Morphocontinuity in the work of Eero Saarinen

Abstract. Continuity as the mathematical tool in the creation of architectural forms is known as morphocontinuity. In the present work, we explain how morphocontinuity appears on the work of Eero Saarinen and discuss its correspondence with its environmental (physical, social and cultural) contexts.

Keywords: Eero Saarinen, mappings, transformations

Introduction

The buildings designed by Eero Saarinen are constituted by continuous forms that have the particular characteristic of unifying the internal and external spaces. The equilibrium of the compositions is achieved by the relationship between the visual spaces and the existent movement between them.

Here we discuss Saarinen's Kresge Auditorium at MIT, the Ingalls Rink at Yale University, and the TWA Flight Center at JFK International Airport in terms of morphocontinuity. The thin shell structures are interpreted as continuous forms and these forms appear mathematically as the membranes of cells [Consiglieri and Consiglieri 2009]. These new forms supersede discrete processes such as breaking up and scattering. Rather, they are characterised by fluidity and unity. Moreover, the architectural design flows into vaster urban areas than mere buildings. This development coincides with our concept of morphocontinuity. The movement and complexity transmitted by the surfaces have a new meaning. The main achievement is then that the building is no longer a closed object but is now an open one. The symbiosis between cities and nature is achieved when no boundaries exist between private and public domains, and between interior and exterior spaces. The form can give rise to a multitude of effects which offer light and sound behaviors in a wide variety of potential directions. The inner spaces, which flow one into one another, are becoming a concern in the reconfiguration of cities. The process of production of the buildings is the result of the interaction between organization and material.

Morphocontinuity is correlated with the morphologic rhythm. It is the four-dimensional framework due to the self-organized life process of a cell [Consiglieri and Consiglieri 2006]. The mathematical cell is a 4D element reflecting cyberspaces. Actually, when we move about in a city, whether inside or outside the buildings, the act of moving brings the fourth dimension into the behavior. This new organization provides the input for the ongoing existence of cities. With the morphologic rhythm, the cell and its correspondent form provide the dynamism for the human living in the different environments.

The present study uses mathematical mappings as a means of analysis. Moreover, their graphical representation shows the visual contrasts of fullness/emptiness and brightness/darkness. Although not all of Saarinen's projects include Euclidean forms, the architectural forms have such differentiability properties that equilibrium is reached. Finally, the aesthetic value of the object is determined by the continuity of the forms and the relationship between inside and outside.

The mathematical framework

In this section, we introduce some definitions for a fixed positive time parameter, $T > 0$.

Definition 1. We say that a set C is a *cell* if it is defined by

$$C := \{(x, y, z) \in \mathbb{R}^3 : f(x, y, z; t) \leq 1\}, \quad (1)$$

with x, y, z representing (as usual) the spatial Cartesian coordinates and f denoting a continuous mapping defined in the three-dimensional Euclidean space and dependent on a time parameter $t \in [0, +\infty[$ (for details, see [Consiglieri and Consiglieri 2006]).

Fig. 1 displays four 2D-cells, namely the graphical representation of circles of radius 1:

(a) at instants of time $t = 0$ and $t = 2$ when $f(x, y; t) = x^2 + (y - t)^2$.

(b) at instants of time $t = 0$ and $t = 1$ when $f(x, y; t) = x^2 + (y - t)^2$.

(c) at instants of time $t = 0$ and $t = 1$ when $f(x, y; t) = |x|^{2-t} + |y|^{2-t}$.

(d) at instants of time $t = 0, t = 1$ and $t = 2$ when

$$f(x, y; t) = \left(|x|^{t^2/2-3t/2+2} + |(2t+1)y - 5t^2|^{t^2/2-3t/2+2} \right) / (1 + t(t-1)).$$

Fig. 1. (a) Euclidean rhythm: translation and juxtaposition; (b) interpenetration of congruent objects; (c) self-organized life cell; (d) movement and shape change

Definition 2. We say that a cell C has a *morphologic rhythm* if

$$\exists R > 0, \quad C(t) \subseteq B_R(0), \quad \forall t \in [0, T],$$

where $B_R(0)$ denotes an open ball centered at the origin $(0,0,0)$ with radius $R>0$.

