Reflections on the Relationship between Perspective and Geometry in the Sixteenth and Seventeenth Centuries

Abstract. Paolo Freguglia examines the relationship between perspective and geometry before Guarini, and more precisely, in the 1500s. The representation of space, which in the pre-Classic age was substantially conceptual and sometimes ideographic, was gradually organised so that it became optical representation, and finally arrived at being able to give a sense of three-dimensions. The techniques of perspective were presented not only as practical rules for drawing in a given manner, in conformation with how observed reality appears to the eye, but were also described according to their geometric underpinnings. Thus were introduced new points of departure for considerations on geometry, as made evident by the work of Desargues and Pascal.

1 Introduction

In this paper I would like to shed some light on the relationship of perspective and geometry before Guarini, and more precisely, in the 1500s. It should be recalled that although Guarini was not a professional mathematician, he was undoubtedly interested in mathematics. In Torino in 1671 Guarini published his *Euclides adauctus et methodicus mathematicaeque universalis*, dedicated to Carlo Emanuele II di Savoia, a work of over 700 pages comprising an introduction and thirty-five books (or chapters). Eclectic in scope, it was mainly intended for teaching purposes. In addition to an analysis of the books of Euclid, there are also explanations of how to calculate with logarithms, how to solve plane and spherical triangles, and classic curves such as conics, quadratics, etc. In the twenty-sixth book Guarini addresses projective geometry, which he had presumably learned and studied during the period from 1662 to 1666 when he was in Paris. It is certain however that he knew perspective techniques (which are themselves different from the geometrical techniques proposed by Desargues and Pascal), which he must have learned in his formative years. We cannot know how much and in what way Guarini was able to connect the theory of perspective to the “projective geometry” of Desargues and Pascal. A marked sensitivity for geometry and, in broader terms, the importance of geometry for architecture, is also found in Guarini’s *Architettura civile*, published posthumously in 1737.

The representation of space, which in the pre-Classic age was substantially conceptual and sometimes ideographic, was gradually organised so that it became optical representation, and finally arrived at being able to give a sense of three-dimensionality. The completion of this technical objective of representation, which took the study of optics into account as well, first occurred during the fifteenth century and was consolidated during the sixteenth, thanks to a large number of treatises written by the artists and architects as well as mathematicians. The techniques of perspective were presented not only as practical rules for drawing in a given manner, in conformation with how observed reality appears to the eye, but were also described according to their geometric underpinnings. Thus were introduced new points of departure for

considerations on geometry. In the first half of the 1600s this aspect was made particularly evident by the works of Gérard Desargues,¹ who was an architect, and by Blaise Pascal,² a mathematician. The geometric results that they presented effectively signaled the birth of projective geometry. “Ideal” elements similar to what we today call “point at infinity” and “line at infinity” were introduced; thus what we have at a conceptual level is an amplification of Euclidean space. The theory of proportions also played a determinant role in these developments. This made it possible to obtain geometric properties relevant to projection, although the context of reference always remains Euclidean. Investigations of the theory of the conics, which was already the object of attentive study in the 1500s, were tied to perspective as well. In this period the conic sections were involved³ not only in perspective (that is, as it was called then, *perspectiva*), but also regarding the so-called burning glasses (mirrors shaped in order to produce combustion) and in “hour lines” (for the construction of sundials). Such topics, which regard the possible applications of the notions inherent in the conic sections, were studied by mathematicians such as Giovan Battista Benedetti, Christophorus Clavius and Francesco Maurolico. To Maurolico (in 1657), Federico Commandino (in 1655) and Giovan Battista Memo (in 1537) we owe translations of the work of Apollonius, which was a review of the theory of conics. It should also be noted that the conics were also examined from a theoretical point of view by Giorgio Valla (1477-1500) and Johann Werner (1468-1522).

2 Treatises on perspective in the Renaissance

Because of their connection with the great developments in figurative art, treatises on perspective flourished during the Renaissance. The representation of objects belonging to a three-dimensional space on a two-dimensional surface is one of the oldest problems of both figurative art and of astronomy. With regard to Antiquity, theatrical creations on one hand and the construction of planispheres on the other offer two highly significant examples: think of Ptolemy’s *Planisferio*, which appeared in Latin in 1507.

