Geometry and Rhetoric: Thinking about Thinking in Pictures

Abstract. Thinking about thinking is tricky business. Pitfalls include a tendency to confuse our metaphors with the act itself, difficulties attendant to discredited notions of introspection as a source of evidence and the twin unreasonablenesses of reductive scientists and mystical humanists. Engaging geometry and rhetoric in a common frame presents the opportunity, especially in the context of architecture, to consider discourse and image in ways that are mutually reinforcing.

Does not 'the eye altering alter all'?
William Butler Yeats

Introduction

On good days, thinking about thinking with images, I am stuck between a rock and a hard place. On bad days, I am completely paralyzed. The problem of representation in thought processes is important because of a conflict between what I believe and what I know. What I know is that I think using images and, by extension, that humans engage the world imagistically as a basic cognitive competence. My conviction is that images of everything – from landscapes, to faces, to geometrical diagrams – form the most basic stuff of mental states and processes; that thinking in pictures is different from thinking in words; and that images can condense broad discursive passages into dense points that can be arranged (and rearranged) in ways different than strings of words. On the other hand, I believe that there is no such thing as a mental picture. If, as Maurice Merleau-Ponty (1908-1961) observes, “consciousness cannot cease to be what it is in perception” [1945: 50] and it is accurate there are no representations in perception due to the “homunculus fallacy,” then at the very least the role of representation in thinking is suspect, if not thoroughly discredited. Significantly, artists and other visually inflected makers have not taken much notice.

Architects, designers, painters, poets – iconophiles of all stripes – may be surprised to hear that there is a controversy in philosophy and psychology “over the nature of, and even the existence of, mental images” [Dennett 1978: 174]. This controversy, raging since the 1970’s, has many philosophical precedents, ancient and modern,⁴ but none so acute as that of Rene Descartes (1598-1650). Descartes, due in part to his discoveries spanning optics, meteorology and geometry collected in the *Discourse on Method* (1637), identified what has since come to be known as the homunculus fallacy, or “the fallacy of the little man in the head.” As characterized by Alva Noe:

It is incoherent to suppose that seeing an object depends on the resemblance between a picture in the eye and the object, for that presupposes that there is, as it were, someone inside the head who perceives the resemblance, this would lead to a regress, as there is no less difficulty explaining how the interior observer can see the interior picture.

The source of the fallacy of the little man in the head is the idea that the retinal picture functions as a picture, as something *perceived* [2004: 44].

Descartes's fundamental insight has been affirmed in many ways over the last 372 years and yet the introspective impression of mental manipulation of pictures in thought remains overwhelming – even as the controversy rages anew.⁵ It is as if each new generation needs to learn again that there are no pictures in the brain.

Were I to characterize this dilemma with more nuance, I would say there is a disjunction between how I think I think and what I think I know. This essay aims to capture the interleaving of geometry and rhetoric in cognitive operations that fold perception into thought. Perhaps the place to begin is with a common misapprehension.

Mind and brain

An accepted initial distinction here is between what might be characterized as spiritual and material modes of interpretation at the seat of cognition, which I reject as a simple but pernicious distortion.⁶ As Humberto Maturana and Francisco Varela observe, denying the homunculus fallacy engenders an “outlook which asserts that the only factors operating in the organization of living systems are physical factors, and that no non-material vital organizing force is necessary” [1980: 137]. This “biological” definition of mechanism, at the root of autopoiesis, is an originary principle of much current thinking about thinking, which I accept.⁷

What is necessary are ideas that respect hard-won introspective evidence while circumventing definitions that support vitalist or even idealistic debates. Here, Andy Clark's angle on “mind's-eye” and “brain's-eye” views, is promising. Why? Because Clark constructs a cognitive model on the basis of complementary mental processes. His distinction between mind and brain centers on tasks performed by deliberate mental effort and tasks that we perform quickly and fluently without conscious awareness. What is refreshing about Clark's argument is that he argues for the integration of mind and brain.

Just how does Clark manage this integration? The juice, of course, is in the argument but just the rhetoric is significant. There is a mind's-eye view of cognition – how we think we think – and there is a brain's-eye view – what we have learned about how we think, based on scientific evidence. Of course, it is not just about thinking. It is about perception and association and emotion. What is important is that we recognize the tangle of competing ideas: that what we think about how we think is not necessarily accurate; that just because thinking about thinking is often comically misguided, it does not follow that there is no place for what we conclude about how we think; that what we have come to accept as the way the brain works has very little to do with the introspective evidence of the mechanisms of thought; and, finally, that it is fitting that the mechanics of perceiving, thinking, feeling etc., are mostly unavailable to the organism performing these acts. In the beginning, just the recursiveness of these phrases explodes the dichotomy of mind/brain into a framework that recognizes the strengths and weaknesses of various forms of cognition and shifts the argument from objects – brains and minds – to networks of relations.

Mind's-eye view

The mind's-eye view is just what you think; it is the mind's own understanding of how it works. This comes in at least two flavors, the professionals' view of the matter and

a popular view. The professionals in this instance are philosophers, psychologists, and/or cognitive scientists – anybody whose job it is to figure out the how behind the why of mind. It seems that maybe a hundred years ago philosophers had a special purchase on these issues. That privileged position seemed to give way to psychology as standards of evidence took on an increasingly empirical flavor. Later still, as the purpose of the inquiry shifted from experimenting to modeling, those who in a different age might have been philosophers or psychologists came to call themselves cognitive scientists. All these professionals utilize different methods towards different ends but in broad strokes they share several common notions.

They share at this point in time what Clark calls as “classical cognitivism,” which are characteristics that might be glossed as follows: 1) There is an individual and a world and though they may be related in any of a variety of ways they are distinct; 2) there is an algorithmic character to thinking – a language of thought – that is discoverable, verifiable and most importantly subject to modification from within the symbolic system.

Regarding the relationships of individuals and worlds, it makes no matter if one takes an idealist, a realist or even some scholastically nuanced position with regards to this association, it is always possible to draw a line between the world and the individual. The algorithmic character of thought breaks down into two sub-clauses: Thinking is a function and as a function it includes symbols yielding tangible structure that is often a literally physical pattern. Furthermore, the symbol structures are subject to instructions that are themselves expressible in symbols. Alan Newell and Herbert Simon assert, “the necessary and sufficient condition for a physical system to exhibit general intelligent action is that it be a physical *symbol*/system” [Newell and Simon 1976, quoted in Clark 1989: 11, emphasis mine]. Elsewhere Simon goes so far as to identify information processing with physical dynamical theories expressed as sets of linked differential equations [Simon 1976].

