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Research

Perspective versus Stereotomy: From Quattrocento Polyhedral Rings to Sixteenth-Century Spanish Torus Vaults

Abstract. Quattrocento perspective and Spanish sixteenth-century stereotomy share a number of concepts, problems and methods, although there seems to be no direct substantial connection between them. This suggests the existence of a common source, but it is not easy to identify it. Neither classical geometry nor the mediaeval practical geometry tradition include a word about orthographic projections, rotations or projection planes. Thus, mediaeval construction shop practices furnish the most probable common source for perspectival and stereotomic methods. Curiously, these practices are seldom mentioned in the exhaustive literature on perspective; even the use of orthogonal projection, although impossible to deny, is not often stressed. On the other side, Gothic tradition is recognised, at least in Spain, as an important source of Renaissance stereotomic methods. By contrast, the role of perspective and Italian and Italianate artists, which has been downplayed so far, should also be taken into account as a source of Renaissance stereotomy.

Introduction

When trying to assess the sources of the geometrical knowledge of Renaissance stonemasons in his now classic book *L'Architecture à la française*, Jean-Marie Pérouse de Montclos made a startling suggestion: taking into account the lack of a clear description of orthographic projections in Mathes Roriczer's booklets, stonemasons could have taken their geometrical expertise, at least in part, from the scientific description of space in perspective treatises, ranging from Euclid's *Optics* to Nicéron's *Perspective curieuse* [de Montclos 2001: 184-185].

At first sight, the idea seems to be rather far-fetched. The central problem in classic masonry construction – the division of an architectural member into voussoirs and the geometrical control of the dressing process by means of templates and other methods, known from the seventeenth-century on as stereotomy – is always solved by means of double or multiple orthographic projections (see for instance [Palacios 2003; Sakarovich 1997; Rabasa 2000]). The stonemason starts the construction process of a complex arch or vault by preparing a full-size tracing involving at least two orthographic projections of the member, a plan and an elevation or a cross-section. At this point, the mason can choose between two basic methods. When using the direct method (*taille par panneaux, labra por plantas*), the stonemason must prepare full-scale templates of the voussoir faces, either by rotation or development of the distorted faces of the voussoir. Alternatively, the mason can use the squaring method (*dérobement, équarrissement, labra por robos*). In this method, orthographic projections are essential, since the mason materializes the

projection planes of the lines in the tracing by means of the square, reversing the projection process, in order to dress the voussours [Palacios 2003: 18-21; Calvo 2003].

In any case, both the direct and the squaring methods start from orthographic projections, that is, from a particular case of parallel projections. Of course, this is not the case in linear perspective, since all projecting lines converge in the station point or centre of projection, in the same way that the generatrices of a cone converge in its vertex. To put it into Albertian terms, all the rays that form the visual pyramid converge in the eye of the observer. Thus, linear perspective belongs to central or conical projection, as opposed to parallel or cylindrical projection.¹

Fig. 1. Drawing of a chalice. Uffizi 1758 A

However, there is at least a connecting point between Renaissance perspective and stereotomy. Piero della Francesca [c. 1480: 37v-41 v], takes great pains to explain the construction of an unusual figure, a wooden polyhedral ring (*mazzocchio*, *torculo*), whose only function seems to have been to support the clothes of some elaborate headgear of the period (see figs. 7, 8, 9 below). A number of perspectival studies for rings in the Uffizi and the Louvre, as well as *Chalice* surrounded by three rings, also in the Uffizi suggest that this figure acted as a typical Renaissance perspective exercise (fig. 1; see also figs. 5, 6 below). These rings also appear in some *intarsia* or inlaid wooden panels of the period, in particular in the *studioli* or private cabinets of Federico de Montefeltro at Urbino and Gubbio, playing the role of a symbol of the geometry amongst the liberal arts. They are also to be found around the necks or over the heads of some characters in Paolo Ucello's paintings, such as the *Flood* in the Chiostro Verde in Santa Maria Novella (see fig. 10 below) or the *Battle of San Romano* (see also [Davis 1980; Kemp 1990: 32-33, 44; Evans 1995: 173-175; Raggio 1996; Davis 2001; Roccasacca 1998; Roccasacca

2000; Roccasecca 2001a; Roccasecca 2001b; Roccasecca 2001c; Field 2005: 168; Esteve 2007]).

Geometrically speaking, these *torculi* are akin to toruses; that is, surfaces of revolution generated by the rotation of a circumference around an axis that lies in the plane of the circle but does not intersect it. During the sixteenth century, a number of torus vaults were built in ashlar masonry in Spain; the most significant ones are the vault in the inner chamber or *recapilla* of the chapel of Gil Rodríguez de Junterón in the Cathedral of Murcia (see fig. 2) and the annular vault around the courtyard of the palace of Charles the V in Granada.

Fig. 2. Chapel of Gil Rodríguez de Junterón in the Cathedral of Murcia. 1525-c. 1545.
Photograph by David Frutos

A number of Italian or Italianate figurative artists were connected to these vaults, at least in the first design stages. Gil Rodríguez de Junterón asked for permission to build his chapel on March 27, 1525. At that moment, the master mason in the Cathedral of Murcia was Jacopo Torni, known in Italy as *L'Indaco vecchio* and in Spain as *Jacobo Florentino*, a painter trained in Ghirlandaio's workshop; later on, he worked with Pinturicchio and Michelangelo. Torni died in 1526, so he cannot have carried substantive work in the chapel; instead, the supervision of the works was probably carried out by Torni's successor, Jerónimo Quijano, since he was listed as a creditor for the chapel's altarpiece in Junterón's will [Gutiérrez-Cortines 1987: 164-167; Vilella 1999]. As for the palace of Charles the V, the general plan and the direction of the first phases of construction were entrusted to Pedro Machuca, a painter who had stayed in Italy for a number of years, probably in Raphael's circle. However, Pedro Machuca died in 1550 and the annular vault was built by his son Luis between 1562 and 1569 [Rosenthal 1985: 96-97, 116-118]. Although it is usually taken for granted that Luis Machuca had made a trip to Italy, the circumstances of this journey are not clear.

It is also worthwhile to remark that both polyhedral rings and torus vaults seem to have played a certain role as icons of geometry, perspective and stereotomy. We have already mentioned the ring in the panelling of the cabinet of Federico de Montefeltro in Urbino, placed among the symbols of the liberal arts as an emblem of geometry. Daniel Barbaro included as chapter headings in *La pratica della prospettiva* a fair number of toruses of different sorts, both in perspective and orthographic projection [Barbaro 1569:

3, 25, 43, 129, 159, 197] (fig. 3). The title page of one of the extant copies of Alonso de Vandelvira's stonemasonry manual, *Libro de trazas de cortes de piedras*, prepared by Felipe Lázaro de Goiti [Vandelvira 1646], includes two stereotomic tracings,² placed symmetrically at both sides of a central panel, and a third one below it (fig. 4). The tracing at the right of the central panel is a general scheme that can represent a number of stereotomic problems solved by squaring; by contrast, the motif at the left depicts the template-construction method for a horizontal-axis torus vault. Thus, the torus vault plays the role of an emblem of one of the basic methods in stonemasonry, just as toruses act as icons of geometry and perspective in Urbino panelling or Barbaro's treatise. All this suggests that Quattrocento polyhedral rings and sixteenth-century Spanish torus vaults can furnish an interesting case study on the connections between perspective and stereotomy and, in a more general way, between science, architecture and the figurative arts in the Early Modern period, both in Italy and Spain.

Fig. 3 (above). Torculo as a chapter heading. Daniele Barbaro, *La pratica della prospettiva*, 1568

Fig. 4 (right). Copy of Alonso de Vandelvira, *Libro de trazas de cortes de piedras*, by Felipe Lázaro de Goiti. 1646. Detail of frontispiece

Polyhedral rings as an exercise on perspective

According to a well known passage in Vasari [1568: 269], Paolo Uccello used to spend days and weeks alone, “*solo e quasi selvatico*”, solving beautiful and difficult perspective problems about rotating arches and ribs, scaffoldings, or round columns placed at building corners, which brought him poverty rather than fame. One day he showed Donatello a number of sketches of rings depicted in perspective from different

angles and a 72-face ball; the sculptor felt sorry for his friend and replied that these perspectival exercises were useful only to marquetry specialists.

A number of drawings of toruses in the Uffizi, such as 1756A (fig. 5) and 1757A (fig. 6), have long been ascribed to Uccello [Kern 1915], taking into account these passages, and also the actual ring in Uccello's *Flood* in the Chiostro Verde in Santa Maria Novella (see fig. 10 below). However, this attribution has been contested recently [Roccasacca 1998; Roccasacca 2001c], at least for 1757A, ascribed by Roccasacca to the Sangallo circle, together with another torus drawing in the Uffizi, 832Ar. As for the *Chalice*, Roccasacca [2000] suggests a later date, in the seventeenth century at the circle of Evangelista Torricelli and the Accademia del Disegno in Florence. Thus, we will leave aside these drawings for the moment and come back to them after dealing with Piero's methods.

Fig. 4. Drawing of a mazzocchio. Uffizi 1756 A

Fig. 5. Drawing of a mazzocchio. Uffizi 1757 A

At the beginning of the third book of Piero della Francesca's *De prospectiva pingendi*, after a number of introductory problems, Piero explains in great detail how to construct a perspective of a torus with eight horizontal parallels, divided in twelve sections by means of meridians [Della Francesca c. 1480: 37v-41v; Field 2005: 168].³ As in drawings 1756 A and 1756 B, Piero substitutes an octagon for the meridian, while he depicts the parallels as circles, at least in the first stages.⁴

He starts his construction preparing a plan (fig. 7) and a profile (fig. 8) of the torus; he includes also in both drawings the station point and a line representing the picture plane of the perspective he is planning to draw [cf. Talbot 2006, suggesting the use of plan and elevation in the Uffizi *Chalice*]. To begin with, he constructs the cross-section of the torus in the shape of an octagon, inscribing it in a square "*de la quantità que tu vole fare grosso il torculo*", that is, the lesser diameter of the torus [Della Francesca c. 1480: 37v];⁵ this octagon, drawn in the profile, will act as a generatrix or meridian of the torus. Next, he constructs a number of circular parallels or directrices of the surface in

the plan, taking into account “*la quantità che tu intendi fare grande il torculo*”, that is, the larger radius of the torus, and the horizontal projections of the corners of the octagon, which he transfers from the profile to the plan using the compass. He explains dutifully that since the ring is “*giacente piano*”, that is, lying horizontally, he only needs to trace four parallels. However, in the next step, he divides the parallels in twelve parts; each of the twelve division points gives the position of a meridian.

