Fractal Geometry in Muslim Cities: How Succession Law Shaped Morphology

Abstract. Islamic succession law has deeply affected the urban fabric of Muslim cities. Properties were subdivided according to a refined and elaborate system of shares that were prescribed by jurists. Successive iterations of subdivision over the course of decades or even centuries gave a fractal character to the cities and thus becomes the main source of their complexity. Most property was subject to successive subdivisions until minimum but functional parts were arrived at. However, this fragmentation should not be understood in isolation from other mechanisms which sometimes resulted in a reunification of the fragments, and thus established a dynamic equilibrium in the urban fabric. The paper will present the mechanisms of subdivision according to laws of succession, and illustrate by hypothetical example their direct impact on the urban morphology, as a means of understanding the complexity of old Muslim cities.

Introduction

The urban fabric of Muslim cities has been an enigmatic issue for scholars. Early studies of orientalists concluded that Muslim cities were lacking order due to the absence of institutions and the excess of freedom of action that characterized their communities. The absence of straight lines, the interlocking of buildings and encroachments on public realm were seen as evidence of this disorder. According to this view, this was mostly due to the laxness of the authorities and officials, who were mainly interested in harems and holy wars [Sauvaget 1934; Letourneau 1985; Raymond 1994].

It was only in the middle of the twentieth century that a new generation of scholars changed the paradigm of research, finding in Islamic law and institutions determinant factors for this complexity [Barbier 1900; Brunschwig 1947; Hakim 1988]. Recent studies showed that the irregularity of geometry and urban fabric in these cities is not a sign of disorder and chaos in the pejorative sense, but rather a sign of high order and complexity [Akbar 1988; Ben-Hamouche 2004, 2009a, 2009b].

Chaos and fractal theory provides an efficient tool for analyzing the urban fabric of old Muslim cities. It presents a new instrument for understanding the complexity of these cities, and thus displaces the Cartesian approach and Euclidean geometry that have long dominated the studies of urban fabric and morphology.

Succession law intervenes at turning points in life, that is, death and birth. Social interactions, regeneration and shifts of property provide an opportunity for the application of the theory of chaos and fractals to the domain of social science. However, this present study will be limited to the causal relationships between succession as a field of Islamic law and the morphology of urban fabric in Muslim cities. Such relationships are believed to be a major factor behind complexity in Muslim cities. A symbiosis of arithmetic for the calculation of heirs shares and geometry for the subdivision and

partition of assets has generated the urban morphology of these cities; without taking this into consideration, any analysis of their urban fabric would be a mere *tatonnement*.

The aims of this study are to contribute to the advancement of the theory of chaos and fractals through an enlargement of its scope, and to provide a new instrument for academicians and professionals for understanding and analyzing the urban fabric of old Muslim cities. Such a contribution would lead to a shift in academic fields of research, as well as education regarding the study of these cities. On a practical level, reconsidering succession law in urban regulation and municipal systems would have direct implications for the urban management and preservation of these cities.

Fractals: back to the roots

The terms “fractals” and “chaos” are organically interwoven, and together form a single, integrated theory. They are usually used concomitantly [Devaney 1990; Briggs 1992; Szemplinska-Stupnicka 2003]. They are concerned with irregular forms and complex objects that show self-similarity at each scale of magnification when they undergo a certain process of iteration. However, they sometimes have distinct connotations and applications in different fields of knowledge. Consequently, the present study is limited to the geometric aspect of the theory.

Literally, the term fractal, coined by Benoit Mandelbrot in 1975, was derived from the Latin *fractus*, meaning “broken” or “fractured”. Mathematicians developed fractal dimension D to measure self-similarity through scale invariance. It is defined as the power relation between the number of pieces and the reduction factor: $A = 1/(s)^D$. Fractal dimension D could thus be defined as: $D = \log(a)/\log(1/s)$ [Bechhoefer and Bovill 1994: 27].

