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Research

Continuity versus Discretization

Abstract. The threefold interest in architecture, biology and mathematics motivated us to examine and justify new architectural forms. We discuss some notions of rhythm: Euclidean, morphogenetic and morphologic. Contemporary relationships between structure and form are based on the generation of shape by technological processes, thus the resulting objects are restricted to their material expression. Here a phenomenological organisation of form and its continuity with the landscape arise out of the mathematical and architectural creativity. The use of the computer is applied from outside to inside instead from inside to outside; this means that we are dealing with the organisational processes via continuous methods instead of evolutionary processes given by computer simulations, known as *genetic algorithms*, where the resulting configurations are reduced to a matter of routine. The role of design as an aesthetic component innovates the theoretical framework of structural engineering to establish the architectural environments.

Introduction

Recent theories of physics and biology suggest the existence of organic forms in architecture [Alati, et. al. 2005; Harris 2007] in opposition to the static Greek theory. The present work promotes a rehabilitation of the concept of continuity as a requisite to find another system of theoretical mathematics in order to read the artistic values through emotion as an analysis of the sign. The present work promotes a rehabilitation of the concept of continuity as a requisite to find another system of eurhythmy. It introduces the notion of continuity as the basis of a new mathematical concept of the architectural form and of a new mathematical interpretation for the analysis of the sign, i.e., in order to read the artistic values of the objects. In fact, mathematics should not be only a technical tool for design science. It is in this sense that the alterations of the classical principles, which had been laid down in order to find the object's value (see below), and their relations with the continuity law should be determined.

Until today, the theories of configurations of forms were formulated following the geometrical principle of Plato and in harmony with the natural numbers. The concept of configuration was achieved through a rejection of dynamic movement. In fact, the Pythagorean School had established that all things were joined with a number without movement. Later Aristotle conceived a method for the knowledge of transformations, rejecting the mechanical movement once again. Thus, Greek science and philosophy have one major limitation: the rejection of the act of passing from one state to another as a rational explanation of the world. This Hellenistic idea obstructed the development of mathematics in the culture of architecture. There were two restraints: the incapacity to realize the concept of a function, and a disregard for the study of the natural phenomena.

In spite of the rigorous study of the movement of the stars, essential for navigation at the end of the Middle Ages, the alteration of these principles was not as easy as it might seem, because the Greek philosophical spirit, established by the Platonic and Aristotelian systems, was deeply rooted. The mathematical tool for organizing images, relating space and shape, was based on the golden rectangle and its variants. The images were sets of families of one variable where the continuity between similar objects did not exist. We can understand the application of movement in architecture through the work of Klee [1961]. Since then, the laws of arithmetical and geometrical progressions could be overcome, leading us to examine the contribution of continuity and its properties to a new process of architectural composition. Moreover, to explain the value of an object in this context, we must take into account how objects belonging to the same family are related according to a law of growth. This interpretation, given by several theories (for instance, the phenomenological theory), becomes one of the objectives that must be applied in architecture. Bearing in mind that movement is an essential part of the growth of cells, the law of continuity can serve as a tool for the composition of architectural objects. It is in this perspective that the concept of Leibniz's infinitesimal and differential calculus can be applied to art. According to the concept of continuity, each object is a cell and its growth creates the final object [Consiglieri and Consiglieri 2006]. Thus, a new concept appears that substitutes the quantitative method with an analysis of continuous dynamics [Marchetti and Rossi 2006], which we will call *morphocontinuity*.

The outline of this work is as follows. First, we state three different notions of rhythm. The sections that follow are in accordance with aesthetic reasonings relative to the mathematical approaches under examination. The final section is devoted to some conclusions and open questions.

Notions of rhythm (mechanical versus organic)

Euclidean rhythm determines the characteristics of the final object by means of discrete modules arranged linearly and additively. No individual module can be changed directly in a continuous way in order to obtain an object, because it results from repetitions of its subparts proceeding to the next levels without continuous changes.

The morphogenetic rhythm is based on the concept of genes and the universal code of information in DNA, affirming the statement of Ch. von Ehrenfels at 1890 (see for instance [Guillaume 1937]) that the form as a whole is more than the sum of its elements. The impact of the theory of emergence in architecture demands new form-finding methods and a seamless integration to replace the conventional separation of design and material production [Weinstock 2006]. The process of self-organization consists of the algorithmic generation of a pattern according to local organizing principles at the cellular level. As opposed to Newtonian space (governed by the laws of mechanics), there arises what is designated as organic. Like Euclidean rhythm, organic space is made up of separate well-defined modules, and the final object is constituted by the dynamic connection between the parts, thus resulting in an open system.