The popular view of the matter is the impression that we direct the course of our own thoughts and that this intentionality is discoverable in words and deeds. The purpose of Clark’s exposition of professional and popular views is to show that the popular view is not just a less competent version of the professional view but an approach to thinking about thinking that solves entirely different problems. What is important for the argument in this essay is that both approaches imply some executive function that directs thought and perception. The popular view of the matter posits, appropriately, a homuncular agent that has the “goal of making the behavior of others intelligible to us to just the degree necessary to plot their moves in relation to our needs and interests” [Clark 1989: 50]. The professional view, much more concerned with mechanisms of thought, has long suppressed the little man in the head but still needs an executive to direct the flow of thoughts, words and deeds. The homunculus-cum-ego preserves the executive without notions, discredited since Descartes, of representation and infinite regress, while at the same time allowing the machinations of minds their algorithmic character.⁸

Brain’s-eye view

We might remember that the discussion in *Microcognition: Philosophy, Cognitive Science, and Parallel Distributed Processing* (1989) took place in the midst of a technological revolution: the application of previously undreamed of computing power being used to model brain functions. A concern with the modeling of cognition, dubbed artificial intelligence, had appeared at the heart of many disciplines – and for good reason (see [Miller 2003] and [Varela, Thompson and Rosch 1991]).

Early attempts to reproduce mental and perceptual processes using computers, drawing on philosophical models of symbolic processing in a classically cognitivist frame, tried to create general problem solving routines that reflected our “faith in the mind’s own view of the mind” [Clark 1989: 4]. A specific example of this approach, drawn from robotics, concerns problems of navigation.

Initially, the design of navigation routines aimed at constructing representations of the spatial envelope – and the objects it contained – that a robot’s central processor could draw on to move from one location to another. This approach is roughly consistent with the mechanism we think allows us to fetch a glass of water from the next room: grasp the glass, stand up by pushing back the chair so that it does not scrape the wall, move through the hall managing not to fall down the stair or trip on the carpet, negotiate the door frame, manipulate the faucet, fill the glass and then back, without spilling water or walking into walls. Conventional introspective evidence suggests that we manage such a feat because we have a mental map of spaces and obstacles that we negotiate in order to reach our goal. It turns out, however, that this does not work very well as a strategy for programming robots to move about.

Rodney Brooks, whose work at MIT in the 1980s helped explode the old paradigm, observes:

[We] have built a series of autonomous mobile robots [and] we have reached an unexpected conclusion (C) and have a rather radical hypothesis (H).

(C) When we examine very simple level intelligence we find that explicit representations and models of the world simply get in the way. It turns out to be better to use the world as its own model.

(H) Representation is the wrong unit of abstraction in building the bulkiest parts of intelligent systems.⁹

These early robots did not so much navigate through a space as bump their way through an obstacle course on the basis of a series of local interactions and equally local course corrections. Stochastic as this navigational result appears to observers, it works as a strategy for moving from A to B. In the process Brooks claims to have discredited the notions of minds as general problem solvers and enfranchised brains as (albeit very complex) computational, nonrepresentational, problem solvers based on overwhelmingly local interactions.

The mechanism of such systems are often characterized as neural networks, and they offer progress modeling perceptual and sensorimotor tasks including real time sensory processing, the integration of input and output modes, the capability to deal with incomplete and/or inconsistent information, mechanisms of three dimensional ranging, to name a few of the kinds of problems on which researchers began to make progress. A general characterization of such competences is called pattern recognition (or pattern completion) and Herbert Dreyfus offers this description of how the simulated neural network mechanisms work:

Fortunately, however, there are now models of what might be going on in the brain of an active perceiver ... that do not introduce brain representations. Such models are called simulated neural networks. Simulated neurons are generally called nodes. Networks consist of a layer

of input nodes, connected to a layer of output nodes by way of a number of intermediate nodes called hidden nodes. The simulated strengths of synaptic connection between neurons are called weights. The output of a neuron is called its activation. Running such a net means specifying the activations of the input neurons and then calculating the activation of the nodes connected to them using a formula involving the weights on these connections, and so on, until the activation of the output is calculated [Dreyfus 2005, 132-3].¹⁰

What these networks effectively explain, by modeling, is our ability to match patterns, learn and perform tasks requiring sensory information without explicitly formulated propositional rules. To understand why propositional encoding of a set of general rules does not work very well for modeling automatic (i.e. sensorimotor) tasks, take an example from the language of color. How do we know that emerald is a kind of green? According to propositional rules, for emerald to be a kind of green it is supposed to be some combination of yellow and blue. But, as Leonardo da Vinci (1452-1519) noted, there is no mixture of yellow and blue pigments which yields the particular visual characteristics ascribed to emerald. Therefore, in a propositional system one would need an exception to the rule that green is a species of color determined by the combination of yellow and blue. The result of this necessity, as observers become more attuned to the subtleties of color, is that there have to be more and more explicitly formulated exceptions in order to take into account effects that are, for one reason or another, ambiguous.

As another example, turquoise is either green or blue or neither of the two, depending on context. In a rules based system there would need to be several rules, each accounting for specific instances of the appearance of turquoise, i.e., as a kind of warm blue, a cool green, or perhaps neither, as its own particular color experience. Oddly, such a rules-based system could produce a program in which there are more rules than there are effects. On the other hand, neural network approaches (which Clark calls "connectionist") "buy you a profoundly holistic mode of data storage [and robust] hypotheses about the pattern of regularities in the domain." He calls this a superpositional storage of data and notes that as the system "learns about one thing, its knowledge of much else is automatically affected"; the system learns through pattern matching "gradually and widely" [Clark 1989: 111].