Fig. 7. Plan of a mazzocchio. Piero della Francesca, *De prospettiva pingendi*, c. 1475

Fig. 8. Profile and plan of a mazzocchio. Piero della Francesca, *De prospettiva pingendi*, c. 1475

As a result of this construction, the plan is placed in top of the profile, in the same sheet of paper (fig. 8). The use of double orthographic projection and the superposition of elements in plan, elevation and profile are characteristic traits of masons' tracings, at least north of the Alps; when Dürer [1525: 84v]⁶ explains how to prepare a perspective using a plan and an elevation of a cube, he ascribes the use of double orthographic projection to masons. In fact, transfers between orthographic views play a central role in the method used by Mathes Roriczer to construct an elevation starting from a plan, the “secret” of late mediaeval masons [Frankl 1945; Shelby 1977]. In Roriczer's booklet about the geometric control of pinnacles, *Büchlein von der Fialen Gerechtigkeit*, [1486: 5r-8r] the issue involves two different problems: first, how to determine the height of a given member, following a number of geometrical rules that rely for the most part in rotated squares; second, how to maintain the correlation between the horizontal and vertical projections of each point. In manuals of descriptive geometry, as well as in stonemason's drawings from the mid-sixteenth century on, this correlation is assured by the use of reference lines connecting the horizontal and vertical projections of each point. However, Roriczer does not use reference lines, but rather transfers measures from the plan to the elevation, using as a guide a vertical line placed at the axis of symmetry of his pinnacle, much in the same way as Piero does with his ring.

Roriczer's explanation is far from clear; possibly this is why Pérouse de Montclos [2001: 184-185] surmised that masons could not have taken the method from Roriczer,

besides the obvious fact that the great majority of French and Spanish stonemasons at the beginning of the sixteenth century could not read German; in fact, a fair number could not read in their mother tongue [Marías 1989: 476].

Of course, Piero could not have taken his method from Roriczer, since the *Booklet on Pinnacles*, which in any case enjoyed little circulation, was published in 1486, when Piero was blind; nor could Roriczer have taken his method from Piero, because *De prospettiva pingendi* was not published before 1899. Rather, we should consider that both Piero and Roriczer were using a standard drafting method; references to such practices appear in *De re aedificatoria* and a well-known letter to Leo X, attributed to Raphael and Baldassare Castiglione [Alberti 1966: 99; Sanzio c. 1510; see also Lotz 1977;⁷ Toker 1985; Di Teodoro 2001; Di Teodoro 2002; Di Teodoro 2003]. Besides, Piero [Della Francesca c. 1480: 38v] is in fact reversing the stonecutters' typical method and extracting the plan from the profile, although in a later step he will construct the meridians in the profile starting from their horizontal projections, thus extracting the side elevation from the plan.

While jumping from plan to profile and the other way around, Piero painstakingly places a string over a line joining each point in the torus to the station point, 96 points in total; at the same time, he transfers the intersection points of these lines with the picture plane to a set of waxed paper rulers or *righe*. Of course, each of these lines is a ray of the visual pyramid, to put it in Albertian terms,⁸ and the intersection of each of these rays with the picture plane furnishes the perspective of a point of the torus. In order to draw the final perspective (fig. 9), the plan allows Piero to determine whether the perspective of a particular point in the ring falls to the right or to the left of a vertical line passing through the center point; however, it gives no information about the height of any point. Thus, in order to compute the height of a point in the perspective, Piero uses the profile, which allows measuring this height easily; in turn, the profile gives no information about the horizontal position of the point.

Fig. 9. Perspective of a mazzocchio. Piero della Francesca, *De prospettiva pingendi*, c. 1475

Piero transfers these points, both from the plan and the profile, to a separate sheet of paper by means of the waxed paper rulers and another set of wooden rulers. In fact, these rulers are used as a gauge, a typical stonecutters' instrument. Piero instructs the reader to make marks on the ruler measuring the distances for each point to a vertical line passing through the centric point, much in the same way as stonemasons used their gauges to transfer measures from their tracings to the stones that were to be dressed [Bechmann 1993: 61; Calvo 1999: I,132-133].

It may seem surprising to see Piero engaging in such complex operations, instead of using simpler techniques, such as vanishing and distance points. Piero himself gives two different reasons in the introduction to the third book of *De prospectiva pingendi* [Della Francesca c. 1480: 32r]. He deals in this book with solids bounded by different surfaces and “*diversamenti posti*”, that is, rotated with regard to the picture plane. To tackle these difficult problems, he introduces in this book a new method, which will be easier to explain and understand than those used in the first two books in his treatise, while allowing him to avoid “the great multitude of lines that these bodies would need if using the first method”.

This last explanation does not sound convincing; in order to draw the ring the draftsman should trace 96 lines⁹ and transfer the intersections of these lines to the perspective. However, we must take into account that a number of perspective techniques, now familiar to us, were not known at Piero’s time. The “distance vanishing point” [see García-Salgado 2003],¹⁰ that is, one of a pair of points placed at the horizon line so that their distances to the centric point are equal to the distance between station point and perspective plane, is not used as an operative method in Piero’s treatise, as Field [2005: 148-150] has remarked. Although his use of the diagonal of a square to compute the position of transversal lines in books I and II of *De prospectiva pingendi* is a significant leap in this direction, he still needed to construct an enclosing rectangle using projection and intersection to trace the diagonal, which in fact does not reach the horizon line and the distance vanishing point [Della Francesca c. 1480: 7r-17 r; see also Panofsky 1927; Klein 1961; Vagnetti 1980; Field 2005, 148-150].¹¹

Besides, Piero did not know a general method for the placement of the vanishing point of oblique straight lines lying in a horizontal plane at a given angle to the picture plane. Of course, he knew the solution for a particular case of this problem; namely, the vanishing point of orthogonals, that is, lines forming an angle of 90° to the picture plane, which is the Albertian centric point. In a particular occasion he used the “distance vanishing point”, that is, the vanishing point of lines forming an angle of 45° to the picture plane, to solve a rather far-fetched problem, although this is an exception in his treatise and not a general method [Field 2005: 148-150]. The exact solution to the general case of the problem of the vanishing point of oblique horizontal lines was put forward only by Guidobaldo del Monte [1600: 43-44; see also Kemp: 1990, 89-90]; Piero mentions nothing of the sort.

On the other hand, the empirical use of the “distance vanishing point” to put into perspective a set of parallel lines was known in the Quattrocento, as the *sinopia* of Uccello’s *Nativity* in San Martino alla Scala suggests, and is ubiquitous in the treatises of Pelerin and Cousin. That is, the “distance vanishing point” was frequently used in the fifteenth and sixteenth centuries not only to measure distances along orthogonals, but also as a “vanishing point” when drawing horizontal lines lying at an angle of 45° to the picture plane, just as the centric point is used as the “vanishing point” of the orthogonals to the picture plane [Klein 1961; Sindona 1980; Kemp: 1990, 37-38; Raggio 1996; Roccasecca 2001a; Pelerin 1505: VIIr, VIIIr; VIIIv, XVIIr, XXr, XXv, XXIr, XXVIr; Serlio 1545: 60r-62v; Cousin 1560: 7r, 8r, 8v, 26 r, etc].

However, these empirical solutions are not mentioned in *De prospectiva pingendi*. In fact, the layout of Piero’s ring, with twelve meridians, seems to be chosen to put forward a case that cannot be solved by these methods. A ring with eight parallels and eight meridians, an obvious choice, could have been solved with two sets of orthogonals, two

sets of transversals and four sets of lines meeting at both “distance vanishing points”, a notion used, at least empirically, by Ucello in San Martino alla Scala. By contrast, a twelve-sided ring cannot be solved through these empirical methods, showing thus the power of Piero’s “other method”, which is even more striking taking into account that Del Monte’s construction was not known in the period.

All this means that Piero knew a general perspective method, based on orthogonal projections and explained in his third book, that was powerful enough to tackle any perspectival problem in Piero’s view. He also dealt in the first and second books of *De prospectiva pingendi* with a number of specific techniques, such as the use of orthogonals meeting at the centric point, the diagonal of a square to transfer measures from a transverse line to an orthogonal, and auxiliary vertical planes to compute the height of a point above the ground plane. These methods were useful for constructing simple figures, such as polygons, cubes, columns, wells, pedestals and even groin vaults. However, when confronted to complex figures, starting from the torus or polyhedral ring and progressing to rotated cubes, vaults, capitals and human faces, the general method showed all his power.