In this study fractal geometry does not entirely adhere to this conventional meaning, nor to the instruments and measurements rules as used in many previous studies [Bovill 1996; Bechhoefer and Appleby 1997; Eglash 1999; Ostwald 2001]. However, it shares the same philosophy and roots of the term. In technical terms, it is related to the arithmetic meaning of “fraction”, that is, a quotient number that represents part of a whole. Applied to succession law, it reflects the endless process of fragmentation of inherited properties into smaller shares and the resulting effect on the urban fabric. This (re)definition is not necessarily opposed to the essence of the theory, but broadens its scope. Bypassing its present technical aspect, which is imprisoning it in mathematical formulae, would nurture its philosophy and help its diffusion.

The scope, source and nature of Islamic succession law

Islamic succession law is believed to lie at the basis of the birth and development of both algebra and geometry in Islamic civilization. Al-Khawarizmi (780-850 A.G.), from whose name the term “algorithm” was derived, introduced algebra to modern mathematics through examples from Islamic inheritance laws [Huma 1997]. In geometry, techniques and tools for the calculation of areas on the ground and shares of heirs were also made available to jurists and dividers [Cami-Efendi 2000; Ca’fer Efendi 1987].

Measuring systems, calculation methods and geometric techniques were developed in Islamic civilization as a result of the pressing need to partition and calculate irregular shapes. In other words, developments in mathematics, and in particular in algebra, were a direct consequence of Islamic laws of inheritance [Cami Efendi 2000: 117-118, 123-

130]. A body of hypothetical science involving tricks and cunning was also developed as an entertaining branch of mathematics [Cami Efendi 2000: 131-134].

Socially, Islamic succession law is deeply rooted in Muslim communities. To colonial authorities during the nineteenth century, it was one of the key themes for understanding social structure and managing administrative affairs of the local societies. Many manuscripts were translated into English and French, the languages of the two major ex-colonial powers, in order to ease the mission of their administration. *Hedaya* is an example of a translated manual that was concerned with generalities in Islamic law [Hamilton 1989], while the *Al-Sarajjiyya* manual [Rumsey and Jones 1869], on which most jurists of the Hanafi school of thought in the Indian continent relied for succession law, was one of the famous documents that was edited by the British lawyer Almaric Rumsey (1825-1899).¹

Islamic inheritance law is considered to be a branch of jurisdiction. It is a sole prerogative of a judge, *Qadi*. An unpublished manuscript that goes back to the eighteenth century, recently annotated and edited by the present author, on which part of this study is based, is *Riyadh al qassimeen*, the “Garden of the Dividers” [Ben-Hamouche 2000]. The author, Cami Efendi, who lived between 1649 and 1723, was an Ottoman judge who used it as a manual for the daily affairs of partition and urban disputes in courts of Cairo and Bursa, Turkey.

Methods of partitions and rules applied to movable objects as well as real property. According to jurists, any property that belonged to a deceased person is considered to be an inheritable asset to be subdivided, no matter how small or large. However, many objects such as wells, mills, wind-towers, ovens, and small pools were considered to be undividable. This was also the case of objects that would become useless or not functional after partition [Cami Efendi 2000: 51-60]. Passages, and roads, courtyards and staircases were subject to this same principle.

A partitioning of usufruct, called *Muhayat*, was applied in the case of properties that could not be subdivided, due either to the large number of heirs, or the small size, or the nature of the object, as will be seen in sections below. Another alternative to physical partitioning of properties would simply be to sell the inherited property and subdivide the income according to the same rules of subdivision.

As a principle, it is recommended that after subdivision each partner enjoys his share separately, with his own road and drainage. Otherwise, the partners must agree to enjoy their respective shares with all rights and immunities in common. If such conditions are not met, the partition must be annulled and made anew [Hamilton 1989: 572].

After subdivision, and according to the number of the heirs, a rule of thumb is applied to determine the share of each heir for the sake of impartiality and pleasure. However, the partition is acceptable if the heirs reach a consent without the intervention of authorities. Once the partition is determined and each member receives his share, it cannot be cancelled.

Islamic succession law is often criticised because of its position regarding women, who are seen as disfavored. A sister, for instance, is given half the share of her brother from the estate of her deceased father. This is, however, only one social case among others. In some circumstances, she receives a share equal to that of a man, as in the case of the brothers and sisters of a deceased man. In some others, she has more than the males' shares [Rumsey and Jones 1869: 6]. In other cases she gets a share of a subscribed

heir while her male counterpart is entirely excluded from the partition. On a large scale, the traditional social system was organized so that males had financial duties towards children, wives and parents. Females were entirely exempted [Yekini 2008: 56-60; Abdul-Jamal 2005].

