Tomás García-Salgado

Facultad de Arquitectura, UNAM
Ciudad Universitaria
Coyoacán, México
tsalgado@perspectivegeometry.com

Keywords: Andrea Palladio, Villa
Rotonda, Design analysis,
geometric analysis, perspective

Research

A Perspective Analysis of the Proportions of Palladio's Villa Rotonda: Making the Invisible Visible

Abstract. Because of differences between Palladio's architecture as built and the ideal architecture represented in the *Quattro libri*, many analyses have been performed in order to bring to light the proportions that underlie the beauty of the architecture. This present paper proposes a method of analysis based on perspective grids laid out on photographs to reveal how perspective is used to heighten the spectator's perception of the forms.

Introduction

Many architects and scholars agree that the Villa Rotonda epitomizes the architectural ideals that Palladio set forth in his *Quattro libri* [1570]. Palladio's villas have been profusely copied, but often cheerlessly or with no understanding of their cultural and temporal context. In England, Palladio's architectural style was adopted after the English translation of his book and soon after the same happened in America. The fact that the drawing of the Rotonda published in the *Quattro Libri* does not correspond exactly with its actual built conditions has motivated the elaboration of detailed drawings for this building, such as those by Bertotti-Scamozzi [1776-83]. More recently, Semenzato has obtained more accurate plans by using internal polygonals [1990]. Artistic drawings of all kinds have also been made, especially in watercolors, such as those of Giovanni Giaconi [2003]. Models assisted by computer technology have also been produced, as those developed for all of Palladio's villas by Lawrence Sass at MIT [2001]. However, I am convinced that superimposing grids on photographs of the villa could lead to a new approach towards finding out its proportions as they are perceived in perspective, and thus tell us something more about its singular composition.

Palladio's drawings versus the actual building

Palladio's woodcut engravings for the *Quattro libri* [1570] were probably intended to express his architectural ideals, not the buildings as they were actually built. As Sass pointed out, "Palladio created the drawings and text in the *Four Books of Architecture* after the buildings were built" [2001: 6]. The Rotonda was begun in 1550, whereas the first edition of the *Quattro libri* was published in 1570, meaning one of two things: either he chose to publish an idealized drawing instead of a real one—if such a "real" drawing ever existed—or he deliberately prepared an ideal drawing aimed at presenting his canons. Curiously, the history of the ownership of the Rotonda's estate is known down to the slightest detail, but little or nothing is known regarding the villa's proportions. Whatever the answer may be, we still have to deal with some dissimilar proportions between the ideal drawing and the actual building. For instance, in the *Quattro libri*, the northeast and southwest vestibules are shown with a width of 6 *f* (*f* being one Vicentine foot), and those of the northwest and southeast with 7½ *f* (as deduced in fig. 1), whereas the latest surveys indicate 10 ½ -11 *f* in reality. This dissimilarity changes the proportions of the main rooms from 15 x 26 *f*, to 15 x 24- 24¼ *f*. How do we explain that?

Fig. 1. Palladio's drawing of the Rotonda in the *Quattro Libri Dell'Architettura* (Plate of page 19, *Libro Secondo*). In the elevation, the two windows of the loggia at the *piano nobile* indicate this is a lateral view of the plan, not the frontal one as the drawing arrangement suggests

In plate number 19 in the *Quattro libri* (fig. 1), the thickness of the walls and the height of the central vault are not indicated; only the proportions of the rooms are given. The height of the columns of the loggias is indicated, but their diameters, which is important for verifying the 12 x 30 *f* proportions of the loggias, as we will see shortly. In the same plate, we see how the central intercolumniation appears slightly larger than the others, suggesting an arrangement in accordance with the precepts of eustyle and diastyle. Obviously, it would take much more than a single plate to accurately depict the architectural elements of the loggias alone, not to mention the many more needed to render the whole building. It seems that Palladio was more interested in justifying the motivation for his design for the loggias¹ than he was in explaining the geometrical layout in detail.

