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Research

Using Key Diagrams to Design and Construct Roman Geometric Mosaics?

Abstract. The complexity shown by some geometrical patterns of Roman mosaics and the high quality of their realization lead to think that for such patterns, unlike scenes with human or animal figures, a model of the general pattern was certainly not sufficient to guide the setting up; in order to answer this question one is led to conjecture the existence of diagrams (key diagrams) with which the craftsman, by looking at them, is able to identify (and/or remember) the geometrical structure of a basic element of the general pattern, as well as a way for constructing it – and possibly the whole pavement – with his usual instruments. This hypothesis is applied to some patterns which were well spread over the Roman world. The present study aims at showing how a given key diagram can apply to varied patterns and, conversely, how the making of a given complex pattern can rely on several articulated key diagrams.

Introduction

Archaeologists who specialize in the study of mosaics generally do not show much interest in “geometric” ones, and prefer to focus their research on those representing “living” scenes, i.e., those featuring gods, humans or animals. However, geometric mosaics are not devoid of interest, since they may give insights into the craftsmen’s geometrical knowledge and processes,¹ from which it is sometimes possible to ascertain links between practical and theoretical knowledge. Based on my personal experience [Parzysz 2007], I would advocate that mathematicians interested in the history and the epistemology of geometry work with archaeologists, in order to increase our knowledge about these points and to help them see facts that they might otherwise miss. And, in a more practical way, this can also lead to making better restorations of damaged mosaics (see for instance [Bar-Shay 1995: 124]). Of course, the geometry involved in such studies is quite elementary, of the type taught at high school level, but studying geometrical mosaics from a geometrical point of view – which seems an obvious thing to do – has not aroused much interest until now, even if some attempts must be mentioned, such as [Prud’homme 1975], [Tebby 1994], [Hanoune 1994] and [Bar-Shay 1995, 2005].

In this paper I would like to deal with a general question about mosaics: starting from the fact that identical geometrical patterns – which are sometimes quite complex – were made through time and space in the Roman world, one comes to the conclusion that some means for passing them on from one craftsman (*pictor*) to another necessarily existed. But, since there is only little archaeological evidence of this phenomenon,² we are bound to make hypotheses about what these means were, and in this paper I will develop one such hypothesis on some chosen examples (among many others).

1 Posing the problem

With regard to mosaics with figures, some specialists think that Roman mosaicists probably made use of sketchbooks of models that served a double purpose: on the one hand, they enabled prospective customers to choose their favourite subjects; on the other hand, they could be used as models for reproducing patterns. Thus, for instance:

It is almost certain that, at the time we are concerned with, mosaics with figures were neither original works nor real creations. They were products which were copied, modified, adapted, or composed from sketchbooks in possession of the mosaic workshops, from which the customer selected the subjects he wanted to be represented in his home [Daszewski & Michaelides 1989: 14] (my trans.).

Others disagree with that point of view and think that there were no such sketchbooks. For instance,

Nobody has ever seen any sketchbook and no text or image can vouch for their existence [Bruneau 1987: 154] (my trans.).

The “sketchbooks” hypothesis is of no use: it is not necessary to suppose their existence to explain the passing on of the iconographic tradition [Balmelle & Darmon 1986: 246] (my trans.).

Nonetheless, the fact remains that the passing on of an iconographic tradition has taken place through time and space, either in the form of sketchbooks or of teaching in the “guild” type:

It seems to us that the main part of the passing on must have taken place in a living way by a passing on from master to disciple, during the pictor’s training [Balmelle & Darmon 1986: 247] (my trans.).

At any rate, the existence of such a passing on appears as the most important element, whatever the vectors used to maintain it.

Regarding the geometric mosaics, let us now consider the following question: How is it possible, on the sole basis of a picture, either drawn or mental, to be able to realize a geometric mosaic with a complex pattern in a satisfactory way?

If one is provided with a drawn representation, the first question is about its nature: it may be a precise drawing, made with instruments, or a mere sketch made only by hand, like those on the medieval architect Villard de Honnecourt’s sketchbook [Erlande-Brandebourg et al. 1986]. If it is a mental image, it will be necessary to concretise it on the ground with the usual instruments: line and ruler, and perhaps – but not necessarily – square and compass.