Another criticism made of the system is its impact on the agricultural land, claiming that Islamic succession law leads to its being fragmented and thus inefficient in terms of productivity and economy of scale [UN-Habitat 2005: 9]. This vision, mostly based on the pro-capitalist standpoint, seems to attribute the weak economy of less developed countries to land tenure through an unfair comparison between rich and poor countries, and disregards other less visible problems underlying economic weakness. It also contests the distributive philosophy behind the succession law, which aims at socializing means and capitals and preventing monopoly and the accumulation of wealth in the hands of a few rich people. Redistribution of material resources, including real estate, is the core of the modern economy and a key concept for social justice, offering equal opportunities to access wealth and means to wealth [De Soto 2000].

The mathematics of inheritance

Fig. 1. An example of a family structure of a deceased man and the share of the heirs according to their kinship

Islamic law of succession is fundamentally a combination of arithmetic, sociology and geometry. Shares are well defined according to kinship and family relationships. On the basis of a few fundamental texts, jurists of different schools of Islamic law developed a complex system of inheritance that is largely applied in most Muslim countries today.

First the heirs are determined, then complex methods of calculations are applied in theory. At a third stage, assets such as houses, stores and lands are subject to a process of subdivision. The sections below will outline the system, enumerate the heirs, and explain the way the assets of a deceased person are subdivided among them (fig. 1).

The heirs apparent

The first class of heirs apparent or sharers, called *As'hab al furudh*, comprises the circle of relatives closest to the deceased defined explicitly in the fundamental texts [Rumsey and Jones 1869: 4-7]. The shares, depending on their family status, are defined in terms of fractions to which they are entitled. Fractions are multiples of either the $1/2$ or the $1/3$. The ones stated in the *Quran* are the $1/2$, the $1/4$, the $1/8$, the $1/3$, the $2/3$ and the $1/6$ (Table 1). Heirs entitled to these shares are twelve persons; four males and eight females. In the first subclass we find the father and the true grandfather or other male ancestor, however far up in the paternal line, and the brother by the same mother and husband. In the second subclass we find the wife, the daughter and the son's daughter or other female descendant however far down, the sister by one father and mother, the sister by the father's side, and the sister by the mother's side, the mother and the true grandmother, that is, she who is related to the deceased without the intervention for a false grandfather² [Rumsey and Jones 1869: 4; Yekini 2008: 43].

The exclusive presence of this class makes subdivision a matter of a simple arithmetic calculation that consists in defining the shares as percentages of the sum total of assets. Often a lowest common denominator is found for this purpose. The daughter who is the only heir receives $1/2$ of her father's estate. A wife is entitled to $1/4$ of the property of her deceased husband if he does not leave children, and only $1/8$ if he does. The husband receives $1/2$ the property of his deceased wife if she doesn't leave any children, and $1/4$ if she does.

However, if within the class of similar heirs, such as many daughters and/or sisters, the share is further subdivided equally among them. For instance, wives of a deceased person in the presence of children from him would share the $1/8$ among them. If only girls are left and number two or more, they are entitled to $2/3$ of their father's property. This share is further subdivided equally among them equally. For instance, if there are five daughters, each is entitled to $2/15$ of the initial share, which was $2/3$ of the estate. The remnant, i.e., $1/3$, is divided among the other relatives. Table 1 gives the list of shareholders classified according to fraction of share.

In some cases, a partition among the primary heirs may lead to a residual part. For instance, the estate of a deceased person who has two sisters ($2/3$) and a mother ($1/6$) will lead to a surplus part of $1/6$. Another process of partition, called return or *radd*, is then applied to the subdivide the remaining $1/6$, where each primary heir is entitled to a new portion of the residue in proportional to his or her prescribed fraction. The initial share is then altered after adding a new portion of the remaining part. Other jurists stipulate that the remaining part is to be taken by the public treasury [Rumsey and Jones 1869: 27].