The theory of perspective construction should be distinguished from the theory of natural vision, which can be traced back to technical studies in Euclid’s *Ottica* and *Catoptrica*, that is, we have to distinguish between what was called *perspectiva communis* and *perspectiva artificialis* or *perspectiva pingendi*. The rules regarding techniques of painting, studied and applied by painters, and during the design phase, by sculptors and architects of the Renaissance, take into account, in addition to Euclidean geometry, the so-called “practical geometry”, manuals⁴ which had been developed since the Middle Ages. Between the fifteenth and the sixteenth centuries we therefore find a series of works that testify to the great interest, not only applicative for speculative as well, with which subjects related perspective were studied with the spirit of the age. Here we give a list, in chronological order, of the most significant of these works in order to provide some idea of what constituted the scientific output in this direction during the Renaissance. As can be seen, some of these works are “commentaries” on works of antiquity:

- L. B. Alberti, *Della pictura* (1435, printed in 1511)
- Piero della Francesca, *De prospectiva pingendi* (1482, 1487)
- Leonardo da Vinci, *Trattato della pittura* (printed in 1651)
- J. Pélerin, *De artificiali perspectiva* (1505)
- A. Dürer, *Istitutionum geometricorum libri* (1525, 1533)
- S. Serlio, *Prospettiva* (1545)
- F. Commandino, *In Planisphaerium Ptolomaei Commentarius* (1558)
- J. Cousin, *Livre de Perspective* (1560)

D. Barbaro, *La pratica della Perspectiva* (1569)
E. Danti, *La prospettiva di Euclide* (1573)
G. B. Benedetti, *Diversarum speculationum [...] liber* (1585)
G. U. Del Monte, *Perspectiva* (1600)
S. Stevin, *Perspectiva* (1605).

The majority of the authors cited above had a special sensitivity to and knowledge of mathematics, and they examined the theme of perspective with a mathematical bent and zeal. Among these, and certainly one of the most important of them was Piero della Francesca (ca. 1410-1492) from Borgo Sansepolcro, master artist, whose works profoundly influenced the history of art. We are not interested in the aesthetic aspects of Piero's work here, but rather in the theoretical underpinnings of perspective as found in his *De prospectiva pingendi*. For Piero perspective is necessary for painting, in the sense that it isn't possible to draw correctly without it. It is almost like a syntactic scheme: its violation means that the pictorial "discourse" is ill expressed; or again, insisting with this analogy, when the rules of counterpoint are violated in a traditional tonal musical composition the result is discordant. It is thus an indispensable technique for the artist. Piero writes in the first book:

Painting consists in three principle parts, which we say are drawing, commensuration, and colouring. By drawing we mean profiles and outlines that contain the object. By commensuration we mean profiles and outlines that are placed proportionally in their locations. By colouring we mean to apply the colours as they are in the objects they show, light or dark according to the changing lights. Of the three parts I intend to treat only commensuration, which we call perspective...⁵

In his turn Leonardo da Vinci wrote:

Perspective is the demonstrative proof that is confirmed by experience that all things communicate their likenesses to the eye by pyramidal lines; and that the pyramids of bodies of equal magnitudes form a larger or smaller angle according to the kind of distance that there is between one and the other...⁶

This kind of statement is quite similar to those of other artists, such as, for example, Leon Battista Alberti, who writes in Book I of *Della pittura* regarding to the use of the pyramid in optical perspective:

The pyramid is a figure of a body from whose base straight lines are drawn upward, terminating in a single point. The base of this pyramid is a plane which is seen. The sides of the pyramid are those rays which I have called extrinsic. The cuspid, that is the point of the pyramid, is located within the eye where the angle of the quantity is. Up to this point we have talked of the extrinsic rays of which this pyramid is constructed. It seems to me that we have demonstrated the varied effects of greater and lesser distances from the eye to the thing seen.⁷

As though he were stating postulates, Leonardo wrote that:

The eye only sees by means of pyramids;⁸

The path of the pyramidal lines caused by the objects and terminating in the eye are necessarily straight lines;⁹

The resemblance of the figures and colour of each body are transferred from one to the other by means of pyramids.¹⁰

He then gives a proof:

The small object from nearby and the large [object] from far away, being seen in equal angles, will appear of equal size.¹¹

Leonardo also recorded his thoughts on some geometric-perspective definitions. For example, regarding the point, he follows the geometers in saying “it is that which cannot be divided into any parts” but the point is also “that which, located in the eye, receives all the points of the pyramids”.