In other words, color in a connectionist model ceases to be a handful of primary colors arranged on a primitive geometric shape, the combination of which accounts for the gamut of color experience. There is no such thing as a primary color, or even a propositional systematization of the attributes of color perception such as hue, brightness and saturation (or their analogues from any system you prefer). Instead, colors in the visual field are superimposed with other visual effects to yield a range of optical experiences, inflected by color, through which we navigate our worlds. In such an environment, learning does not inscribe propositions, it matches patterns.

Mechanisms of robotic navigation, or what Clark calls three dimensional ranging and data retrieval through pattern completion, are among the exciting discoveries that neural networking or connectionist approaches yield.

In the connectionist approach ... the meaningful items are not symbols; they are complex patterns of activity among the numerous units that make up the network. ... In the connectionist approach ... meaning is not

located in particular symbols; it is a function of the global state of the system and is linked to the overall performance in some domain, such as recognition or learning

In other words, symbols are not taken at face value; they are seen as approximate macrolevel descriptions of operations whose governing principles reside at a subsymbolic-level [Varela, Thompson and Rosch 1991: 99-102].

What the word “subsymbolic” captures so well is that *how we think we think* is built on a neural substrate of which we have no experience and to which we have no access. What we are describing are the unconscious processes which ground symbolic cognitive operations but which, unlike the psycho-therapeutic unconscious, are operations to which we cannot have access (for implications and extensions, see [Varela, Thompson and Rosch 1991: 48ff]).

To summarize the brain’s-eye view: thinking is continuous with perception, which is non-representational (neither propositional nor imagistic), built up from local interactions, which are aptly characterized as pattern matching and are, finally, overwhelmingly transparent to the organism.

Multiplex forms of explanation

In contrast to the neural networking approach, the mind’s-eye view of thinking is a conscious and deliberate act, very much in the philosophical tradition characterized as “serial reasoning tasks of logical inference, temporal reasoning tasks of conscious planning” [Clark 1989: 127]. Euclidean geometry may be the most succinct example of the mind’s-eye view of reasoning. There are collected in one place, the thirteen books of *The Elements*: five explicit assumptions (the common notions); twenty-three symbols (the definitions); five postulates (the rules) which together in various combinations are used to deduce 465 accurate conclusions (the propositions).¹¹

Descriptions of such reasoning tasks are not well captured by connectionist, brain’s-eye models. Rather than draw on Clark’s examples of high-level symbolic characterizations (chess playing, the architecture of von Neuman machines, mathematical proofs, etc.), I will present examples related to the perception of space and the encoding of memories. The goal here is to maintain some symmetry with the examples discussed above. I draw these examples from literature on the role of perspective in the construction of space and from the “memory palace” tradition in rhetoric. These examples are not as “hard-core” as the classical conceptual-level representations favored in the literature of cognitive science, and they are tuned to the subject at hand, geometry and rhetoric. I select them because they provide excellent examples of processes in which there are both brain’s-eye and mind’s-eye descriptions of thinking about thinking, descriptions in which geometries play a central role.

The construction of depth through visual means, *perspective*, is an especially interesting case, partly because it is such a contentious domain in the history of art, philosophy and culture. For the discussion at hand, we can sidestep the bitter part of these disagreements by granting both biological and cultural components to the construction of space in representation. In fact, my argument hinges on the differentiation of symbolic and subsymbolic parts of the construction of space. As is so often the case in contentious domains, gradations of class membership complicate the argument. For instance, perspectival reproductions in painting, drawings and

photographs in which there is “a rigorous two-way, or reciprocal, metrical relationship between the shapes of objects as definitely located in space and their pictorial representations” [Ivins 1973: 9] may or may not be relevant to any one particular representation. At times, perspective is merely implicit, as in drawings where only the relative size of visual subjects is depicted, suggesting depth but not forcing a unified viewpoint. At others, perspective is purposely suspended to create representations for which there are multiple points of view as in synthetic cubism. Finally, and of course in many cases of visual art in the late twentieth century, it is not even warranted to speak of visual works as representations at all, since that implies a subject-object dichotomy, reinscribed in perspective, which is not at all germane.¹²

The point is, from a mind’s-eye view and as mechanisms of symbolic processing, there are many different ways to characterize the point of view in images, some of which conflict to the point of being mutually exclusive. Perspective figures in some of these ways of thinking about representations but not in others, and the measure of just how conceptually bound by it we are can be taken by how radical one thinks Brooks thesis is in “Intelligence without representation.”

For me, it changes everything. It is no longer appropriate to think of perceptual space as a continuous medium in which we are embedded. It is no longer possible to think of navigation as movement in reference to a mental map, however dynamically constructed. Corollary to these ideas for humans (but not necessarily relevant for robots) we can no longer build up our visions of the world from monocular or static points of view. These are all comments that are relevant to perspective as a symbolic system – a story many minds have made up to explain why we are susceptible to the illusion of depth in depthless representations.

Since Bishop Berkeley (1685-1753) we have known that depth is not given in the optic array. The easiest way to make this argument is to imagine a monocular, stationary viewpoint and grant a distinction between scenes and images. Under those conditions a point in the field of view may vary on *x*- and *y*-axes but its location on the *z*-axis (in depth) is “underdetermined.”

Scenes are three dimensional distributions of matter whereas images are only two dimensional distributions of gray tones. Thus images necessarily underdetermine scenes. For the remarkable Bishop Berkeley this was reason to draw the conclusion that it is impossible to see scenes. We deduce (representations of) scenes and we may be *wrong* in our deductions [Koenderink et al. 2000: 3].

Now allow the viewpoint to move. This turns the problem of deducing depth in scenes into a question of structure-from-motion. The details are technical and quite fascinating (see [Koenderink and van Dorn 1991]). The short version is that the visual system is adept at constructing depth, given a processing of only four points in three consecutive views.¹³ In this scenario we construct depth from motion using mechanisms that are elemental and liable to connectionist modeling. Though the brain’s-eye (subsymbolic) modes of visual perception are far from being worked out in detail such structure-from-motion modeling for depth perception suggests that what we call perspective is a symbolic (mind’s-eye) manifestation of other, more elemental (brain’s-eye) processes.¹⁴

Take another, more conjectural, example.¹⁵ We have two eyes and yet the overwhelming impression is that we perceive scenes from a unified point of view, the so-called cyclopean eye.¹⁶ Therefore, there must be some mechanism that combines views from two sensors into a single image. Furthermore this mechanism must be hardwired into visual perception in such a way that, though we are aware of the result of fusion, we are not aware of the mechanism. This mechanism would be accurately described as a geometrical transformation that represents a field with two foci as a field with one focus, fusing information from two eyes into a unified point of view.