Conversely, sometimes the total sum of the assigned shares of the heirs becomes greater than unity. A subtraction process, called *al-awl*, is then applied. A deduction is made from each share in proportional to its relation to the whole so that the deficit is divided fairly. For instance, a deceased woman may leave competing primary heirs, such as a husband who is entitled to $1/2$, two full sisters who are entitled to $2/3$, and a mother who has a right to $1/3$ of the property. In this case, since the sum of the shares is greater

than the whole estate, all the shares are reduced proportionately. The fractional shares are consequently changed to a new common denominator, that is equal to the sum of the numerators (Table 2).

#	The basic fractions	The sharers*	Conditions
A	1/2	1-The husband, 2-the only daughter, 3-the son's daughter, 4-the sister, 5-the father's sister.	1-if the wife doesn't have offspring 2-if she has no brother or sister 3-if there is no other son or son's son 4-if there is no brother, father, son or son's son. 5-if there is no father's brother, and as in (4)
B	1/4	1-The husband, 2-the wife (wives)	1-if the wife has no offspring 2-if the husband has no offspring
C	1/8	1-The wife/widow (wives)	1-if the husband has offspring
D	2/3	1-Two daughters or more, 2-the two sisters or more, 3-the two father's sisters or more, 4-the two son's daughters.	1-if there is no son 2-if there is no brother, son(s),son's son or father 3-if there is no father's brothers plus the conditions above (2) 4-if there is no son or son's son.
E	1/3	1-The mother, 2-the brothers' mother, 3-the grand-father.	1-if there is no son, son's son and brothers 2-if they are numerous and there is no father, no grand father or son and no son's son 3- If there are brothers and the 1/3 is the highest than in any other alternative subdivision
F	1/6	1-The mother, 2-the grand mother, 3-the father, 4-the grand-father, 5-the mother's brother, 6-the son's daughter, 7-the father's sister.	1-if there is a son, a son's son, or two or more brothers 2-if she is unique (otherwise grand-mothers share the 1/6) 3-If there is a son or son's son 4-If there is a son, a son's son and no father 5-if he is unique, and no son, son's son, father or grandfather 6-if there is only one daughter without her brother, nor her uncle's son 7-if she has a sister and has no brother or father and there is no son's son.

Table 1. The prescribed shares of heirs apparent according to their status and under some conditions. Source: adapted and translated from [Amrani 2000: 62-63]

	Heirs	Old share	New Share
1	The husband	1/2.= 3/6	3/9
2	The Two sisters	2/3.=4/6	4/9
3	The mother	1/3.=2/6	2/9
4	Total	(3+4+2)/6=9/6	9/9

Table 2. Applying the method of deduction on prescribed shares

The residuary heirs

The second class mainly comprises the male agnate relatives to the deceased, known as *Ossba*. They are heirs whose right to inherit could be lost through the birth of a nearer relative. They are of three kinds: the upwards relatives (i.e., the ancestors however far up the line), the downwards relatives (i.e., the sons and the son's sons however far down the line), and the lateral relatives of the deceased and his parents (i.e., the brothers and uncles). In general these heirs have no subscribed shares as do those in the first category; their shares are known only after the first class heirs have taken their prescribed shares. If these latter are deceased, the heirs presumptive have the entire estate.

An order of priority, based on a principle of exclusion called *Hujb*, which literally means screening and/or excluding, is applied to this class and the class below. The aim is to give the closest relative priority over the other heirs, who in this case are partially or totally excluded. A simple form of screening consists of an heir who is excluded due to the presence of other, higher heirs. For instance, a father fully screens the grandfather, and the son fully screens the grandson. However, the screening could be partial, as in the case where heirs are shifted from a position to a lower one. If a deceased man leaves brothers and a mother, the mother will pass from the share of 1/3 to 1/6 due to their presence.