Let's return to the work of Piero della Francesca.¹² The treatment develops along the pattern of mathematical treatises, although not always with the same kind of rigour. However, as a preliminary we have to underline the fact that perspective is not projective geometry; rather, it constitutes a “Euclidean model” of a technical apparatus that serves to represent – to “see” – a three-dimensional reality in two dimensions. We might say paradoxically that a perspective representation constitutes a Euclidean model of the projection. This can be best clarified with an example. As we know, two straight lines in the Euclidean plane remain as such in a representation; let's say that they have the same direction, and let's add conceptually that they meet in a point at infinity, but that this point does not belong to the Euclidean plane and therefore we can't represent it with any point on this plane. If instead we do choose a point (the vanishing point) in the Euclidean plane and make it so that straight lines that we hold to be and conceive of as parallel converge at that point, then the point is the conventional-perspective representation in the Euclidean plane of a point at infinity.

We can see that graphic rules and equalities between relations form the basis for perspective drawing. The latter, that is, equalities between relations, is based on the theory of proportions.

Piero's work was given a great deal of consideration by authors of perspective in the 1500s. We need only mention the treatise of Daniele Barbaro, who in spite of the fact that he gives just a passing mention to “Pietro dal Borgo S. Stefano” in the ‘Proemio’, draws on Piero's work a great deal, as has been often pointed out, in the rest of the treatise.

Going back to what was said earlier concerning the relationships between perspective and projective geometry, it is worthwhile to underline the fact that the vanishing points are representations in the Euclidean plane of points at infinity. The perspective plane can thus be seen as the Euclidean representation of the projective plane whose line at infinity, understood as the set of its points at infinity, is the straight line on which we have represented all of the vanishing points and which can be intuitively equated to the horizon.

The treatises on perspective continued to develop autonomously in the 1700s with technical refinements and mathematical methods that were increasingly defined, as can be seen, for example, in the works of Brook Taylor and J. H. Lambert.¹³ Descriptive geometry and projective geometry, which owe a great deal to perspective, were given their definitive treatment by, respectively, Gaspard Monge (*Géométrie Descriptive*, 1798) and J. V. Poncelet (*Traité des propriétés projectives des figures*, 1822). However, as Poncelet himself acknowledged, we owe to Gérard Desargues (1593-1662) and Blaise Pascal (1623-1662) the study of the geometric properties, above all of the conics, which although in a different theoretical context than the one introduced by Poncelet, concern projective geometry and were inspired, as we shall see, by the theme of perspective.

3 Towards projective geometry

In some of the propositions, and more precisely from 129 to 145 of the *Collectiones mathematicae* (ca. 320 A.D.) by Pappus of Alexandria, we find a kind of theory of transversals and notions which regard, for example, what we would today call conditions of alignment between points or the invariance of cross-ratios.¹⁴

However, it was the work of Desargues that played the crucial role for the themes of projective geometry. Although his results did not enjoy either at the time in which he was developing them nor in the periods that followed the success they deserved, it must be admitted that the notions he set out were of great significance historically and conceptually. They effectively broadened the understanding of fundamental Euclidean notions, gave special consideration to the concept of projective transformation, and shed light on the invariance of such transformations of relations among points, as in the case of the involution among points, an idea explicitly introduced by Desargues. The relevant works of Desargues are the *Première proposition géométrique*, as presented by Bosse in 1648 and the *Brouillon Projet d'une atteinte aux événements des rencontres d'une cône avec un plane et aux événements des contrariétés de entre les actions des puissances ou forces* of 1639. In the first Desargues presents the well-known theorem of homologous triangles.

The 1639 *Brouillon Projet*, although essentially an outline, is characterised by an expository organisation and a concise method of geometric investigation that are particularly novel. In this work (as in *Première proposition*) the Ptolemy-Menelaus theorem plays a crucial role in the proofs of the fundamental results in terms of projective geometry, and Desargues specifically mentions it in the *Brouillon Projet* in connection to the case of the plane. Let's begin by presenting the definitions of Desargues regarding what we call elements at infinity and thus of the notion, also introduced by Desargues, of involution.

We say that several straight lines are of the same ordinance (*ordonnance des lignes droites*) if they are either all parallel or if they all meet in the same point. In both cases these straight lines tend to a point. ... The point to which the straight lines tend in the two cases seen above is called the "goal" of the straight line of a given ordinance (*but d'une ordonnance de droites*). ... We say that several planes are of the same ordinance (*ordonnance de plans*) if they are either all parallel or they all meet in the same straight line. In both cases these planes tend to a straight line. ... The straight line to which the planes tend in the two cases seen above is called the "goal" of the planes of a given ordinance (*but d'une ordonnance de plans*).

