one kind of voussoir, as a diagram in Villard's portfolio, attributed to the anonymous draftsman known as Hand IV, suggests [Villard de Honnecourt c. 1225: 21v; 2009: 13, 150; Branner 1963; Shelby 1969; Bechmann 1991].

Later, from the mid-thirteenth century on, masons multiplied the ribs, starting with a pair of tiercerons and a lierne at the Amiens crossing (fig. 4); gradually, such networks of ribs grew in complexity, including double tiercerons, ridge ribs and many kinds of straight or curved liernes. Along with its decorative function, such a network of ribs makes it possible to place the stones in the severies directly over the ribs, dispensing with any kind of centering for the panels. However, such complexity required new methods of geometric control. In a square or rectangular quadripartite vault, involving only diagonal ribs and wall or transverse arches, the symmetry of the vault guarantees that both diagonal ribs meet naturally in space. By contrast, in complex Late-Gothic vaulting, a series of sophisticated procedures were required to assure that ribs meet precisely at the keystones, both in plan and elevation.

The main section of Palacios and Martín's contribution to this issue focuses on these problems. Taking as a case study a number of vaults built by Juan de Álava in the cathedrals of Santiago de Compostela and Plasencia, the Blackfriars convent of San Esteban in Salamanca, and the San Marcos priory of the military order of Santiago in León, they examine the methods used by Álava to control the spatial layout of the vaults while keeping the radius of all ribs equal where possible. In particular, they classify Álava's vaults in four types. First, they deal with *rampante llano* or almost horizontal ridge rib vaults, such as the ones at the cloister of the cathedral of Santiago de Compostela (fig. 8). After this, they analyse the nave vault in San Esteban and the vault in the presbytery of Plasencia cathedral, which use horizontal longitudinal ridge ribs, while transverse ridge ribs feature a remarkable slope; the resulting effect resembles a pointed barrel vault. Next, they study the vaults in the cloister of San Esteban, featuring strictly horizontal ridge ribs, akin to the ones in English Gothic.

Palacios and Martín finish their typology with a peculiarly Spanish kind of vault. While in French or English churches it is customary to place the choir at the head of the church, behind the crossing, in Spanish and Southern French cathedrals it is usually placed before the crossing. In small churches with small or no aisles, this location is impractical and the choir is placed usually at the foot of the church, in a mezzanine placed over a surbased vault. The authors deal with a pair of such vaults, located at the *sotocoros*, or under-choirs, of San Marcos in León and San Esteban in Salamanca, remarking that the use of three-centre arches allows Alava to build extremely surbased vaults through the use of powerful geometrical methods.

* * *

Ana López Mozo's paper, "Ovals for Any Given Proportion in Architecture: A Layout Possibly Known in the Sixteenth Century," tackles a significant problem regarding the empirical methods of builders and architects during the Renaissance. As Palacios and Martín stress in their contribution, the three-centre basket-handle arch was used widely along the Late Gothic period. To construct the directrix of such arch is relatively easy; besides, since it involves arcs with only two different radii, it is quite convenient for practical stonecutting: the voussoirs for such an arch can be dressed using only two kinds of false squares with a curved arm, known as *biveaux* or *baiveles*. However, to adjust a three-centre arch to a given span and rise is not so easy. Renaissance masons mastered a number of methods for the tracing of four-centre ovals, such as the four well-known solutions explained by Serlio and other similar procedures included in the manuscripts of Hernán Ruiz and Vandelvira; of course, these methods can be applied both to the layout of oval vaults and the tracing of basket-handle arches [Serlio 1545: 17v; Ruiz c. 1550: 24v; Vandelvira c. 1585: 18r; Huerta 2007]. Each of these procedures is only valid for a given proportion between span and rise. With the onset of the Renaissance, such restrictions began to be felt as unacceptable; the need for a tracing method that made it possible to construct a surbased arch of a given span and height was seen as an urgent issue. The problem was tackled frequently through the use of an affine transformation of the circumference. Although masonry literature furnishes a number of different solutions to this problem, stonecutters essentially started by tracing a circumference whose radius was coincident with the rise of half the span of the arch they were intending to build, and adapted it to the other dimension by means of the affine transformation. Although such constructions furnish points of an ellipse, masons were usually not interested in such abstract concept; in fact, they usually joined these points by means of arcs of a circle passing through three consecutive points [Dürer 1525; Serlio 1545: 13v-15r; L'Orme 1561: 12r-13r; Ruiz c. 1550: 37r-37v; Vandelvira c. 1585: 18v; Martínez de Aranda c. 1600: 1-2].