There are various ranks of *‘asib*, a person who has no prescriptive share. The first category is the sonship (i.e., son, grandson, etc.), next comes the fatherhood (father, grandfather, etc.), then brotherhood (full and paternal brothers, and their sons), and finally, unclehood (germane uncle, consanguine uncle, and their sons) [Yekini 2008: 50]. According to rules of priority for inheritance, Islamic law also installs a system of partial and full exclusion. For instance, the existence of a higher rank *‘asib* cuts off the heirship of those of lower ranks. A son cuts off brothers, while a father cuts off uncles, whereas a son does not cut off a father [Yekini 2008: 50].

In the presence of male and female relatives such as sons and daughters, the rule that applies to the subdivision of inheritance among them is that the males are entitled to double the share of the females. For instance, a son inherits a share equivalent to that of two daughters, a full (germane) brother inherits twice as much as a full sister, a son's son inherits twice as much as a son's daughter, and so on.

In some cases, an overlap occurs between the first and second classes. An heir who is in a given category in the presence of some other heirs, could be shifted to another category in their absence. For instance, if the deceased left only one daughter or agnatic granddaughter (the daughter of the son), her share is fixed as 1/2. However, the presence of the deceased's sons (i.e., her brothers) would shift her to the second category, where she is entitled to a share half that of her brother. If there are two or more daughters or agnatic granddaughters without brothers then their share is two-thirds. Two or more daughters will totally exclude any granddaughters. If there is one daughter and an agnatic granddaughter, the daughter inherits 1/2 and the agnatic granddaughters inherit the remaining 1/6, making a total of 2/3. If there are agnatic grandsons amongst the heirs, then the principle that the male inherits a portion equivalent to that of two females applies.

The distant kindred

This third class consists of larger circle of relatives. It comprises the descendants of the deceased, like the daughters' children, the upper grandmothers and grandfathers, the sisters' and brothers' descendants, and the parental aunts and uncles by the same mother only, as well as maternal uncles and aunts [Rumsey and Jones: 34-35]. Schematically with regards to the deceased, they could be organized into the roots, if directing upwards, the branches if directing downwards, and the far lateral relatives on both sides.

Not all jurists agree about this class in the subdivision. Some maintain that the distant kindred are not entitled to any share at all, as they are not explicitly mentioned in the fundamental texts. According to this opinion, in the absence of the previously stated heirs the property of the deceased is transferred directly to the public treasury. In some

cases, the public treasury becomes a member of the heir classes of the deceased. This opinion has also an effect on the process of subdivision of the property.

The importance of such a class depends on the two previous ones. In the presence of the two previous ones, nothing is left to the heirs of this class. However, cases where the two classes disappear are not exceptional. History shows that masses of people died due to epidemics and earthquakes, leading to the extinction of entire families who resided in the same city or a region, and survival of their far relatives in other areas [Ben-Hamouche 2009b: 94-96].

In the majority of cases, most subdivision processes are limited to two or three generations covering a time span of, say, from 60 to 120 years. However, jurists elaborated methods and legal opinions that cover up to four generations. There are exceptional cases, such as a deceased who leaves two cognate far grandfathers from both his mother and father who are the only remnant heirs. The estate in this case is subdivided equally between them (fig. 2).

Fig. 2. Example of upwards distant kindred

The presence of many overlapping generations of distant kindred in the absence of the two previous classes leads to a very complex combination of rules that define each heir's share, as highlighted by Rumsey and Jones [1869: 36-48]. The results of the subdivision may turn into a series of many fractions with large denominators, ranging from tens to thousands, depending on the number of heirs.

The logic of subdivision could be mathematically modeled and turned into a simple software. The three classes of heirs as defined above would be the major stages of decision-making in an algorithm that helps lawyers as well as the layman define the shares of each heir depending on the data entered.

The arithmetic that comes from the algorithm could then be displayed graphically through the simultaneous representation of all shares in shapes within a square of 100 x 100 pixels using the cellular automata system that reflects the stages of iteration.³

The geometry of subdivision: uncovering the mystery of complexity

While mechanisms of subdivision acted continuously on the morphology of the city in congruence with the life cycle of social entities (i.e., family units and individuals), they did so together with other mechanisms in an integrated system. When segregated, each one appears to have a negative effect on the morphology. For instance, the pre-emption principle, *Shufa'*, gives the partner in a property or the neighbor in an adjacent property the right to get back the shares of other partners when these latter put them up for sale on the market. Consequently, this mechanism partially acted to counter succession, as shares were unified in one new property. Other mechanisms, such as *waqf*, *Irtifaq*, *Ihiya al-mawat*, have similar effects of countering the fragmentation of properties [Ben-Hamouche 2009a, 2009b].