Morphologic rhythm provides a mechanical interpretation, where the object shares the space and time as integrated elements and, basing itself on a continuous process, results in an architectonic state of equilibrium. We say that a cell C has a *morphologic rhythm* if

$$\exists R > 0, \quad C(t) \subseteq B_R(0), \quad \forall t \in [0, T],$$

where $B_R(0)$ denotes an open ball centered at the origin $(0,0,0)$ with radius $R>0$, and C represents a *cell* defined by

$$C := \{(x, y, z) \in \mathbb{R}^3 : f(x, y, z; t) \leq 1\}, \quad (1)$$

with x, y, z representing (as usual) the spatial Cartesian coordinates and f denoting a continuous function defined in three-dimensional Euclidean space and depending on a time parameter $t \in [0, +\infty[$ (for details, see [Consiglieri and Consiglieri 2006]).

Serial discretization

A composition can be governed by different methods. The serial system creates a new code for the production of structures according to a recursive process, without necessitating the return to the generator module. The serial theory appeared as a technique for constructing new conceptual and structural realities and was only later consolidated into a theory. The form is not a pre-existent thing and a series becomes a mode of polyvalent method [Boulez 1975]. We can state that the serial discretization is an additive process.

To understand the present situation of how the series is an attempt to make the final objects more physically intimate with Nature, we refer to the works of Peter Eisenman, where are only considered sequences of sums of translations and rotations of generative elements. (fig. 1).

Fig. 1. Studies and Max Reinhardt Haus at Berlin by Peter Eisenman

Discretization of the theory of emergence

Generative theories change the concept of the shapes of an object in art. The creation of form can be processed through the integration of self-organization. By means of the interaction of each subpart of the whole, we obtain a unified image. For example, in fig. 2, a form embodies an “infinite number” lines. This concept can be found in the science of design [Kappraff 2001].

Thus the method of the periodicity is still totally accepted and interpreted as a philosophical act in accordance with a triangular scheme: beginning, middle and end.

Fig. 2. Organic structural studies for cylindrical forms by Michael Hensel

Continuity of the forms (the negation of the number)

Let us explain the notion of morphocontinuity in the establishment of families of unities with different dimensions. We define a cell as in (1): a unit whose dimension is variable with time. In a phenomenological sense the cells allow, as whole, a modification in the specificity of we want to observe. The dimension is never represented by a number, but constitutes a variable within multiple architectonic aspects; in another words, a geometrical representation is a growing phenomenon that we must study in connection with its context. Thus, because we know the continuous properties of the mathematical functions, we can comprehend the variation of the analogies of the object. The object is a family of cells defined by continuous functions that depend on parameters of space and time becoming a dynamic object. This new concept establishes a new interpretation of form that till today belonged to the harmony of the numbers.

We define a *form* as the subpart of the boundary of the cell defined by

$$F := \{(x, y, z) \in A \times \mathbb{R} \subseteq \mathbb{R}^3 : z = f(x, y; t)\},$$

where f is a continuous function defined in A depending on a time parameter $t \in [0, +\infty]$.

Although the process of morphocontinuity is different from that of morphogenetics, the essence of both theories translates the final form in a 4D-environment (3D-space and 1D-time), which does not happen with Euclidean theories [Alati et.al 2005] (fig. 3).

Fig. 3. (a) FresH2O pavilion (Netherlands, 1998) by Nox team and Kas Oosterhuis, (b) its discretization, (c) the graphical representation of a form under the continuous function $f : A = [-2, 2]^2 \rightarrow \mathbb{R}$ defined as $f(x, y) = \sin(\pi x / 3) \cos(\pi y / 7)$. The graphic visualization is produced using Maple 4

Fig. 4. (a) Modern Art Museum (South Korea, 1996), (b) Chiat House (Colorado, 1997), projects by Frank Gehry

In the struggle against Kant's *a priori* forms, which are known as ideal Euclidean forms of contemplation and that do not reflect real (sensible) space, the characteristics of Einstein's concept of space/time have been confusedly applied in the nexus between mathematics and architecture. The interactive cyberspaces for cultural activities in public buildings are a first step in comprehending the notion that time is an indispensable condition of our life and, consequently, is inseparable from space and from what exists

outside us. For our purposes, a compartment exists in space-time where time is an independent entity.

In the works of Frank Gehry presented in fig. 4, the deformations of the cells serve as a basis for the systems of analogy of the form. The object is no longer a result of periodic schemes, and the image can be perceived through the sensations of the deformed shapes. The form is not the consequence of Euclidean geometry and its variants such as the serial theory, but is rather the consequence of the relationships of continuity that determine the structure of the unity. Gehry's geometrical forms also radically contest the morphogenetic process, although, structurally speaking, the construction of the form could have been made with an emergent cells patterns scheme.

