Alexandra Paio

Departamento de
Arquitectura e Urbanismo
ISCTE-IUL
Lisbon University Institute
Av. das Forças Armadas
1649-026 Lisbon PORTUGAL
alexandra.paio@iscte.pt

Benamy Turkienicz

SimmLab - Laboratory for the
Simulation and Modelling in
Architecture and Urbanism
Universidade Federal do Rio
Grande do Sul
Rua Sarmento Leite 320
Porto Alegre, RS, BRAZIL
CEP 90020-150
benamy.turkienicz@gmail.com

Keywords: shape grammar;
Portuguese urban design;
proportion; symmetry

Research

An Urban Grammar Study: A Geometric Method for Generating Planimetric Proportional and Symmetrical Systems

Presented as a poster at Nexus 2010: Relationships Between
Architecture and Mathematics, Porto, 13-15 June 2010.

Abstract. This paper is part of ongoing research to explore (a) descriptive and generative potentials of shape grammars to unveil the order of urban morphological complexity underlying Portuguese treatises, and (b) the knowledge embedded in Portuguese urban cartographic representations produced from the sixteenth to the eighteenth centuries for purposes of architecture and military engineering. The shape grammar made it possible to infer the grammatical rules of composition based only on the written descriptions provided by the Portuguese treatises, as well as to decode the geometry of ideal urban plans and built cities. This paper suggests that these historical cities, both built and ideal were based on a structured knowledge-based process from where it is possible not only to retrieve a generative parametric urban grammar but also to construct a computational model, UrbanGENE, capable of iteratively generating Portuguese planimetric proportional and symmetrical urban systems. The paper suggests that the knowledge and computational tools achieved could be successfully deployed in the teaching and learning of architectural history.

1 Introduction

In the last decade, urban history has been very important for the dissemination of knowledge about Portuguese urban design. These studies have developed hypotheses of morphological continuity in Portuguese urban design from the sixteenth to the eighteenth centuries based on shared composition rules and procedures. Architects and historians have demonstrated that Portuguese urban design evolved from planned, regular and erudite principles of precision and standardization [Araujo 1992, 2000; Horta-Correia 1997; Carita 1999; Teixeira & Valla 1999]. This is in contrast to previous studies in which where the lack of planning rules was considered a pervasive characteristic of Portuguese urban design [Holanda 1936; Azevedo 1956; Smith 1955; Santos 1968].

Along with these two contradictory assumptions, there are other issues to be tackled. One of these is related to the description of the process mediating the theoretical knowledge held by urban makers and, at the same time, the representation of this knowledge in urban historical cartography produced from the sixteenth to eighteenth centuries.

To describe this process, we have to infer the genetic and generative principles of Portuguese urban design, as well as understand the form-making logic. The understanding of relationships between morphological elements, categories and classes of

basic relations, supports the recognition of the city as a structured order with repeated composition rules.

The historical documentation suggests that Pythagorean-Euclidian geometry is a crucial ingredient for understanding Portuguese urban design-thinking and urban design-making. To unveil this genesis, a descriptive method, shape grammar [Stiny & Gips 1972; Stiny, 1980] has been adopted. As a method, shape grammar supports the analysis of the form-making logic and has proved to be powerful in the shape analysis, description, interpretation, classification, evaluation and generation of design languages [Stiny & Mitchell 1978, 1980; Knight 1981, 1989; Buelinckx 1993; Brown 1997; Turkienicz et al, 2005, 2006, 2007; Colakoglu 2005] and to provide a gateway to define parametric urban shape grammars [Beirão & Duarte 2005; Steino 2005; Duarte et al, 2006].

The present shape grammar study permitted the analytical/descriptive morphological decomposition of the sixteenth to the eighteenth century's historical cartography of Portuguese urban designs and has helped to unveil the geometric principles that characterize Portuguese treatises and practice in this period. As it is known, shape rules [Stiny 1980] have the potential to bridge the gap between traditional drawing techniques and modern computational methods of urban design. Following this potential, it is possible to devise a geometric method for the automatic generation of planimetric proportional and symmetrical urban systems. Taking this idea as departure point, the goal of this research is to develop a parametric urban grammar computational model to generate new Portuguese towns during the sixteenth to eighteenth centuries. The model should enable the grammar's user to interpret, manipulate and generate simultaneously various geometric constructions as representations of Portuguese historical urban designs.

