Buthayna H. Eilouti

Department of Architectural
Engineering
Jordan University of Science
and Technology
POB 3030
Irbid 22110, JORDAN
buthayna@umich.edu

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Research

A Formal Language for Palladian Palazzo Façades Represented by a String Recognition Device

Abstract. This article represents an effort to reveal a new interpretation of the expression ‘the architectural language of Palladian designs’ that is closer to real linguistic paradigms than it usually means. Palladian designs exhibit a highly ordered and well articulated formal language comprised of a finite set of vocabulary elements in plan, elevation and volumetric treatment, together with an implicit set of mathematical rules for the arrangement of these rudimentary elements. The scope of this article is limited to the morphology of the façades of the first nine buildings shown in the second book of Palladio’s treatise, specifically the palazzo designs that he presents in chapter three. The morphology is described in terms of a symbolic encoding system that is represented textually and graphically as a finite state automaton, the concept of which is borrowed from theories of formal languages and computation. The system helps to emphasize commonalities in façade languages and to propose a prototype for generating Palladian palazzo façade designs. The automaton-based encoding system may be developed to function as a base for a computerized façade encoder and decoder.

Regular language processing and recognition

Studies of formal languages, and their structural and computational representations in linguistics, computer science and related fields as well as their applications, are well-established (e.g., [Chomsky 1964]; [Aho and Ullman 1972]; [Salomaa 1973]; [Harrison 1978]; [Hopcroft and Ullman 1979]; [Linz 1997]; [Revesz 1983]). Furthermore, there are some research efforts that aim to expand their applicability to explain architectural design morphology (e.g., [Eilouti and Vakalo 1999]; [Eilouti 2005]; [Eilouti, 2007]). However, the applicability of formal language manipulation techniques to explain architectural language composition and processing is still inadequately explored. New layers of these techniques can be further investigated and employed to classify stylistic families of architectural artifacts, as seen in this article, which follows in the tradition of previous efforts to analyze Palladian designs based on methods related to computer science theory and practice (e.g., [Stiny and Mitchell 1978]; [Mitchell 1990]; [Hersey and Freeman 1992]).

In the context of formal languages, regular languages epitomize one of the simplest forms of language definition and processing. They consist of strings (words) constructed from a given alphabet of symbols (letters). A specific regular language consists of a set of accepted strings, where any given string is either accepted or rejected according to stipulated criteria that determine language membership. In the literature on computational application of regular languages, some simple devices are defined to help classify strings into languages. One of these devices is known as a finite state automaton (FSA).

A finite state automaton is an extremely restricted and simplified model of a computer that functions as a language recognition device. An FSA consists of a finite number of states, and specified transitions between states, with one state typically designated the initial state. The FSA is initialized with an input string, which it reads one symbol at a time. The automaton is in the initial state when it reads the first symbol, and then it enters a new state that is dependent on the current state and the symbol just read. The process of reading an input symbol and transitioning from state to state is repeated. This transition process can be both iterative and recursive, as states can be repeated as many times as required and can be called internally from any state to re-enter that state or enter another. When the symbols of the input string are exhausted, the reading process stops at one of the states. These terminal states can be either final states or not. Strings that end at final states are considered acceptable as members of the given language, and in this case, the automaton indicates its approval of the string it has read. Otherwise, if the string stops at one of the non-final states, the string is rejected and the automaton declares its disapproval of the string's membership to the language at hand. The language accepted by an automaton is the set of all strings it accepts. Any accepted string is said to be a member of the language that is defined by the automaton and any rejected string is labeled as not a member of that language [Lewis and Papadimitriou 1981].

One of the powers of FSAs is that they help generate enormous unforeseen strings. Another powerful aspect of FSAs lies in the capability to take the union, intersect and complement of languages in order to derive new languages. FSAs can manipulate symbols in order to enable users to model problems, test solutions and transform sets of symbol combinations into algorithms for which step-by-step procedures can be described. They have applications in computer algorithm building and processing as well as in the lexical analysis phase of a compiler.