We can see right away that Desargues highlights the fact that from a projective point of view points and straight lines in infinite case both have the same characteristics as the corresponding entities in finite case; in fact, like the latter, the former can be obtained as intersections (at infinity) of straight lines and planes. Desargues then presents the generation of a sheaf of straight lines (analogous to a sheaf of planes) in this way:

Supposing an infinite straight line having a fixed point moving along its entire length, we see that in the various positions that it assumes during this motion it gives the various straight lines of a single sheaf whose centre is the fixed point.

This way of generating sheafs brings to mind the notion of direction of motion of the first kind in the sheaf of lines, where the straight line generatrix is able to move in two

opposite directions starting from an initial position. In this same vein we also have the concept that if a fixed point, the centre of the sheaf, is in the finite case, then each point of the line that is different from the centre which moves, generates a circumference; while in the infinite case the fixed point generates a line perpendicular to the line that moves. Thus Desargues is able to identify a connection of sorts between the infinite straight line and the circle. He likewise introduces the notion of involution between six points on a straight line considered as a set of points, based on and developed from the theory of proportions. What follows shows how Desargues reasons and how he presents and studies the notion of involution.

Given a point O on a straight line and the three pairs of segments (called branches) Oa, Oa'; Ob, Ob'; Oc, Oc' (see figure), if the rectangles constructed on the three pairs of segments are equal, that is, if

$$Oa \cdot Oa' = Ob \cdot Ob' = Oc \cdot Oc' \quad (3.1)$$

then the six points (called nodes) a, a'; b, b'; c, c' are said to be in involution. Let's see now how the involution between three points can be characterised by relations that do not involve point O (called the trunk of the involution). Let's begin with the second part of (3.1), which we rewrite as:

$$Ob' : Oc' = Oc : Ob \quad (3.2)$$

From (3.2) we obtain:

$$(Ob' + Oc) : Oc = (Oc' + Ob) : Ob$$

that is, summing and then exchanging the means:

$$b'c : bc' = Oc : Ob \quad (3.3)$$

Comparing (3.2) and (3.3) we have:

$$Ob' : Oc' = Oc : Ob = b'c : bc' \quad (3.4)$$

Using the same procedure Desargues also obtains:

$$Oc' : Ob = Ob' : Oc = b'c' : bc \quad (3.5)$$

Multiplying (3.4) by (3.5) member by member, we have

$$Ob'/Ob = (b'c \cdot b'c')/(bc' \cdot bc) \quad (3.6)$$

Starting from $Ob' : Oa' = Oa : Ob = b'a : a'b$ and $Oa' : Ob = Ob' : Oa = b'a' : ab$, multiplying member by member we have:

$$Ob'/Ob = (b'a \cdot b'a')/(a'b \cdot ab) \quad (3.7)$$

Comparing (3.6) and (3.7) we obtain:

$$(b'c \cdot b'c')/(bc' \cdot bc) = (b'a \cdot b'a')/(a'b \cdot ab) \quad (3.8)$$

In an analogous way Desargues establishes that:

$$(c'b \cdot c'b')/(cb \cdot cb') = (c'a \cdot c'a')/(ca \cdot ca') \quad (3.9)$$

$$(a'b \cdot a'b')/(ab \cdot ab') = (a'c \cdot a'c')/(ac \cdot ac') \quad (3.10)$$

Thus it is established that:

(Theorem): Let there be six points $a, a'; b, b'; c, c'$ on a straight line. Equations (3.8), (3.9), (3.10) characterise each involution of the six points in question.

Desargues thus resorts, as we said, to the Ptolemy-Menelaus theorem and applies it as a lemma to two theorems of signal importance for projective geometry: the so-called theorem of complete quadrilaterals, and the demonstration of the invariance of involution under projection.

4 The conics according to Desargues and Pascal

In the *Brouillon Projet* Desargues applies his results relative to involution and complete quadrilaterals to the study of conic sections. He defines the conics as projections of the circle but, unlike the Greeks, he was able to see general projective properties. According to Desargues the generation of the conics occurs when the following basic conditions are taken into account. If a straight line that has a fixed point moves along a circumference there are two possibilities:

1. the fixed point is in the plane of the circumference and so the straight line with its successive positions describes a sheaf that can be with the centre at infinity or no, depending on whether the fixed point is at an infinite or finite distance;
2. the fixed point is outside the plane of the circle at some finite or infinite distance, so that we obtain a solid that we call cone if the point is at a finite distance, and cylinder if the point is at an infinite distance.