The technical term for such a geometrical transformation is projection, in this case transforming ellipses into circles (transforming optical fields with two foci into perceptual fields with one focus). In other words, the cyclopean eye is the result of a fundamental perceptual competence that integrates binocular fields through projective transformations analogous to the way that projective geometry is the general case and perspective the special case.¹⁷ Our susceptibility to the impression of depth in perspective representations with vanishing points (a mind's-eye view) becomes the unintended consequence of the process that fuses data from two sources (a brain's-eye view). In short, perspective is a symbolic level manifestation of a subsymbolic projective competence. Andy Clark's position – and this is the point of the exposition to date – is that we require both symbolic and subsymbolic paradigms to account for the richness of human cognition.

Mixed models thus require multiplex forms of psychological or computational explanation. Not just different cognitive *tasks*, but different aspects of the same task now seem to need different kinds of algorithmic explanation. Since humans must frequently negotiate some truly rule-governed problem domains (e.g., chess, language, mathematics), some form of mixed model may well be the most effective explanation. ... A virtual symbol processor provides guidance and rigor; the [connectionist] substrate provides the fluidity and inspiration without which symbol processing is but an empty shell. In words that Kant never used: subsymbolic processing without symbolic guidance is blind; symbolic processing without subsymbolic support is empty [1980, 174-5].

I turn now to a mixed model that requires “multiplex forms of psychological *and* computational explanation” on the way to suggesting the lamination of geometry in mind and brain.

A mind's-eye view of geometry and rhetoric

A mind's-eye view of rhetoric (thinking about thinking as a conscious and deliberate act) is perfectly captured in the tradition of the arts of memory, a family of strategies for mnemonic storage and recall developed in ancient Greece, and codified in the second-century A.D. pseudo-Ciceronian treatise *Rhetorica ad Herennium*.¹⁸ These practices, which included the technique of *imagines et loci* (images and locations, involved a kind of mapping where the text to be remembered is superimposed on (usually) architectural elements and committed to memory as an ordered whole. The sequence of textual images, often structured by the topography of a building, facilitates recollection. As recounted by Frances Yates [1966], elements of the art include rules for places (*loci*) and rules for images (*forma*), followed by a discussion of memory for things and memory for words. *Loci* are the placeholders, usually an architectural ornament or structure, which are committed to memory in a sequential fashion. *Loci* are as detailed as possible, for example, neither too large nor too small, neither too bright nor too dim, neither overly

ornamented nor too plain. *Forma* are images applied to *loci*. The *forma* must be as vivid as possible: active not vague, exceptional in some way (grotesque, comic, or beautiful), and revisited often.

There is a bifurcation in the rules for images, between memory for things (*res*) and memory for words (*verba*). Memory for words (*verba*) concerns the memorization and recitation of lengthy discursive and poetic works. In this situation, a practitioner of the memory art walks mentally through the *loci*, “reading” off the text to be performed. Rules for things (*res*) promotes a more associative, less literal, ordering of what is to be remembered, offering a method of ordering and accessing discrete yet related chunks of information.

The obvious and often-voiced criticism of this particular memory art is that a practitioner must commit to memory more than is actually being recalled (recall the propositional, symbolic, thinking about thinking discussed above). Not just one datum needs to be encoded but three (or more) data have to be committed for each word or thing to be recalled. This suggests something interesting about human information processing and, by extension, the wisdom built into mnemonics: there is no “grandmother cell” – no specific location in the brain where individual memories are stored. Where the literal application of the “memory palace” technique may be preoccupied with the careful selection and placement of *res* or *verba* to facilitate recollection, a more process-oriented interpreter will note that the practice also (and perhaps more importantly) supports dynamic recall by its linked trail of visual and rhetorical associations. This is plausibly the story a mind would tell itself about the way thinking transpires.

As with all thinking, good memory practice is cultivated and, as Umberto Eco observes, corruptions of memory are resisted with strategies that are constantly reengaged by new technologies that are remarkably constant across technological divides [Eco 2004]. According to tradition, upon hearing of the invention of writing the Pharaoh Thamus observed that the written word is not an aid for memory but a tool for forgetting. He recognized that writing allows us to forget by allowing us to ignore what is known. Quite literally, out of sight, out of mind. What the Pharaoh failed to foresee is that we still need a memory for things written, if only to find where that which was forgotten has been stored. And so it goes – wax tablets give way to parchment, which in turn give way to printing from movable type, etc. In each circumstance humans require a strategy for remembering. More importantly, at each technological divide the art of memory is recreated with more elaborate techniques; by the Renaissance it had become permissible to express anything in terms of everything.

Again, this dance plays out at the technological divide between print and electronic media. The literal strategy of encoding *res* and *verba* on storage media is remarkably similar to the strategy of the memory palace: The data to be recalled are located at specific locations on a very long sequential medium (disk drive, CD, DVD, etc.). It is also consistent with the history of the memory tradition that the rules governing the encoding are ever more elaborate (i.e., abstract), requiring a file allocation table (an intermediate and gigantic *loci*) to reassemble *res* and *verba* into recognizable *forma*. Perhaps the most telling argument is the standardization of desktop computer architectures around serial processing (so-called von Neumann machines). There are many reasons for this, one of them being the historical prejudice for serial processing and sequential storage, as embedded in the literal version of the memory arts tradition.

To summarize the mind's-eye view, by analogy to memory: there exist symbolic representations – *res* and *verba*; structural relations – rules for *loci* and *forma*; and operations that arrange and rearrange symbols on mental structures – the memory arts. One interpretation, the literal one, supports sequential processing of symbolic content, similar to the way we think we think. Another interpretation suggests how, as in the case of perspective, a mind's-eye process is actually the result of a more elemental operation evolved to support an entirely different process, as in a model of mental operations that is subsymbolic.