Fig. 5 shows how by means of the continuous functions we can obtain dynamic effects of vibrant excitation without entering in disorder.

Fig. 5. The graphical representation of forms using $A = [-2, 2]^2 \setminus \{(0, 0)\}$, for the time parameter $t > 0$ and $f(x, y; t) = \frac{x^4 y^4}{(x^2 + y^4)^t}$: (a) $t=2$, (b) $t=3$; and for the initial time parameter $t=0$: (c) $f(x, y) = \frac{x^3 y - xy^3}{x^2 + y^2}$, (d) $f(x, y) = \frac{x^2 y}{x^4 + y^2}$.

The graphic visualizations are produced using Maple 4

Singularities

As a consequence of the concept that the universe is matter in constant transformation, the regularities and the irregularities of the phenomena result in free will in the conception of architectural forms.

Differential calculus permits us to determine for each cell the relationships that correspond to singularities and sets of singular points. Then, the object is defined including

dominant supporting points. For instance, the triangle has three singular points (the vertexes) whose expressive character should be taken advantage of in architectural practice.

Fig. 6. Auditorium and Technological College (Finland) by Alvar Aalto

Fig. 7. (a) NY tower (USA) by Frank Gehry, (b) Euskotren tower (Spain) by Zaha Hadid

In Modernism this theme was applied in an essential way, where these representative singular points correspond to what we find in the works of Alvar Aalto (see fig. 6). On the one hand, there exist the border points between lateral surfaces and the roof that as a whole constitute an ensemble of singular points. On other hand, there exists a lower vertex that reveals an entry as a dominant supporting point. The object is a unified form that is harmonized through the curve constituted by the unique (non-differentiable) points, privileging the unique point of minimum height in the determination of the image. This

point provides a sublime angle that heightens the impact of the surprise of the object on the principal façade.

The actual interpretation of the object is the result of the sensations and qualities of multiple images, that is, the form loses the principal façade. In fig. 7, the singular points are elements forming differentiable curves. These reflect the expression of tension [Kandinsky 1987] where the singularities are established by an exterior force.

Fig. 8. Kunsthaus at Graz (Austria) by Jan and Tim Edler

Determination of the aesthetic value of the object (emotion and sensations)

When we analyze an object, in the first place we sense its characteristics, we perceive the rhythms of full and empty, clear and dark, color and texture, and so forth, and only afterwards do we become conscious of the value of that form. Sensations are considered the dominant elements in the determination of the images [V. Consiglieri 2007]. To comprehend objects we observe their shape, color, material and content. Thus these characteristics specify the object through a set of sensations and notions. These properties are not in the objects, but in ourselves. The emotion, as we observe it, organizes the objects in the architecture and the qualities of the objects that correspond to our senses. We conclude by saying that the aesthetic value is a consequence of the references given by the above properties and that it does not exist by itself, that is, the value of the objects exists in the sense of our spirit and do not have any reality outside this. Or simply, the object has no aesthetic value if humankind does not exist.

To understand the value of the mathematical curves referring to the objects that are shown in fig. 8, we should consider a scale of intensity for the sensations. Consider the scale of the sensations of the color vermillion, which is constituted by a variety of gradations going to their absence. When we study the notion of vermillion as a color, we study the successive (infinitesimal) segments of division, we compare them with other imperceptible tones of the same scale, and finally we verify the existence of the scale which is delineated such that it increases or diminishes the reality of the color. In the philosophical point of view, the emotion corresponds to the continuum (unity) of the sensations and it gives a unique value to an object. This new relationship between the elements of the objects is not determined by a number, as it was determined by the methods of the proportions, but by a variable. The aesthetic value is given to an object when we perceive the relationships between the curve and its tangent, which philosophically correspond to the character of the perception [Deleuze 1968], that is, the form that represents the desired object gives voice to its better perceptible quality as the distance between curves and its tangents tends to the intersection point. This principle represents a new concept for achieving beauty, like the golden number was used to achieve harmony in classical theories.

The linguistic essence

In linguistics, the substitution of a single letter can completely change the meaning of a word. In the case of the parallelepiped, the alteration of its length, height and depth can change its aesthetic value. Fig. 9 shows how images can be integrated through the positioning of small unities and their relationships in different cultural moments, namely, architectural rationalism (a), expressionism (b) and deconstructivism (c).

The studies connected with the grammar of a structural system may follow two approaches. First, the generative grammar corresponding to the discretization of the form pays attention to the sounds or letters that transform a material word into an abstract word, where the psychological process is not described. This recursive process in architecture formalizes an algorithmic strategy in the modeling of a pattern. Second, the grammatical method of Noam Chomsky corresponding to the continuity of the form, where the harmonic unity is not a word as it was for Ferdinand de Saussure and other linguists, but corresponds to the unity of phrase that represents a reasoning or proposition. This gives us a new pragmatism in the composition. Similarly, the architectonic composition can be analyzed through the execution of its form.