The above examples take one ball in 2D as a motif (see [Consiglieri and Consiglieri 2006] for 3D examples) under a rigid body motion where the transformation is such that the two balls act as the unique set (1) that has a free parameter denoting the time. In order to represent the juxtaposition (fig. 1a), choosing $t=0$ we get the initial ball and choosing $t=2$ it represents the translated ball. If we choose smaller instant of time, for instance $t=1$, we obtain an interpenetration of the congruent ball (fig. 1b).

The self-life of a cell may be clarified as follows. If we take the circle of radius one, its shape can freely change maintaining the quality of its boundary, the circumference, in separating its interior from its exterior, e.g., becoming a square (cf. fig. 1c). Moreover, the morphologic rhythm can have the following interpretation: the ball can even perform motion and shape changes simultaneously (cf. fig. 1d).

Definition 3. We define a *form* as the subpart of the boundary of the cell given by

$$F := \{(x, y, z) \in A \times \mathbb{R} \subseteq \mathbb{R}^3 : z = h(x, y, t)\},$$

where h is a continuous mapping defined in A depending on a time parameter $t \in [0, +\infty]$ (for details, see [Consiglieri and Consiglieri 2009]).

The mathematical space is the infinite set of points given in reference to the three dimensional Cartesian coordinate system. However, the architectonic space can be understood as an urban space given by the continuum 3D-space plus the time which are embedded in the 4D (space-time) coordinate system (see fig. 2).

Fig. 2. The graphical representation of a form using $A = [-2, 2]^2 \setminus \{(0, 0)\}$, for the time parameter

$$t > 0 \text{ and } h(x, y, t) = -\frac{x^4 y^4}{(x^2 + y^4)^t} : (a) \ t=2; (b) \ t=3.$$

The equilibrium should be understood as the aesthetic harmony of the visual and physical movements of the curves of the surfaces, i.e., of the properties of continuity and differentiability of the forms. At the moment, there is no a unique solution to a perfect equilibrium if the equilibrium is interpreted in terms of the dynamical point of view just described. In order to explain the present interpretation, a brief look back at the past is helpful. The Greeks aimed to obtain unifying effects, with the objective of abolishing and balancing the ascendant and descendant forces, without destroying the perception of the volumes. Using the same principles, the Roman edifices were more complex and appeared to be lighter, notwithstanding the fact that they were still massive buildings. During the Gothic period, the structures are dominated by arches and buttressing elements. These systems of equilibrium were structurally stable due to the application of proportions. With the dome of Santa Maria del Fiore cathedral in Florence by Brunelleschi [Argan 1955], the fusion between constructive technique and aesthetic emotion is managed by load methods. The movement, dynamism and complexity transmitted by the surfaces in the Baroque period were given by the successive repetitions of fragmented pieces, but plastic expression was an entity disassociated from the functionalities of the building. However, even in this era the essential characteristic of exuberance in architecture could be achieved by infinitesimal analysis [Frankl 1981: 177]. As long as the structure of a building was conditioned by the weight of the volumetric masses and the distribution of the loads, the form does not express its structures. The form is subordinated to the balance of its volumetrics. Only in the modern era does the structure become the form itself. It stands up to the plasticity. The plasticity introduced in architecture affords the possibility of realizing new configurations.

The Kresge auditorium

Extracted from the Euclidean object, the sphere, the concrete roof is the one-eighth of a sphere anchored at hidden abutments at three points [*L'architecture d'aujourd'hui* 1956; Wright 1989] (fig. 3).

Fig. 3. Eero Saarinen, Kresge Auditorium, MIT. Drawing by Teotónio Agostinho

Under the morphocontinuity approach, the concrete shell can be understood as a form and the building as a cell (see Definitions 3 and 1, respectively). This simple form is one solution for the problem of roofing of large spaces. At the same time, it engages both

the senses and the intellect. It achieves an aesthetic value, although some critics argue that the visual form is not in correlation with its function (for instance, Bruno Zevi, quoted in [*L'architecture d'aujourd'hui*: 50-51]).