Geometry and Rhetoric in the mind's eye

I confess to being utterly fascinated by Giordano Bruno's (1548-1600) geometry of thought as interpreted by Arielle Saiber in *Giordano Bruno and the Geometry of Language*. Saiber sensibly contextualizes such a move this way:

A geometric reading of literature is not meant to replace other forms of textual analysis, as it does not claim to be a master key that unlocks all of a text's meaning and value. Its aim is to uncover important links between philosophical, aesthetic, or scientific notions of space and language's explicit and implicit demonstrations of these notions. The ways the geometry of space and form reveal itself in literature does, in fact, suggest much about how the imagination—individual and societal—orients and locates itself [2005: 26].

How the imagination orients itself is particularly suggestive in Bruno on two accounts; in Bruno's memory theatre and in the transmutation of geometric figures and rhetorical figuratives.

Bruno's mnemonics graduate from memory system to memory palace on the authority of Frances Yates, who concludes that in Bruno, the memory theatre was understood to be a method for printing "archetypal images on the memory, with the cosmic order itself as the 'place' system, a kind of inner way of knowing the universe" [1964: 191].

Yates' interpretation has been challenged, by Saiber on whether or not Bruno was a magus (Saiber holds not, see [2005: 92]), and by Rita Sturlese on whether letters recall images (Yates) or images recall syllables [Sturlese 1991 and Eco 2004]. While these details are significant they do not really alter the fact that Bruno was seeking "to establish within, in the psyche, the return to unity through the organization of significant images" [Yates 1966: 228]. In Bruno's words, by the act of imprintation "you may gain possession of a figurative art which will assist, not only memory, but all the powers of the soul" and "you will arrive from the confused plurality of things at the underlying unity" [Yates 1964: 199]. That this underlying unity is accomplished through figuration, has inspired a whole generation of hypermedia theorists.

Saiber elucidates Bruno's art in a manner that I see as complementary to Yates through a geometric analysis of Bruno's rhetoric. It is an inspired performance.

Bruno saw geometry's figures as equivalent to language's figuratives, and he used both kinds of figurations to signify, refer to one another, and indicate an integrated vision of the universe and all that is in it. In his writing, geometric space/form and verbal space/form were used together to strengthen one another. As such, they worked together to fulfill a task of

extreme importance to the Brunian world view—demonstrating the interconnectedness of the celestial, terrestrial, and human worlds, and the importance of not breaking these divine bonds [2005: 17].

Saiber's exposition is organized as a progression of increasingly perfect geometric forms: lines, angles, curves.¹⁹ These geometric figures are juxtaposed in analogy to rhetorical figures: brachylogia, systrophe and hyperbaton (lines); synciosis, chiasmus, traductio (angles) and hyperbole, ellipsis, circumlocution (curves). I find these most satisfying analogies. Significantly, geometry and rhetoric are presented on par with each other. This is markedly different from the work of Leon Battista Alberti (1404-1474) who, for instance, insisted in his exposition of perspective, "I beg you to consider me not as a mathematician..." [Alberti 1976: 43]. Bruno would not have written such a thing for he believed above all in mathesis, "[a] symbolic ordering system that is not a mere mental abstraction, but a fusion of perception (thoughts) and the perceived (things in the world)" [Saiber 2005:52]. Furthermore, if Rensselaer Lee is correct in *Ut Pictura Poesis*, Alberti devised the categories of his theory of art by adapting rhetorical categories to painting.²⁰ I have always suspected that such an interpretation reflects a logocentric bias but this is not the place to argue this point. What is important for this argument, and for Bruno, is that "mathematics is more of an 'existential problem' than an 'abstract intellectual exercise'" (Hilary Gatti, in [Saiber 2005: 46]). Such an attitude allows Bruno latitude that is both breathtaking and confusing.

The idea that figures of geometry and figurations of rhetoric are reciprocal is confusing to minds whose habit is disciplinary. Rhetoric as geometry violates our contemporary sense of the order of things. Disciplines were not proprietary for Bruno as they are for us and Saiber's exposition brings this home:

The trope of hyperbaton reverberates through the *Candelaio* as a device to illuminate the intimate workings of rectilinearity by inverting and diverting order. Rectilinearity is problematic when it is rigid and closed—when it is unidirectional. It is useful to the philosopher, on the other hand, when it allows for aperture and fluidity. As Bruno envisioned a universe in which space and thought are elastic, he created a language that exhibits such a trait. There is a plasticity to Bruno's writing, to his way of extending a discourse into a 'formless' format or, rather, into a linear format with multiple directionality implicit within it. Whether he is using brachylogia, systrophe, hyperbaton, or other devices to create a sense of exaggerated accumulation or diversion, his syntax and semantics both imply a sort of 'hyper-linearity.' The one-directional-sentence-on-the-page bursts into radiating vectors of potential and possible knowledge [2005: 86].

Bruno's "plasticity" is not just a matter of expression but a habit of mind, one we can better appreciate the more we cultivate the reciprocity of rhetoric and geometry.

That which is confusing is also, always, a source for that which may be breathtaking or revelatory. Geometry is conventionally thought of as the paradigm of deductive logic by reference to Euclid's *Elements*, wherein an edifice built of 465 true propositions has stood the test of time. That geometry is the model of reasoning—from a few general, self-evident propositions to the ensuing conclusions—is obviously important in the history of thinking about thinking. We must also, however, consider the flip side of the geometric coin. Working one's way through a proof where the result is not initially obvious, there is

often a moment before which nothing makes sense and after which the conclusion is perfectly obvious. This “eureka” moment is more or less intense depending on the magnitude of our initial confusion. What I would suggest is that in the eureka moment we find a twin to the “geometry as logic” model, which might be called “geometry of revelation.” This eureka paradigm is an important model for thinking about thinking because it is the face of geometry that encodes methods and processes for dependably reproducing revelatory experiences.²¹ Giordano Bruno exists squarely in a tradition where geometry and rhetoric are reciprocal, often revelatory means of expression. Saiber’s exposition of the spatiality of his rhetoric and Yates’s exposition of the language-like geometry of his memory theatre demonstrate this, but at the end of the day Bruno’s geometry is a story minds tell themselves about the ways they work.