Fig. 9. (a) Garden City at Meaux by Le Corbusier, (b) Rebstock Park urbanism project by Peter Eisenman, (c) urbanism unities project by Daniel Libeskind

The message results from the meaning of the words and sentences and proposes, in the socialization process, a rationalization of the culture inherent to the panorama of the world. In the case of the house, considered as a proposition, it is an isolated house with one or more stories. It has recourse to a tautological character. While doing the architectonic analysis in the specification of the proposition and in the establishment of values, we have to consider the expression and the content. The content constitutes the combination of the spatial and formal elements in a rational sense and must have a useful value. The expression is neither a subjective attitude nor an individual feeling. It consists of what we give to the character of the image, a prerogative of critical reasoning, and constitutes a continuum of effects and of elements that belong to the structure of the object, entailing the form to a sign

Thus this tautology characterizes the knowledge of the proposition at the conceptual level of ideas, and the value of the propositions are interconnected to the symbolic codes, such as the functional typologies and the syntaxes that belong to each cultural moment.

The investigation of general configurations goes from images to signifiers with values. The forms are altered under a syntactical character, marking new physical and expressive aspects in the scales of proportions and light which introduce a real sensibility. In short, the emotion acts like a linguistic organization of logical and synchronic thought.

Conclusions

In the first place, we must recognize that the behavior of different forms depends on deterministic and indeterministic theoretical trends.

Following Heisenberg's Uncertainty Principle on which quantum physics is based, research into the manifestations of chaos interacts with the arts and other intellectual spheres [Mandelbrot 1982], according to the philosophical essay on probabilities.

To express mathematically the principle of determinism in accordance with the new laws of physics and biology, the principle of determinism introduced by Laplace must be redefined [Dahan-Dalmedico et al. 1992]. The new concept has to include the idea that any physical system is predictable, in the sense that at least after the previewed occurrence had happened, it is possible to verify that it was determined by the system state. This theoretical structure reflects an order that contains in its essence a fragmentation of the universe that gives us a perspective of its totality. In fact, it represents a primordial role on the new notion of determinism.

This present work is centered in this conflict between artistic principles, between the organic and the analytical. The object is presented as a cultural subject under different manifestations of human thought. We can synthesize the analytic approach proposed here in the following way:

First, the object has a life of its own during its existence, maintaining a constructive stability either in the internal spaces or through the external manifestations produced by natural accidents.

Secondly, the basis of the object stands in a physical structure determined by a state in which the supporting forces are organized, producing equilibrium defying the law of gravity.

The emotion of a form is perceived in three separate aspects: the visceral, the experimental and the behavioral. In spite of this, there is still no concise theory about emotion. There are recent works that try to address these problems, showing how in our world man comprehends things by means of the senses. The theories of emotions, whether from an individual or a general point of view, both start from a neutral intensity. One direction indicates what is not desirable, and from the same starting point, the other direction gives us the degree of pleasure of the relationship between the object and the observer. The method consists of a framework organized such that it attains a continuum of the different sensations. The values of the object appear as the result of the volumetric essences of surfaces, lines and elements, giving us formal comparisons in an ensemble of binary oppositions that is, between interior and exterior, vertical and horizontal, full and empty. So the formation of rational determinative relationships of the configurations is a theory whose main objectives is to make state precisely the concept of the form and to determine the efficacy of the positioning of the areas and volumes.

In conclusion, we are entering into two new theories, one of emergence and one of morphocontinuity, that correspond respectively to the perception of the infinite and the difference of the sensations, a possible step in the appreciation of the harmony that until today and for centuries was supported by the arithmetical and geometrical proportions with geometry as the only theory.

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Victor Consiglieri received a B.S. degree in architecture from Escola Superior das Belas-Artes de Lisboa (ESBAL) in 1956, and a Ph.D. degree in morphology of architecture from Faculty of Architecture of Technical University of Lisbon in 1993. He was in Paris 1964-65 on a scholarship from Centre Scientifique et Technique du Bâtiment (CSTB). He was in Câmara Municipal de Lisboa 1956-62, Ministério do Ultramar 1962-66, Caixa da Previdência 1966-76, Faculty of Architecture of Technical University of Lisbon 1976-97 and as invited professor in University of Évora 2004-05. He realized many projects for public buildings such as kindergartens, elementary schools, institutions for youth, and centres for the elderly. He was member of Associação dos Arquitectos and of Ordem dos Arquitectos 1956-2004. His actual interest is on contemporary aesthetic.