Fig. 9. Anet. Chateâu, Hemispherical dome over the chapel. Philibert De l'Orme, c. 1550

During the intensive research on the vaults of the Escorial for her Ph.D dissertation [2009], López Mozo found a number of basket-handle arches that do not fit in the solutions furnished by Serlio, Hernán Ruiz and Vandelvira for three-centre half-ovals. After performing a thorough survey of oval-tracing methods in French and Spanish stonecutting literature, she remarks that no procedure for the tracing of an oval of a given proportion is documented in stonecutting literature before the treatise of Tomás Vicente Tosca [1707-1715; the section on stonecutting was published in 1712]. However, according to López Mozo, it is not realistic to suppose that the Escorial basket-handle arches were traced as half-ellipses. Along with the practical considerations about *baiveles* and voussoirs, she remarks that arches such as those in the Basilica narthex include a moulding that is parallel to the edge of the arch, and there is no practical solution for tracing a curve that maintains a constant distance from an ellipse. Thus, the presence in the Escorial of ovals with different proportions, in particular in the main church narthex, leads her to posit the hypothesis of a general tracing for three-center arches of any proportion between span and rise, used by the stonecutters at the Escorial. Such a method is perfectly possible in theory, and is in fact included in Tosca's treatise, although there is no direct evidence of its use in the Renaissance.

At the same time López Mozo points out the presence of a drawing about conic sections in the correspondence of Juan de Herrera, the architect of the Escorial in its later phase; besides, the presence in his personal library of Federigo Commandino's translation of Apollonius's *Book on Conics* [Apollonius Pergaei 1566] shows he was interested in the theory of the conic sections. All this leads us to an interesting issue. Although such abstract mathematical concepts as conic sections and ellipses are apparently of no interest to stonecutters, there are some exceptions. First, Dürer [1525/c. 2008: 16r-18r] furnishes a sensible procedure for the construction of conic sections, thirty years before the publication of Commandino's translation, based on the use of orthographic projections; quite significantly, he mentions explicitly that he is using stonecutters' methods, as I remarked earlier. Although the method is exact in itself, the results are flawed; in particular, the ellipse is not symmetrical about two axes, and rather resembles an ovoid. Some years later, the manuscript of Hernán Ruiz reproduces Dürer's drawings, managing to present a more than acceptable result – especially considering the precision of the drawing instruments of the period – using reference lines, as in his stereotomic drawings.

That is, masons mastered a powerful geometrical tool – orthographic projection – which can be used to solve not only the practical problems of stonecutting, but also to tackle the abstract theorems of learned geometry. The two worlds, practical and theoretical, were beginning to make contact at this period, but still were separate realms; the proof of such state of events is that, as far as we know, neither Hernán Ruiz nor Herrera used the abstract geometry of conics for practical purposes, although it can be argued that both men were grappling for such solutions.

* * *

Fig. 10. Seville, Cathedral. Elliptical vault over the Chapter Hall.
Hernán Ruiz el Joven and Asensio de Maeda, c. 1560-1592

Giuseppe Fallacara, Fiore Resta, Nicoletta Spalucci and Luc Tamboréro have contributed a paper entitled "The Vault of the Hôtel de Ville of Arles" regarding an outstanding example of French stereotomy: the flat vault at the ground story at the Arles Town Hall (fig. 12).

Spanning an area 16 m x 16m with a rise of only 2.4 m, it includes two separate vaults, meeting at a hidden reinforcing arch; each of these vaults is penetrated by a number of lunettes. This remarkable achievement has always been seen as a masterpiece of the art of masonry. At the same time it poses an interesting case study of the power struggles between the corporations of artisans and the Parisian academy. After a number of construction failures and consultations with architects and masons, the direction of the execution was taken over from Dominique Pilleporte, mason, and Jacques Peytret, painter-architect, by Jules Hardouin-Mansart in 1673. However, Hardouin-Mansart had to leave for Béziers and he instructed for a month Peytret on the technical details of the vault construction, leaving him a number of drawings and templates. Thus, the local practitioners played the role of executors of the designs of the architect; although Peytret solved a number of important execution details and contributed a number of personal solutions, all in all he kept the general design of the vault furnished by Mansart, as Richard Etlin [2009] has shown recently.