As we can see, a cone and a cylinder are seen in the same way. This is followed by an exposition of the conic sections that is very similar to that by Apollonius. It is worthwhile noting that the cylindrical sections had already been studied between the sixteenth and the seventeenth centuries,¹⁵ but with Desargues we find another way of looking at the subject. Thus the theorem about the quadrilateral inscribed in a conic and cut by a transversal is stated in this way:

Given a quadrilateral inscribed in a conic, a secant line which does not pass through any of the vertices of the quadrilateral cuts the conic in two points that are joined in the involution to which belong the three pairs of points marked on the same straight line by the pair of opposite sides of the quadrilateral (Desargues, *Brouillon Projet*, Theorem 3).

Desargues says that “when well understood, this can be applied on numerous occasions” [Taton 1951: 137 ff]. Thus, as Taton observes, the author observes “the determining role of the projective method that serves him as a guide” [1951: 137 ff] for the investigations proposed in the *Brouillon Projet*.

As is well known, in his *Essay pour les Coniques* of 1639/1640 Blaise Pascal studied some remarkable properties of conics, making reference to the methods of Desargues. Historians of mathematics are almost unanimous regarding Desargues’s influence on Pascal, to the point of considering Pascal’s results as a follow-up, an amplification and a partial evolution of those of Desargues. Following some definitions, there is Pascal’s well-known theorem of a hexagon inscribed in a conic. Pascal precedes this with the following definition: “when several straight lines converge to the same point or are parallel among themselves, the these straight lines are said to be of the same order and the same ordinance...”. This is of course thoroughly analogous to what Desargues had said regarding *ordonnance des lignes droites*.

5 Conclusions

I wish to conclude with a few brief historical considerations on the relationships between Descartes and Desargues and Pascal, who knew each other thanks to the centre of attraction and cultural discussion that P. Mersenne created. In a letter from Descartes to Desargues dated 18 June 1639, he refers to the *Bouillon Projet*, which Mersenne had sent to Descartes. In the first part of the letter Descartes disagrees with Desargues's choice of introducing new mathematical terminology (as example of which, and perhaps the only term which remains in use today, is involution) inasmuch as scholars conventionally referred to the classical traditions, which in the case of conics went back to Apollonius; if instead Desargues's intention was write for those who practiced perspective and architecture in a technical sense, then these terms might enjoy a certain success, because they were more closely linked to the French language and to a practical sense. But in this case, the mathematical task that was outlined in the *Projet* would be but little appreciated. Descartes's opinion is typified by the following part of the letter, in which we can clearly see not only that he is suggesting the use arithmetic for doing geometry, but also that he has little appreciation for concise methods. He writes:

As far as this is concerned, it seems to me that, in order to make your proofs simpler, using arithmetical terms and operations would not be out of place, as I did in my *Géométrie*; because there are many more people who know what a multiplication is and what a composition of three ratios is, etc.

Descartes concludes by saying that he thinks that Desargues's proposal of unifying the notion of points in the finite case and in the infinity case is valid. In a letter dated 12 November 1639 Mersenne had written to Descartes in extremely flattering terms regarding the precocious and brilliant mathematical mind of Pascal. In April 1640 Descartes had a copy of the *Essay* in his hands and considered it to be a unqualified outgrowth of Desargues's concepts.

As I said, Desargues's formulation and methods were not successful in the mathematical world at the time, except for the interest they aroused on the part of Abraham Bosse (1611-1678) and Philippe de la Hire (1640-1718). The attention they deserved would only arrive in the nineteenth century. It was instead the methods of Descartes, undoubtedly due in part to their technical potential, that characterised the developments of geometry and mathematics in the second half of the seventeenth century.

Translated from the Italian by Kim Williams

Notes

1. For a more in-depth analysis of the work of Desargues, cf. [Field 1997]; [Freguglia 1982]; [Poudra 1864] and [Taton 1951].
2. For Pascal's contribution to geometry, and especially his *Essay pour les coniques*, cf. [Taton 1951: 190-194], [Pascal 1904-1914] and [Gehrhart 1892].
3. Cf. [Clagett 1965-1984], in particular vol. IV, A Supplement on the Medieval Latin Traditions of Conic Sections (1150 – 1566) published in 1980, and [Maierù 1996].
4. For more on Medieval and Renaissance treatises of practical geometry, cf. the various texts edited and commented in the form of notebooks by the Centro Studi per la Matematica Medioevale of the University of Siena.
5. *La pictura contiene in sè tre parti principali, quali diciamo essere disegno, commensuratio et colorare. Disegno intendiamo essere profili et contorni che nella cosa se contiene. Commensuratio diciamo essere essi profili et contorni proportionalmente posti nei luoghi loro.*

Colorare intendiamo dare i colori commo nelle cose se dimostrano, chiari et oscuri secondo che i lumi li devariano. De le tre parti intendo tractare solo de la commesuratione, quale diciamo prospettiva Regarding the work of Piero della Francesca in general, cf. [Panofsky 1993; 1939; 1974]; for *De prospectiva pingendi* cf. [Piero della Francesca 1984].