According to Roccasecca [1998], the ring at the Louvre presents pricked holes on most corners of the figure, as well as a fair number of lines incised with a stylus, all of them underlying the actual drawn lines. This suggests that the drawing could have been prepared using Piero’s technique, since the rulers should make any auxiliary construction unnecessary. However, the perspectival methods applied to this problem seem to have evolved during the first half of the sixteenth century. In the Uffizi drawings of rings, as well as in the *Chalice* [Kern 1915; Roccasecca 1998; Talbot 2006], a number of incised lines are placed at the intersection of the meridian planes with the horizontal planes for each parallel. Of course, for each parallel, all these lines intersect at the center of the parallel; these centers are placed at the axis of the torus, at different heights. All this suggests that the draftsmen of these rings, quite probably in the Sangallo circle for 1737A and 832 Ar, and possibly for 1736A, used a different technique, based in the rotation of a horizontal plane to place it at a frontal position, thus allowing the parallel to be constructed in true shape and then transferred to the perspective.¹²

Thus, in the fifteenth century Piero solved the problem of the *torculo* using projections from plan and elevation, while in the sixteenth century some draftsmen, probably from the Sangallo circle, complemented the use of plan and elevation with sophisticated methods to tackle this difficult problem again. All this explains the role played by the polyhedral ring as an emblem of Quattrocento perspective; it allowed the real masters of perspective to show all their cunning.

Polyhedral rings in Italian Renaissance painting and marquetry

The rings drawn in all the studies we have seen so far share a common trait: the axis of revolution of the surface is vertical; in Piero’s terms [Della Francesca c. 1480: 37v], the figure is set lying on a plane or “*giacente piano*”. As a result, during surface generation, each point describes a circle lying in a horizontal plane; thus, at the first stages of the perspective construction, while tracing the plan of the ring, the draftsman can use the compass to draw easily the plan of each of these circles, as Piero states clearly [Della Francesca c. 1480: 37v].¹³ Also, in this position the torus has a horizontal plane of symmetry, and thus each of the eight horizontal sections of the ring lies above another section; as a consequence, the draftsman only needs to draw four circles and use four rulers [Della Francesca c. 1480: 37v].¹⁴

However, most extant polyhedral rings in Quattrocento paintings and marquetry panels, such as all three panels of Paolo Uccello's *Battle of San Romano*, the *Flood* in the Chostro Verde in Santa Maria Novella, also by Uccello (fig. 10), or the inlaid panels in the Urbino cabinet, do not lie in horizontal planes.¹⁵ Thus, each point in the parallels will describe a circle in a sloping plane during surface generation, and the parallels will be projected as ellipses in the plan.

Fig. 7. Paolo Uccello. Detail of the torus from the *Flood*. Chostro Verde, Santa Maria Novella, Florence

This simple detail poses a formidable problem for the draftsman, right at the beginning of the perspective construction. To begin with, the notion of the ellipse was not widespread in the fifteenth century; the first translation of Apollonius of Perga's *Conics*, by Federigo Commandino, was not published until 1566. It is true that the construction of conic sections explained by Dürer [1525: 16r; see also Roccasacca 1998] was known in the Sangallo circles and that Spanish stonemasons found an empirical way to construct the projections of circles lying in sloping planes along the sixteenth century, as we shall see later, but Quattrocento literature mentions nothing of the sort.¹⁶

Piero was aware of this problem; in fact, he states explicitly that “We shall put the *torculo* lying flat; however, when lying in another way, it will be necessary to make as much rulers as circles are contained in the *torculo*”.¹⁷ That is, Piero is considering the case of the sloping ring, advising the reader that in this case the circles do not stand one above the other, since there is no horizontal symmetry plane, and instructing the reader to use as many rulers as circles are contained in the figure.

Piero does not explain in so many words how to construct this sloping ring, but the next section in *De prospectiva pingendi* deals with a rotated cube [Della Francesca c. 1480: 42r-43v; see also Evans 1995: 153-154]. The position of this section within the manuscript is somewhat surprising, since it stands between the flat-lying ring and the base of the column, which includes a fair number of flat-lying toruses.¹⁸ Thus, the placement of the rotated cube in the text of *De prospectiva pingendi* hints strongly that

Piero included it as a simplified demonstration of the method he was proposing to solve the problem of the rotated torus.

In order to rotate the cube so that “*nisuno suo lato sia equidistante al termine posto*”, that is, that no face or edge of the cube would lie parallel to the picture plane, Piero follows a complex, although quite logical, procedure in four steps [Della Francesca c. 1480: 42r-43v]. First, he draws in plan and elevation a cube lying on a horizontal plane (fig. 11); however, the plan of the cube is rotated with regard to the reference lines that connect plan and elevation; thus, the elevation shows an oblique view of the cube. In the second step (also shown in fig. 11), Piero rotates the elevation of the cube around a horizontal axis passing through one of its lower corners. Piero easily manages to construct the plan of the rotated cube, taking into account that each point will move during the rotation on a plane that is orthogonal to the rotation axis, that is, a frontal plane; also, each point of the plan will be joined to the rotated elevation by a reference line.¹⁹ That is, once more Piero reverses the typical method of mediaeval masons and extracts the plan from the elevation.

Fig. 11. Plan and elevation of a flat-lying and a rotated cube. Piero della Francesca, *De prospettiva pingendi*, c. 1475

However, the cube is not yet in its final position, since Piero intends that “no edge should be parallel to the picture plane”, as we have seen; in order to do that, he simply rotates the picture plane of the perspective. Now, he can place the point of view of the perspective and trace all the visual rays in order to compute the horizontal position of each point in the perspective, as he did with the polyhedral ring (still in fig. 11).

Once this is done, he cannot perform directly the same operation starting from a vertical projection, since he has still no profile, but only an elevation, which is not even parallel to the picture plane of the perspective. Thus, Piero constructs on a separate sheet (fig. 12) a profile, that is, an orthogonal projection on a plane that is perpendicular to the picture plane of the perspective. In order to do so, he starts from the rotated plan and transfers the height of each point from the rotated elevation to the profile; once this is done, he can trace visual rays in the profile view, find their intersections with the picture plane and transfer the resulting vertical positions to the perspective in order to complete it (fig. 13).

Fig. 12. Plan and profile of a rotated cube. Piero della Francesca, Fig. 12. Plan and profile of a rotated cube. Piero della Francesca, *De prospettiva pingendi*, c. 1475

Fig. 13. Perspective of a rotated cube. Piero della Francesca, *De prospettiva pingendi*, c. 1475

This method can be easily extrapolated to the polyhedral ring, although the number of lines and point transfers involved is quite daunting (fig. 14). It can be also used to rotate heads, as Robin Evans [1995: 153-154] has suggested; although Piero does not explain the technique, Albrecht Dürer does so in *De symmetria partium in rectis formis humanorum corporum* [Dürer 1532: 117v-118v, 126v-127r, 130v, 132r-134r].²⁰

All this brings us back to Vasari's [1568:269] mysterious comment on Uccello "rotating arches and ribs". Both Piero [Della Francesca c. 1480: 29r-30r; see also Kemp 1990: 29] and Sebastiano Serlio [1545: 45v-46r] explain how to construct the perspective of a groin vault (fig. 15), but they make no use of rotations of any kind.²¹ By contrast, these rib rotations were standard practice in late Gothic stonecutting practice. Hernán Ruiz [c. 1550: 46v], Philibert de L'Orme [1567: 108v], Alonso de Vandelvira [c. 1580: 96v], Alonso de Guardia [c. 1600: 85b], Gelabert [1977: 281] and a good many other Renaissance writers on stonecutting explain how to compute the curvature of the ribs of a tierceron vault and the height of the secondary keystones (fig. 16), rotating diagonal ribs and tiercerons around a vertical axis, in order to bring them to the plane of the transverse arches and depict them as circular arcs (see also [Rabasa 1996]).

Fig. 14. Piero's construction for a rotated cube applied to a mazzocchio. For the sake of clarity, only the extreme instances of each set of reference and projecting lines are shown.

Drawing by José Calvo-López

Fig. 15. Groin vault. Sebastiano Serlio. *Tutte l'opere di architettura*, 1600

Fig. 16. Rib vault. Alonso de Vandelvira, *Libro de trazas de cortes de piedras*, c. 1580

This ingenious method, quite probably of late Mediaeval origin, avoids constructing a standard orthographic projection of the diagonal rib and the tierceron, which would have appeared in such a projection as elliptical arches.²² German masters went much further, since they rotated not only single ribs; they unfolded a whole path of ribs in their *Netzgewölbe* or net vaults, to construct a virtual rib, the *Prinzipalbogen* or principal arch, extending from the springer to the main keystone [Müller 1990; Tomlow 2009]

However, both the simple rotations of French and Spanish masters and the “principal arch” technique involve rotations around a vertical axis, while Piero’s method uses rotations around a horizontal axis. Rotations of this kind appear much later in stonecutting. An interesting example can be found in another stonecutting manuscript of the period, *Cerramientos y trazas de montea*, by Ginés Martínez de Aranda [c. 1600: 11, 15, 40, 46]; the author rotates the intrados joint of a number of skew arches and rear-arches in order to compute the angle between the intrados joint and the face joint. At that moment, however, both horizontal and vertical axis rotations were standard practice in perspective treatises, as Jean Cousin [1560: 27r, 28r, 32 r, etc.] makes clear.

All this refers to the rotation of the objects depicted in orthographic projections, while projection planes stand still. By contrast, when Piero constructs the rotated profile of his cube, the last step in the perspective construction process, he is performing a typical operation of descriptive geometry, known much later as change of projection plane [Della Francesca, c. 1480: 43r-43v; see also La Gournerie 1860: VI-VII]. That is, he leaves the cube standing still and rotates the projection plane of the elevation in order to construct a profile. This operation is quite usual in French and Spanish stonecutting treatises and manuscripts of the sixteenth century; it appears in Hernán Ruiz [c. 1550:

47v], Philibert de L'Orme [1567: 113r], Alonso de Vandelvira [c. 1580: 24v, 80r, 81r, 90r, 90v, etc.], Ginés Martínez de Aranda [c. 1600: 7, 9, 10, 72, 76, 78, 86, etc.], and others, and even in De L'Orme's carpentry treatise, the *Nouvelles inventions pour bien bastir et a petits frais* [1561: 13r]. Excepting this later treatise,²³ all stonecutting texts use the same method employed by Piero; the mason should trace reference lines from all relevant points in the plan, and once this is done he should transfer the heights of each point in the original elevation to the new elevation or, in Piero's case, to the new profile.