Luc Tamboréro is a member of one of these corporations of ancient craftsmen, *Les Compagnons du Devoir*; he even uses such a traditional nickname as *La perseveránce d'Arles*. In order to finish their training period and be accepted as *compagnon fini*, the members of these corporations must present a model of an outstanding work, known as *chef-d'oeuvre*. Tamboréro fulfilled this requirement with a model of the Arles vault in stone, at 1:5 scale; this allowed him to study the stereotomic layout of the vault [Tamboréro 2003], stressing its connections with carpentry procedures. However, a number of questions remained to be answered.

Fig. 11. Paris, Abbaye de Val-de-Grace. Groin vaults with oval cross-section on the transverse vaults. François Mansart, Jacques Lemercier, Pierre le Muet and Gabriel Leduc, c. 1650

Tamboréro teamed up with a number of researchers from the Politecnico di Bari led by Giuseppe Fallacara, who have conducted a state-of-the-art survey of the vault, using a 3D laser scanner, presented in their contribution to this issue.

The detailed study of the vault carried out by the Bari team, together with Tamboréro's practical expertise, has allowed them to reach interesting conclusions. For example, they have compared the general dimensions of the ground floor room and the three façades of the Town Hall with traditional Provençal measurements in *cannes* and *pans*, while also hypothesizing that it may correspond to a composition based on the Golden Section. Second, they point out the idiosyncratic layout of some capitals of the columns along the walls, which are reminiscent of similar solutions by Juan Caramuel y Lobkowitz [1678]. Third, they stress the combined use of vertical plane curves, such as the intersections between the corner lunettes and the main vaults, and warped curves, at the junctions between the large vaults and the axial lunettes, as well as at the union between both main vaults.

This seems to indicate that the geometry of the Arles vault stems from two different traditions: the Gothic paradigm, where simple linear curves provide a network over which complex surfaces are laid out, which, according to Tamboréro [2003], reaches seventeenth-century masonry construction through carpentry; and the mainstream Renaissance system, carried on into the Baroque period, which starts from relatively simple surfaces that lead to complex non-planar intersections (see also [Calvo-López 2000a] for the alternating use of both systems on lunettes). A detailed image of the geometry of the vault, carried out starting from the data gathered with the scanner, together with the analysis of the archival documentation, leads Fallacara and his colleagues to hypothesize that the corner lunettes were set at the start of the tracing process; that is, planar curves were used at the beginning of the design and warped curves at the end.

Fig. 12. Arles. Town Hall. Surbased vault on the ground story.
Jules Hardouin-Mansart and Jacques Peytret, 1674

From this starting point, the authors analyse the issue of stonecutting methods used in the construction of the vault. First, they assert that the vault cannot have been dressed by squaring, since in this method there is no need for the detailed instructions given by Hardouin-Mansart to Peytret; besides, in this particular vault the "loss of stones" pointed out by De l'Orme would have been quite significant. Next, they discard also the possibility that the vault was made using templates, since all intrados surfaces are warped, and finally they hypothesize that it may have been done using bevels, known in French as *sauterelles*. Although there is no mention of bevels in the French stonecutting literature of the period (with the exception of Bosse, who was rejected by practising masons), the authors remark that the bevel is present in Diego López de Arenas' [1633] carpentry treatise; it is also quite frequent in Alonso de Vandelvira's stonecutting manuscript [c. 1585]. Of course, the centuries-old connections of Southern France with Spain [Pérouse de Montclos 1982: 200-212] support this theory.

* * *

Rocío Carvajal's paper, "Stairs in the Architecture Notebook of Juan de Portor y Castro: An Insight into Ruled Surfaces," deals with the last significant example of masons' personal notebooks, dated 1708 and prepared by Juan de Portor y Castro; this mason was apprenticed in Santiago de Compostela [Taín 1998, 67-68, 269] and also connected with another important stonecutting center, Granada. Many solutions in Portor's notebook recall the methods of Ginés Martínez de Aranda [c. 1600], and in fact Portor [1708: fol. 22; see also Gómez Martínez 1998: 38] mentions a contest for the post of master mason in Granada cathedral, won by Juan de Aranda Salazar, nephew and disciple of Martínez de Aranda.