6. *Prospettiva è ragione dimostrativa, per la quale l'esperienza conferma tutte le cose mandare all'occhio per linee piramidali la lor similitudine; e quali corpi d'eguali grandezza faranno maggiore o minore angolo a al lor piramide secondo la varietà de la distanza che sia da l'una a l'altra...* [Leonardo da Vinci, MS. A, fol. 3r].
7. *La piramide sarà figura di un corpo dalla cui base tutte le linee diritte tirate in su terminano ad un solo punto. La base di questa piramide sarà una superficie che si vede. I lati della piramide sono quelli razzi, i quali io chiamo extrinseci. La cuspide, cioè la punta della piramide, sta drento all'occhio quivi dove l'angholo delle quantità. Sino ad qui dicemmo de i razzi extrinseci da i quali dia conceputa la piramide; et parmi pruovato quanto differentii una più che un'altra distanza tra l'occhio et quello che si vegga* [Alberti 1956: bk. I, 46-47].
8. *L'occhio non vede se non per piramide* [Leonardo da Vinci, *Codex Arundel*, fol. 232v].
9. *Il concorso delle linee piramidali causate da li obietti e terminate nell'occhio è necessario essere rette linee* [Leonardo da Vinci, *Codex Arundel*, fol. 9v].
10. *Le similitudini delle figure e colori di ciascun corpo si trasferiscono dall'uno all'altro per piramide* [Leonardo da Vinci, *Codex Arundel*, fol. 232r].
11. *La cosa piccola da presso e la grande da lontano, essendo viste dentro a equali angoli, appariranno d'eguale grandezza* [Leonardo da Vinci, *Ms. A*, fol. 8r].
12. For more details, cf. [Boffito 1993: 134-135].
13. Cf. [Freguglia 1995]; [Andersen 1991]; [Lambert 1981].
14. The cross-ratio is the ratio between two simple ratios: given three distinct points A, B, P on an oriented straight line r , it is possible to identify the ratio $h = AP/BP$ (with the condition that $AP = -PA$ and $BP = -PB$), which, it turns out, depends on neither the orientation of r nor on the unit on measure chosen on r ; simple ratio h is conventionally denoted as (ABP) . If instead we now consider four points A, B, C, D (in a finite case and at infinity) on r , the cross-ratio b of the four points considered in a given order is:

$$b = (ABCD) = (ABC) : (ABD) = (AC/BC) : (AD/BD).$$
 It can be proven that b does not change if two of the elements are interchanged, as long as the other two are also interchanged.
 Further, the four points of an oriented straight line r , taken in a given order, form a harmonic set as long as their cross-ratio in that order is equal to -1 . To be precise, if A, B, C, D are the four points in given order, then

$$(ABCD) = -1,$$
 from which, among other things, it happens that

$$(ABC) = -(ABD).$$
15. Maurolico wrote an important manuscript on cylindrical sections in 1534, entitled *Sereni cylindricorum libelli duo*, in which he reconstructs a treatise by Serenus of Antinoupolis (II sec. A.D.) which arrived to the West through the work of Francesco Filelfo (1398-1481) and translated in part by Giorgio Valla; cf. [Tassora 1995].

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Paolo Freguglia is Full Professor at the University of L'Aquila, where he teaches mathematical biology and history of mathematics, which are also his fields of research. He has been visiting professor at several universities outside Italy. He is a chairman and member of the editorial board of several international journals. His scientific production (in all, approximately 150 works between books, essays and articles) principally regards the following areas of research: the construction and analysis of several mathematical models in physics (betatronic motion and geometrical optics) and biology (biomathematical models of neo-Darwinian concepts); the history of mathematics in the sixteenth and seventeenth centuries (regarding Bombelli, Tartaglia, Stevin and in particular François Viète); the history of logic and of mathematics in the nineteenth and early twentieth century, including Peano and his school, and the related themes of the algebra of logic, mathematical logic, the foundations of geometry, and geometric calculus (G. Bellavitis, C. Burali Forti, W.R. Hamilton, etc.).