The Mathematical and Graphical Representation of String Processing

Finite state automata of regular languages are generally classified into two types, the deterministic and the nondeterministic [Hopcroft and Ullman 1979]. While the transition from a state to another in the deterministic automaton is completely *determined* by the input symbol, the operation on some of the input symbols in the nondeterministic is probabilistic. In other words, while on any given input symbol a deterministic automaton must proceed from the current state to one and only one designated state, a nondeterministic may move to more than one state.

The essential mathematical representation of a deterministic FSA of a regular language recognition process is formulated as follows [Aho, Sethi and Ullman 1986]; [Lewis and Papadimitriou 1981]:

A FSA is a quintuple $M = (K, \Sigma, \delta, S, F)$, where

K: is a finite set of states (including final and non-final states)

Σ : is an alphabet of symbols

$S \in K$: is the initial state at which the recognition process starts

F: is the set of final states (subset of K) that determine acceptable strings

δ : is a transition function (from $K \times \Sigma$ to K) that maps a current state and a symbol to a new state.

A rule according to which the automaton M moves from a state ($q \in K$) to another state ($q' \in K$) is encoded into the transition function δ in terms of the input symbol and two states. For example, if the current state is q , the input symbol is σ , and the new state is q' , then $\delta(q, \sigma) = q'$.

FSAs are represented graphically as attributed state diagrams. The pictorial representation of the finite state diagram is a directed graph, where descriptive text is incorporated into the graphical components. States are represented by nodes, where a circle signifies a state of string processing, and double concentric circles signify final states which are also known as states of string acceptance. Arrows which connect two state circles are used to show the direction of string processing. The initial state where the process of string recognition begins is usually denoted by the symbol $>$ (see fig. 1).

To demonstrate how FSAs function, let us consider the following example. Take the alphabet to consist of two symbols, X and Y . Let $\{X,Y\}^*$ denote all possible combinations of the rudimentary alphabet elements (i.e., all possible words made up of X 's and Y 's). Suppose we want to produce a finite state automaton M that generates the language $L(M)$ consisting of the set of all strings in $\{X,Y\}^*$ that have an even number of Y 's (including zero) and optionally any number of X 's before, after or in between the Y 's.

The finite state automaton M is illustrated by fig. 1. According to this FSA, at the start state q_0 , the input symbol ' X ' can be added as many times as required (including zero times). Then, upon the addition of the symbol ' Y ' the automaton moves to the state q_1 ; as at state q_0 , zero or more ' X ' symbols can be added. Next, the automaton can go back to the state q_0 if another ' Y ' symbol is added. The FSA transitions from q_0 to q_1 , or vice versa, only upon the addition of a ' Y ' symbol. When the string has been fully read, symbol by symbol, the process terminates. If the string contains an even number of Y 's, the FSA will terminate in state q_0 , which is a final state in this example (denoted by the double circle), so that the string is accepted. If the string contains an odd number of Y 's, the process terminates in q_1 , a non-final state, and so the string is not accepted.

Fig. 1. A finite state automaton of the language consisting of the strings of $\{X,Y\}^*$ that include an even number of Y 's

Examples of accepted strings, which thus belong to $L(M)$, are: X , $XXYY$, $XXXYYXXY$, and $XYYYXX$. They all have an even number of Y 's. Examples of rejected strings are: $XXYYY$, $XXXYYXXYY$, and $XYYYX$. These strings have an odd number of Y 's and do not belong to $L(M)$. This language can be represented as $L(M) = (X^*YX^*Y)^*$. As above, an asterisk (*) means to repeat the symbol directly to the left (or group of symbols within parentheses) as many times as desired (≥ 0 times).

The four input/output values for the FSA's transition function $\delta(q, \sigma)$ are listed in the four rows of Table 1 (the first two columns are the input and the last is the output). Note that each input/output value of the transition function corresponds to an arrow from state to state (or from a state to itself) in the FSA.

Q	σ	$\delta(q, \sigma)$
q0	X	q0
q0	Y	q1
q1	X	q1
q1	Y	q0

Table 1. The table of input/output values for the transition function for the automaton of fig. 1

In the following sections, the concept and representations of the finite state automaton will be applied to a selected set of Palladian façades to help explore new layers of their underlying compositional and procedural languages.