Multiplex forms of computational explanation: a gloss on geometry and meaning

Recently, Dominic Widdows has addressed how thinking in discursive frames may be expressed in geometric terms, how search engines work, and the ways that words may be said to be similar [2004]. As a software engineer for GoogleTM, Widdows is well-positioned to explain the mechanics of search engines: *Geometry and Meaning* centers on the “union of geometry and logic in natural language processing” [2004: vvii], signaling something not unlike the union of geometry and analysis in analytic geometry. Meanwhile, “natural language processing” is what it says – the algorithmic expression of the ways humans intuitively use language. Like Saiber, Widdows is concerned with the spatiality of language, which is implicit in the main body of the text and wonderfully glossed in a series of asides that appear as “threads of supplementary material.” At the heart of his presentation is an exposition of similarity,²² which has a strong geometric flavor.

Like equality, similarity is a relationship between two (or more) things, although it expresses a looser, fuzzier, reckoning than direct equivalence. Geometrically speaking, similarities are qualitative relations consisting of translations, rotations and/or scaling operations. In natural language processing, similarity is a rather more sophisticated operation, if only because a relationship in one domain (language) is expressed as a quantifiable ratio in another domain (mathematics). What is merely implicit in Euclidean geometry (the idea of length) must be made explicit as the geometry of language is rendered progressively more analytical.

A rigorous definition of measurable distance must be established in order for a concept of linguistic similarity to even exist. Trading on the notion that “similar” words are in some sense “closer” than others, Widdows turns to Felix Hausdorff’s (1868-1942) elucidation of the conditions necessary for metric spaces to give similarity an algorithmic basis. Given a collection of things (a set) and a collection of numbers (real numbers) one can map a set onto the reals in such a way that there is a measurable distance between all members, if the following conditions are met:

1. The distance between two members of a set (in geometric terms two points) is expressible as a non-negative real number.
2. Distance between two points is zero, if and only if the two points are equal (coincide).
3. The distance between point *a* and point *b* is the same as the distance between *b* and *a*.
4. For three objects included in the set, the distance from *a* to *b* plus the distance

from b to c has to be greater than or equal to the distance from a to c .

The first two conditions above are usually combined into the first “rule,” known as the identity axiom for distance in a metric space. The third is the axiom of symmetry, and the fourth is referred to as the “triangle inequality.” When all of these conditions are met, then and only then is it possible to measure length in some space.²³ What is most interesting are the techniques developed in natural language processing to preserve metric space while adapting it to the ways humans process relations among collections of things, words, and ideas.²⁴

What we must come to terms with are the ways human information processing is *not* metrical. Regarding the identity axiom, it is quite common that humans confuse or misattribute identical things. The perception of color is a case in point. Everyone knows the degree to which colors are dependent on their environments and how easy it is to misidentify identical colors under different conditions, belying the applicability of the identity axiom. To take another simple example, consider the interaction of time and distance. We often perceive the outbound leg of a trip to differ in subjective duration from the homeward bound leg, even when the paths taken are identical. In this case our introspective evidence violates symmetry. To understand the way triangle inequality is relaxed in the space of human cognitive processing requires a short digression.

Widdows relates the triangle inequality axiom to transitivity. As Euclid wrote “things equal to the same thing are equal to one another” (first common notion). In the context of similarity, however, matters are somewhat different. For example, a door handle is related to a door and a door is taken to be part of a room, but we would not typically say a door handle is part of a room. In other words, door handle is proximal to door and door is proximal to room in ways that things-one-grabs are not necessarily related to envelopes or locations.²⁵

Although these examples of violations of metric space in human mental processing are all too brief (and consequently subject to quibbling) the general point is obvious. Human information processing is not intrinsically metrical. What is of importance are the ways computational linguistics preserve metric spaces while adapting them to decidedly non-metric human information processing. Let us consider only one case; the manner in which distance is adapted to actual numeric measurement of similarity. Remember that similarity is a relationship between two or more things and that distance and similarity are converse (i.e., words close together are more similar and words further apart are less similar). Widdows expresses this as a ratio: the similarity between a and b is equal to one, divided by the distance from a to b . The potential problem with this expression is that it would be possible for the distance between a and b to be zero (if a and b are identical), which would be undefined in a mathematical sense.²⁶ It is better in this circumstance to tweak the similarity measure by adding one to distance between a and b in the denominator. Similarity is thus simply expressed as:

$$\text{sim}(a,b) = \frac{1}{1-d(a,b)} \quad (4.8)$$

Which would ensure that $\text{sim}(a,a) = 1$ and that for any other object b , $\text{sim}(a,b) < 1$. If desirable, this may allow us to interpret $\text{sim}(a,b)$ as the probability that a and b are interchangeable [Widdows 2005: 103].²⁷

Note the functionality effected by this simple expediency in the denominator. Given the relationship between two members of a set (words, concepts), what is a plane space in geometric terms becomes rather more conceivable as ratio ranging between one and zero. There is no real mathematical difference because, as we know from Georg Cantor (1845-1918), there is exactly as much space between one and zero as there is in an unbounded plane. Jumping among mathematical domains to solve specific problems is a characteristic of Widdows' book *Geometry and Meaning* and, more generally, of some kinds of mathematical innovation.²⁸ Such techniques allow Widdows to migrate to whichever domain facilitates solution of the problem at hand, preserving the expediency of a field of words while implementing analytic solutions as probabilities.

After developing the concept of similarity through a series of more sophisticated geometrical images and numerical expressions, Widdows offers the most radical part of *Geometry and Meaning*. Using a toolbox of techniques (including the definition of metric space and several measures of similarity) and a group of processes for deducing and creating, relationships among collections of words, documents and concepts, he demonstrates how (quantum) logic can be used to extend the range of standard (Boolean) logic. Building on the notion of similarity and adding word vectors, normalization, clustering and vector negation he establishes the following:

In Grassmann's algebra of conceptual structures in higher and higher dimensions, spaces of different dimensions are nested inside one another in a kind of lattice. This enables us to use lower-dimensional subspaces representing more specific concepts to be contained in higher-dimensional subspaces representing more general concepts. This brings some of the benefits of taxonomic classification, and it is a lot more flexible—whereas a node in a taxonomic tree has a unique path to the root, subspaces in vector spaces can trace their lineage via many different routes. For example, a line is contained in infinitely many different planes—thus, each of these planes could potentially be regarded as a 'parent concept' for the line. Grassmannian geometry naturally enables concepts to be cross-referenced [Widdows 2005: 236].