Thus, it seems clear that Piero and other authors on perspective, such as Cousin, shared with stonecutting literature not only the use of orthographic projections, but also a number of rather sophisticated techniques that much later were to find a place in descriptive geometry, such as rotations and change of projection planes. We shall tackle the problems of the formation of these techniques and the direction of influences between perspective and stonecutting in the final section of this article, once we have dealt with the stone rings of the Spanish Renaissance.

Torus vaults in Spanish Renaissance architectural practice

The oldest of these built toruses is the vault of the inner chapel or *recapilla* of Gil Rodríguez de Junterón in the Cathedral of Murcia (figs. 2, 17, 18), begun in 1525 and completed around 1543 [González Simancas 1905: 151-152; Gutiérrez-Cortines 1987: 164, 170]. Junterón had spent some years in Rome during the papacy of Julius II, serving in the curia as protonotary. When he asked for permission to build his chapel on March 27, 1525, the master mason in the cathedral was Jacopo Torni, a Florentine painter apprenticed in Ghirlandaio's workshop.

Fig. 17. Chapel of Gil Rodríguez de Junterón. Plan of the inner room. Survey by Miguel Ángel Alonso and Ana López Mozo

Fig. 18. Chapel of Gil Rodríguez de Junterón. Longitudinal section of the inner room. Survey by Miguel Ángel Alonso-Rodríguez and Ana López-Mozo

Later on, Torni had worked with Pinturicchio, at least at the Stanze Borgia, and carried on a number of commissions on his own, until Michelangelo requested his help to paint the Sistine ceiling. His brother may have worked with the Sangallos; this is quite significant for our purposes, taking into account Roccasecca's attribution of some polyhedral ring drawings to the Sangallos' circle, although the connection is indirect [Vera 1993:16-17; Roccasecca 1998; Roccasecca 2002b].

Vasari [1550: 528-529; 1568: 524-525] stresses Torni's qualities as a draftsman, while pointing out his laziness; however, his short Spanish career suggests that Vasari's portrait may be biased. He arrived in Spain not later than 1520, starting work at the Royal Chapel in the Cathedral of Granada, where he received the commission for the Annunciation group over the vestry door, three panels for the Santa Cruz altarpiece and a fair number of decorative works; he also carried on a restoration of the painting of the Virgen de la Antigua in the Cathedral of Seville [Velasco c. 1564; Gómez-Moreno 1925b; Calvo Castellón 1994, 218-222]. On March 29, 1522, the chapter of the Cathedral of Murcia asked him to accept the post of master mason of the cathedral; at the same time they offered him a number of commissions at the behest of Pedro Fajardo, first Marquis of Vélez, such as the main altarpiece in the cathedral and a number of sculptures for his family chapel [Gutiérrez-Cortines 1987: 64-65; González Simancas 1905: 87]. His main duty as master mason was the supervision of the great bell-tower of the cathedral, whose construction had begun three years before under the direction of another Italian artist, known as Francisco Florentino. In less than four years, Torni completed the first story of the tower, which includes an ornate sacristy with two remarkable stereotomical pieces, a sail vault and a skew passage, while holding the post of

master mason of San Jerónimo in Granada before his death in Villena, probably when inspecting the construction of some works in the church of Santiago or its parish house [Gutiérrez-Cortines 1987: 61-66, 112-129, 135-136; Calvo 2005a].

Although no documentary evidence connects Torni with Junterón's chapel, historians have traditionally attributed the general design of the chapel to him, taking into account a number of Italianate traits in the chapel and some similarities with the cathedral sacristy and other documented work by Torni. We should take also into account that Junterón seems to have been a political client of Fajardo, since he undertook a difficult mission in the war of the Comuneros on his behalf [González Simancas, 1905: 156-157; Gutiérrez-Cortines 1987: 161-164].

In any case, since Torni died ten months after the beginning of the chapel's construction, it is clear that he was not in charge of the actual execution of the chapel. This task was quite probably undertaken by Jerónimo Quijano, a Spanish sculptor trained in the entourage of Philippe Vigarny. He had carried on works with Torni and his father-in-law, Juan López de Velasco, and held the post of master mason in the cathedral after Torni's death; much later, he was listed as a creditor in Junterón's will for the altarpiece in the chapel [González Simancas 1905: 156-157; Gutiérrez-Cortines 1987: 164; Villella 2002].

Cristina Gutiérrez-Cortines and Marzia Villella have stressed the traits that connect Junterón's inner chapel with the first project of Julius II's tomb, such as the general plan, a rectangle terminating in two semicircles or the use of niches between pilasters [Gutiérrez-Cortines 1987: 176-177; Villella 1999]. However, the most singular member in the chapel, the vault over the inner room, has no connection with Julius II's tomb nor with Quijano's other work. When confronted with a similar problem in Santa María in Chinchilla, Quijano used a straightforward solution: a barrel vault terminating in two quarter-sphere vaults [Gutiérrez-Cortines 1987: 200-211; Calvo 2005b; Calvo 2008].

By contrast, the vault over Junterón's inner vault is a horizontal axis torus vault; so to speak, one of the rings in the heads of the contenders in the Battle of San Romano or around the neck of the survivors of the Flood has completed its rotation, reaching a vertical plane. However, only the fourth part of the surface is actually built, since the upper half of the outer part of the surface is enough to span the chapel area. To be precise, less than half a quarter of the entire surface is used; in fact, the survey by Miguel Ángel Alonso and Ana López Mozo (figs. 17, 18) has shown that the generatrix is somewhat smaller than a semicircle, in order to increase the width of the arch around the relief of the *Paradise* in the tympanum of the back wall of the chapel and the span of the entrance arch. The treatment of this surface shows a fair understanding of the geometry of this figure. Instead of giving the same value to meridians and parallels, as Uccello or the cabinet-makers at Urbino had done, the builder of the chapel stressed the radial pattern of the generatrices, thus bringing the focus of the whole architectural composition of the chapel to the reliefs of the *Paradise* and the *Nativity* in the chapel's altarpiece. In contrast, the builder broke the continuity of the directrices at both sides of each generatrix; in this way, the directrices adopt an asymmetric pattern, made almost invisible by the rich array of grotesque sculpture in each compartment (fig. 19). At the same time, the portion of the vault included between two consecutive generatrices acts as a course, and the breaking of the directrices prevents the voussoir from slipping, a sound constructive practice [Calvo 2005c: 152, 161-163].

Fig. 19. Vault over the inner room in the Chapel of Junterón.
Stonecutting diagram by José Calvo-López

We shall come back to these issues in the next section; for the moment, we shall discuss another solution for the torus vault built over the lower gallery around the courtyard of the palace of Charles V in Granada. Quite remarkably, this piece shows a number of parallels with the inner chapel of Junterón, while choosing opposite solutions in a number of crucial points. A wide consensus supports the authorship of Pedro Machuca for the project of the building, although another architect, Luis de Vega, was consulted and perhaps introduced a number of changes; in contrast, Manfredo Tafuri has argued pervasively for a project by the hand of Giulio Romano, although this hypothesis is grounded on stylistic terms only [Rosenthal 1985: 11-12; Tafuri 1987].

Lázaro de Velasco, the Spanish son of Jacopo Torni, mentions an Italian sojourn of Pedro Machuca in the introduction to his translation of Vitruvius, remarking that he arrived more or less at the same time as Torni [Velasco c. 1564]. However, nothing is known about Machuca's Italian activity. Taking his cue from Velasco, Earl Rosenthal [1985:16-17] has speculated with the possibility that Machuca may have frequented in Italy the same circles as Jacopo Torni. However, this hypothesis is at odds with the general Raphaellesque character of Machuca's paintings and the stylistic details that connect his architectural work with Giulio Romano. On the other hand, Machuca was in fact connected with Torni shortly after they arrived in Spain; each of them painted three

panels for the Santa Cruz altarpiece, flanking the central piece by Dierick Bouts [Gómez Moreno 1925b; Calvo Castellón 1994: 218-222].

Further, both Machuca and Torni were connected to the Mendoza family, whose patronage was crucial in the introduction of the Renaissance in Spain. Machuca directed the construction of the palace in the Alhambra as *escudero* or client of Luis Hurtado de Mendoza, a prominent member of the family, while Torni worked in the chapel of Nuestra Señora de la Antigua in the Cathedral of Seville, the funerary chapel of Diego Hurtado de Mendoza [Rosenthal 1985: 12-13; Velasco c. 1564; Morales 1992: 185-187]. Both Diego Hurtado de Mendoza and Pedro Fajardo, the political patron of Junterón and artistic patron of Torni, had been educated by Pietro Martire d'Angheria, an Italian humanist under the patronage of the father of the Captain-General of the Alhambra, the first Marquis of Mondéjar, Íñigo López de Mendoza, father of Luis Hurtado. It is also worthwhile to remark that Angheria has been included in the group of humanists that suggested to Charles the V that he build the palace, along with Baldassare Castiglione.²⁴ Besides, two prominent members of this group of humanists in the entourage of Charles V during his stay in Granada, Alfonso and Juan de Valdés, were to belong to the chapter of the Cathedral of Murcia later on [López de Toro 1953: 64-66, 77-78, 82-84, 198, 200, 202-205, 211-212, 276-277, 386-387, 413-415, 421-422; López de Toro 1955: 30-32, 258-259; 268-269; Rosenthal 1985: 10; Rosenthal 1988; Meseguer 1957]. It is also noteworthy that an important foreman of the building team in the palace of Granada, Juan de Marquina, had worked with Torni's predecessor in the Cathedral of Murcia, Francisco Florentino, and had lived in Murcia during the 1520's; he was probably aware of Junterón's plans for his inner chapel [Gutiérrez-Cortines 1987: 52, 61, 328-330; see also p. 95, note 111 and p. 356, note 141; Rosenthal 1985: 53-54]. Thus, a densely woven net of connections between patrons, artists and builders ties together the chapel of Junterón and the palace in Granada.