Given the bulk of Portor's notebook, including more than one hundred stereotomic problems, Carvajal's contribution focuses on straight stairs. First, the author stresses masons' idiosyncratic methods of representation and analyses their evolution. Some drawings included in Vandelvira's manuscript may seem at first glance ordinary elevations; however, detailed inspection shows that all stair flights are shown in a side view, while according to present-day orthodox technical drawing conventions some flights should be shown in front view. Although Vandelvira's representation strategy may be considered highly unorthodox by current standards, it has clear benefits, allowing the mason to control easily the tracing of the stair and the dressing of its voussoirs. A century later, Portor follows an eclectic trail: he includes an orthodox elevation, constructed by means of reference lines drawn from the plan, but the real stonecutting procedure is based on a disassembled set of side views of the flights, just as Vandelvira's was.

Carvajal's paper focuses on a particular kind of stairs, those with straight-profile flights. Actual built examples of such stairs are quite scarce in Spain; the problem is usually solved by means of curved-profile flights, exemplified by the lost staircase of the Capitole at Toulouse and its Iberian precedents [Pérouse 1982: 168-169; Gómez-Ferrer 2009] and the stairs at the convent of Santa María de la Victoria and the Chancillería in Granada, mentioned by Vandelvira [c. 1585, 59r], which Portor must have known well. In contrast, straight-profile flights are much less usual, since they involve complex problems during execution. In fact, two of the most significant examples, the stair at the Casa de Contratación in Seville and the one at the convent of Santa Catalina in Talavera are considered important stereotomical archetypes, and are cited both by Fray Laurencio de San Nicolás [1639: 118v] and Portor [1708: fol. 15].

Carvajal explains that the intrados of such stair flights, in particular the Seville type, is a hyperbolic paraboloid, since it materialises a surface that passes through two straight lines that are neither parallel nor concurrent; of course the masons did not use such a name, nor did they think in terms of generatrices and directrixes. Portor explains, however, a number of different solutions to the same problem. These stairs can be drawn using longitudinal courses, drawn in parallel with the enclosing walls, which lead to transversal joints between the voussoirs in the same course; however, the problem can be solved the other way round, using transversal courses and longitudinal joints. As for curved profile stairs, transversal joints are usual in the earliest examples, in particular in the Catalonia and Valencia area, but reaching as far as Toulouse in the north and Lorca in the south; by contrast, longitudinal courses are typical of Andalusian examples, such as the Chancillería and Santa María de la Victoria. However, while Vandelvira focuses on longitudinal courses for curved profile stairs, in the Andalusian tradition, he solves straight-profile stairs with transversal courses, and in fact the most significant built examples of straight-profile stairs, the ones at Seville and Talavera, are solved with transversal joints.

In contrast, Portor tries to furnish both solutions for straight-profile stairs, longitudinal and transverse courses. In particular, in the transverse course solution he uses a hyperbolic paraboloid for the flight and a plane for the landing, trying to achieve continuity between the intrados surfaces of the stair. This poses a fairly sophisticated problem: the paraboloid can be laid out using a line in the plane as the edge generatrix, in order to achieve first level continuity between both surfaces; however, since the paraboloid is a warped or non-developable surface, the tangent plane at the edge generatrix is variable, while the tangent plane at the landing is of course the landing plane itself. In any case, as Carvajal remarks, masons were not hindered by such difficulty; they achieved continuity in a practical way, retouching at will the intrados surface along the junction between flight and landing.

Thus, this episode exemplifies an essential trait of stonecutters' approaches to geometrical problems: they used remarkably advanced notions, such as the hyperbolic paraboloid, were fully aware of the difference between developable and warped surfaces, and even tackled such complex problems as geometrical continuity, anticipating results that entered the realm of learned geometry only during the eighteenth century, as we shall see in the papers by Marta Salvatore and Snezana Lawrence. Anyhow, stonemasons' methods remained in the purely empirical realm, since their interest was not focused on abstract problems, but rather on practical execution.

* * *

At first glance, the paper entitled "Graphical Tools for an Epistemological Shift: The Contribution of Protoaxonometrical Drawing to the Development of Stonecutting Treatises," by Miguel Alonso, Elena Pliego and Alberto Sanjurjo, departs from the main thread of this issue, the connections between stereotomy and orthogonal projection, and focuses instead on the forerunners of axonometric drawings included in stonecutting treatises. However, the increasing presence of this particular kind of parallel projections in the treatises and manuscripts of De l'Orme [1567], Vandelvira [c. 1580], Martínez de Aranda [c. 1600], Derand [1643], De La Rue [1728] and Frézier [1737-1739] poses an uncomfortable question: since stonecutting is so closely tied with orthographic projection over the centuries from Villard [c. 1225] to Monge [1795, 1796], as we have seen, why are protoaxonometrics frequent in stonecutting literature? As the authors stress, the purpose of these representations is clearly didactic, in contrast with the strictly operative nature of orthographic diagrams in these treatises and manuscripts, which reproduce stonecutters' full-size tracings literally. In other words, since stonecutters' diagrams are devoid of any representative power, protoaxonometric drawings must supplement them, so that treatises can be understood by a wide, non-mason readership, as a result of an epistemological shift that was turning a closed craft practised by artisans into a branch of mathematics.