Palladio devoted his first book to a detailed description of the classical styles (*toscano, dorico, ionico, corinthio, composito*), so perhaps for this reason he did not feel compelled to do the same for his own works; after all they can be inferred from the orders. There are no descriptions at all of how his buildings were built, nor are working drawings for this purpose included, if any were indeed ever elaborated. It seems obvious that the stonemasons required some sort of drawings to carve the capitals and the stone frames for the doors and windows, to shape the entasis of the columns, and the moldings of the pediments. In other words, the challenging question here is how Palladio managed the geometry of his buildings during construction, to which we have no answer. It seems that Palladio wanted us to learn the ideals from his treatise and the reality from his works, leaving our imagination to bridge the gap between his theory and practice. The perspective analysis I am proposing here intends to bridge, in part, that gap.

Squares within squares

The geometry of the *square* seems to rule the configuration of the Rotonda's plan, since several imaginary squares can be identified in a concentric position alternating with other squares, which are rotated 45° from the main plan. I discussed this particular feature in a previous article, showing how these squares relate to one another, from the stairways to the nucleus of the building [2004]. Based on this approach, here I will describe its spatial meaning to link it with the discussion in the next section. Thus, to proceed in an orderly fashion, I will refer first to the analysis of the plan, and later to its volumetric interpretation in perspective.

As can be noticed in fig. 2 (*A* through *H*), the arrangement of the plan has mirror symmetry along both the longitudinal and the transversal axes. We had to assume a hypothetical value for the thickness of the walls to redraw the plan, since Palladio's drawing lacks this data. Lawrence Sass has estimated the wall thicknesses to be 18 inches at the *piano nobile* [Sass n.d.: 8]; according to my calculations equals approximately 1-1/4 *f*, almost the same as the dimension estimated by Sass, although the surrounding walls could be thicker, as much as 1½ *f*.

The sequence of the squares *A-B-C* can help us understand the spatial integration between the villa and its environment (see fig. 2, *A, B, C*). We will begin by demarcating the ground plan at the four stairways as square *A*. This square has a spatial meaning that is not easy to grasp until one surveys the villa from different vantage points, wondering why it looks bigger than it actually is. This is followed by Square B surrounding the projecting loggias of the building that characterize its unique formal expression. Square *C* integrates the composition of the whole toward which the invisible squares *A* and *B* converges.

Fig. 2. (A through H). Squares within squares. Drawings by Ambar Hernández and author

On the interior, square *D* is formed by a continuous perimeter passing throughout all the doors of the rooms that surround the central hall. This feature of spatial continuity could have been helpful during construction for checking the position of the inner walls by simply stretching a cord along the perimeter of square *D*. Square *D* crosses all the chambers either at the center of their doors, or at one side of them; this seems to have been planned from the beginning for this purpose at the *piano terra*. In its turn, square *E* holds the transition between the twelve rooms (including the halls) and the circular hall, besides the stair system connecting all the floors vertically.

Now, the first square rotated by 45°, *F*, is obtained by inscribing the four corners of the building, passing from the plinth of the extreme columns of the loggias until the center of the stairways is reached. Standing at the center of the first step of a stairway, the relationship among squares *A*, *B*, and *C* makes sense, since no other loggia can be seen in it but the one in front of us. Another rotated square *G* can be traced from the first step of the loggias running across the main chambers, inscribing at the same time square *D*. Finally, square *H* joins at the thresholds of the four main entrances, inscribing the nucleus of the house while crossing at one side of the halls' doors. Some other interesting relationships can also be found by extending both diagonals of any of the main chambers, the same being true for the diagonals of the small chambers.

The significance of “the whole” and “the parts” in Palladio’s architecture becomes clearer and more comprehensible when it is perceived in perspective, as in the hyperrealistic fig. 3. Here, an imaginary geometry of volumes *A*, *B*, and *C*, on a photograph taken from the north, allow us to visualize why the villa looks bigger than it actually is, as we said before. Certainly, after walking for a while in the gardens, one can perceive the building within a virtual volume, extending its dimensions beyond its walls, and vice versa, and how the landscape embraces it gradually.

Fig. 3. A hyperrealistic representation of the virtual squares *A* and *C*. The reader will be able to imagine where should square *B* lay —between *A* and *C*. Photograph and superimposed drawing by the author

Laying out grids in perspective ...