In the case of a precise scale drawing the first idea which comes to mind is to reproduce it at real size on this sole basis. Then a double problem arises, linked both with scale and reproduction. At first sight, the question of shifting to real size, from measures on the drawing or by reading dimensions, can be solved in two different ways: either with a calculation of proportionality or by making use of a proportion scale. It is well known that the Roman numeration system, since it is not positional, is not very good for calculation and implied the use of *abaci*, especially when multiplications and divisions were needed, as

in this case. That is why making use of a proportion scale is more likely. But this technique makes three operations necessary for each length:

1. measuring the original length in the drawing;
2. transforming it with the scale;
3. transferring this new length *in situ*.

Besides the tediousness of this process, each one of the three above operations generates some kind of imprecision:

1. (*measurement*) imprecision of the drawing and of the instrument with which the length will be transferred;
2. (*transformation*) imprecision of the scale, of the transfer on the scale, of the reading;
3. (*transfert*) imprecision of the instrument.

Used this way, this technique would give an inaccurate result, but to some extent this drawback could be contained with a guiding grid:

In marking land divisions, cumulative error was prevented by establishing a taut grid framework, which was done either by subdividing the surface from equidistant points along vertical and horizontal axes, that is working from the centre outwards, or by dividing the perimeter of a square or rectangle into equal units and connecting the points on opposite sides, that is working from the outside inwards. The latter procedure would be more practicable for laying out a mosaic pavement [Tebby 1987: 277].

Nevertheless, such a “servile” method will quickly show its limits, because of some internal geometrical relations between the components of the pattern. Here is an example taken from a “multiple pattern” mosaic found in Migennes, France [Bassier, Darmon & Tainturier 1981]. These two square panels (figs. 1, 2) are basically filled with the same pattern, described as a “star of eight lozenges inscribed in a circle” [Balmelle et al. 2002; pl. 289].

Fig. 1

Fig. 2

This description is formal, and it does not quite fit the real mosaic, since the shapes of the stars on the two panels are not identical, and the so-called “lozenges” look more like rhomboids. The corresponding theoretical pattern (fig. 3) results from a simple geometrical construction beginning with that of a regular octagon inscribed in a circle, then linking every third vertex in order to obtain a star octagon (fig. 4).

Fig. 3.³

Fig. 4.

It seems obvious that this is not the process implemented in Migennes, and we can say that, when designing the pattern, the craftsman was not conscious of the subjacent star octagon: he contented himself with putting eight adjacent quadrilaterals around the centre of the circle. This is far from being always the case: see for instance fig. 5, from Adana, Turkey [Balmelle et al. 2002: pl. 289c], on which the alignments are much better respected. Anyhow, and beyond any supposed “clumsiness” of the mosaicist, this example shows how the difference between the craftsman’s having a geometrical or a non-geometrical view of the pattern can be detected.

Fig. 5

Another process can be thought of, which is both easier to use and more efficient than just using a guiding net. This process is essentially similar to the one used “naturally” in the case of scenes with human or animal figures, that is:

- identifying *signifying elements* (figures, parts of the body...)
- discerning an *internal logic* in the work, in order to articulate these elements (composition of the scene, disposition of the body parts...).

For a geometrical pattern, the basic unit will be an element for which a geometrical structure and a possible construction process will have to be determined. And, in case this element is duplicated, a repetitive pattern (i.e., a transformation group) will have to be identified.