This paper has four sections. The first section shows, briefly, some traditional views based on iconographic evidence and historical data. The second section describes the process of inferring and structuring the urban grammar. The third, introduces a geometric method for generating Portuguese planimetric proportional and symmetrical urban systems. The final section discusses the partial results of the research.

2 Knowledge-based Portuguese urban design

The complexity of Portuguese urban design from the sixteenth to eighteenth centuries has been the subject of different authors [Ribeiro 1962; Horta-Correia 1997; Lamas 1993], under different analytical methods. Bodies of knowledge as diverse as economics, sociology, history, geography and architecture have been used to interpret Portuguese urban form of the cited period from different point of views.

Three basic concepts have emerged from the explanation and description of the historical Portuguese planned city. The first concept arises from the ideal city models present in Portuguese treatises. Several scholars have argued, interpreting the knowledge present in available treatises, that the existing theory and the pragmatism from Portuguese urban makers constituted predominant factors behind the planning and the construction of the Portuguese city [Chicó 1956; Carita 1999; Bueno 2003; Horta-Correia 1997; Moreira 1982; Araujo 1992, 1998; Murteira 1999; Paio 2007, 2009].

The second concept identifies morphological elements and its correspondent structural relationships [Lamas 1993; Rossa 2002; Teixeira & Valla 1999; Reis Filho 1962; Rhoden 1999]. For these authors, the identification of discrete morphological elements (street, square, institutional buildings, blocks, lots) constitutes the basis for the

analysis, description and the understanding of categories of basic relations and, additionally, for the understanding of the city as the result of a structured order with repeated rules.

The third and final concept begins with the influence of the topographic features [Ribeiro 1962; Gaspar 1969; Salgueiro 1992; Amaral 1987; Teixeira & Valla 1999] in the genesis of Portuguese cities. The city is then described according to the ability of the Portuguese urban makers to adjust or to superimpose a planed urban geometric structure onto the chosen site to build the city.

These three approaches advocate that Portuguese urban morphological order combines theory and the urban maker's knowledge: knowledge not only to design, but to actually build cities. Authors argue that it is precisely this know-how that allowed Portuguese urban makers to successfully manipulate geometric structures and morphological relationships to promote the city as a structured system of forms and spaces. The awareness of geometry has given birth to pragmatic procedures, enabling the Portuguese effectively respond effectively to a wide variety of conceptual problems and to use urban design to efficiently control their territory according to a set of pre-defined objectives.

Authors were also concerned about linking particular solutions to individuals or excessively fragmenting or classifying urban designs according to historical periods [Teixeira 2008]. Portuguese urban design studies [Rossa et al. 2006] have tried to structure the whole picture without pinpointing general compositional rules and procedures. As Duarte put it "the traditional studies in urban and architecture history only describe a style without saying how it is possible to generate new projects in that style" [2007:20]. In other words, Portuguese urban history lacks of a description based on genetic and generative principles explaining a plurality of design solutions that characterized the sixteenth to eighteenth centuries. This paper represents an effort to overcome this problem by using the descriptive and generative morphological decompositions of shape grammar in order to unveil genotypic similarities beyond the apparent diversity present in the *Corpus* (fig. 1, fig. 2).

3 The Portuguese urban morphogenetic genesis

... the principles of practical geometry are necessary for Military Architecture intelligence and *Fabrica* ... who was not good as a geometer and as an arithmetic, will fail without knowing his mistakes and only will discover them when the object is built ...

Pimentel, 1680 [1993: 3].

In order to analyze the design knowledge embedded in Portuguese cities, we defined a corpus of thirty Portuguese treatises (practical geometry, military architecture, military engineering, etc.), used in military classes of Portuguese schools from the sixteenth to eighteenth centuries (fig. 1), and a corpus of seventy-five samples of Portuguese urban historical cartography (fig. 2).

The analysis of the treatises has furnished, in two steps, the primary genetic and generative principles of Portuguese urban design. The first step disclosed a Pythagorean-Euclidean geometric genesis, the geometric operations necessary to build the syntax of the Portuguese urban plans. The second step has revealed the Vitruvian urban genesis present in the treatises, the theoretical urban models.