Now, to analyze the volumetric composition of the building in perspective, we will lay out some virtual grids on its façades, combining squares and rectangles, and see that several of their diagonals make explicit the proportions of the windows, doors, and the loggias. Of course, I am not suggesting here that Palladio literally drew grids on the façades in order to place the elements. Those who have been involved in architectural practice know how geometry is applied while building; how dimensions and proportions are as easily adjusted by moving cord-lines in the field, as lines are moved with an straightedge on a drawing table.

According to the *Quattro libri*, the original proportions of the Rotonda are as follows: in plan, $12 \times 30 f$ for the loggias, $15 \times 26 f$ for the main rooms, $11 \times 15 f$ for the small rooms, $6 f$ for the access halls, and a $r = 30 f$ for the circular room. In elevation: the height of the columns is $18 f$, that of the main rooms is $21\frac{1}{2} f$, the attic is $7 f$, and there are $10 f$ from the *piano terra* to the *piano nobile*. The question now is how to visualize these proportions in three dimensions.

We have already established that Palladio's drawing does not provide enough information about the elements of the façades. This notwithstanding, he must have decided how to manage its proportions *in situ* during construction, somehow applying a geometrical procedure (at least, this is my hypothesis). I cannot otherwise explain the many coincidentally related elements that become evident when a virtual grid is superimposed on the façades – in this case by means of photographs. In a way, these grids can reconstruct the design process of placing the elements of the façades in reverse, revealing their geometrical proportions. Thus, windows, doors, columns, and pediments can be analyzed either individually or in conjunction with each other. In support of my hypothesis, I will quote here Palladio's own concept of beauty, which states, *La bellezza risulterà dalla bella forma, e dalla corrispondenza del tutto alle parti, delle parti fra loro, e di quelle al tutto: conciosia che gli edificij habbiano da parere uno intiero, e ben finito corpo...* [1750: I, 6]. In my opinion, the meaning of "correspondence" goes beyond the words, especially when we try to visualize it geometrically in a real building.

... On the windows

Just as a horizontal plane is suitable for establishing the width and length of a room, a vertical plane can be used to set the width and height of all the vertical elements. In practice, Palladio used both methods, as we will see. Despite the fact that the windows of the main rooms appear to be in the center of the walls in his elevation drawing – between the corner of the building and the interior face of the loggia's lateral arch –, they were actually built differently. Viewing the villa from the west (see fig. 4), it is evident that the window of the main room (a) is not placed at the center of the wall due to the fact of being next to the fireplace. In addition, this room has two more windows, one facing the interior of the loggia (b), and the other one around the building's corner (c). Therefore, the question to elucidate is how these windows were proportioned on the walls during construction. By tracing the diagonals of the wall (on fig. 4), it is remarkable that they intersect at exactly the left frame and upper corner of the window (a), and that this intersection point coincides with half of the height of the building. The same is true when the diagonals extend from top to bottom on the wall. Notice that the diagonals in both cases are taken from the interior face of the lateral arch of the loggia.

Fig. 4. A view from the northwest façade showing the windows' proportions as deduced in perspective. Photograph and superimposed drawing by the author

It appears more likely that the width of windows (a) and (b), and thus the fireplace in between of them, was proportioned along the large wall of the main room from its interior side; while the wrought-iron lattice of the window (b) run parallel to its diagonals.² In its turn, window (c) was put at the middle of the shortest wall in the same manner. This procedure may have been foreseen, since the walls of the *piano terra* were raised. In contrast, the heights of the windows seem to have been proportioned from the exterior, otherwise they would align with the lintel of the interior doors, below the halfway line of the building's height. The window sill also aligns with the lateral arch seat, as the grid makes evident. Once again, in fig. 4, if window (a) were placed above the crossing diagonals of this wall, then the human scale would be compromised, suggesting that only giant people could live there, and on the other hand, if it were placed below this diagonal, then the *piano nobile* would appear higher, suggesting that a window is missing, or that an entire floor is hidden. It would have simply been disastrous to give 1/8 of the wall's height above, or below its diagonals to set the window's height. Therefore, the intersection point of the diagonals is the one that strikes the perfect balance between the window and the wall, or *delle parti fra loro*, in Palladio's terms.