The Palladian Design Study Sample

For the purposes of this study, the scope of analysis is limited to the designs illustrated in chapter three of the second book of Palladio's 'Four Books of Architecture' [Palladio 1997]. This chapter includes palazzos (houses in the city or town houses) that are designed by Andrea Palladio. It is noteworthy to point out that one of the designs illustrated in this chapter represents a villa rather than a palazzo. About the inclusion of this villa, which is the house of Monsignor Paolo Almerico (Villa Rotonda) in Vicenza, with the city houses explains Palladio "because it is so close to the city that one could say it is in the city itself" [Palladio 1997: 94].

All of the Palladian houses, exemplified by the ones selected in this study, exhibit a high level of order and consistency in their geometric language. A visual study of the palazzo designs reveals that they consist of a finite set of vocabulary elements in plans, elevations and massing. The components are placed together in a systematic format that is based on well articulated proportional and rhythmic systems. The architectural language underlying the Palladian villa plan designs was the focus of some pioneering studies of the shape grammars (e.g., [Stiny and Mitchell 1978]). Although the languages of all Palladian designs in two- and three-dimensional representations are significant for the visual studies and architectural research, the focus of this paper will be on a new representation of the morphology of Palladian palazzo façade designs.

The palazzos in this study sample consist of nine buildings. These include the houses of:

1. Signor Floriano Antonini in Udine (P1)
2. Count Valerio Chiericati in Vicenza (P2)
3. Count Iseppo De Porti in Vicenza (P3)
4. Count Giovanni Battista Della Torre in Verona (P4)
5. Count Ottavio De Thiene in Vicenza (P5)
6. Counts Valmarana in Vicenza (P6)
7. Monsignor Paolo Almerico in Vicenza (P7)
8. Signor Giulio Capra in Vicenza (P8)
9. Count Montano Barbarano in Vicenza (P9)

In the following sections, the string development and recognition systems introduced in the previous sections will be applied to a selected set of the façades of Palladio's palazzos to highlight their formal language. As such, façades of the above listed nine houses will be analyzed according to their lexical and syntactic structures, and then encoded in the form of FSA representations.

The Palladian Façade Language

The architectural elements underlying the Palladian palazzo designs reveal clear ingredients of a formal language that can be assembled into a finite alphabet and a finite set of compositional rules (e.g., rules for symmetry and rhythm). The major concern of this section will be to highlight the morphology of this language as enunciated in Palladio's palazzo designs, as well as the process of composing the planar visible configurations of these designs as manifested in a selected set of their various façades.

The main vocabulary elements shared by the façades of Palladian palazzos consist of the wall units (solid, with door, with window, with two windows, with door and window, or negative which represents a space between walls or columns), columns (Doric, Ionic, Corinthian, and Composite), pediments, entablatures (that vary according to their cornice, frieze, and architrave details), roofs (pitched or domed), and sculptural units. The hierarchy of the vocabulary elements of the Palladian façade language is illustrated in fig. 2. The sculptural units will not be discussed in the morphology analysis as they are out of the scope of this article. Among the remaining vocabulary elements, the emphasis will be on wall, opening and column units. Examples of the architectural vocabulary components of the Palladian façades are shown in fig. 3. These components are the ones referred to in the FSA encoding of the front façade of Palazzo Antonini, which is detailed in the following section. The Palladian vocabulary set consists of:

W: wall unit:

- W0: space between walls or columns.
- Ws: solid wall without openings.
- W1: wall with window. It has the variations W11 (example in P1), W12 (example in P3) and W13 (example in P3).
- W2: wall with two windows. It has the variations W21 (example in P1) and W22 (example in P6).
- W3: wall with door. It has the variations W31 (example in P2) and W32 (example in P4).
- W4: wall with door and window. It has the variations W41 (example in P1) and W42 (example in P9).

C: column unit:

- C1: Ionic (example in P1).
- C2: Composite (example in P1).
- C3: Doric (example in P2).
- C4: Corinthian (example in P8).