Fig. 1. The corpus of thirty Portuguese treatises (practical geometry, military architecture, military engineering, etc), used in military classes of the Portuguese's schools from the sixteenth to the eighteenth centuries

Fig. 2. The corpus of seventy-five samples of Portuguese urban historical cartography from the sixteenth to the eighteenth centuries

The knowledge embedded in the treatises confirmed the ability of Portuguese urban planners to configure geometric proportional structures along with invariant features of many Portuguese planimetric urban systems. These invariances have made it possible to develop a parametric shape grammar capable of capturing the genetics of Portuguese urban plans from the sixteenth to the eighteenth centuries.

3.1. Pythagorean-Euclidean geometric genesis. The analysis of Portuguese treatises and compendiums helps us to identify the structure of the rationale of urban planning taught to apprentices and, additionally, to decode the urban design logic supported by this rationale. In the treatises present a particular way of conceptualizing and systematizing knowledge.

Geometry, in its operative dimension, becomes a preparatory science, a mental discipline, an abstract discourse enabling models to be built from visual reasoning processes. In this sense, form theory is mathematical in nature and comprises a rich vocabulary, which unambiguously describes geometric forms (such as points, lines, circles, triangles, squares, pentagons, etc.) and geometric qualities (such as symmetry, proportion, hierarchy, rhythm, etc.). As Alberti said “design a firm and graceful preordering of the lines and angles, conceived in the mind and contrived by an ingenious artist” [Alberti 1955: 1-2].

Formal axiomatic Euclidean geometry, that is, geometric constructions based on the compass and an unmarked straightedge, was fundamental for Portuguese urban planners. These were essentials to the abstract mental process and visual reasoning in the construction of formal models, exact representation and construction on the site.

3.2. Vitruvian genesis. Portuguese treatises reveal a strong component of classical, Vitruvian origins [Carvalho 2000; Bueno 2003; Paio 2007]. Pimentel’s remarkable Portuguese treatise on fortification design and city planning, *Método Lusitânico de Desenhar as fortificações das Praças Regulares e Irregulares* (1680), shows the complex relationship between abstract, formal considerations and pragmatic military issues. This military compendium reveals not only a confident assimilation of Vitruvian planning ideas but also many other European military treatises from the seventh century [Carvalho 2000] (fig. 3). Pimentel described in detail the important components, relations, and dimensions of the ideal city, but didn’t provide any illustration of it. To “formalize” the written description, a formal language – shape grammar – has been used, making possible a graphic interpretation of the text, which can then be used to generate of some urban plans (fig. 4, fig. 5).

Based on Pythagorean-Euclidian structure it was possible to deduce a geometric method of design starting from a center (the point of the compass), a direction (essential to define symmetry), and a limit (a circle), which expresses domain and periphery [Critchlow 2001] (fig. 4). Alberti commented, “... Platforms should all terminate within a circle, and indeed from a circle is the best way of deducing them” [Alberti 1955: 194]. A circle that is divided in three equal parts defines the generative shape of the planimetric system. After using a geometric transformation operation, rotation, it was possible to establish the beginning of two triangle-based proportions, one based on the hexagon and the other based on the root-three rectangle. These two triangle-based proportional structures would ensure a repetition of ratios and shapes throughout the design [Kappraff 2002; Scholfield 1958], as well as adapt to different scales and hierarchies. These two options will generate two basic planimetric proportional and symmetrical systems, a grid whereby urban elements such as streets, squares, urban blocks, and main buildings emerged (fig. 4, fig. 5).

Fig. 3. Tree diagram to illustrate the theoretical urban models present in European military treatises from the seventh century

Fig. 4. Pimentel's ideal city (TPF_i10_1)

3.3. Geometric and generative principles. Through the analytical decoding of the corpus' grammar, it was possible to progressively depict geometric and topological attributes and to establish two sets of categories and classes [Mitchell 1998]. These were geometric-configurational (GC) and topological-functional (TF) (Table 1). Each category was divided into four classes. The geometric-configurational (GC) category is strongly Euclidean knowledge-based and is constituted by seven different elements: Position (GC_ps), Direction (GC_drc), Limit (GC_lim), Generative Shape (GC_gene.shape), Diagonal (GC_dng), Proportion (GC_prop) and Symmetry (GC_sym) [Paio and Turkienicz 2009]. The topological-functional (TF) category is related to urban elements: Streets (TF_s); Urban Blocks (TF_ub); Main Buildings (TF_mb) and Squares (TF_sq) (Table 2). The two categories are related so as to make it possible to associate the category's four classes to each other. Classes of the geometric-configurational (GC) category were used to generate symmetrical and proportional layouts where the geometric structure has been deployed to generate streets and urban blocks. The definition of the point (geometric center) and the direction (vertical and horizontal axes) permitted operations such as rotation and, further on, the positioning of the church (ch), town hall (th), or military buildings (mlt) and the location of one or more squares (sq).