... On the lateral arches of the loggia

A rectangular grid successively divided through its diagonals and superimposed on one of the lateral arches of the loggia reveals the geometrical construction of all of the arch's elements (see fig. 5).

Thus, the opening of the arch and its pilasters, the abutment and imposts, and the lower arch at the *piano terra* can all be determined. Of course, an exact match would be exceptional since measures ultimately depend on the building materials; for instance, the section of the pilasters depends on the size of the bricks. There are two rectangles in fig. 5, one from the top of the arch to the floor level, and the other from the top to the ground level. If the first one is divided into thirds vertically, its upper third would align at the spring of the imposts.

Obviously, such a division into thirds would coincide all along the shaft of the column, its section being one-ninth of the height of this rectangle, meaning that the proportion of the columns is 1:9. The diagonals of the second rectangle intersect at the spring line of the lower arch, fairly close to the pilaster section at one-fourth of its width. In the same manner, several other relations can be found in fig. 5 (facing page). What this grid is telling us is how the parts correspond to each other geometrically, and how the proportions were obtained by subdividing the whole into parts, as Palladio probably did in accordance with Vitruvius's definition of symmetry: *La symmetria è l'armonico accordo tra le parti di una stessa opera e la ripondenza dei singoli elementi all'immagine d'insieme della figura* [Vitruvius 1990: 23].

... On the loggias

The loggias, as projecting elements of the main body, are fundamental for the volumetric composition of the villa; without them the villa would be a simple hollow box. In addition, the stairs enhance the perspective of the visitor, leading him to a transitional space between outdoors and indoors. Thus, stairs, loggias, and main body were successfully combined in perspective as never before. Just like a pawn in a chess game has an important role to play, sometimes making the difference between winning or losing, the *piano terra* plays an important role in the Rotonda.

Fig. 5. A view of the loggia's arch from the northwest. Here, both the loggia's lateral size of $12f$ and the columns' section of $2f$, were corroborated. The grid outlining makes the proportions among the parts comprehensible. Photograph and superimposed drawing by the author

If Palladio had not conceived a new architectural program for the villas, exploiting the the *piano terra* to free the *piano nobile* from the unsightly service spaces, and to avoid humidity as well, then you would not be reading this paper. The *piano terra* was precisely the key to raising the villa from the ground, making it appear more graceful from afar, while permitting a magnificent view of the landscape from the loggias.³

The incremented width of the central intercolumniation of the loggias makes the building appear more dynamic when one walks toward the stairs, while framing the access door at the same time; the lateral windows of the *piano nobile* appear centered in their walls (see fig. 6).

Fig. 6. The incremented breadth of the central intercolumniation of the loggia makes the building appear dynamic when walking towards the stairways, framing the access door while the lateral windows of the *piano nobile* appear to be centered in their respective walls. Photograph and superimposed drawing by the author

It might be naïve to ask what makes the loggias so magnificent, when we know that beauty has always been a controversial subject and depends on no specific formula. However, when my assistant Ambar Hernández and I were analyzing the northwest loggia, we found something that might help others to understand its beauty objectively, without trying to discover its formula. Common sense suggests that the columns must have been set along its transversal axis, which means that we cannot perceive their real size because our sight passes tangentially to both sides of the column shaft, instead of passing at the end points of their diameters.

In other words, a column in perspective looks a little bit wider than it actually is; this effect is, of course, more noticeable at close range. Therefore, we decided first to lay out a grid along the axis of the columns according to its intercolumniation sequence, and second, to lay out another grid tangential to the faces of the columns in order to investigate which type of sequence it could fit visually.

Thus, to establish the sequence of the former, according to Palladio's book, and our fig. 5, we have: 9d (18 *f*) for the height of the column, 6d (12 *f*) for the depth of the loggia, and 2 *f* for the columns diameter.⁴ Therefore, the intercolumniation sequence along its axis is as follows:

$$4 \frac{1}{4} + 2 + 4 \frac{1}{4} + 2 + 5 + 2 + 4 \frac{1}{4} + 2 + 4 \frac{1}{4} = 30 \text{ } f.$$

Now as a rule, to lay out the grid it has to be equally divided into as many parts as the sequence has feet. Therefore, the northwest loggia was divided in 30 *f* horizontally, and in 9d (18 *f*) vertically, as it is shown fig. 7a. Here, some vertical lines of the grid do not align with the shaft of the columns as expected, due to the sequence containing fourths of foot.