The vault in the lower gallery also shares a parallel history with the chapel in Murcia. Laying aside Vega's proposal for half-columns and pillars, Pedro Machuca had laid out the foundations for a ring of free-standing columns and prepared a model of the palace, which included part of the colonnade. However, as in the case of Torni and the Murcia vault, Machuca did not live to see the vault erected; as we have seen, it was built by his son Luis between 1562 and 1569; at that moment, however, Marquina was either dead or retired [Rosenthal 1985: 99-100, 112-119; Rosenthal 1988].

In other aspects, the vault in Granada reverses a good number of design traits in the Murcia archetype. First of all, its axis of revolution is vertical, just like Piero's toruses or the rings around the Uffizi's *Chalice*. Thus, the generatrices or meridians are semicircles laid out on vertical planes, while the directrices or parallels are horizontal circumferences. Following the same constructive logic as the Murcia vault, here the roles of these curves are reversed. The horizontal directrices act as bed joints, and thus are almost uninterrupted along the huge circumference of the gallery, while the vertical meridians are broken to prevent the voussoirs from slipping. In another striking parallel with the Murcia vault, the generatrix is not strictly a circumference, since the first course of the vault in the outer side, springing from the corridor wall, is set at a lower height than the first course in the inner side, resting on the columns that separate the corridor from the courtyard. By contrast, the treatment of the surface in the Granada wall is completely opposite to the Murcia one; instead of being hidden below a host of grotesque sculpture, the vault in the palace displays a completely nude intrados, making a great display of the

network of joints and predating the characteristic *nudité d'intrados* in seventeenth century French vaults [Pérouse de Montclos 2001:108-111; Etlin 2009].

Rosenthal cites a number of classical precedents for this vault, such as the annular vault at the seventh level of the Praeneste sanctuary, the Maritime theater in Hadrian's Villa and the deambulatory in Santa Constanza [1985: 217-218; see also Rosenthal 1988]. He also suggests that Pedro Machuca could have been thinking of another constructive solution for the vault, such as a concrete vault in the Roman tradition; that he finally adopted an ashlar vault under pressure from Luis de Vega and other Castilian advisors; that it was not easy for him to maintain his original idea of free-standing columns, since the Castilian consultants favoured pilasters and half-columns; and that Luis Machuca was planning to have the rough tufa surface of the vault plastered and covered by frescoes [1985: 118-119]. Except the discussion about free-standing columns or pilasters, which dates from around 1530 and is well documented, the rest are hypotheses, since the documents from the 1560s are scarce. However, it seems clear that there was a great concern about the constructive feasibility of the solution. Besides, in the eyes of everybody, except maybe the Machucas, the aristocratic ideal of magnificence was tied to the use of ashlar, and the courtiers would hardly accept another material for such a visible element in an Imperial palace [see for example Díez del Corral 1992 or Barbé 1990].

All this suggests that either Pedro or Luis Machuca could have conceived an annular vault starting from sources in Antiquity, direct or indirect; we should remember that Serlio's third book presents a quite expressive drawing of the interior of Santa Constanza, which was included in the Spanish translation by Francisco de Villalpando [Serlio 1544, XX; Serlio 1552, III-XIIv]. However, taking into account the number of connections between Granada and Murcia, it is rather puzzling that Rosenthal does not connect the Granada vault with the Junterón one, although he makes much of Machuca's relationship with Torni [Rosenthal 1985:13, 16-17]. At least, Junterón's vault must have played in Granada the role of a feasibility check; that is, the vault in Murcia made clear that it was possible to build such unusual shapes in ashlar. However, influences in the opposite direction cannot be discarded. Either the Valdés brothers, who belonged with Junterón to the Murcia chapter, or Pedro Fajardo, who kept up a lively correspondence in Latin with Pietro Martire d'Angheria, could have informed Junterón about the early projects of the Granada palace.

It is quite easy to understand why the vault in the Granada palace, about twenty meters in internal diameter, has no derivatives. The vault in Murcia has given birth to a modest number of heirs. It is almost literally split in half at the crossing of the nearby church of Santiago in Orihuela: two niches flanking the central space, probably built by Quijano, are covered by surfaces generated by a quarter of a circle rotating around a horizontal axis and thus, the result is the eighth part of a full torus [Gutiérrez-Cortines 1987: 260; Palacios 2003: 225; Calvo 2005c: 43]. The treatment of the surface lies halfway between the vault in Murcia and the one in Granada, since it is divided into coffers; thus, it eschews the rich grotesque sculpture of the Junterón chapel, but does not show the nudity of the intrados of the Alhambra vault. This eighth-of-a-torus surface is also used in a wall arch in the crossing of the church in the convent of San Francisco in Baeza, by Andrés de Vandelvira. This is quite interesting, since Alonso de Vandelvira, son of Andrés, wrote the only manuscript that deals with the construction of torus vaults; we shall deal with his explanations of stereotomical procedures applied to torus vaults in the next section.

Torus vaults in the manuscript of Alonso de Vandelvira

Usually, stonecutting treatises or manuscripts describe the problems they are going to deal with in geometrical or constructive terms. To point out an extreme example, Ginés Martínez de Aranda [c. 1600, 191-193] includes in *Cerramientos y trazas de montea* such an elaborate piece as the “Puerta en torre redonda contra capialzado cuadrado desquijado de arco en torre cavada”, that is, a doorway in a convex wall confronting a square rear-arch joined to an arch in a concave wall. However, a small number of stonecutting problems, such as the *Vis de Saint-Gilles*, a spiral staircase covered by a sloping barrel vault or the *Arrière-voussure de Saint-Antoine*, a rear-arch with curved joints, take their names from the cities or places where a particularly significant example of each problem is built. These geographical denominations or *appellations d'origine*, in Pérouse de Montclos' words [2001, 202-204], are also used in Spain. Vandelvira's manuscript includes the *Ochavo de La Guardia*, a coffered octagonal vault; the *Caracol de Mallorca*, a spiral staircase with a winding newel; the *Bóveda de Cuenca*, a square coffered vault, and the *Bóveda de Murcia*, a horizontal axis torus vault, quite similar to the vault in the *recapilla* of Junterón [Vandelvira c. 1580: 103v, 51r, 97v, 69v].

We should stress two points about these appellations. First, the small number of geographical denominations indicates that these problems and built examples are seen as exceptional challenges, as we remarked when dealing with Goiti's treatment of the torus vault as an emblem of stonecutting. Second, the appellations are applied to geometrical problems, rather than to actual built examples. This is quite clear in France, where *Arrière-voussures de Marseille* or *Pendentifs de Valence* are not difficult to find. A small detail in Vandelvira's manuscript hints in this direction; when talking about the *Vis de Saint-Gilles* he points out that “it is executed in Saint-Gilles in France” and not “it is in Saint-Gilles”.²⁵ This is essential for our purposes, since Vandelvira's drawing and explanation of the *Bóveda de Murcia* depart in a number of small but significant details from the built example in the chapel of Junterón [Vandelvira c. 1580, 52v, 69v].

Vandelvira starts his construction (fig. 20) by tracing the outline of the plan of the vault. He then divides the end semicircles in an odd number of voussoirs and draws a number of parallels of the surface, starting from the division points. This involves a first point of departure from the Murcia example, since the end sections of the springing of the vault in Junterón chapel are not exactly semicircles, as we have seen. Thus, it is clear that Vandelvira is not including in his manuscript either a survey or a project of the Murcia vault, but rather a general solution to the problem of horizontal-axis torus vaults; therefore, he leaves out a trait that stems from the adaptation of an abstract model to the particular circumstances of Junterón's chapel.²⁶

Since the parallels in Vandelvira's tracing run from one end of the vault to the other, the joints are not broken between one course and the next, as in the actual Murcia vault. Of course, this layout does not prevent the voussoirs from slipping; Vandelvira is aware of the fact, since he makes a passing remark about an alternative solution involving “ligazones”, that is, the breaking of joints between one course and the next, but he does not explain this solution in detail.

Once this is done, Vandelvira draws the elevation of the vault, tracing reference lines from the plan until they meet the springing line in the elevation. Since the intersections with this line allow Vandelvira to measure the diameters of the longitudinal section of the vault and each parallel, he can easily construct the section and the parallels. He can also divide the section in an odd number of voussoirs and trace a number of lines from the

midpoint of the springing line to the division points; this operation furnishes an array of lines converging in the central points of the springing line, which represent the meridians of the vault.²⁷

Fig. 20. Horizontal-axis torus vault. Alonso de Vandelvira,
Libro de trazas de cortes de piedras, c. 1580

Next, Vandelvira constructs the horizontal projections of these meridians in the plan. Although the meridians are semicircles, they are projected as half-ellipses, since they lie on sloping planes. To perform this complex operation, Vandelvira draws reference lines starting from the intersections of each meridian with each parallel in the elevation, and brings them to the plan; where each of these lines meets the corresponding parallel, he can place a point belonging to a meridian. Once all points of a meridian are placed, he joins them by means of arcs of a circle, taking them in groups of three points.²⁸ That is, here Vandelvira inverts the traditional Gothic procedure of extracting the elevation from the plan and places the horizontal projections of the points of the meridians starting from the elevation, much in the same way as Piero had done with the parallels in his ring or with his rotated cube.