Fig. 5. Tree diagram to illustrate the generation of some Portuguese urban plans from the Pimentel's ideal city

Table 2. Geometric-configurational (GC) and topological-functional (TF) categories

Table 1. Theoretical framework

4 An urban grammar. A geometric method for generating Portuguese planimetric proportionate and symmetrical systems.

The proposed historical urban design grammar is a parametric shape grammar defined in the algebras U12 V12 W12. The generation of an urban derivation develops over four stages: two to generate the planimetric proportional and symmetrical system and two to generate a planimetric urban system (Table 2). These stages are: (1) define position, direction, limit and generative shape; (2) define the rules of proportion and symmetry; (3) define streets and urban blocks; (4) insert main buildings and squares. Each stage has a specific set of shape-rule schemata. These stages are sequential, using a step-by-step process, enabling the generation of plans similar to the Portuguese urban plans from the sixteenth to the eighteenth centuries. Transitions between sequential rules application and stages are controlled by the Shape-Grammar Meta-Language (SGMT) descriptors [Liew 2004]. SGMTs established an alternative method to write grammars for design, introducing seven descriptors for shape grammar language. These explicitly determine the sequence in which a set of rules is applied. At the same time, the method restricts the rule's application through a filtering process using the context to guide the rule matching process [Liew 2004]. The descriptors modify the conditions (rule selection, drawing state, matching conditions and application method) surrounding the process of applying a rule in shape grammar.

4.1. Stages of the urban grammar developing process. The urban grammar presented is still limited in that it only partially describes the range of elements of Portuguese urban plans. Specifically, it represents urban planimetric proportional and symmetrical systems, orthogonal streets (s), urban blocks (ub), main buildings (mb) (churches, town halls, priest's house, governor's house, director's house and military buildings) and squares (sq).

Other elements, such as lots, houses and topographical features, have been omitted. Since the grammar is designed as a sequence of stages, the omitted elements can be inserted into the grammar at a later stage. In order to demonstrate the urban grammar developed so far, an example of the process of deriving a Portuguese urban plan is illustrated and described (fig. 10).

4.1.1. Initial Shape. The initial shape for the Portuguese urban grammar consists of a single labeled point O, with a pair of coordinates (0,0) (rule: GC_0_position) (fig. 6, fig. 10).

4.1.2. Stage 1: Define position, direction, limit and generative shape. In the first stage of the urban grammar, the user defines the basic geometric generative principles of the planimetric proportional and symmetrical system. This stage is composed of seven rules, that permit the user to manipulate dimensional parameters of the Limit and the Generative Shape. The first step inserts the geometric axis1 (x, y) to symmetry and proportion and indicates that the user is in stage 1 (rule: GC_1_direction). The application of rule GC_2_lim_wrkout inserts a circle, which will be divided into equal parts defining the generative shape. In this step the user has parameters to define the circle radius (r_limwrk) and limits to work (lim_wrk): work inside (wrksnd), work out and inside (wrkoutsnd) and work outside (wrkout). These parameters are associated with various labels: Fortification (frt) or Public Square (sq). In this case the parameter used was labeled as sq, which will authorize the user only to work out (lim_wrkout) of the generative shape (gene.shape). The final step inserts the generative shape, in this case an equilateral triangle labeled (sq) (rule: GC_3_gene.shape_trg) (fig. 6, fig. 10).