Also, a subdivision is not always the sole or compulsory recourse for partners. A partition of usufruct, *muhayat*, as seen earlier, is an option open to the heirs as long as they reach an agreement. The asset in this case is kept physically intact, and each heir enjoys the use of the thing held in partnership either by turns or simultaneously, where this is possible. For instance, a courtyard could be shared as a subdivided asset, where each partner has his own access and rooms. This is also the case of small shops, public baths, houses and cultivable land [Cami Efendi 2000: 94-101; Hamilton 1989: 576-578].

Passages and roads that are collectively owned may be subjected to subdivision in case of a dispute among the partners, and if this is practicable. If partners disagree regarding the extent of the road, that is regarding the height and breadth which each ought to have, the judge must regulate their proportions according to the breadth and height of the doors of their respective houses, as that is sufficient. According to [Hamilton 1989: 572] the advantage of this arrangement is that if any of them is desirous of making a projection or terrace from his house over the road he may do it above the height of his door but not below it, and the road will still remain in common. Quite often territories of houses along a road are marked by physical elements, such as arches and signs, that have unique architectural significance, but appear as mere aesthetical elements to unknowing visitors, tourists and outsiders.⁴

Basically, an asset, depending on its size as well as the number of the heirs, should be judiciously subdivided in response to the legal shares in terms of areas, as well as autonomy and conditions of use. "Conditions of use" mainly refers to access, drainage of rain water, and provision of light.

In contrast to what appears to be the basic unit in the city, that is, the house, for succession law the spatial unit in a subdivision is the room, or *bayt*, defined as the single roofed place surrounded with walls, with a door or entry. A tenement that is composed of different rooms, a roofed court, and a kitchen, so that a man may reside there with his family, is called *manzal*. This is also the case of the tenement with an open court, termed *dar* [Hamilton 1989: 572]. A direct impact on the morphology of such a derivative concept of domestic space is the setting of property lines after each new subdivision. Following the construction of walls and the lines of interior partitions, only rooms and other undividable elements appear to maintain their integrity in the urban fabric. Two adjacent rooms that once both opened onto the same courtyard may, after partition, have opposite entrances, each of which opens onto a different courtyard. Understanding this may make it possible to decipher the dynamics and complexity of urban geometry at the domestic level in old Muslim cities. By way of recurrence, many houses, after removing

the lines of subdivision, could be reconstructed into their initial state, in which, for instance, they had larger courtyards and simpler shapes as blocks, and were surrounded by larger roads.

Winding streets, cul-de-sacs and dead-end streets are often enigmatic urban elements. A portion of land that is located in the heart of the initial property would necessitate a dead-end street and/or a narrow winding alley to provide access, a feature that forms an important portion of the road network in old Muslim cities [Raymond A. 1994]. Consequently, it is evident that a dead-end street and a winding alley that were mainly described by Western scholars as signs of an *a priori* geometry and voluntary irregularity in Islamic cities, are in fact *posteriori* events that resulted from a realistic response to an existing situation.

The partitioning of many houses and tenements located in different places is a source of many controversial legal opinions. The divergence stems from differences in the market value of the property due to the difference in location in the city (or cities), and to different functions, such as shops, stores, gardens, or houses. For instance, if a deceased man left several houses held in partnership or coparcenaries in one city, the grand jurist Abu Haneefa maintained that, for the sake of fairness, each house must be separately divided among the heirs [Hamilton 1989: 570]. For him, the value of the houses may differ greatly depending on the cities, lanes or neighborhood in which they are situated. Their proximity to or distance from water or a mosque makes it impossible to achieve equality in the partitioning without dividing each house separately.

However, other jurists, with a more realistic view, hold that if it is convenient, the whole of the houses would better be united in one general partition and not divided separately. The share of each partner must be assembled as far as possible in one of the houses, so that it may be solely his property. Implicitly, a system of compensation would be devised in order to cover the differences of market value between the different assets.