The crucial steps in the process start at this point. Vandelvira has used orthogonal projections as the foundation of the stereotomic method; now he will apply a different geometrical technique, involving an approximate development of the torus surface, for the preparation of the intrados templates of the voussoirs. In order to construct these templates, Vandelvira uses a number of imaginary cones, acting as substitutes for the torus surface, which is non-developable. These cones have their vertexes placed at the axis of the torus; thus, each cone passes through two consecutive parallels. He first traces a generatrix of the cone, joining two consecutive division points in the springing line, carrying it over until the line meets the axis of the torus; the intersection point will be the vertex of the cone. Then, Vandelvira develops the cone using a well-known technique: he traces two circular arcs with their centres placed at the vertex of the cone; this will give him the edges of the intrados template of the voussoir corresponding to the parallels, while the generatrix stands for one of the meridians. In order to complete the development of the intrados face of the stone, Vandelvira should compute the angle between both developed meridians, taking into account the length of the circular arcs that represent the parallels; however, he makes no mention of this calculation, although he draws the final meridian.²⁹

These cones seem to be indirectly related to Alberti's pyramid. When explaining the same procedure, Alonso de Guardia [c. 1600: 87v] advises the reader to "place the straightedge passing through two points of the hemispherical belonging to the course whose template you are trying to construct, drawing the *visual lines* marked with C, until you reach the perpendicular line marked with D" (our italics).³⁰ The connection of this term with perspective seems even clearer in Ginés Martínez de Aranda [c. 1600, 67]. He uses the cone-development method to construct the intrados templates of a peculiar arch with a spherical intrados, the *Arco en vuelta de horno por la cara*; when talking about the generatrix of the cone he lacks a word for this line and dubs it again as "visual line". Since the axis of this cone is horizontal, as in the "Bóveda de Murcia", it is easy to understand why the generatrix is associated to one of Alberti's rays, while the intersection point with the axis, that is, the vertex of the cone, corresponds of course to the station point.

An interesting variation of this scheme is the *Bóveda de Murcia por cruceros*, [Vandelvira c. 1580: 70v] that is, a coffered torus vault in which the coffering is built as a network of ribs, adapting the mainstream Gothic construction method to classical coffering. This scheme, which as far as we know was never actually built, shows a striking resemblance to the hollow rings depicted in Leonardo's *Codex Atlanticus* or the treatises of Barbaro and Lorenzo Sirigati [Esteve 2007]. In any case, there is a closer source for Vandelvira's coffering, a number of works by Diego de Siloé such as the vaults over the crossing and the presbytery in San Jerónimo in Granada and the tunnels between the presbytery and the ambulatory in the Cathedral of Granada.

Compared to the explanation of the stereotomic procedure for the Murcia vault, Vandelvira's references to the annular vault in Granada are somewhat disappointing. First, he quotes it when explaining the *Vis de Saint-Gilles*, a staircase covered by an annular ascending vault, in these terms:

This method is also useful to construct a vault around a circular courtyard, as it is done in the Royal Alcázar in the Alhambra in Granada; however, ... you should take from the second end the same portion of the block that you take from the first end, since this vault lies all at the same level [Vandelvira, c. 1580: 53r].³¹

That is, an annular vault such as the one in Granada can be built using a simplification of the procedure proposed by Vandelvira for the *Vis de Saint-Gilles*, taking into account that the vault in the palace of Charles V is not sloping.

Fig. 21. Round courtyard with columns. Alonso de Vandelvira, *Libro de trazas de cortes de piedras*, c. 1580

Later on, Vandelvira includes a section specifically devoted to the problem of vertical-axis torus vaults, under the heading “Patio redondo con columnas”, including a drawing of a round courtyard with a perimetric corridor [Vandelvira, c. 1580, 111r]. Although the sketch (fig. 21) resembles the central space in the palace of Charles V, once again there are a number of important differences: the columns are joined by arches, not lintels, and the cross-section of the vault is an oval, not a segmental arch. The written explanation is quite short. After stressing the stereotomic difficulties of the warped arches, Vandelvira deals with the vault in two lines, stating that:

This vault should be understood by any one who knows the *Vía de San Gil*; however, it is easier, since the *Vía de San Gil* is a sloping vault and this one is set at the same level; so the stones should be set square [...] and you should take the same portion of stone from both ends taking into account the bonding as the templates [...] show [Vandelvira, c. 1580: 111r].³²

Thus we should turn our attention to Vandelvira's explanation of the *Vía de San Gil* [Vandelvira, c. 1580: 52v-53 r], focusing on the basic tracing technique, and leaving aside the steps in the tracing and carving process that are connected with the ascending movement of this spiral staircase. The mason is to trace the plan of the courtyard, including the outside wall and the column line; then he should construct a cross-section of the vault, connected to the plan by means of reference lines, and divide it into voussoirs; once this is done, he should enclose each voussoir in a rectangle with horizontal and vertical sides.

Fig. 22. Barrel-vaulted spiral staircase. Ginés Martínez de Aranda, *Cerramientos y trazas de montea*, c. 1580

Fig. 23. Barrel-vaulted spiral staircase according to Martínez de Aranda. Stonecutting diagram by José Calvo-López

These enclosing rectangles are the trademark of the squaring method for voussoir dressing; rather than relying on templates, as in the “Bóveda de Murcia”, this method relies heavily on the orthographic projections of voussoir faces, materialised at the carving phase by means of the square. However, when explaining this step, Vandelvira focuses on the ascending movement of the vault, and does not give clear directions for the essential dressing process. Fortunately, Ginés Martínez de Aranda is a bit more explicit when dealing with the *Vía de San Gil* [1600: 231-233]. Following his directions (fig. 22, 23), the mason should inscribe a wedge shape in the upper face of the voussoir and remove the material outside the wedge by means of the square; doing so, he is in fact reversing the orthogonal projection process and materialising the projecting planes of all voussoir edges with the square. Once this is done, he is to place at both sides of the voussoir an

auxiliary template with the portion of the cross-section of the vault that corresponds to the voussoir, to trace these templates and to remove the material outside the shape of the template. That is, he should take out four wedges, corresponding to the intrados, the extrados and both bed joints, controlling the execution of the edges between voussoir faces with the help of a *cerce*, or curved-edge ruler; this process should give as a result the finished voussoir.

Fig. 24. Round courtyard without columns. Alonso de Vandelvira, *Libro de trazas de cortes de piedras*, c. 1580

When discussing the *Vis de Saint-Gilles*, Vandelvira remarks that the squaring method is quite difficult; in a well-known passage, Philibert de L'Orme stresses that it causes "*gran perte de pierres*" [Vandelvira, c. 1580: 52v; De L'Orme 1567: 73v]. In fact, Vandelvira hints that the template method can be used in a level annular vault, although he does so quite indirectly. Immediately before the *Patio redondo con columnas* he includes another design, the *Patio redondo sin columnas* (fig. 24), featuring again a torus vault, cantilevered from the external wall of a round courtyard. Although the intrados surface of the vault features coffering, it is not executed as an independent network of ribs, as in the "Bóveda de Murcia por cruceros", but rather as a superficial decoration, independent from the voussoir-division scheme, as in the Orihuela niches [Vandelvira, c.

1580: 110v]. The *Patio redondo sin columnas* does not include the usual written explanation, but Vandelvira makes a passing reference to this design in the next section, the *Patio redondo con columnas*: “Since the preceding courtyard is quite clear, I have not included any writing, for it is no more than a hemispherical vault and you should leave a lantern as wide as the opening in the courtyard demands...” [Vandelvira, c. 1580: 111r].

In fact, the vault is not a portion of a hemispherical vault, but clearly a part of a torus surface. Thus, we should understand that the reference to the dome applies mainly to the stereotomic technique, and that Vandelvira is suggesting that the cone development method he used in the hemispherical vault can be used in the same way in the *Patio redondo sin columnas*.³³

Orthographic projections in perspective and stereotomy

As we have seen, Vandelvira uses the squaring method, which makes heavy use of orthographic projections, for the most difficult stonecutting traits, such as those involving warped surfaces [c. 1580: 26r, 46r, 52v, 58 v, 60r]. In the same way, Piero used his “other method”, involving direct projection from plan and profile, for the “most difficult shapes” [Della Francesca c. 1480: 32r; see also Di Teodoro 2001; Di Teodoro 2002]. However, the general structure of both *De prospectiva pingendi* and the *Libro de trazas de cortes de piedras* strongly suggests that a painter or a mason dealing in practice with moderately complex problems, such as columns, pedestals, wells, trumpet squinches, skew arches or hemispherical vaults, should not use methods that rely heavily on orthographic projection, but rather centric points, Piero’s antecedent of the distance vanishing point construction, rotations or developed templates. These particular methods are used where adequate, but cannot be considered general methods, since they cannot solve all problems; for example, the centric point is useless when trying to construct the perspective of an oblique column. In the same way, Vandelvira cannot use true shape templates in the *Vis de Saint-Gilles* or the groin vault, taking into account the complex nature of these surfaces, and needs to resort to orthographic projections and the squaring method [Vandelvira, c. 1580: 25r, 52v].³⁴ Thus, both in perspective and stereotomy, orthographic projections furnish a general operative method, although in both fields, a number of special purpose methods furnish more efficient solutions to specific problems, such as the centric point, the distance point, vanishing points, rotations, rabattements and developments.

However, this is not the only use of orthogonal projections in stereotomy and perspective. As we have said, Vandelvira gives two different solutions for the *Arco en torre redonda y cavada*, an arch opened in a curved wall [Vandelvira, c. 1580: 22 r, 24v]. When explaining the first solution, using full-scale templates, he fears that the result will befuddle the reader. To leave any doubt aside, he advises the reader to make a model of the arch by squaring: “if you want to prove it, made a model by squaring, as I will teach you further on, and apply the templates to the model, and you will find that the templates are correct” [Vandelvira, c. 1580: 22r]. Four pages later, he explains the procedure to dress the voussoirs of the same arch by means of squaring, almost excusing himself for using such stone-consuming method: “I will show now the arch in a round wall by squaring, because I had promised to do so, and also to cast light on other pieces that can be done only by squaring”. Thus, if Vandelvira applies the squaring method to this arch, it is because he grants it stronger empirical evidence, brought off by means of models [Vandelvira, c. 1580: 24 v].³⁵

Empirical evidence plays a central part in the historical developments in perspective, although it was usually furnished by other means. As Filippo Camerota [2001] has suggested, most probably the strange profile of the upper edge of Brunelleschi's panel of the Piazza della Signoria allowed him to superimpose the profile over the Signoria skyline, verifying in a most empirical way the exactitude of his perspectival constructions.