However, if a gifted craftsman can easily reproduce by mere sight a subject with human or animal figures, the complexity of some geometrical patterns – which had to fit within a given surface (rectangle, square, circle...) – made it difficult to set them up by just imitating the finished pattern. Of course, as we have seen above, mosaics for which you can say the craftsman did not really understand the geometrical structure can be found, because, in comparison with a theoretical model, they show irregularities that cannot be attributed only to the workman who laid the *tesserae*. In a geometrical pattern, any property not taken into account will result in a small deviation which, once repeated throughout the mosaic, will increase and possibly make a makeshift repair necessary in some place, as can sometimes be observed. From this, and the fact that some geometrical mosaics have a perfect design, one is led to conjecture that the *pictor* had a good geometrical knowledge, including in particular one or several diagrams, either mental or drawn, of the basic element of the pattern, allowing him to undertake a geometrical construction of this element as well as of the global pattern. I will call such a diagram a *key diagram*. When drawn, a key diagram does not necessarily have to be made with great precision; it may even be drawn by hand, the condition being that it has to be “readable”, i.e., that by looking at it you are able to see its geometrical properties and a way to construct it with simple instruments. The existence of key diagrams, which are situated at a junction point between learned and practical geometries, is of course a hypothesis, but it explains, on the one hand how complex patterns could be obtained from simple diagrams, and on the other hand how some mosaics could attain an astounding degree of complexity and precision. Some quite common key diagrams could be learnt and memorized by the *pictor* during his training, in the same way as high school students learn and memorize how to draw a perpendicular bisector with ruler and compass nowadays. Some other, more specific complex patterns, could result from a personal elaboration and possibly be noted down if the mosaicist, finding them nice and interesting, wished to be able to re-use them later. Moreover, as we shall see, a key diagram could be used, through variations, to produce different patterns.

In order to illustrate this, I will give here below some examples based on two elementary figures inscribed in a square (a tilted square and a regular octagon), together with their use in the making of some complex geometrical patterns.

2 Tilted square within a square

This part is about a pattern which can be found in France, both in the southwest (Montcaret) and in the northeast (Metz, Liéhon), and also in Germany (Kreuzweingarten). These mosaics are not quite identical – they differ from one another in their ornamental details – and not contemporaneous (they were dated between the second and the fourth centuries A.D.), but their geometrical structure is quite the same (fig. 6).

Fig. 6.

This pattern has been described as “a geometrical pattern [...] of *peltae*⁴ (Amazons’ shield) [...] inserted in an outline of imbricated lozenges”⁵ ([INRAP 2005], describing the mosaic of Liéhon).

An first examination of this pattern shows that it is built on a square grid (fig. 7). A more thorough investigation reveals a new structure, made of lozenges and squares (fig. 8).

Fig. 7.

Fig. 8.

Setting the *peltae* and their volutes aside, a basic element of the pattern, which is easily obtained by dividing the sides of a square into three parts, can be identified (fig. 9).

The composition of this pattern, as well as a way for constructing it, can easily be read on the diagram. The lozenges appear by reflections, with sides of the square as axes of symmetry; iterating such symmetries will allow the setting up of the pavement (fig. 10).

Fig. 9.

Fig. 10.

On this basis, the setting up of the pattern can be operated in three steps:

1. Drawing the square grid (fig. 11);
2. Drawing the tilted squares within “blocks” of 3×3 unit squares, and small “poised squares” in the centre of each block (fig. 12) by using symmetries with lines of the grid as axes;
3. Drawing the *peltae* with half-circles constructed from knots of the grid (fig. 13).

Fig. 11.

Fig. 12.

Fig. 13.

Then all that is left to do is to draw (freehand) the volutes at the extremities of the *peltae*.

The drawing in fig. 10 makes it possible to carry out the first two steps of the construction, because it indicates not only the position of the tilted squares in the square grid but also their relative positions (symmetries). This is the reason why I consider it as a key diagram for the pattern in fig. 6. We may also remark that by itself this diagram makes up the pattern of a mosaic pavement, as for instance in Lyon, France, and Itálica, Spain [Balmelle et al. 2002: pl. 408a, b].

As for the *peltae* and their volutes, they can be considered as decorative elements, added in order to adorn with curves the otherwise stern rectilinear pattern. As mentioned above,

peltae are a common design in Roman geometric mosaics and can be associated with various polygons (in particular squares, rectangles and lozenges).

3 Regular octagon inscribed in a square

3.1. Constructions

The regular octagon inscribed in a square (fig. 14) is a design frequently used in Roman geometric mosaics. Two “classical” construction processes – at least – can be thought of. They are represented in fig. 15 and fig. 16, which can be “read” easily and detailed below, and used as key diagrams.

Fig. 14

First process

fig. 15

1. In a square, draw the two diagonals.
2. Draw the four arcs of circle centred at the vertices of the square and going through its centre.
3. These four arcs intersect the sides of the square at eight points, which are the vertices of the octagon.