4.1.3. Stage 2: Define proportion and symmetry. In the second stage of the urban grammar, the user manipulates shape rules to create a symmetric and proportional structure, based on a generative shape selected in stage 1. The sequential steps emulate the similitude to the operation with the compass and the straightedge. In order to clarify the following steps, an example shows set operations (steps) that can be manipulated by the user: Step 1. The application of the rule chg_axis_stage2 carries the derivation into stage 2; Step 2. Define the entire axis (sym_axis_trg1; sym_axis_trg2) of the equilateral triangle symmetry (rule: GC_4_symmetry_axis_trg); Step 3. The user has to decide the type of symmetry: Bi-axial or Multi-axial. In this case user applies the Multi-axial that generates a radial configuration. Step 4. Decide the triangle-based proportions needed to work: hexagon or root-three rectangle. The derivation shows that the user repeated the hexagon based on a variation of a geometric progression $1:\sqrt{3}$ (tPG:3). The user has several labeled and parameterized rules to choose from. The user has chosen the repetition-division V (3) $dhxg2a = 0.10hgx2$ and $dhxg3a2 = 0.35 hgx3$; Step 5. The regulating structure is defined by lines originating in the triangle-based proportion and results in a particular grid whereby streets (s), blocks (ub) and main buildings (mb) will emerge in the following stage. The result is a consistent geometric structure that regulates the planning of the Portuguese urban layouts at many scales (fig. 7, fig. 10).

Fig. 6. Rules to define position (GC_0_position), direction (GC_1_direction), limit (GC_2_lim_wrk_out) and generative shape (GC_3_gene_shape_trg)

Fig. 7. Rules to define symmetry (GC_4_symmetry_axis_trg) and proportion (GC_5_proportion_hexagon)

Fig. 8. Rules to define stage (CHG_axis_streets), streets (tf_3_s1) and urban blocks (TF_3_urbanblocks; TF_4_urbanblocks)

Fig. 9. Example of rules for locating main buildings (tf_7_mb_ch_5; tf_8_mb_th_4; tf_8_mbrqt_join; tf_10_mb_HprstandHgvn_1) and squares (rule tf_12_sq_ch_10)

DEFINE A PLANIMETRIC PROPORTIONATE AND SYMMETRICAL SYSTEM

STAGE 1: POSITION (GC_ps), DIRECTION (GC_drc), LIMIT (GC_lim) AND GENERATIVE SHAPE (GC_fgr_grd)

STAGE 2: PROPORTION (GC_prop) AND SYMMETRY (GC_sym)

DEFINE A PLANIMETRIC URBAN SYSTEM

STAGE 3: STREETS (TF_st) AND URBAN BLOCKS (TF_ub)

STAGE 4: MAIN BUILDINGS (TF_mb) AND SQUARES (TF_sq)

Fig. 10. The derivation of a Portuguese urban design Sample (RC_pln8)

4.1.4.Stage 3: Insert streets and urban blocks. In the third stage, urban generative elements emerge according to set-rules as follows: Step 1. The application of rule *chg_axis_streets* leads the derivation into stage 3; Step 2. The application of the rule *tf_3_s1* defines first structure streets (s); Step 3. The same rule application defines the final structure of streets. After the user has dimensional parameters to apply ($60 > Wms > 70$ palmos) and hierarchy (if applicable); Step 4. The recursive application of rule *TF_3_urbanblocks* define urban blocks (ub); Step 5. Erase the streets structure and geometric structure no longer necessary (rule: *TF_4_urbanblocks*). This final step will highlight the urban blocks grid and the main axis, a fundamental step towards the final stage of the grammar (fig. 8, fig. 10).

4.1.5. **Stage 4: Locate main buildings and squares.** The application of rule *chg_axis_mb* leads the derivation into stage 4. The user has, again, set-rules options to locate the main buildings (mb): churches (ch), town hall's (th), governor's house (gvr_house), priest's house (prt_house) and military buildings (mlt) and set-rules to locate the squares (sq) with various formal shapes. All set of shape rules transmit the relation between the main axis and the main buildings (mb) and these with the squares. In this final stage the user has to define main axis to insert the main buildings (mb). The application of rule *LT_4_mb_maxis* defines the *axisx_street* as main axis. The derivation shows one way to insert the church (ch) and town hall (th) (rules *tf_7_mb_ch_5*; *tf_8_mb_th_4*; *tf_8_mbrqt_join*). The application of the rule *tf_10_mb_HprstandHgvn_1* inserts the governor's house and priest's house. After the rule *tf_12_sq_ch_10* locate the associated church square (sq). This stage has set of *OPRTV_rules* to join, to divide or to stretch that permit manipulate de urban blocks (ub). The final step of the stage and the grammar is the erasure of the main axis (fig. 9, fig. 10).