Fig. 2. A hierarchical chart of some vocabulary elements of the Palladian palazzo façades

Fig. 3. The vocabulary elements of Palazzo Antonini (cf. [Palladio 1997: 80])

Encoding the Palladian Façades by Finite State Automata

In this section, selected front and courtyard façades of the aforementioned nine houses will be represented by strings of symbols. For each palazzo, a finite state automaton diagram will be illustrated. Both forms of representation, the symbolic and the graphical, will describe the Palladian language of façade compositions. The first palazzo will be described in more details than the others. All representations introduced for the first palazzo may be applied to all of the others. However, for space management the representations of the remaining eight palazzos will be kept to minimum.

1. The Palazzo of Signor Floriano Antonini in Udine (P1):

In terms of its compositional language, the façade of Palazzo Antonini consists of five major horizontal levels (see fig. 4). These are: L1) the first floor elevation, L2) the decorative level that separates the walls and columns of the first floor from the second floor level, L3) the second floor level, L4) the decorative level that crowns the second floor walls and separates them from the roof level, and finally L5) the roof level in which pitched masses close the spaces underneath.

The composition of each façade level can be thought of as an input tape that is read from left to right, in order to encode the level as a string of symbols. The process of composing each string is similar to the string processing sequence for regular languages using FSAs, discussed earlier in this article. The initial state corresponds to the leftmost edge of the building, and transitions between states occur as the FSA reads the symbols that correspond to the architectural elements. The process of composing the strings of each level in Palladian palazzo façades moves through a maximum of seven states, Q0–Q6, shown in fig. 4. The FSA for Palazzo Antonini's façade is illustrated in fig. 5.

For example, for the first level, L1, the procedure starts with the initial state Q0, adds a wall unit with window (W11) and moves to state Q1. Upon the addition of a column unit (C1), the automaton remains in state Q1, but upon the addition of a wall unit with two windows (W21), it transitions to Q2. Similarly, a column unit keeps the state at Q2 whereas a wall unit with two windows moves it back to Q1. The FSA moves between Q1 and Q2, and vice versa, adding a column and a wall with two windows each time, until half of the columns are processed. The Q1–Q2 sequence thus adds three columns and two wall units. A wall with door and window (W41) is then added through the movement from state Q1 to state Q3. Cycles between Q3 to Q4 are directly analogous to cycles between Q1 and Q2, since they are mirror images around a vertical axis that passes through the center of the door. Thus the Q3–Q4 sequence also adds three columns and two wall units. Upon the addition of a wall with one window, the automaton moves from state Q3 to state Q5 to conclude the symmetric part of the façade. Finally, the rightmost asymmetric wall (W11) is added. This last step transforms the FSA from state Q5 to the final state Q6.

With some change in the input symbols, this FSA can be applied identically in level L3. The major difference in the input symbols lies in the column order, which is Ionic (C1) in L1 and Composite (C2) in L3. In the ornamental levels L2 and L4, states Q2 and Q4 can be dropped. Similarly, in the roofing level L5, the states of Q1 through Q4 are not needed.

The strings for each level of the Palazzo Antonini façade are:

L1 = W11 C1 W21 C1 W21 C1 W41 C1 W21 C1 W21 C1 W11 W11

L2 = E1 E2 E1 E1 (these elements vary according to their level of detail in decoration)

L3 = W11 C2 W21 C2 W21 C2 W41 C2 W21 C2 W21 C2 W11 W11

L4 = E3 E4 E3 E5

L5= R1 R2 (these elements vary according to their shapes)

Fig. 4. The main levels (L1–L5) and states (Q0–Q6) of the front façade of Palazzo Antonini (see [Palladio 1997: 80]). States are indicated on the façade just before they transition to the next state, which results in Q1–Q4 being positioned at the columns

Fig. 5. The finite state automaton for the front façade of Palazzo Antonini

The graphic interpretations of the symbols in the previous strings are illustrated in fig. 3. For the first level L1, the transitions are:

$Q0 \rightarrow Q1 \rightarrow Q1 \rightarrow Q2 \rightarrow Q2 \rightarrow Q1 \rightarrow Q1 \rightarrow Q3 \rightarrow Q3 \rightarrow Q4 \rightarrow Q4 \rightarrow Q3 \rightarrow Q3 \rightarrow Q5 \rightarrow Q6$.