Second process

fig. 16

1. In a square, draw a diagonal.
2. Draw two arcs of circle centred at the extremities of this diagonal and going through the other two vertices of the square.
3. The parallels to the vertices of the square going through the points of intersection of the arcs with the diagonal intersect the sides of the square in eight points, which are the vertices of the octagon.

One is led to think that such construction processes could be memorised under the form of a diagram associated with their making, this diagram being *de facto* a key diagram of the corresponding configuration.

3.2. Eight-lozenge star

Let us now consider the pattern represented in fig. 17, which is widely spread throughout the Roman world. It is described as an “outlined orthogonal pattern of tangent eight-lozenge stars forming squares and smaller poised squares” [Balmelle et al. 2002: pl. 173b]. At first this pattern may appear complex, and one can easily understand that its accurate setting up might cause some problems to the craftsman. I will hereafter consider two possible “readings”, both of them based on the octagon inscribed in a square. One may remark that this elementary pattern does not appear explicitly in the mosaic.

Fig. 17

4 Readings

4.1. First reading

Focussing on the so-called “stars”, adjacent regular octagons can be distinguished. They delimit poised squares between them (figs. 18, 19) and constitute an “outlined orthogonal pattern of adjacent octagons (forming squares)” [Balmelle et al. 2002: pl. 163a].

Fig. 18

Fig. 19

Fig. 20

Each star can be inscribed in a square belonging to a square grid, according to the key diagram of fig. 20, the making of which, starting from an octagon inscribed in a square, can be described as follows:

1. Draw the medians of the square;
2. Draw the circle going through the points where the diagonals of the octagon parallel to the sides of the square intersect.

Thus, all the points needed for drawing the star are constructed.

One just has to divide the surface of the pavement into squares and draw the key diagram of fig. 20 in one of the squares to be able to set up, by transferring some lengths, the orthogonal grid (“tartan grid”)⁶ with which the whole pattern will be drawn. This grid can be set up in two steps:

1. Setting up the tartan grid which will make it possible to draw the octagons (fig. 21);
2. Setting up the additional lines which will make it possible to draw the poised squares (fig. 22).

Fig. 21

Fig. 22

Note. To set up the tartan grid, one may for instance realise key diagrams in the squares at the corners of the square grid, then draw lines joining corresponding points by lines parallel to the grid.

4.2. Second reading

Focusing now on the large squares in fig. 17, one may distinguish another orthogonal pattern, based also on regular octagons, but which are secant (figs. 23, 24).

fig. 23

fig. 24

fig. 25

The new key diagram (fig. 25) starts from the octagon inscribed in a square as well. It can also be easily read:

1. Draw the diagonals and the medians of the square;
2. Draw two squares, the vertices of which are the middles of the sides of the octagon (thick dotted lines). This gives all the points necessary for drawing the pattern.

Note. This key diagram is more visible in mosaics which show a single octagon, shown in fig. 26, found in Bignor (Cornwall, UK) [Balmelle et al. 2002]: pl. 391b). The associated key diagram, quite apparent in the pavement, is clearly similar to fig. 25.

Fig. 26.

Fig. 27.

Fig. 28.

Fig. 29.

To set up the whole pattern on this basis it is possible, as with the first reading, to begin with the construction of the tartan grid associated with the octagon-in-a-square (fig. 27) and, after drawing the octagons, draw additional lines joining the intersections of neighbouring octagons (fig. 28). The pattern will then be completed with the medians of the squares circumscribed about the octagons, which will make it possible to draw the poised squares (fig. 29).

Note. A simplified – and frequent – variant of this pattern consists in replacing the first tartan grid by a regular square grid. Hence irregular octagons, and “lozenges” which are in fact parallelograms. In this case a single key diagram is sufficient to set up the whole pattern (fig. 30). But this form is not often associated with a pattern of eight-lozenge stars; it is

more frequently used to get a pattern of secant octagons. In this case, the key diagram coincides with the pattern itself.

fig. 30

This example shows that several key diagrams can be associated with the same geometrical pattern. Since they are attested in simpler forms of the pattern (here: a pattern of adjacent or secant octagons), all these diagrams have some likeness. Moreover, in both cases the linking of two key diagrams (an octagon in a square followed by a set of octagons constructed on the tartan grid) allows the setting up, by a rather simple process, of a pattern which may look complex. Such a pattern could certainly not be obtained by just copying a drawing (even accurate) of the whole pavement, because it would not be possible – except by chance – to get regular octagons or lozenges.