Mathematically, 1/8 of a sphere can be represented as the form defined by:

$$F = \{(x, y, z) \in \mathbb{R}^3 : x, y \geq 0, z = \sqrt{R^2 - x^2 - y^2}\},$$

where the constant $R > 0$ denotes the radius. The cell correspondent to the auditorium is not 1/8 of a sphere, but is rather the set

$$C = \{(x, y, z) \in \mathbb{R}^3 : x, y, z \geq 0, x + y + z \geq R, x^2 + y^2 + z^2 \leq R^2\}.$$

This cell represents the closed set constituted by the inner space surrounded by the (1/8 of the) sphere and the planar section (fig. 4) and its boundary, namely the portions of the sphere and the plane. In order to identify the set C with the auditorium in accordance with Definition 1, the set C is rotated such that the triangle becomes the ground floor, i.e., xy -plane ($z=0$), under the rotation over the axis located at $(x, R-x, 0)$, for all real x . The projection of the boundary of the spherical surface is no longer the triangle after rotation, but is rather the union of three arcs of ellipses.

Fig. 4. (a) 1/8 of a sphere; (b) planar section

Moreover, the building is characterized by its volume, which is calculated as the difference between the volume integral under spherical coordinates that represents the volume of 1/8 of the ball and the one that represents the volume of the tetrahedron

$$V = \int_0^{\pi/2} \int_0^R \int_{\pi/2}^{\pi} \rho^2 \sin \varphi \, d\varphi \, d\rho \, d\theta - \int_0^R \int_0^{R-x} \int_0^{R-x-y} 1 \, dz \, dy \, dx = \frac{\pi}{6} R^3 - \frac{1}{6} R^3.$$

At a first interpretation, the spherical structure separates the inner from the outer spaces. However, the internal environment changes in accordance with different situations of the two systems: doors and windows. The sensation of space-time depends on the location of the doors, and the communication with exterior depends on the dimensions of the windows. Besides, the interconnection of the interior with the exterior

is quite different if the openings are located at a lower or a higher level. At the superior level, the relationship with the exterior is panoramic. If it is located at the ground level, it creates movement toward the different directions and human activities. It reflects that the outside is always another inside, as argued by Le Corbusier [Consiglieri 1994: 95].

Here, the transparency of the glass walls dematerializes the boundary between the interior and the exterior spaces, dissolving the traditional dichotomy between inside and outside.

The Ingalls rink

Fig. 5. Eero Saarinen, Ingalls Rink, Yale University. Drawing by Teotónio Agostinho

Given the couple composed of the rectangular rink and the seats that surround it, which forms the basis for the architectural object, the principal facade is surprisingly constituted by an awning with the boundary edge measuring 73.0 m (fig. 5). The main purpose of this shell is to funnel the audience from outside into inside through the mediating point, which is the entrance. This entrance, which faces onto and simultaneously ends at the lateral walls, flows into the walls of the parking and the external arrangements. The axis of symmetry of the suspended vault coincides with the axis of the rink and the corresponding lateral seats, establishing the unity of the inside with the outside. Thus we can say that the building is constituted by two twin cells, each of which is highlighted by a form under the continuous mapping:

$$h : A = [-1,1] \times [-1,1] \rightarrow \mathbb{R}$$

defined as:

$$h(x, y) = \sin(a x) \cos(b y)$$

for some constants $a, b \in \mathbb{R}$ (compare to the FresH2O pavilion, Netherlands, 1998, by Nox team and Kas Oosterhuis [Consiglieri and Consiglieri 2009]). The intertwining of functional spaces and facades recreates a cyclical and shared feeling around the building.

The TWA Flight Center

The roof of the TWA terminal is constituted by four free-flowing shell structures suggesting flight and having a thickness that varies from 20 cm to 1.10 m (fig. 6). This variation reveals the development of new structures and formal systems. The flying shells are continuous forms of the kind of *non-poids* in architecture theorised by modernist European architects [Robichon 1965]. The various interior levels are linked by stairs and sculptural galleries. This fluid space transforms our visual perception. Indeed, the created cells are provided by an emotional movement because the images change under our visual perception. This movement can be mathematically interpreted by the time parameter in eq. (1). The organization is commonly compared to the cyclical nature of the Möbius strip folding back onto itself. The application of topological deformation of a surface can lead to the intersection of external and internal planes in a continuous morphological change and insert differential fields of space and time into an otherwise static structure [Emmer 2004: 65].

Fig. 6. Eero Saarinen, TWA Flight Center, JFK International Airport.
Drawing by Teotónio Agostinho

In addition to the glass in this building acting to dissolve the boundary between interior and exterior, as it did in the other two buildings considered earlier, here the

different sized surface units are distributed such that the modulation of the interior and exterior spaces becomes irrelevant. Moreover, they organize the internal environment and maximize the penetration of sunlight. The final object is constituted not only by different forms but also by different cells, which include the interior of such forms.