The input/output values for the transition function are shown in Table 2. As in Table 1, each row corresponds to an input/output value of the transition function, where the first two columns are the input and the last column is the output.

The FSA of the front façade of Palazzo Antonini is formally represented by $M = (K, \Sigma, \delta, S, F)$, where:

K is the finite set of states = $\{Q0, Q1, Q2, Q3, Q4, Q5, Q6\}$

Σ is the alphabet = $\{\emptyset, W11, W21, W41, C1, C2, E1, E2, E3, E4, E5, P, R1, R2\}$

$S \in K$ is the initial state = $Q0$

F is the set of final states (subset of K) = $\{Q6\}$

δ is the transition function (from $K \times \Sigma$ to K).

For example, the transition functions of level L1 are (see Table 2):

$\delta(Q0, W11) = Q1$,
 $\delta(Q1, C1) = Q1$,
 $\delta(Q1, W21) = Q2$,
 $\delta(Q1, W41) = Q3$,
 $\delta(Q2, C1) = Q2$,
 $\delta(Q2, W21) = Q1$,
 $\delta(Q3, C1) = Q3$,
 $\delta(Q3, W21) = Q4$,
 $\delta(Q3, W11) = Q5$,
 $\delta(Q4, C1) = Q4$,
 $\delta(Q4, W21) = Q3$,
 $\delta(Q5, W11) = Q6$.

q	σ	$\delta(q, \sigma)$
Q0	W11	Q1
Q1	C1	Q1
Q1	W21	Q2
Q1	W41	Q3
Q2	C1	Q2
Q2	W21	Q1
Q3	C1	Q3
Q3	W21	Q4
Q3	W11	Q5
Q4	C1	Q4
Q4	W21	Q3
Q5	W11	Q6

Table 2 (right). The table of input/output values for the transition function of level L1 of Palazzo Antonini's front façade

The same transition function works for level L3, with C1 replaced by C2. Of course, the transition functions for levels L2, L4, and L5 would be considerably simpler.

All the representations produced for the Palazzo Antonini façade can be applied to the remaining eight palazzos. However, formulas and graphs for only one level of each of these palazzos will be demonstrated in the following sections. The number and locations of states (Q0–Q6) may vary for the following houses, as can the assignment of initial and final states. However, commonalities among these will be discussed later in this article.

2. The Palazzo of Count Valerio Chiericati in Vicenza (P2):

This house is built upon a piazza in Vicenza and raised five feet above the ground level. The order on the first level is Doric and on the second is Ionic. The state locations and additional vocabulary elements are illustrated in fig. 6. The level analyzed for this façade is L1. Note that the fourth column from the left (only half of which is visible) is located in a different plane (its axis is shifted behind the main plane of all others), and so it is not included in the analysis (the same is true for the fourth column from the right).

The FSA in fig. 7 represents L1 of this palazzo. Columns of this level are of the Doric (C3) order. The same FSA applies to L3 with a change of column style into Ionic (C1).

Fig. 6. The state assignment of the front façade of Palazzo Chiericati (cf. [Palladio 1997: 83])

Fig. 7. The FSA of the front façade of Palazzo Chiericati

3. The Palazzo of Count Iseppo De Porti in Vicenza (P3):

This house has two main façades, each of which includes an entrance from a public street. The façade analyzed here is the front one, half of which is shown in fig. 8. The following automaton (fig. 9) represents L3 of this palazzo. Columns of this level are of the Ionic order. Ws stands for a solid wall. W12 and W13 symbolize wall with window units, the former being crowned with an ornamented triangle, and with an ornamented arch in the latter. Note that the FSA is nondeterministic: state Q2 together with symbol W13 can lead to either state Q1 or state Q4.