5 *Wreath-like pattern of squares in an octagon*

A single key diagram can be found under different forms.

The pattern that we will now study (fig. 31) is described as “wreath-like pattern in an octagon and around a circle, of eight poised lateral squares, these motifs contiguous to each other, forming lozenges and creating the effect of an eight-pointed star” [Balmelle et al. 2002: pl. 315]. The associated key diagram stems directly from the key diagram in fig. 25.

The octagon-in-a-square being constructed, this diagram (fig. 32) can be read as this:

1. Draw the diagonals and the medians of the square;
2. Draw two squares, the vertices of which are the middles of the sides of the octagon (thick dotted lines);
3. From the intersecting points of these two squares, draw lines parallel to the sides and the diagonals of the initial square.

Thus, all the points needed for constructing the pattern are obtained.

Fig. 31.

Fig. 32.

We can see that fig. 32 and fig. 25 stem from a common pattern, which is none other than fig. 33, composed of a regular octagon inscribed in a square and two squares, the vertices of which are the middles of the sides of the octagon.

Fig. 33.

We may then consider fig. 33 as a “basic” diagram, which could possibly be completed in different ways to give rise to more “sophisticated” diagrams making it possible to achieve more specific patterns. This leads to conjecture the existence of several articulated levels of key diagrams of increasing complexity, such as, for instance:

Fig. 34.

6 Conclusion

Studying the geometrical properties of a complex pattern leads, on the one hand, to identifying a basic element which is eventually replicated throughout the mosaic pavement by way of geometrical transformations (translations, symmetries, rotations...) and, on the other hand, to looking for possible constructions of this basic element with instruments which were commonplace when the mosaic was made. Such constructions had to be passed on from one craftsman to another under the form of a diagram on which the basic pattern, as well as its construction, is clearly “readable”. This pattern may be pretty general (like the octagon-in-a-circle) or more specific (like the eight-lozenge star), and we have seen that a complex pattern can result from articulating several key diagrams of increasing complexity.

We have also seen that various key diagrams may possibly generate a single pattern, and conversely a single key diagram may be used in the construction of several patterns. The study of Roman mosaics shows that two mosaics which at first look identical are very seldom quite similar; most of the time, even if their constitutive elements are the same, their composition shows greater or lesser differences, as if they were local variations imagined by the *pictor* from a general basic formula. Nonetheless, without any archaeological evidence, which key diagrams were used to make a given mosaic can only be conjectural, as is the very reality of such diagrams, even if it seems quite likely.

Notes

1. For instance, in Besançon (France) a detailed geometrical study allowed me to explain that some apparent mistakes of the craftsmen could in fact be justified and how some real mistakes occurred [Parzysz 2007].
2. Some scarce traces of lines are sometimes found under a mosaic (see for instance [Chantriaux et al. 2007]), but they are far from being sufficient to give an accurate idea of how it was designed.
3. The diagrams of this article were made with the Cabri® II Plus software.
4. This is a very common pattern, consisting of three half-circles organized as in fig. 6.
5. We may remark that the truncated lozenges are usually named *scuta*, which were the Roman legionnaires' shields.
6. I call “tartan grid” an orthogonal grid in which the distance between two successive parallel lines can take two values.

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About the author

Bernard Parzys is Professor of mathematics – now emeritus – at the University of Orléans (France). His teaching was essentially devoted to math teachers' training and one of his research topics was – and still is – the teaching and learning of geometry. His interest in geometric mosaics was aroused in November 1994 when a Roman *domus* was excavated not far from his flat in Metz, where he was working at the time; the local newspaper reported the fact and highlighted the find of a mosaic (analysed in § 2 of this article). This led him to study the geometric pattern of this mosaic in a paper published by a local journal. He has been working extensively on this topic since 2004, trying to find out the models implemented by the antique mosaicists and group them into “families”, in order to uncover the constructive processes put into play.