5 Discussion

This paper suggested a possible application of shape grammar techniques to urban design history. A structural and structured knowledge-based research was essential to recovering the elements of genetic foundations and represent these as constituents of visual reasoning processes and, additionally, to develop a generative parametric Portuguese urban grammar interpreter, UrbanGENE. The results showed that shape grammars can constitute a valuable basis for the understanding of the process of Portuguese urban design. The resulting planimetric system ends up corresponding to a basic compositional procedure supporting the implementation of designs and working as a scale. Once the planimetric system is established, the urban plan incorporates proportions and symmetry in a surprisingly balanced way.

From the preliminary results obtained, it can be said that it is possible to create a useful tool to be used in the learning process of historical urban design. The descriptive and generative character of this tool will allow the user to both interpret and simulate new designs simultaneously based on theoretical knowledge, as well as to manipulate and generate various Portuguese colonial parametric urban design solutions. Since the major part of pre-design stages in urban design are devoted to the study of precedents as strategy for producing new designs, the experience may offer a new incentive to improve the teaching of geometry and urban design precedents in the schools of architecture and urbanism.

References

- ALBERTI L. B. 1955. *Ten Books on Architecture*. J. Leoni, trans., J. Rykvert, ed. London: Alec Tiranti
- AMARAL, I. 1987. Cidades Coloniais Portuguesas. Notas preliminares para uma geografia histórica. *Povos e Culturas* 2: 193-214.
- ARAUJO, R. 1992. *As Cidades da Amazônia no século XVIII. Belém, Macapá e Mazagão*. Porto: FAUP.
- . 2000. *A Urbanização do Mato Grosso no século XVIII. Discurso e Método*. Ph.D. dissertation, Lisboa: UNL.FCSH.
- AZEVEDO, A. 1956. Vilas e cidades do Brasil Colonial: ensaio de geografia urbana. *Boletim da Faculdade de Filosofia e Ciências e Letras*, n. 208.
- BEIRÃO, J and J. DUARTE. 2005. Urban Grammars: Towards Flexible Urban Design. *Proceedings of the eCAADe Conference*, Lisbon: 491 - 500.
- . 2007. Urban design with patterns and shape rules. *Proceedings of the 2nd International Seminar on New Town Simulation*: 1-11.

- BROWN, K. 1997. Grammatical Design. IEEE Expert, Special issue on *Artificial Intelligence in Design*: 27-33.
- BUELINCKX, H. 1993. Wren's language of City church designs: a formal generative classification. *Environment and Planning B: Planning and Design* 20: 645-676.
- BUENO, B. 2003. *Desenho e designio: O Brasil dos engenheiros militares (1500-1822)*. Ph.D. dissertation, São Paulo: Faculdade de Arquitectura e Urbanismo da Universidade de São Paulo.
- CARITA, H. 1999. *Lisboa Manuelina e a formação de modelos urbanísticos da época moderna (1495-1521)*. Lisbon: Livros Horizonte.
- CARVALHO, J. 2000. Luís Serrão Pimentel, O Método Lusitano e a fortificação. MSc. Dissertation, Lisbon: Universidade Lusíada.
- CHICÓ, M. 1956. A "cidade ideal" do Renascimento e as cidades portuguesas da Índia. *Garcia de Orta*: 319-328.
- CRICLOW, K. 2001. *Islamic Patterns. An Analytical and Cosmological Approach*. London: Thames & Hudson.
- COLAKOGLU, B. 2005. Design by grammar: an interpretation and generation of vernacular hayat houses in contemporary context. *Environment and Planning B: Planning and Design* 32: 141-149.
- DELSON, R. 1979. *New towns for Colonial Brazil. Spatial and Social Planning of the Eighteenth century*. Dellplain Monographs in Latin American Studies 2. Syracuse, NY: Syracuse University.
- DUARTE, J.P., G. DUCCLA-SOARES, L. CALDAS and J. ROCHA. 2006. An Urban Grammar and for the Medina Marrakech. *Design Computing and Cognition '06*: 483-502.
- DUARTE, J. 2007. *Personalizar a habitação em série: Uma Gramática Discursiva para as Casas da MALAGUEIRA do Siza*. Lisboa: FCG-FCT.
- GASPAR, J. 1969. A morfologia urbana de padrão geométrico na Idade Média. *Finisterra* IV, 8: 198-215.
- HOLANDA, S. 1992. *Raízes do Brasil* (1936). Rio de Janeiro: José Olympio.
- HORTA-CORREIA, J. 1997. *Vila Real de Santo António. Urbanismo e Poder na Política Pombalina*. Porto: FAUP.
- Kappraff, J. 2002. *Beyond Measure*. Singapore: World Scientific.
- Kruger, M. and Silva, C.2000. A Gramática da Forma das igrejas cistercienses. *Actas do Colóquio Cister: Espaços, Territórios, Paisagens*. Lisboa: 309-335.
- KNIGHT, T. 1981. Languages of design: from Known to new. *Environment and Planning B: Planning and Design* 8: 213-238.
- . 1989. Mughul Gardens Revisited. *Environment and Planning B: Planning and Design* 17: 73-84.
- LAMAS, J. 1993. *Morfologia Urbana e Desenho da Cidade*. FCG-JNICT.
- LIEW, H. 2004. SGML: A Meta-Language for Shape Grammar. Ph.D. dissertation, Massachusetts Institute of Technology.
- MARCH, L. 1976. *The Architecture of Form*. Cambridge: Cambridge University Press.
- MARCH, L. and G. STINY. 1985. *Spatial systems in architecture and design: some history and logic*. Environment and Planning B: Planning and Design 12: 31-53.
- MARSHALL, S. 2009. *Cities, Design & Evolution*. London: Routledge.
- MAYER, R. 2003. *A linguagem de Oscar Niemeyer*. MSc. Dissertation. Federal University of Rio Grande do Sul.
- MITCHELL, W. J. 1998. *The Logic of Architecture*. Cambridge, MA: MIT Press.
- MOREIRA, R. 1982. *Um tratado português de arquitectura do século XVI*. Master dissertation, Faculdade de Ciências Sociais e Humanas da Universidade Nova de Lisboa.
- MURTEIRA, H. 1999. *Lisboa da Restauração às Luzes*. Lisboa: Ed. Presença.
- OXMAN, R. 1997. Design by re-representation: a model of visual reasoning in design. *Design Studies* 18: 329-347.
- PAIO, A. 2007. Knowledge of geometrical design and composition in a Portuguese approach to urban layout. *Proceedings of the ISUF XIV International Seminar on Urban Form, XIV*: 212 - 232.
- PAIO, A. 2009. Geometry, the Measure of the World. *Nexus Network Journal* 11, 1: 63-76.