As perspective methods were developed, a number of physical devices provided this empirical evidence, serving at the same time as operative methods, such as Alberti's *velo*, Dürer's and Keser's frames, nails and ropes, Vignola-Danti machines or the *camera obscura* [Alberti 1435, II-31; Dürer 1525, 90v, 91r; Vignola 1682:56;³⁶ Kemp 1990: 167-203]. However, orthogonal projections stand between empirical evidence and the ingenious centric point method. Following Alberti's trail, Piero gave a proof of the role of the centric point as the vanishing point of the lines that are orthogonal to the picture plane, and did so using double orthogonal projection [Della Francesca c. 1480: 6r-6v; Elkins 1987; Field 2005: 141-146].³⁷ In a similar way, in Vandelvira's manuscript, orthogonal projections stand between the purely empirical evidence furnished by reduced-scale models and the more elaborate template method.

Thus, Pérouse de Montclos was following a sound trail: both in perspective and stereotomy, orthographic projection plays the dual role of a general operative method and a connection between the empirical basis of both fields and their particular specific-purpose methods. Perspective and stereotomy have also in common a number of techniques based on orthogonal projections, such as the use of rotations and changes of projection plane, which were much later to be accepted between the methods of descriptive geometry, in the strict sense of the term.

However, we must not jump to conclusions and surmise that the geometrical knowledge of Spanish Renaissance masons derives from perspective treatises. Alberti's works on perspective were not translated into Spanish until the late eighteenth century, while Piero still awaits his turn [Rejón de Silva 1784]. The main sources on perspective available to Spanish Renaissance masons were Serlio's second book, not translated into Spanish but enjoying a certain circulation, and Pedro Ambrosio de Ondériz translation of Euclid's *Optics*, dated 1585, while Vandelvira was writing his manuscript [Serlio 1545; Euclid 1585]. Hernán Ruiz includes both perspective and stonecutting problems in his manuscript [c. 1550: see for instance 46v-47v, 51r-58v] but some of his perspectival methods are not connected with the tradition of Brunelleschi, Alberti and Piero, since the orthogonals do not always meet at a single centric point, as Gentil [1998] has stressed. There are references to "visual lines" in Alonso de Guardia [c. 1600: 87v] and Ginés Martínez de Aranda's [c. 1600: 67] manuscripts, which seem connected to Alberti's pyramid, although they could also derive from Euclid's *Optics*, by way of the translation by Ondériz, and this is about all. Of course, a number of problems and methods could have been introduced into Spain by Italian or Italianate artists such as Torni, Machuca or Siloé, in particular the use of torus vaults, but this can hardly account for the use of cone developments in Seville at such an early date as 1543 [Ruiz de la Rosa: 2002].

Thus, Quattrocento perspective and Spanish sixteenth-century stereotomy share a number of concepts, problems and methods; however, there seems to be no direct substantial connection between them. This suggests the existence of a common source, but it is not easy to identify it. Neither classical geometry [Euclid c. -250: books XI-XIII] nor the mediaeval tradition of practical geometry, spanning from Hugh of Saint Victor [c. 1125] to Mathes Roriczer's *Geometria Deutsch* [c. 1490] include a word about

orthographic projections, rotations or projection planes.³⁸ In fact, orthographic projection is almost completely lacking from architectural drawings and full-scale tracings in Antiquity, and only appears clearly in the High Gothic period [Sakarovitch 1997: 21-31, 35-51]. Thus, mediaeval construction shop practices furnish the most probable common source for perspectival and stereotomic methods. Curiously, these practices are seldom mentioned in the exhaustive literature on perspective; even the use of orthogonal projection, although impossible to deny, is not often stressed. On the other hand, the Gothic tradition is recognised, at least in Spain, as an important source of Renaissance stereotomic methods, although this debate is still open; by contrast, the role of Italy, Italian and Italianate artists and perspective in this field is almost taboo.

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Notes

1. It can be argued that cylindrical projections can be understood as a particular case of conical projections, with the vertex of the cone placed at infinity. However, when seventeenth-century treatises, such as Jean-Charles de la Faille’s *Tratado de la arquitectura* [c. 1640] compare the vertex positions in cylindrical and conical projections, they do so to stress the differences between the two kinds of projection. Although the works of Girard Desargues, also dating from around 1640, hint at a common foundation for cylindrical and conical projections, the idea was fully developed only by nineteenth-century projective geometry. Thus, if we understand the idea of connections between perspective and stereotomy in general terms, it seems rather anachronistic at first sight.
2. The use of stereotomic tracings in frontispices was not new; it had been put into practice by Philibert de L’Orme in *Le premier tome de l’architecture* [1567]. However Goiti gave it a new turn, since both side tracings act as emblems of the two basic stereotomic methods.
3. Field offers a precise and detailed explanation of Piero’s perspective techniques ranging from [2005: 129-173], but deals with the torus in a just a few lines [2005: 168].
4. Roccasecca [2001b: 95-96] has pointed out that the construction of the rings in drawings 1756A and 1757A does not follow exactly Piero’s method, since in 1756A and 1757A there are compass marks at the center of the meridians and inkless lines joining these centers with the corners of the meridians; also, in 1757A there is an inkless line following a parallel.
5. To be precise, here Piero seems to be using the side of the enclosing square, which equals twice the apothem of the octagon, as an approximation to the lesser diameter of the torus, which should equal the radius of the octagon.
6. Page numbers are taken from the electronic facsimile in Bibliotheca Perspectivae.
7. See in particular the “Postcript” to the English translation in pages 39-41, where he explains that taking into account the 1966 edition of *De re aedificatoria*, the letter to Leo X must be read as an explanation of a passage in Alberti’s text explaining orthographic projection, mistranslated in Bartoli’s edition.
8. It does not make much sense in this case to classify these rays as centric, median or extrinsic. To start with, the whole perspective falls below the centric point. Besides, when Alberti differentiates between median and extrinsic rays, he is thinking of a figure with a simple contour or, as topologists put it, a simply connected figure. However, the ring is a double connected figure, since it has a hole in the middle. Thus, there are two contours in the perspective, given the particular eye point Piero has chosen, and two sets of extrinsic points, one external and one internal.

9. Each of the 96 points has a different projecting line. However, the torus is symmetric about a horizontal plane and about a good many vertical planes, including a plane that is parallel to the projection plane of the profile. Thus, each point in the plan is superimposed to another point and the same thing happens in the profile; as a result, the draftsman should trace 48 lines in plan as well as 48 lines in the profile. However, he must transfer each of these intersections twice to the perspective, since each intersection represents two different points; this amounts to 192 transfers. It is not surprising, then, that Piero advises the reader to use a large set of rulers to avoid being misled by such a multitude of points.
10. Garcia-Salgado's terminology, although unusual, is useful here, since a number of studies (for example [Panofsky 1927; Klein 1961; Field 2005]) use the term "distance point" in the sense of "distance vanishing point" while others claiming the use of "distance point" in Alberti or Piero must mean the "distance to the perspective plane".
11. There is an interesting exception. In fact, in fol. 11 r., when solving a rather far-fetched problem – how to cut a square from a rectangle of unknown proportion –, Piero gives sound instructions on how to use the "distance vanishing point" in order to construct a square lying on the ground plane; he does so by placing the "distance vanishing point" on the horizon line at the correct distance and tracing a diagonal through the "distance vanishing point", without using projection and intersection in the profile. However, when he tries to present the proof of this construction, his demonstration is unsound. It is quite remarkable that Piero does not use this construction again; thus, he seems to grasp the notion of "distance vanishing point" on an empirical basis, but he is not sure enough of its foundation to use it as an operative method and in fact this passage is lacking from the Latin version of the manuscript; see [Field 2005: 148-150].
12. In fact, Uffizi 832 Ar includes a plan of the ring that could have been used for this purpose, while Richard Talbot [2006], working independently from Roccasecca, has posited such a solution for the *Chalice*. It is worthwhile to remark that similar methods are suggested by Alberti [1435: II-34], present in Serlio [1545: 33r, 34r, 35r] and ubiquitous in Cousin [10r, 21r, 22v, 26r, etc.]. However, this technique does not solve all the problems involved in the construction of the rings or the chalice. Once the draftsman has constructed the intersection of a meridian plane and the ground plane, he must raise it to its real horizontal plane; this problem can be solved using the method explained by Piero in his second book [Della Francesca c. 1480: 18r-24v; Field 2005: 156-159]. After this, the artist must place each corner of the ring in the corresponding line. Roccasecca has remarked that Uffizi 1757A includes an inscribed elliptical line connecting all the corners at the middle level of the ring, while Uffizi 830Ar, labelled "*Per fare uno mazzocchio*" includes Dürer's [1525: 16r] well-known construction for a conical section. Roccasecca [1998] suggests that the draftsmen of these toruses could have used a number of ellipses to place the corners of the ring along the horizontal lines, constructed maybe using an ellipsograph; in fact, 832Ar includes a singular ellipsograph as well as a scheme for the gardener's ellipse. In any case, there is a final problem to be solved, which is not mentioned by Roccasecca: the center of the resulting ellipse is not the projection of the center of the original circle. In our case, the draftsman can overcome easily this difficulty, since he can place both ends of the shorter axis of the ellipse and divide the resulting segment in two equal parts to place the center of the ellipse. However, it is not so easy to find the ends of the longer axis of the ellipse, in order to use the ellipsograph, since they do not correspond to a diameter of the original circle; at this point the draftsman of 1757A probably used some technique based on trial and error.
13. Della Francesca [c. 1480: 37v]: *Poi piglia il sexto et circula la quantità che tu intendi fare grande il torculo, et il suo centro sia M, et il circulo sia circulo A; poi tira MA linea recta, et con lo sexto piglia la quantità de FB del quadrecto e polla su la linea MA principiando da A, et dove termina l'altro pie del sexto su la dicta linea AM segna B; poi piglia il sexto e poni un pie sopra M et l'altro stendi perfine ad B et descrivi il circulo, che sia il circulo B, and so on until all four circles are constructed.*
14. Della Francesca [c. 1480: 37v]: *benché sieno proposti octo circuli, in questa demonstratione faremo con quactro, perchè poremo il dicto torculo giacente piano. Ma quando giacesi altrimente, siria necessario che fussino tante righe quanti circuli (e) in esso torculo se contene.*