Complicated partitioning can also occur in the case of buildings with more than one floor. In a case of a house with two floors, the same school of law has three diverging opinions regarding the value of each floor. Abu Haneefa maintains that the ground floor has a value double that of the upper floor, a fact that affects the subdivision. The subdivision into two equal parts of a property with two floors, considering the value of a square meter of the lower floor to be twice that of the upper one, is shown in fig. 3.

Fig. 3. The subdivision of a two-storey house, where the value of the lower floor is twice that of the upper one

The impact on urban geometry

The sections below show the method of calculation of shares, the subdivision process and the impact on the urban geometry. The first one will present the method applied to a simulation case, whereas the second one will present evidence on the ground through a semi-simulation case of an urban fabric in old Algiers.

Alternatives designed by Salmane Ben-Hamouche

Fig. 4. A hypothetical subdivision of a land parcel of 60 x 120 m². Alternatives and shares displayed graphically: blue for father, purple for mother, green for wives, yellow for sons, and brown for daughters

A hypothetical property that devolved on a number of heirs best explains the methodology and the process of subdivision and the way fractal geometry is generated. A deceased man leaves a piece of land measuring 60 x 120m² that is accessible from four sides. The asset had devolved on his heirs, who are his two parents, his two wives, and his children, who are four boys and four girls. The subdivision of the property considers the two classes of heirs. The first consists of the sharers who have predetermined parts, and the second is that of the children, who will subdivide the remaining share according to the rule that males have twice the share of females (fig. 4).

The system of subdivision may be further complicated if the frontage elevation is taken into consideration during the operation. A portion of land that gives onto a street

would have a higher commercial potential than the one located inside the block or on a dead-end street. The frontage could then be quantified in terms of metrics and subdivided in proportional to the shares, a fact that further increases the complexity of calculations and geometry.

Fractals, iteration and bifurcation

Subdivision is initially a collective decision-making process that consists of agreeing, eliminating alternatives, and maintaining the most convenient one. In addition to his legal status, in this case the divider person would also be a witness and arbitrator. After having defined the shares through calculations, many possibilities figure on the ground to the divider as well as the heirs. Depending on the size of the property as well as the number of shares, a decision is first made about the best alternative of subdivision, one that considers fairness as well as conditions of autonomous use of each part. It is only after this step of agreement is reached that shares are distributed among heirs. Specific allocation of shares to each heir is then made either through another agreement or by drawing lots.

Fig. 5. The impact of succession law on the urban geometry in old Muslim cities. Fractality in lines of urban fabric and the process of land subdivision

Fig. 6. The impact of succession law on the distribution of water and the morphology of desert cities, Tamentit, South of Algeria

According to the Islamic succession law, subdivision is also an iteration process *par excellence*. Iteration in mathematics refers to the process of applying a function repeatedly, using the output from one iteration as the input for the next. When taken over a long time span, say decades and centuries – that is, the time of development and growth of cities – it means an act of repeating an operation with the aim of defining new shares with a combination of prescribed and deduced fractions. An initial simple function of iteration consequently produces complex morphology at later stages, as is known in chaos theory [Batty 1991, Ben-Hamouche 2009a]. The scale of the process may be best realized when we consider the urban dimension and the number of operations made on the daily basis, which is, ironically, related to the rate of deaths in the city. In other words, private assets that form the largest number of the urban space are almost all subject to subdivision. One exception is made for the few public buildings and endowments assets, *waqf*, that are protected from alterations due to the religious clauses of donators. In terms of fractal geometry, a subdivision process is continuously a point of bifurcation that gives rise to many tree-like possibilities of fragmentation, that all heirs establish. However, while in theory such an iteration of subdivision is endless, in reality it stops when it reaches a certain threshold of minimum “workable” shares (fig. 5).