Conclusions

In order to interpret the chosen works conveniently, we consider two types of interpretations for the structure:

1. the structure is understood in terms of its physical characteristics and mathematical methods. In this point of view, the structure is defined by the characteristics of the materials and the loads;
2. the structure loses its importance in relation to the final object and is driven by mathematical precision and clarity.

The works studied here have roofs of different forms. Although they are all limited by the use of concrete, the three buildings show that the dynamic search is possible. We can conclude that the study of eurhythm is based on the concepts of continuity and differentiability of the object itself and continuous movement between interior/exterior.

As far as contemporary interpretations of architectural space (namely utopian space [Roisecco 1970: 52-53, 364-445], existential space [Norberg-Schulz 1977: 430-433], emergent, biomorphic or organic spaces [*Architectural Design* 2008]) attempt to imitate the biological behaviour of self-organization, along with the simultaneous, ongoing search for new materials that attends these new concepts, these concepts are still included in our definition of morphologic rhythm and consequently in morphocontinuity.

Saareninen's oeuvre shows that designing morphogenetic buildings inspired by fractals is not the only way to create a perfect symbiosis between nature and functional behaviour. At a macroscopic level, the concrete structures seem to be modeled with the same plasticity as contemporary architectural objects using the new materials inspired by nanotechnology. Structurally, concrete is relatively economical, in that it affords strength through curvature and form. The goal is to enhance the surface's contact with air and sun through characteristics such as a maximisation of the surface-to-volume ratio. Although these three buildings were built before the invention of today's biomaterials, they still demonstrate the capacity to affect the flow of rainwater and wind as well as to absorb solar energy. Moreover, because the buildings cannot successfully exist in isolation, the aesthetic value of the objects discussed here is still influential today, since the objects are situated in the dynamic centre of even more dynamic surroundings.

The equilibrium of the composition of the object is achieved through the continuity and differentiability of its elements. The main difference between the equilibrium of the auditorium and that of the other buildings is that the auditorium is based on an Euclidean form, while the cells of other buildings are non-Euclidean. However, the equilibrium of their spatial elements of all three buildings is based on the fluidity between the inside and the outside.

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References

- L'architecture d'aujourd'hui*. 1956. Auditorium du M.I.T., Cambridge, U.S.A., Eero Saarinen et Associés, Architectes. *L'architecture d'aujourd'hui* **64**, 27 (Mars 1956): 50-53.
- Architectural Design*. 2008. Neoplastic design. *Architectural Design* **78**, 6.
- ARGAN, G.C. 1955. *Brunelleschi*. Arnoldo Mondadori Editore (BMM 415), Veronesi.
- CONSIGLIERI, L. and CONSIGLIERI, V. 2006. Structure of phenomenological forms: morphologic rhythm, *Nexus Network Journal* **8**, 2 (October 2006): 101-110.
- . 2009. Continuity versus Discretization. *Nexus Network Journal* **11**, 2 (2009): 151-162.
- CONSIGLIERI, V. 1994. *A morfologia da arquitectura 1920-1970*, vol. 2. Ed. Estampa, Lisbon.
- EMMER, M. 2004. *Mathland: from flatland to hypersurfaces*. Birkhäuser, Basel.
- FRANKL, P. 1981. *Principios fundamentales de la Historia de la Arquitectura. El desarrollo de la arquitectura europea: 1420-1900*. Ed. Gustavo Gili, S.A., Barcelona 1981. Translated by *Die Entwicklungsphasen der Neueren Baukunst*. Ed. B.G. Teubner, Stuttgart 1914.
- NORBERG-SCHULZ, C. 1977. *La signification dans l'architecture occidentale*. Pierre Mardaga éditeur, Bruxelles. Translated by *Significato nell'architettura occidentale*. Electra Editrice, Milano 1974.
- ROBICHON, R. 1965. Vers le non-poids en architecture. In *Structures nouvelles en architecture*. Conservatoire National des Arts et Métiers, Paris (Avril 1965), 10-11.
- ROISECCO, G. 1970. *Spazio, evoluzione del concetto in architettura*. Mario Bulzoni Editore, Roma.
- WRIGHT, S.H. 1989. *Sourcebook of Contemporary North American Architecture: From Postwar to Postmodern*, Van Nostrand Reinhold, New York.

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