Fig. 8. The state and vocabulary assignment of one half of the façade of Palazzo De Porti (cf. [Palladio 1997: 85])

Fig. 9. The FSA of the façade Palazzo De Porti

4. The Palazzo of Count Giovanni Battista Della Torre in Verona (P4):

This house in Verona has a long and narrow rectangular plan, for which Palladio provides two interior courtyards. The house faces a principal street with one of its narrow sides. The façade Palladio shows is the one resulting from a section through the inner courtyards, illustrated in fig. 10.

Fig. 10. The state assignment of the court façade of Palazzo Torre (cf. [Palladio 1997: 87])

The following FSA (fig. 11) represents L1 of this palazzo. Columns of this level are of the Ionic order (their order is Composite in L3). Since transition from state Q1 with the symbol W0 could be to either Q2 or Q3, this FSA is nondeterministic (as are those for P5, P7, and P8 below).

Fig. 11. The FSA of the court façade of Palazzo Torre (Verona)

5. The Palazzo of Count Ottavio De Thiene in Vicenza (P5):

This house is located in the middle of Vicenza city near the Piazza. As in the previous palazzo, the façade of interest here is the one facing the inner courtyard that is illustrated in fig. 12.

Fig. 12. The state and level assignment of the court façade of Palazzo Thiene (cf. [Palladio 1997: 89])

The following FSA (fig. 13) represents L3 of the façade facing the courtyard. The courtyard is encompassed from all sides with loggias of pilasters. The order of court's L3 columns is Composite. W41 symbolizes a wall with door and window unit, and W32 symbolizes a wall with arched opening in it.

Fig. 13. The FSA of the court façade of Palazzo Thiene

6. The Palazzo of Counts Valmarana in Vicenza (P6):

This house consists of two parts divided by a middle court. The façade studied here is the front one (fig. 14).

Fig. 14. The state and level assignment of the court façade of Palazzo Valmarana (cf. [Palladio 1997: 92])

The following FSA (fig. 15) represents L1 of the front façade. The scale of C2 (column of Composite order) is multiplied by 2 to extend over two levels (for six major columns). W22 symbolizes a wall with two windows, one square and the other arched. W32 is an arched gateway.

Fig. 15. The FSA of the court façade of Palazzo Valmarana

7. Monsignor Paolo Almerico City Villa in Vicenza (Villa Rotonda) (P7):

Although this famous building is a country villa upon a hill close to Vicenza, Palladio classified it with city houses because of its site's proximity to the city [Palladio 1997: 94]. The FSA of this example is developed for its typical front portico illustrated in fig. 16.

The following FSA (fig. 17) represents the loggia at the front of each side of the symmetric four façades.

Fig. 16. The state assignment of the Façade of Villa Rotonda (cf. [Palladio 1997: 95])

Fig. 17. The FSA of the Façade of Villa Rotonda

8. The Palazzo of Signor Giulio Capra in Vicenza (P8):

This house is located on the principal street of Vicenza. The shape of its site is irregular in plan. However, its façades exhibit a language that is consistent with the others analyzed so far (fig. 18).

Fig. 18. The state and level assignment of the upper floors of the façade of Palazzo Capra (cf. [Palladio 1997: 97])

The following FSA (fig. 19) represents L3 of the front façade. The columns in this level are of the Corinthian order (C4). There are two columns on each side of the symmetry axis. As a result (in all façades with an even number of columns on each half of a symmetric façade), the FSA moves from Q2 to Q4 instead of Q1 to Q3 (with an odd number of columns on each half).

Fig. 19. The FSA of the façade of Palazzo Capra

9. The Palazzo of Count Montano Barbarano in Vicenza (P9):

In this façade, columns of the Composite order (C2) are scaled to extend over two stories (fig. 20). In this case L1 and L3 can be combined in one FSA representation (fig. 21). W42 symbolizes a wall unit with rectangular window and arched gateway.