The physical results of subdivision on the geometry of urban fabric can be readily noticed. Initially, regardless of topographic constraints, no oblique partition lines are made in undeveloped buildable land. After construction, subdivision lines follow exterior walls and partitions that mostly intersect at right angles. Dead-end alleys emerge as an outcome of subdivision in response to the location of some shares deep within residential blocks away from streets. Over time, their number increases as a consequence of the number of subdivision operations. In most cases, courtyards tend to be square in shape and centrally located in the plot. Despite the intensive use of the courtyard house as a preferred typology, because it insures privacy and provides a good response to harsh climatic conditions, sharers tend to compete with each other to have portions of elevations on the streets. Consequently, resulting shares tend to be deep, with smaller elevations along streets.

Succession law has also had an impact on the subdivision of water, irrigation systems and water networks in desert cities. To a large extent, channels irrigating palm trees and gardens reflected the subdivision of water into shares. Delta elements were constructed as points of bifurcation for the diverging water channels [Bonine 1979; Ben-Hamouche 2008]. Their number along the path of water from the source to its destination reflected the stages of subdivisions. In case of conflicts of passages and crossings, canals were constructed like overpasses that could go as high as three levels. Tamentit, south of Algeria, provides an excellent example of such water subdivisions and the extent to which water systems shaped the landscape and the urban geometry of the city (fig. 6).

Postscript: Succession law and urban regeneration policies

At present, most old Muslim cities are suffering from a continuous degradation due to various and interacting factors. The failure to absorb modernity and the overwhelming new ways of life, which are based on private cars, electronic devices such as washing machines, electricity, and gas, are considered determinant factors. The bright lights of big, new cities have also driven most of original populations out of the old cities, in search of a decent life. Consequently, this has turned the old cities into slums for poor classes of inland migrants. Politically, the interest of public authorities is mostly directed to new developments, which are used by the politicians in electoral campaigns as proof of achievement and signs of progress, which has heavily marginalized the development of the old cities.

Succession law itself, which for centuries was a source of urban dynamics, has in a way turned into a negative factor and a source of degradation. Its abolishment led to an institutional crisis. During the pre-colonial period, most cities were equipped with an institution for subdivision of inheritance. This was the case of the Ottoman institution called *shughl al mawarith al makhziniyya*, which was in charge of the liquidation of properties of deceased and lost persons. Subdivision was made as soon as a person was buried [Ben-Hamouche M. 2009b]. Properties were listed, sorted out, and then subdivided among heirs in the presence of a representative of the court. All too often the public treasury was the main heir for all the devolved assets during major catastrophes such as wars, droughts, and epidemics.

In the absence of these old institutions, most buildings and properties that socially fall into inheritance are witnessing a form of dispersed ownership and thus often becoming no man's land. Over decades of unsolved problems of inheritance, rights of generations of heirs have accrued over most of the old decaying buildings. Achieving an agreement among these generations became an impossible mission for co-owners, most of whom are

either absent or disinterested. Tenants who are still present in the place make no effort to improve or maintain the old buildings, which have a low turnover and suffer from advanced states of decay. An initial step in a further study would have the goal of detecting the number of buildings in old cities that suffer from such a problem and investigating in the feasibility of re-inventing the institution of subdivision of inherited properties.

Conclusion

This study has highlighted the system of Islamic succession law and its direct impact on the physical environment. The morphology of old Muslim cities cannot be analyzed without understanding the mechanisms of such a system. Mathematics, sociology and engineering were combined under this system and generated the dynamics of these cities.

It is also evident that any preservation or regeneration policy aimed at improving the situation in old cities must necessarily pass through the reconsidering of the Islamic succession law and the re-establishment of the institutions in charge of its implementation.

Notes

1. In its origin it was authored by the jurist Siraj Al-deen Muhamed Al_sajawandi, who died around the 13th c. A.D., while Professor Almaric Rumsey (1825-1899), of King's College, London, was the author of many works on the subject of the Muslim law of inheritance and a barrister-at-law.
2. A false male ancestor is where a female ancestor intervenes in the line of descent.
3. It is the intention of the author in future research to automate the succession law and further highlight the impact on the physical environment.
4. Examples of such elements are given in B.S. Hakim [2009: 29-41].
5. Although it is possible to find real examples of subdivision, it is beyond the scope of this paper as it requires a social survey of heirs, a collection of legal records over decades, and a physical survey on the ground of the subdivided assets.

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