- PAIO, A. and B. TURKIENICZ. 2009. A generative urban grammar for Portuguese colonial cities, during the sixteenth to eighteenth centuries. Towards a tool for urban design. *Proceedings of the eCAADe27*: 585-592.
- . 2010. A Grammar for Portuguese Historical Urban Design. *Proceedings of the eCAADe28. Future Cities*: 349- 358.
- PIMENTEL, L.S. 1993. *Método Lusitano de Desenhar as Fortificações das Praças Regulares e Irregulares* (1680). Lisboa: Direcção do serviço de fortificações e obras do exército.
- REIS FILHO, N. 1962. *Contribuição ao Estudo da Evolução Urbana do Brasil (1500/1720)*. São Paulo: Pioneira Editora.
- RHODEN, L. 1999. *Urbanismo no Rio Grande do Sul: Origens e Evolução*. Porto Alegre: Edipucrs.
- RIBEIRO, O. 1962. *Portugal, o Mediterrâneo e o Atlântico*. Coimbra: Coimbra Editora.
- ROSSA, W, ARAUJO, R., and CARITA, H. 1998. *Actas do Colóquio Internacional. Universo Urbanístico Português 1415-1822*. Lisboa: Comissão Nacional para as Comemorações dos Descobrimentos Portugueses.
- ROSSA, W. 2002. *A Urbe e o Traço, Uma década de estudos sobre o urbanismo português*. Coimbra: Almedina.
- ROSSA, W. and Luisa TRINDADE. 2006. Problems and precedents of the 'Portuguese city': understanding medieval urbanism and it's morphology. *Murphy. Journal of architecture history and theory* 1: 70-109.
- SALGUEIRO, T. 1992. *A Cidade em Portugal. Uma Geografia Urbana*. Porto: Afrontamento.
- SANTOS, P. 1968. Formação de cidades no Brasil colonial. *Actas do V Colóquio Internacional de Estudos Luso-Brasileiros*: 5-114.
- SCHOLFIELD, P. H. 1958. *The Theory of Proportion in Architecture*. Cambridge: Cambridge University Press.
- SMITH, R. 1955. Urbanismo colonial no Brasil. *Anais do II Colóquio Internacional de Estudos Luso-Brasileiros*: 1-42.
- STEINØ, N. and N. E. VEIRUM. 2005. A Parametric Approach to Urban Design, Digital Design: The Quest for New Paradigms. *Proceedings of the eCAADe Conference*, Lisboa: 585 - 592.
- STINY, G. 1978. *Algorithmic aesthetics. Computer Models for Criticism and Design in Arts*. Berkeley: University of California Press.
- . 1980. Introduction to shape and shape grammars. *Environment and Planning B: Planning and Design* 7: 343-351.
- STINY, G. and J. GIPS. 1972. Shape grammars and the generative specification of painting and sculpture. *Information Processing*: 1460-1465.
- STINY, G. and W. J. MITCHELL. 1978. The Palladian Grammar. *Environment and Planning B: Planning and Design* 5: 5-18.
- . 1980. The grammar of paradise: on the generation of Mughal gardens. *Environment and Planning B: Planning and Design* 7: 209-226.
- TEIXEIRA, C. 1996. Portuguese Colonial Settlements of the 15th-18th Centuries. Vernacular and Erudite Models of Urban Structure. *La Ville Européenne comme Modèle*, Paris :15-26.
- TEIXEIRA, M. and M. VALLA. 1999. *O Urbanismo Português, séculos XVIII-XVIII. Portugal Brasil*. Lisboa: Livros Horizonte.
- TEIXEIRA, M. 2008. O Estado da Arte da Investigação Urbana em Portugal: A investigação dos núcleos urbanos de origem Portuguesa no mundo. *Revistas Ceurban* 8: 1-21.
- TURKIENICZ, B. and R. MAYER. 2005. Cognitive Processes, Styles and Grammars. The cognitive approach applied to Niemeyer's free form. *Proceedings of the eCAADe23*: 529-536.
- . 2006. Oscar Niemeyer Curves Lines: Few Words Many Sentences. Pp. 135-148 in *Nexus VI Architecture and Mathematics*, Sylvie Duvernoy and Orietta Pedemonte, eds. Torino: Kim Williams Books, 2006.
- TURKIENICZ, B., E. WESTPHAL, and M. CAVALHEIRO. 2007. The Affordance between Context and Function as a means to stimulate the cognitive process in architectural design. *Shaping Design Teaching*. Aalborg: Aalborg University.