Fig. 20. The state assignment of Palazzo Barbarano (cf. [Palladio 1997: 98])

Fig. 21. The FSA of the façade of Palazzo Barbarano

The Palladian Palazzo Façade Language

By comparing the nine façades via their graphic and string representations, the main structure of the FSA of Palazzo Antonini (P1) as illustrated in figs. 4 and 5 seems to represent the most comprehensive form among all others. All other façades can be represented by different subsets of this general form. In particular, the first six states (Q0–Q5) of the FSA of Palazzo Antonini shown in fig. 5 are common to all the other palazzos except three. The exceptions include the fourth, seventh and ninth houses (P4, P7 and P9). In the fourth palazzo P4, the second, third, fourth and fifth states Q1–Q4 are the only ones kept from the seven states of fig. 5. These states represent a subset of the language that describes the processing of a colonnade surrounding a courtyard. Similarly, the states Q1–Q4 are kept in P7 to represent columns that surround a central gateway. The similar subset of Q1–Q4 represents the alternation of columns and walls in P9. As such, to find the general FSA that recognizes the languages encoded by all the FSAs introduced in this article to describe the nine palazzos' façades, the first palazzo's FSA can be modified to represent all others. The modified version is illustrated in fig. 22. The W and C symbols (in fig. 22) that appear on arrows represent wall and column units that are selected from the vocabulary set. The empty string $\emptyset$ is added to allow skipping the addition of a wall unit at the start or end of compositions. The arrow from Q1 to Q3 is used when the number of columns on each side of the symmetry axis is odd (P1–P2, P4–P7). Analogously, the arrow from Q2 to Q4 is used when the number of columns on each side is even (P3, P8, and P9).

The FSA in fig. 22 can be further simplified to allow the description of the addition of odd and even number of columns on each side of symmetry in one cycle. The FSA in fig. 23 represents the simplified and comprehensive prototypical language that is shared by the Palladian palazzo façade designs. The major distinction between the nine façade designs lies in the transition functions in cycles 1 and 2 and in the number of times each cycle is repeated to produce the correspondent façade. A cycle here represents the addition of a column and a wall (or space), and forms a part of the sequence described in the previous FSAs. The numbers of cycles in the nine façades are listed in table 3. These values represent the number of loops through the cycles (C W) or (W C) as applied for each house.

Fig. 22 (above). The generic FSA of Palladian Palazzo façades

Fig. 23 (below). The simplified prototypical FSA of Palladian Palazzo façades

House	CYCLE 1 (CW) $\delta((Q1, C) \rightarrow Q1 \rightarrow (Q1, W) \rightarrow Q2))$ m	CYCLE 2 (WC) $\delta((Q3, W) \rightarrow Q4 \rightarrow (Q3, C) \rightarrow Q3))$ n
P1	2	2
P2	6	6
P3	3	3
P4	6	6
P5	2	2
P6	2	2
P7	2	2
P8	1	1
P9	3	3

Table 3. The transition functions of the major cycles of the prototypical Palladian palazzo façades FSA of fig. 23. In the transition function for CYCLE 1, Q1 and Q2 can be interchanged; for CYCLE 2, Q3 and Q4 can be interchanged

The common language of Palladian palazzo façades is generated by the nondeterministic FSA $M = (K, \Sigma, \delta, S, F)$, where

K is the finite set of states = $\{Q0, Q1, Q2, Q3, Q4, Q5, Q6\}$

Σ is the alphabet = $\{\emptyset, W, C\}$, where W stands for a wall unit, $\emptyset$ for an empty shape and C for a column

$S \in K$ is the initial state = $Q0$

F is the set of final states (subset of K) = $\{Q1, Q5, Q6\}$; $Q1$ allows the recognition of colonnade compositions

δ is the transition function (from $K \times \Sigma$ to K), comprised of the transitions:

$\delta(Q0, W) = Q1,$
 $\delta(Q0, \emptyset) = Q1,$
 $\delta(Q1, C) = Q1,$
 $\delta(Q1, W) = Q2,$
 $\delta(Q1, W) = Q3,$
 $\delta(Q2, \emptyset) = Q1,$
 $\delta(Q3, C) = Q3,$
 $\delta(Q3, W) = Q4,$
 $\delta(Q3, W) = Q5,$
 $\delta(Q4, \emptyset) = Q3,$
 $\delta(Q5, W) = Q6,$

Given that:

W : a wall (solid, with openings, or negative) unit

C : column of a given order

$\emptyset$: empty string (allows change of state without processing a façade element).