The lack of representative efficiency of the orthographic projections included in stonecutting treatises stems from two different issues. The first one is strictly related to masonry: inscribing a full-size tracing on a floor or a wall is slow and tiresome; thus, masons were prone to leave any unnecessary lines out of the tracing. The extreme economy of masonry tracings is mirrored in books and manuscripts; even such printed treatises as Cristóbal de Rojas's *Teoría y práctica de fortificación* [1598: 98v] dispense with arches' outer profiles. The other reason is more abstract and affects any kind of orthographic projection: as I remarked earlier, any plane that is orthogonal to the projection plane, such as wall faces in horizontal projections or floors and side façades in cross-sections, is depicted as a line. This geometrical fact disqualifies plans and elevations, as such, for volumetric representation. That notwithstanding, a number of complements to orthographic projections, such as shades, shadows and line weight, can lend these non-representations some illusion of volume.

After mentioning the Chinese and Renaissance precedents of axonometric drawing, Alonso, Pliego and Sanjurjo chart the evolution of protoaxonometrical representations in stonecutting literature from the first, scattered examples in De l'Orme [1567], Vandelvira [c. 1585] or Martínez de Aranda [c. 1600], to the frequent use of this kind of representation by Derand [1643] and the mastery of the technique by De La Rue [1728]. They stress such points as the early presence of worm's eye views in a scheme about voussoir dressing in Martínez de Aranda [c. 1600], applied afterwards to vault representation. The authors also explain the systematic procedure used by De La Rue to construct parallel projections, taking coordinates from plan and elevation and transferring them to cavalier perspectives. Such representations may be considered legitimate axonometrics, since De La Rue's technique implies the existence of an axis, although it is not explicitly depicted in the plate, in contrast to nineteenth-century drawing manuals.

The authors mention a particular detail that is quite significant for our purposes. De La Rue's treatise [1728] includes a curious feature in order to improve its didactic efficiency: here and there, some fold-out models in lightweight cardboard are included between the sheets of the book. At first sight, this might recall present-day children's pop-up books, but the intention of such fold-out models is not only didactic: one of them shows the errors in the solutions of De l'Orme [1567], Jousse [1642] and Derand [1643] for the sail vault over a square plan. Up to a certain extent, the empirical geometry of Vandelvira [c. 1585] and Curabelle [1644] is still alive here.

In any case, the detailed analysis of De La Rue's axonometric drawings and its didactic nature brings us back to the initial question: can orthographic projection be used as a representational vehicle to convey an intuitive depiction of volume and space? Probably, Gaspard Monge thought so: as mentioned, De La Rue's axonometric drawings were torn out of the copy of De La Rue's treatise used at the École Polytechnique. This violent reaction against two centuries of evolution towards the use of oblique parallel projection might have been provoked, again, by two different reasons.

Fig. 13. Paris, Church of Saint Sulpice. Flat vault with spiral joint.
Gilles-Marie Oppenord, 1714-1745

First, quite probably these protoaxonometries were considered unscientific by Monge's entourage. As far as I know, in Monge's period no one thought of cavalier or military perspective as projections of any kind (see [Rabasa 1999] for this problem in the nineteenth century). If Monge had conceived De La Rue's cavalier and military perspectives as oblique projections, he would quite probably have rejected them as arbitrary, in contrast to the canonical orthogonal projections. Second, following the tradition of the Mézières engineering school, Monge struggled to endow orthographic projections with the representative power they lack in themselves, through the use of shades and shadows [see Sakarovitch 1997: 81-89]. This explains the presence of this subject in a later edition of Monge's *Géométrie descriptive* edited by Brisson [Monge and Brisson 1820], based on Monge's lectures at the École Polytechnique.