This cross referencing of concepts is easily distinguished through quantum logic where an ambiguous word (suit as in clothing, suit as in legal action) can be divided into constituent parts, any shading of which can be subtracted from a solution set to yield the most relevant match. What is radical is the concatenation of individual points into lines, planes and *spaces*, and their subtraction from even higher dimensional spaces. Expressed in linguistic terms, individual words are combined into very large fields (which in effect are isomorphic with concepts) that are cluttered with misspellings, related ideas, blended terms; in short, myriad ambiguities. It is a fundamental human competence to be able to quickly tease out of this soup a particular meaning relevant to the search (or conversation) at hand. Automating this competence with a nuance that even approaches

what we manage intuitively is very difficult. It turns out that vector negation, in the context of quantum logic, is a promising means of mathematical approximation for what we accomplish naïvely.

This brief gloss on Widdows' work is intended to suggest how a fundamental human competence with language, which at another time would have been considered rhetorical, is adapted to mathematics in general and to geometry in particular. The notion that a rhetorical competence in separating figure and ground in world fields is expressible in rigorously mathematical terms is not really provable in a theoretical frame without the mathematical apparatus that Widdows presents so admirably. What might constitute a demonstration is a working model that accurately deduces the objective of my query, effectively eliminating 98.9% of irrelevant matches while offering 99.8% of all relevant matches. The closest example of this, in contemporary terms, is a good search engine, and what is remarkable is not how relatively poorly they currently function, but that they work at all. I for one am wildly happy to have the web at my fingertips, and *Geometry and Meaning* offers, among other things, a very useful briefing on the current state of the art, its mathematical basis and potential evolution. Meanwhile, we are left with continuous (quantum) and discrete (Boolean) logics, illustrated in geometric terms, which help very dumb machines respond to very sloppy rhetorics.

In conclusion, I suggest there are geometries that function from a brain's-eye point of view in rhetoric that are analogous to geometries that function in the visual perception of space; i.e. adapting a qualitative geometric measure (in this case similarity) to a quantitative purpose, disambiguating words, concepts and languages. While it is remarkable that geometry and rhetoric are commensurable in both mind's-eye and brain's-eye frames, it is even more remarkable that we are so good at performing such translations and transliterations. Even though we may not be consciously aware of the mechanisms we apply in everyday tasks, all of us utilize geometry with remarkable sophistication.

Finally, I suggest that it is brain's-eye geometries that constitute thinking in pictures and that although the stories I may tell myself about the ways I think with images are more or less plausible fictions, the conviction that there is a non-propositional, non-textual, non-representational substratum to the ways we cope with the world, persists. Exactly what that looks like and precisely how it works are matters of ongoing research – in built works as well as in theory. Meanwhile, how I think we think is changing based on what I think we know in experience.

Notes

1. The most obvious confusion of this kind is the misplaced metaphor of the eye as a camera. Eyes do not work like cameras in any important way and the fact that they both share lenses has proved to be most misleading.
2. I am aware of the low repute with which introspection is regarded in psychological literature and, frankly I do not agree. What I take to be introspection is a blend of nuanced observation of our own processes in perception and experimental evidence from psychology, neurobiology and other cognitive sciences. This kind of awareness is a necessity component of the recursiveness fundamental to thinking about thinking. Francisco Varela, Evan Thompson and Eleanor Rosch capture it well:

In this paragraph, we see again how the fundamental circularity with which we began this book comes to the fore. To explain cognition, we turn to investigate our structure – understood in the present context as our computational mind. But since it is also cognition as experience that we wish to explain, we must turn

- back and attend to the kinds of distinctions we draw in experience – the phenomenological mind. Having attended to experience in this way, we can then turn back to enrich and revise our computational theory, and so on. Our point is not at all that this circle is vicious. Rather, our point is that we cannot situate ourselves properly within this circle without a disciplined and open-ended approach from the side of experience [Varela, Thompson and Rosch 1991: 54].
3. We might agree to characterize the reductive scientist as one who insists mental capacities are not real until they can be computationally modeled and that only things that are susceptible to such modeling are to be taken seriously. We might agree to characterize humanists who only consider irrational motivations for human action or theorists blind to everything but talk of desire as “mystical humanists.” The twin unreasonableness of “purely reductive scientists and mystical humanists” is itself a reductive obfuscation and yet, it remains a touchstone of my thinking that both extremes are equally non-productive.
 4. In ancient philosophy Plato’s allegory of the cave may be taken as the myth of origination. A modern incarnation is Arthur I. Miller’s *Imagery in Scientific Thought* [1986], which is good evidence that the contemporary imagistic controversy owes a great deal to the epistemological reflections of modern physicists. Here is a sample from Albert Einstein (1879-1955):

What, precisely, is “thinking”? When, at the reception of sense-impressions, memory-pictures emerge, this is not yet “thinking.” And when such pictures form series, each member of which calls forth another, this too is not yet “thinking.” When, however, a certain picture turns up in many such series, then—precisely through such return—it becomes an ordering element for such series, in that it connects series which in themselves are unconnected. Such an element becomes an instrument, a concept [Miller 1986: 43-4].
 5. I take Barbara Maria Stafford’s work to be a particularly good guide to the varieties of embodied seeing in the context of contemporary cognitive science and art making.