Fig. 7a. Here, a grid equally divided vertically in $30 f$ representing the colonnade sequence ($4 \frac{1}{4} + 2 + 4 \frac{1}{4} + 2 + 5 + 2 + 4 \frac{1}{4} + 2 + 4 \frac{1}{4} = 30 f$) along its transversal axis; does not visually fit with the columns' intercolumniation

Fig. 7b. Now, a grid equally divided vertically in $34 f'$ representing the colonnade sequence ($5 + 2 + 5 + 2 + 6 + 2 + 5 + 2 + 5 = 34 f'$) up to its frontal plane, almost fits perfectly with the columns' intercolumniation

Fig. 7c. This new grid, placed at the wall of the piano nobile, also approximates the positions of the windows and the columns. The conjunction of both virtual planes, 7b and 7c, produces the spatial consistency of the loggia. In 7c, the number of feet totals 66, which allows us to estimate the interior walls' thickness as $1 \frac{1}{4} f$, and the exterior's as $1 \frac{1}{2} f$. Photograph by the author, with superimposed drawings by Ambar Hernández

As we already have pointed out, the viewer does not actually see the extreme ends of the columns' diameters, which is why we set the second grid tangentially to their faces. Thus, the viewer's sight would be captured when passing at both sides of the columns' entasis. So the question is, could such a grid coincide with the colonnade sequence? The answer is yes. According to the precepts of eustyle and diastyle, the intercolumniation is $2\frac{1}{2}$ diameters (d) of a column for the former, and $3d$ for the second. These precepts were well known to Palladio from Vitruvius; as Palladio says, *... mi proposi per maestro, e guida Vitruvio...* [1750: I, 5]. In geometrical terms an intercolumniation like this does not differ too much from the actual one, so we decided to apply it to the second grid and see what happened; unexpectedly it fit pretty well with the colonnade intervals, this is exactly the 'something we found' (see fig. 7b).

The roundness of the columns makes the tangential plane more suitable because it corrects the visual appreciation of the geometrical sequence in perspective. As can be appreciated in fig. 7b, it is remarkable how the eustyle-diastyle grid matches with both empty and solid spaces along the façade of the loggia. Therefore, the grid sequence:

$$5 + 2 + 5 + 2 + 6 + 2 + 5 + 2 + 5 = 34f',$$

is an interpretation of the proportions of the loggia as they are perceived in perspective. Naturally, f' measures a little bit less than f . This discovery leads me to hypothesize that Palladio was aware of these sorts of perspective effects, and corrected the Vitruvian formula as it should be when applied in perspective. It seems paradoxical that $30 \times 18 f$ can transform into $34 \times 18 f'$ while maintaining a ratio of 1.667.

Finally, by laying out a new grid on the building's façade from side to side, similar to that of fig. 7b, but now in true Vicentine feet, it once again makes sense with the eustyle-diastyle sequence, as is shown in fig. 7c. Even though this grid does not fit all the elements in the sequence perfectly (and I did not want to force it), it suggests that it could fit perfectly at certain intervals of distance. As I have learned from my practice in perspective, distance is always involved. Proving this would require more photographs taken from a distance one meter closer or further away. However, both the grids in figs. 7b and 7c fairly depict the spatial consistency of the depth of the loggia, since they echo each other visually. This explains why the loggia appears so well proportioned to the whole from any distance or when we move closer to it, and perhaps it is this invisible echo that results in its beauty.

Conclusion

This is a glimpse of what can be achieved by superimposing grids on photographs in order to analyze the Rotonda's actual proportions. Here, I have only focused on some parts of the whole. A complete analysis of the windows with all their ornamental elements would itself require another paper, and so on for the rest of parts. The grid criteria could be applied in the same way to analyze the constructive system of the building, since walls and vaults were also built in proportion. In particular, the brick vaults of the *piano terra*, which I was lucky enough to capture with my own camera in 2003 (see fig. 8), cry out for us to elucidate their constructive geometry. After all, the *piano terra* was the key to the innovation embodied by the Rotonda, as well as one key reason why this building was included in the list of Unesco World Heritage Sites.