* * *

As we have seen, the first volume of Frezier's three-volume *Pratique de la coupe des pierres et des bois ... ou traité de stéréotomie* [1737-1739], focuses on *stéréotomie*; during this period, the term stood for a branch of abstract geometry dealing with the division of solids. Only after thoroughly surveying this field in the first volume, Frézier deals with practical stonecutting or *tomotechnie* in the remaining two volumes. In particular, *stéréotomie* includes *tomomorphie*, the science of the form of sections, that is, the nature of curves resulting from the intersection of solids; *tomographie*, that is, the technique that makes it possible to draw these sections, either on a plane or on a surface; *ichnographie* and *ortographie*, which deal with the orthogonal projections of solids, either on horizontal or vertical planes; *épipedographie*, the science of surface developments; and *goniographie*, which encompasses methods for computing the real size of angles depicted in orthogonal projection.

Thus, Frézier turned the small, ancillary abstract appendix at the end of De La Rue's treatise into a formidable scientific basis for the craft of stonecutting. At the same time, he laid the foundations for the old and new science of Descriptive Geometry, as Marta Salvatore stresses in her article, "Prodromes of Descriptive Geometry in the *Traité de stéréotomie* by Amédée Francois Frézier". In particular, Frézier combined the graphical methods of previous stonecutting treatises, such as those by De l'Orme [1567], Jousse [1642], Derand [1643] or De La Rue [1728], with the analytical work of such mathematicians as Alexis Claude Clairaut [1731], who had studied warped curves using multiple orthogonal projection. This allowed Clairaut to define these curves by reference to two or three planar curves, since any curve of double curvature can be seen as the intersection of two or more cylinders generated from planar directrixes; it is rather striking to find such an abstract concept explained as an operative stonecutting method in Frézier [1737-1739: II,13].

Taking this into account, Salvatore's contribution focuses on *tomomorphie*, that is, the properties of planar or warped curves resulting from the intersection of a pair of solids, in particular quadric surfaces. For example, given an elliptical cylinder, Frézier endeavours to find the plane that will give circular sections of the elliptical cylinder. In order to do so, he rotates the section plane along an axis of the elliptical directrix until it reaches a position where the second axis of the planar section equals the length of the first axis, which is fixed, since it is also acting as rotation axis. Such an operation is fairly easy in an elliptical cylinder, and in fact it finds precedents in stonecutting literature [Martínez de Aranda c. 1600, 16; Blondel 1673; Gerbino 2005]. However, the same procedure cannot be applied to an elliptical cone, since the centres of the circular sections do not lie on the axis of the cone; the solution to this problem would not be found until Theodore Olivier's period [1843-1844].

Another interesting issue in Frézier's *Tomomorphie* is found in his original contributions to the treatment of warped curves created by the intersection of quadric surfaces. While Clairaut applied analytical geometry to such curves, Frezier dealt with them in terms of synthetic geometry; such an approach was quite unusual at the time. Anyhow, this leads Frézier to explain in detail the nature of three different warped curves, which he dubs *cicloïmbre*, *ellipsïmbre* and *ellipsoidïmbre*, making use of an empirical metaphor: he suggests that the reader imagine a circle drawn in the spine of a book prepared for binding. If the bookbinder plays with the pages, sliding one over the other, the circle transforms itself into a warped curve, called *cicloïmbre* by Frézier, which is in fact the parallel projection of a circle upon a cylindrical surface; parallel projections of ellipses on quartic surfaces give *ellipsïmbres* as a result, while central projections of ellipses over cylinders furnish *ellipsoïmbres*.

Of course, Frézier did not neglect the application of geometry to actual stonecutting. In fact, he deals with practical matters in two-thirds of the treatise; however, in contrast to the empirical approach of older manuals, he always tackles construction problems from the standpoint of abstract geometry, sternly criticising the methods of previous authors, as Salvatore remarks. Thus, Desargues's somewhat flawed attempt is brought back to life: the abstract science of space takes precedence over the practical methods of stonecutters, which is understood rather as a technical application of pure science. However, Frézier's position is more balanced than that of Desargues; while Desargues simply does not take into account the work of De l'Orme, Frézier shows a thorough knowledge of his predecessors, if only to confute their solutions. Thus, Frézier's work stands at a crossroads: on the one hand, it is a rare example of a sound work on synthetic geometry in a century dominated by algebraic methods; on the other hand, he makes the decisive move in order to push construction procedures from the empirical field to the realm of rational geometry, tying a knot between abstract science and practical activity. All these factors make Frézier's treatise a forerunner of Monge's Descriptive Geometry, not only at the epistemological level, but also at the social one, since the sound application of science to the practices of craftsmen was to have the effect of shifting the status of such branch of knowledge upward, and potentially that of the craftsmen themselves, as Monge saw clearly.