All the words that can be composed for Palladian palazzo façades are variations of the general form:

$$W^a (C W)^m C W C (W C)^n W^b$$

for some integers $a, b, m, n \geq 0$.

The number of cycles, m and n , for each of the nine façades analyzed, are listed in Table 3. Note that $m=n$ in all façades. As such, the language becomes:

$$W^a (C W)^n C W C (W C)^n W^b$$

(for the nine façades the integers satisfy $0 \leq a \leq 1, 1 \leq n \leq 6, 0 \leq b \leq 2$).

The middle part of the string, $C W C$, represents the central part of each façade through which the axis of symmetry passes. This part is always a wall (or space) that is surrounded by two columns. It is, in turn, surrounded by an alternation of walls and columns from both sides. For a colonnade, W is replaced with $W0$ (a space).

As the prototypical automaton illustrated in fig. 23 suggests, the first symbol of a Palladian façade string consists of a wall unit that is applied zero or more times (zero meaning that this unit can be skipped in some façades to start the façade strip with a column), followed by a loop that consists of an alternation of a column and a wall unit. The string has always a central word CWC that represents the main gateway unit. Then the first-half sub-string is mirrored around its central word to produce a symmetric sub-string. In P1, an additional unit is added to make the string asymmetric.

The FSA-based representations of the nine Palladian façades as introduced in this article help emphasize the common language that unifies all of them and may help produce new Palladian façades that conform to the same vocabulary elements and process sequence. The illustrated representations demonstrate also that a simple regular language recognition device can be applied to explain the processing of a physical object's components such as those constituting architectural façades. Although such an application is not suggested to be intuitive, it proposes more than an intellectual exercise. Such an added layer of interpretation can highlight the commonalities of different structures. It can classify collections of art products and encode them in short strings that facilitate their comparisons and management.

The pictorial prototypical representation of the FSA (fig. 23) can be further developed to automatically model elevations by feeding series of different symbols iterated as many times as needed. These symbols can then be replaced by graphical units to draw their correspondent building elevations by computer. For instance, such graphical units may be thought of as pre-stored blocks in AutoCAD or library cells in 3DMax. Each block or cell may replace a designated symbol as requested by the user. Furthermore, each symbol may tag a command that linearly adds a graphic unit that may be scaled and assigned to a style as needed.

Conclusion

A selected set of the façades of nine city houses that are designed by Palladio has, in this article, been analyzed to externalize their underlying common formal language. By the isolation of each level of the façade at hand, a word in the architectural language can be conceived as a linear string that can be sequentially processed one symbol at a time. As such, the selected set has been represented by a string recognition device that is known as a finite state automaton. The concepts underlying this automaton are borrowed from formal language studies and their computational applications. The proposed string-based representations are formulated in terms of symbolic formulas and graphical directed diagrams. A prototypical automaton is concluded for the Palladian façades of the study sample instances. It represents the formal language common to Palladian palazzo façades. Upon feeding the right symbols and cycling as many times as ordered by the user, the automaton can regenerate the existing façades and can propose new emergent façades that belong to the same style language. The finite state automata representation is demonstrated to help add a different layer of interpretation to the commonalities and differences of the façade structures and their underlying language. Similarly developed prototypes can be derived to represent other Palladian building types which can be compared to the ones analyzed in this article. These prototypes can be further developed to compare different façade languages and to automatically generate existing and emergent façades.

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About the author

Buthayna H. Eilouti is Associate Professor in the department of Architectural Engineering in Jordan University of Science and Technology. She earned a Ph.D., M.Sc. and M.Arch. degrees in Architecture from the University of Michigan, Ann Arbor, USA. Her research interests include Design Studies, Design Mathematics and Computing, Design Pedagogy, Visual Studies, Shape Grammar, Information Visualization, and Islamic Architecture.