What specifically interests me is how the brain-mind opportunistically seizes the structures it encounters, and how the surroundings, in turn, are repercussive, reflecting back the patterns the viewer seeks. ... nature and mind dynamically operate on one another. [They are] embedded in a mutual sentence [2007: 95].
 6. Sadly, the dualism creeps back in at the “seat of cognition” phrase. In fact, I am not at all convinced that the mind/brain is the seat—but that is a different essay.
 7. I do not believe this definition is reductive because of the feat of balance between consciousness and cognition managed in *The Embodied Mind* [Varela, Thompson and Rosch 1991].
 8. For a developed version of this idea, see [Varela, Thompson and Rosch 1991], especially chapter 4.
 9. [Brooks 1991, 3]; see also [Varela, Thompson and Rosch 1991: 208].
 10. There is a good alternate description in Smolensky:

In the von Neumann machine, memory storage is a primitive operation (you give a location and a contents, and it gets stored); memory retrieval is also a primitive operation. In subsymbolic systems these processes are quite involved: they’re not primitive operations at all. When a memory is retrieved, it’s a content-addressed memory: part of a previously instantiated activation pattern is put into one part of the network by another part of the network, and the connections fill out the rest of that previously present pattern. This is a much more involved process than a simple “memory fetch.” Memories are stored in subsymbolic systems by adjusting connection strengths so that the retrieval process will actually work: this is no simple matter [1987: 5].
 11. The edifice of Euclidean geometry is an impressive accomplishment and justifiably the standard against which all logical argument was compared, up until the nineteenth century when David Hilbert found some critical logical flaws in the assumptions, the symbols, and the rules. See [Reid 1970] for an expert exposition of this story.
 12. What used to be called craft also falls into this domain (see [Paz 1974]). Further, these are but

two of the catalog of thirteen perspective effects given in the work of J. J. Gibson [1950], summarized by Edward T. Hall in his excellent book on social space [1966: 178-182].

13. This excerpt from “Affine structure from motion” gives a sense of a few of the necessary qualifications involved:

It is assumed that you: 1) can identify any given fiducial point in the different views and thus that a correspondence has been established. 2) have access to the full apparatus of spherical trigonometry: you may measure the angles between pairs of visual rays (apparent size) as well as the dihedral angles defined by triples of visual rays. 3) know a priori that the spatial configuration of the fiducial points is rigid. That is, you may assume that the mutual distances between arbitrary pairs of fiducial points in three-dimensional space are equal in the case of all N views. ... Then the problem is to find the spatial configuration of the M points (the shape) as well as the position of the vantage point relative to that configuration for the N views [Koenderink and van Dorn 1991: 377].

14. [Marr 1982] is relevant to this subject.

15. The conjecture alluded to depends on modifications and extensions of Kubovy’s argument [1988]. If warranted they could be subjected to experimental falsification.

16. Evidence for such was discovered by Bela Julesz and reported in *Foundations of Cyclopean Perception* [1971].

17. Nicely captured in the familiar dictum: all perspectives are projective but not all projectivities are perspectival.

18. This quote from Eric Auerbach gives a nice sense of the central importance of this treatise:

The art of the hinting, insinuating, obscuring, circumlocution, calculated to ornament a statement or to make it more forceful or mordant, had achieved a versatility and perfection that strike us as strange, if not absurd. These turns of speech were called *figurae*. The Middle Ages and the Renaissance, as we know, still attached a good deal of importance to the science of figures of speech. For the theorists of style of the twelfth and thirteenth century the *Ad Herennium* was the main source of wisdom [1973: 27].

19. “A note about the progression of the chapters from ‘lines’ to ‘angles’ to ‘curves’ is due. The order is an allusion to the widely held Renaissance belief that the circle (or, in three-dimensions, the sphere) was the most perfect of forms, and that all other figures were inevitably deficient in comparison. A straight line was less perfect than a triangle, a triangle less perfect than a square, a square less perfect than a pentagon, a pentagon less perfect than a hexagon, and so on until a figure reached ‘gon-less’ perfection—in other words, it had become a circle” [Saiber 2005: 4].

20. The long quote from Lee is as follows:

Professor Panofsky has called to my attention the fact that Alberti’s threefold division of painting represents an indirect adaptation, long before Dolce’s direct adaption, of the rhetoricians’ *inventio*, *dispositio*, and *elocutio*: *inventio* being partly included by Alberti under *compositio* (where he speaks of arrangement, decorum, etc.) and mentioned once, in its own name, at the end of his book in connection with his advice concerning literary knowledge; *dispositio*, the preliminary outline of the orator’s discourse, being represented also by *compositio* ... [and] also by *circonscriptione*, the outline drawing through which the disposition of figures in a sketch would chiefly be made; and *elocutio*, the actual performance of the oration, by *receptione di lumi*, the rendering of the picture [1940: 71].

21. It is understandable why mathematicians and philosophers of science are shy of treating geometry as the technology of revelation – it undermines their professional status as hard nosed realists – and it is necessary, in my estimation, to acknowledge the ration of geometry that does not fit. Talk of the eureka moment has been current in the literature for decades, should it not be realized for what it is – a way of dependably reproducing revelatory experiences?

22. With apologies to Dominic Widdows for the unauthorized revision of his chapter heads: Chapter one is about the similarity of numbers and sets; chapter two the similarities among words; chapter three the similarities in taxonomies; chapter four, measuring similarity. Chapters five and six are about the similarities between vector spaces and word fields; chapter seven, discrete and continuous logics; chapter eight, ordered sets and concept lattices.
23. As it appears in Widdows [2004: 100]:
Definition 6 Let A be a set and let $d: A \times A \rightarrow \mathbb{R}$. The function d is a metric on A if and only if the following axioms hold.
 1. $d(a,b) \geq 0$ for all $a, b \in A$, and $d(a,b) = 0$ if and only if $a = b$.
 2. $d(a,b) = d(b,a)$ for all $a, b \in A$. (So d is symmetric.)
 3. For all $a, b, c \in M$, $d(a,c) \leq d(a,b) + d(b,c)$.In this case d is called a metric on A and (A, d) is called a metric space.
... The three metric space axioms, and the term ‘metric space’ (*Metrischer Raum*) itself, were introduced by Felix Hausdorff (1914, Ch 6), ...
24. For a philosophical argument drawing on similar ideas in the context of *image schema* (“a recurring, dynamic pattern of our perceptual interactions and motor programs that gives coherence and structure to our experience”); see [Johnson 1987: 39ff].
25. For a more nuanced discussion, see [Widdows 2004: 114ff].
26. In a linguistic context, this would mean that there is such a thing as infinite similarity, which there may be, but it is difficult to handle mathematically.
27. It should be noted that there are many measures of similarity and that even the practical utility of similarity has been attacked; see [Gärdenfors 2000: 109ff]. Yet the notion that, for instance, yellow and orange are similar by some measure that yellow and black are not, intuitively captures something about the ways we use language that is worth preserving.
28. For Descartes’s jumping between geometry and analysis, see Felix Klein (1849-1925) [1893]; for jumping among different kinds of geometries, see [Rees 2005].

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