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Snezana Lawrence has contributed a paper about a quite interesting subject: "Developable Surfaces: Their History and Application." When dealing with the connections between mathematics and the real, everyday world, we tend to think in terms of "applied science": abstract notions are conceived in the minds of scientists and are applied to practical issues only afterwards. The notion of ruled surface, as many other geometrical concepts, seems to arise from the inverse process; it appears in stonecutting treatises and manuscripts as early as the sixteenth century, but it only reaches the realm of learned mathematics around 1750, through the work of Monge [1769] and Euler [1772]. Euler's interest in surface developments may have been connected with cartography, and thus lies outside of the scope of our subject, but it is clear that Monge was attracted to the topic not only through topography, but also as a result of his experience as professor of stonecutting [Sakarovitch 1992].

Fit. 14. Paris, Church of Sainte Geneviève, now Panthéon. Lunettes, dome and oval vault. Jean-Baptiste Rondelet and Maximilien Brébion, 1780-1790

Lawrence stresses in her article that Monge used developable surfaces to solve a problem that is connected both with artillery and topography: to compute the height a wall should have in order to protect a military position from enemy fire. In the mid-eighteenth century, the solution of this important issue involved long and complex calculations; Monge found a quick way to solve the problem on site by employing a plane that is tangent to the topographical surface and a developable tangential surface. Such a successful solution showcased Monge's outstanding mathematical capabilities and allowed him to rise within the ranks of the Military Engineering School at Mézières, breaking the social barriers due to by his humble origins. However, Monge was prohibited from publishing his discoveries, which were classified as military secrets; he could only explain his findings about developable surfaces in an abstract fashion, leaving out any possible military use of the concept.

As a result of his prestige in Mézières, Monge was granted the position of Professor of the Theory of Stonecutting. He must have felt a keen interest in developable surfaces and their application to stereotomy, since stonecutters had been involved with these surfaces for centuries, as we have seen, using either rigid or flexible templates to control the shapes of voussoir faces. The use of such instruments by masons shows a typical shift from the empirical to the analytical understanding of the problem. In earlier masonry literature, the developable surface is conceived as one where two finitely distant generatrices are parallel or concurrent; by contrast, in the modern notion of developable surface, the generatrices that need to be parallel or concurrent are placed at an infinitely small distance.

Of course, Monge assumes this last position, linking the issue with differential geometry. However, the roots of his interest in this subject lie in practical stonecutting, in particular in the construction of exactly fitting templates for intrados faces and bed joints, as Monge himself states in his *Descriptive Geometry.* This allowed him to manipulate such objects with remarkable spatial insight, obtaining powerful results about developable surfaces; in fact, Lawrence stresses that Monge's visual approach led him to superior results than the analytical methods used by Euler, who was blind by that time.

At the same time, Monge's abstract approach to the issue made him go to great lengths to confirm that the bed joints of an ellipsoidal vault with three different axes were developable surfaces. In order to do this, and at the same time keep this surfaces orthogonal to the ellipsoidal surface, he studied in depth the notion of curvature lines, which led him to adopt sloping joints for the ellipsoidal vault, departing from usual practice. Furthermore, he was enraptured by his discovery; he suggested that the roof of the main hall of the National Assembly should be constructed in the shape of an ellipsoidal vault with three different axes, featuring a network of joints along curvature lines. He stressed that the result would be as beautiful, but much less arbitrary, than the ribs on Gothic churches [Monge 1796]; in some way, he was trying to convert the room into a Temple of Reason, conceived in a mathematical way.

As Sakarovitch [1997: 309-313; 2009a] and Rabasa [2000: 296-302] have remarked, such an approach stretches the conventions of practical stonecutting beyond reasonable limits, since orthogonality between intrados and bed joints, on the one hand, and developable templates on the other, are advisable but not mandatory. In fact, no ellipsoidal vault with bed joints following curvature lines has ever been built, as far as I know. I will come back to the issue of Monge's abstract conceptions against practical procedures in the next section, when dealing with Rabasa's article about the opposition between Monge and La Gournerie, but for now it is worthwhile to recall a striking episode that marks the far end in the evolution of the subject from empirical constructive practice to abstract mathematical concepts: Lawrence points out that around 1900, Lebesgue dealt with developable surfaces that are not ruled, although they cannot be used in the real world in which architecture takes place.