15. An exception is the ring in the Gubbio studiolo; see [Raggio 1996: 23].
16. Of course, all these rings could have been drawn by empirical means, using a *velo* to depict a wooden model, or preparing a perspective of a flat-lying torus on a sheet of paper, applying it to the painting and rotating it until it fitted over the head of one of the fighters in San Romano or around the neck of a character in the *Flood*. However, we hope that the next paragraphs will make clear that these explanations are unlikely, since Piero knew the problem and hints at a different solution.
17. *In questa demonstratione faremo con quactro, perchè poremo il dicto torculo giacente piano. Ma quando giacesse altramente, siria necessario che fussino tante righe quanti circuli (e) in esso torculo si contene* [Della Francesca c. 1480: 37v].
18. Incidentally, these bases, which appear in the *Flagellation* in Urbino and in the *Annunciation* and the *Meeting of Solomon and the Queen of Sheba* in San Francesco in Arezzo are the only instances of toruses or polyhedral rings in the extant paintings of Piero.
19. In order to do so, he places the base line of the cube at a sloping angle at will; then he traces perpendiculars to these sloping lines to represent the edges of the cube that were originally in a vertical position. These lines are now sloping, although they are still parallel to the vertical projection plane, and thus the draftsman can easily measure their lengths. Since the rotation axis is perpendicular to the vertical projection plane, the plane of rotation will be parallel to this plane; as a result, the next face will present no difficulty, since the horizontal projection of each point will move along a line that is perpendicular to the reference lines. Tracing these lines and finding their intersections with the corresponding reference lines, Piero can easily place each point in the plan of the rotated cube.
20. Page numbers are taken from the electronic facsimile in Bibliotheca Perspectivae.
21. Although the title page of the 1545 edition of Serlio is *Il primo libro di architettura, di Sebastiano Serlio, Bolognese*, this volume includes both Serlio's first book on Geometry and his second book on Perspective, and of course the explanation of the perspective of the groin vault belongs to the second book.
22. Leaving aside the tiresome procedure needed to construct the ellipse, which was mastered by Vandelvira but probably unknown to Gothic masters, this rotation furnishes a simple method for showing the diagonal rib and the tierceron in true shape and to control the curvature of these ribs, which is essential in the dressing of springers, keystones and ordinary voussoirs; an elliptical projection would have been useless for this purpose.
23. The technique used by Philibert in the *Nouvelles inventions* [1561] is quite interesting, since he constructs an oblique elevation of a groin starting from a standard elevation, but has no direct connection to our subject.
24. Íñigo López de Mendoza was a key figure in the first Spanish Renaissance. He built the monastery of San Antonio in Mondéjar as early as 1489 and supported his uncle, Pedro González de Mendoza, in his striking decision to build a Renaissance façade in the college of Santa Cruz in Valladolid in 1487. Paradoxically, he was in charge of the supervision of the Gothic building of the Royal Chapel in Granada, although he died before the arrival of Machuca and Torni [Gómez Moreno:1925a; 1925b]
25. Literally, "*está puesto por obra en una villa que se llama San Gil en Francia*" [Vandelvira c. 1580, 52v].
26. Another line, the shorter proportion of the central rectangle in comparison with the Murcia vault, is not so easy to justify, and seems to be related simply to the width of the sheet of paper Vandelvira is using. This small detail is not negligible: as a result, the intrados surface is not a torus from the topological point of view; in fact, if the inner half of the torus were to be materialised, the surface would intersect itself. However, this has no consequences in the actual building process.
27. This meridians are half circles; however, they are depicted as straight lines in the elevation since they lie on plane that is perpendicular to the projection plane of the elevation.
28. This procedure raises two interesting questions, although neither of them can be analysed here in full detail. First, why does Vandelvira bother to construct these ellipses? It is quite usual in stonecutting tracings to leave out any line that is not essential for the ultimate purpose of the tracing: to construct the templates of the faces of the voussoirs or to compute the angles

between voussoir edges. However, the meridians of the *Bóveda de Murcia* are not necessary for the construction of the templates and Vandelvira does not mention the angles between voussoir edges or *salta-reglas*. Quite probably, he tackled this complex problem because he intended to lay out the tracing exactly below the vault to be built and to control the position of the voussoirs by means of a plumb line once they were set in place; in fact, he refers twice to this technique [Vandelvira c. 1580: 23 r., 23 v]. Second, what is the nature of this curve? The horizontal projections of the meridians are half-ellipses, but the resulting curve is not, since it is an assembly of arcs of a circle. However, if the purpose of these curves is just to control the placement of the voussoirs, it is easy to surmise that in a real stonemasonry tracing the masons would not care to join the points, and that in the *Libro de trazas ...* the curve is drawn in order to show the shape of the meridians for the benefit of the reader.

29. The reason for this striking omission can be found in Vandelvira's explanation of the hemispherical vault, or as he puts it, the "beginning and example of all Roman vaulting" [Vandelvira c. 1580, 60v]. Once he has drawn the arcs that stand for the parallels, he advises the reader that "*las cuales dos cerchas cerrarás por do quisieres*"; that is, "you can close these arcs as you will". Such a cavalier attitude is quite understandable if we take into account that in sixteenth century the stone did not usually arrive from the quarry to the construction site in standard sizes; thus, playing with the length of the voussoir allowed the mason to use the stones coming from the quarry in the most economical way. In fact, this technique was used in such a carefully executed work as the dome over the crossing of the Escorial basilica, where the voussoirs of each course show different lengths. However, this approach cannot be used in a horizontal-axis torus vault, since the courses are not placed between two parallels, but rather between two meridians. Thus, the builders of the vault in Junterón's chapel must have resorted to some device for controlling the length of the voussoirs, although Vandelvira does not say a word about this issue.
30. Guardia [c. 1600: 87v]: "*y para sacar las plantas por caras de las hiladas desta dicha pechina pondras la Regla en la buelta de horno en los dos puntos de la hilada que quisieres sacar y tiraras las líneas bisuales señadas con la C que bayan a parar a la línea perpendicular señalada con la D*".
31. Vandelvira [c. 1580: 53 r]: "*Sirve también esta traza para hacer una bóveda alrededor de un patio redondo como está puesto por obra en la alcázar real del Alhambra de Granada, mas antes de trazar las piedras igualmente por entrambos cabos que lo que se roba por la una parte se ha de robar por la otra, por andar a nivel y no capialzar más la una parte que la otra, y así las piezas no han de ser más altas que la tardosa del arco a las líneas de puntos*". Another interesting problem is the source of Alonso de Vandelvira's knowledge of the vault in the palace. According to Galera [2000: 27-28] Andrés de Vandelvira could have seen the model of the courtyard, prepared in 1532. In Galera's opinion, this would explain the differences between the drawing in the *Libro de trazas* and the built courtyard, such as the different height of the springing of the vault in both sides. However, we have seen that Alonso made similar changes in the *Bóveda de Murcia*; this is quite natural, since he was trying to furnish general models for horizontal or vertical torus vaults, and not surveys of the Murcia and Granada vaults; thus, it is easier to assume that Alonso himself could have seen the completed lower gallery, since he describes other works in Granada, such as the Chancillería staircase, completed in 1578 after his father's death.
32. Vandelvira [c. 1580: 52v-53r]: "*Este arco se entenderá por la vía de San Gil aunque más fácil, por ser la vía de San Gil un arco capialzado y éste a nivel y así puestas las piezas en cuadrado como las B.B. demuestran, robarse han igualmente por entrambas partes teniendo consideración a sus ligazones como parecen en las plantas señaladas con las C.C. que miran al centro*."
33. In fact, Vandelvira [c. 1580: 69v, 103v] also puts on equal standing the *Bóveda de Murcia*, a torus vault and the *Ochavo de la Guardia*, a coffered quarter-sphere vault.
34. We are quoting here in reference to the original folio numbers, given by Barbé in round brackets.
35. Vandelvira [c. 1580: 24v]: "*Porque dije de enseñar el arco torre cavado y redondo por robos y porque también sea lumbré para entender otras trazas que no se pueden hacer si no es por*

- robos, pongo ahora éste...*”. Again, the quotation is referenced to the original folio numbers of the manuscript, given by Barbé in round brackets.
36. Page numbers are taken from the electronic facsimile in Bibliotheca Perspectivae.
 37. Elkins states that the proof “relies almost exclusively on a two-dimensional system of proportions” [1987: 220] but this is misleading, since the visual rays, picture plane and perspective projections are not lying in the same plane and Piero uses orthogonal projection to represent them; in fact, later on Elkins mentions side and top views [1987: 224].
 38. Of course, Roriczer’s *Fialenbuchlein* [1486] uses orthographic projections; in fact, it is the first text to explain the transfer of measures from plan to elevation, an antecedent of the reference lines used in descriptive geometry. But this is precisely the point: the *Fialenbuchlein*, deriving from workshop practice and oral traditions, uses orthographic projection, while the *Geometria Deutsch* [1490 c.], which derives from *De inquisitione capacitatis figurarum*, as Shelby has shown [1972: 412-416], does not mention projections at all.

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