Fig. 8. This photo of the southwest passage at the piano terra shows us the constructive system of the vaults with its peculiar interlaced brick pattern. Their proportions and geometry will be the subject of future study. Photograph by the author

Notes

1. *Onde perche gode da ogni parte di bellissime viste, delle quali alcune sono terminate, alcune più lontane, & altre, che terminano con l'Orizzonte; vi sono state fatte le loggie in tutte quattro le faccie...*[Palladio 1570: II, 18].
2. In my opinion, the window-wrought-iron of window (b) was deduced from the proportion of the opening, not by following a predetermined angle.
3. *Però lodo che nella più bassa parte della fabbrica, laquale io faccio alquanto sotterra; siano disposte le cantine, i magazzini da legne, le dispense, le cucine, i tinelli, i luoghi da liscia, o bucata, i forni, e gli altri simili, che all'uso quotidiano sono neccessarij: dal che si cavano due commodità: l'una che la parte di sopra resta tutta libera, e l'altra, che non meno importa; è,*

che detto ordine di sopra diviensano per habitarvi, essendo il suo pavimento lontano dall'humido della terra: oltra che alzandosi; ha più bella gratia ad esser veduto, & al veder fuori [Palladio 1750: II, 3- 4].

4. According to Palladio [1750: II, 4] and [1750: III, 6], it is supposed that the graphical scale representing half a Vicentine foot is drawn at full scale, in which case it measures approximately 35 cm. The note below the graphical scale says: *Questa linea è la metà del piede Vicentino, co'l quale sono state misurate le seguenti fabrique. Tutto il piede si partisce in oncie dodici, e ciascun'oncia in quattro minuti*. According to Semenzato [1990: 92], the diameter of the columns measured at the base is equal to 0.745 m, supposedly equivalent to 2 Vicentine feet, in which case $f = 0.372$ m.

References

- BERTOTTI-SCAMOZZI, Ottavio. 1786. *Le fabbriche e i disegni di Andrea Palladio*. Venice.
- GARCÍA-SALGADO, Tomás. 2004. La Invencción de la Rotonda. <http://www.perspectivegeometry.com>
- GIACONI, Giovanni and Kim WILLIAMS. 2003. *The Villas of Palladio*. New York: Princeton Architectural Press.
- PALLADIO, Andrea. 1570. *I quattro libri dell'architettura*. Venice. Facsimile edition, Ulrico Hoepli Editore Libraio, Milan, 1990.
- SEMENTZATO, Camillo. 1990. L'architettura della Rotonda. Pp. 45-100 in *Andrea Palladio. La Rotonda*, Renato Cevese, Paola Marini and Maria Vittoria Pellizzari, eds. Milan: Electa.
- SASS, Lawrence. 2001. Reconstructing Palladio's Villas: A Computational Analysis of Palladio's Villa Design and Construction Process. Pp. 212-226 in *Reinventing the Discourse – How Digital Tools Help Bridge and Transform Research, Education and Practice in Architecture* [Proceedings of the Twenty First Annual Conference of the Association for Computer-Aided Design in Architecture (ACADIA), Buffalo NY, 11-14 October 2001].
- . n.d. Materializing Palladio's Ideal Villas. Computational Reconstruction through Physical Representation of Digital Information.
http://ddf.mit.edu/projects/PALLADIO/White_Paper_a_Palladio.pdf.
- VITRUVIUS, Marco Pollione. 1990. *De Architectura, Libri X*. Padua: Edizioni Studio Tesi.

About the author

Tomás García-Salgado received his professional degree (1968), Master's degree and Ph.D. (1982-1984) in architecture. He is a formal researcher in the Faculty of Architecture of the UNAM (México), and holds the distinction as National Researcher, at level III. Since the late 1960s, he has devoted his time to research in perspective geometry, his main achievement being the theory of Modular Perspective. He also has several works of art, architecture, and urban design. More information regarding his work is available at <http://perspectivegeometry.com>.