* * *

Last but certainly not least, in "La Gournerie versus Monge," Enrique Rabasa presents a significant episode that shows that the battle between mathematical abstraction and practical construction that had started with Desarges and Curabelle was still blazing in the mid-nineteen century. Olivier, a prominent heir of the Monge tradition, wanted to "enlarge the conquered realm" of Descriptive Geometry, in Gino Loria's [1921] words, and stressed the utility of such geometrical procedures as transpositions of the projection planes. Although present in Monge's original lessons in Descriptive Geometry, such changes were not stressed either by Monge or by his disciples, such as Hachette [1822] and Leroy [1834]. Olivier, in contrast, focused on them as the "fundamental problems" of Descriptive Geometry, using both transpositions of vertical and horizontal planes. Jules Maillard de La Gournerie, who succeeded Leroy at the École Polytechnique and Olivier in the Conservatoire National des Arts et Métiers, remarked that such methods were not new at all, since they were used by Desargues; he approved of changes of vertical projection plane, but remarked that substituting a sloping projection plane for horizontal one is useless in stereotomy, since there is no advantage in tampering with the natural position of the voussoirs inside the masonry. Thus, La Gournerie fired against Monge and Desargues by elevation, passing over Olivier's position; he stood against an abstract conception of geometry that makes the vertical and horizontal planes equal, neglecting the presence of gravity; that is, La Gournerie implicitly approved the secular lore of masons against the abstract conceptions of Monge's school.

Fig. 15. Dresden, Zwinger. Coffered vault, dome over pendentives, skew arch and octagonal vault on the ground access to the courtyard. Gottfried Semper, 1847-1855

Rabasa also emphasizes La Gournerie's reaction against Monge's abstract conceptions of such issues as axonometry and linear perspective, stressing that Monge and his assistants tore the drawings in cavalier perspective out of De La Rue's treatise, fearing that they would lure the students to an *arbitraire* method of spatial representation, which had a long and venerable history dating back to Classical antiquity. By contrast, La Gournerie shows a remarkable interest on axonometry, although he focuses on the abstract, isometric flavor of axonometry invented, for once, by an Englishman, Farish [1822], rather on the simplistic but effective cavalier perspective, which he dubs "rapid perspective", while also stressing that France was less advanced in this field than Britain and Germany.

The issue of linear perspective in La Gournerie is more complex. Monge had proposed as a perspective method the old projection-and-intersection method of Piero della Francesca [c. 1480]; although Monge [1820] explains the vanishing point method as an *abregé* or simplified procedure, it is quite clear that any departure from exact, abstract, central projection would be for him even more arbitrary and unscientific than axonometry. Against such a stance, La Gournerie starts by mocking the notion of an objective, scientific perspective, remarking that, in order to achieve an optimal trompe-l'oeil effect, perspectives should be seen closing one eye, from a particular point that is not marked at all. As a consequence, he underwrites all the licenses painters have taken along the centuries, drawing as circles spheres placed near the corners of the picture, which should be depicted as ellipses according to scientific central projection, or eschewing the fat columns that should be placed at the sides of the drawing, and so on. Although such stances may at first seem contradictory to his initial statement, it is crucial to understand that they are driven by skepticism, since La Gournerie rejects the concept of objective, rational perspective, and instead considers the result acceptable if pleasing to the eye of the observer; Curabelle would have wholeheartedly approved such a stance.

Another pungent issue addressed by Rabasa's article is the social extraction of both contenders. Monge was the son of a small merchant and could not hold any important post in the army of the Ancien Régime, which was open only to the nobility and, arbitrarily enough, to the sons of glass merchants. He climbed the ladder to the highest positions solely on the basis of personal scientific merit, but he must have suffered much scorn along the way. It is no wonder that when the Revolution came, he was a fervent Jacobin. By contrast, La Gournerie was the son of a nobleman who had fought with the Royalists at the Vendée and inherited from him the title of Viscount although, if we are to believe Laussedat [1897], he never used it along with his name. At first sight, a nobleman raising the flag of artisans' methods strikes us as a romantic, nostalgic, and somewhat contradictory figure. However, La Gournerie was not imprisoned in an ivory tower: like De l'Orme, Rojas and Frézier, he had been trained as an engineer in Brittany, working at the Ileaux de Bréhat lighthouse, placed on a rock that was only accessible during low tides. Thus, the key to La Gournerie's position seems to be skepticism, which fits his political conservatism well; like Principe Salina in Giuseppe Tomasi di Lampedusa's novel *Il Gattopardo,* he knew the Ancien Régime was lost forever, but he mistrusted the abstract rationalism of the Enlightenment, both in politics and in the "graphic arts".

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