About the authors

Alexandra Paio is an architect with a Master's degree in urban design from ISCTE-IUL, Lisbon, Portugal. Currently she is a teaching assistant lecturing in Geometry and Architectural Composition at the Department of Architecture and Urbanism, ISCTE-IUL. Since 2007 she has been working on her Ph.D. thesis, entitled "Geometric models of representation in Portuguese urban design during the XV-XVIII centuries," at the same university. She has published papers in national and international journals and conferences about the relationship between geometry, architecture and urban design. She has participated in several workshops including the Shape Grammar Workshop by George Stiny (MIT), Faculdade de Arquitectura-UTL, Lisbon. She is a researcher in ADETTI/ISCTE-IUL.

Benamy Turkienicz holds a degree in architecture (Universidade Federal do Rio Grande do Sul, 1976), an MA in Urban Design (Joint Center for Urban Design - Oxford Polytechnic, UK, 1979), a MSc (Bartlett School of Architecture/University College London, UK, 1981), and a Ph.D. in Urbanism (Chalmers University of Technology, Sweden, 1982). He is a full professor at the UFRGS, teaching in the Department of Architecture of the Faculty of Architecture, the Program for Post-Graduation and Research in Architecture and the Program for Post-Graduation and Research in Design. He is an invited professor at the Master in Urbanism Erasmus Mundus and coordinates a Master Course in Planning Design at the Cape Verde University (cooperation protocol between CAPES/UniCV). He is co-author of several software applications to support architectural and urban design, including CityZoom. Turkienicz has delivered several lectures and conferences in universities in Brazil and abroad, including Nexus 2006 in Genoa, Italy. Currently, he is responsible for Post Graduate evaluation and accreditation in Architecture, Urbanism and Design at Capes, Ministry of Education in Brazil. Since 1995 he has been responsible for the SimmLab – Laboratory for the Simulation and Modeling in Architecture and Urbanism, Universidade Federal do Rio Grande do Sul, Brazil and for the NTU – Nucleo de Tecnologia Urbana/UFRGS, a joint research group which support municipalities and consulting firms in the planning and assessment of settlements and large scale architectural and urban projects.