João Pedro Xavier

Faculdade de Arquitectura
da Universidade do Porto
Rua do Gólgota, 215
4150-755 Porto, PORTUGAL
jpx@arq.up.pt

Keywords: Leonardo da Vinci, representational techniques, bird's eye perspective, cavalier perspective, axonometry, centrally-planned architecture

Research

Leonardo's Representational Technique for Centrally-Planned Temples

Abstract. Leonardo invented a new technique of representation which combines the building plan and a bird's-eye perspective of the whole into a single system. Bird's eye perspective may have developed out of cavalier perspective, and instances pre-dating Leonardo can be found, but not used in the same way as he employed it. Though not pre-axonometric, Leonardo took advantage of axonometric representation's capacity to construct/deconstruct an object into its component parts in order to clarify fitting and functioning. This paper investigates the originality of the technique and special relationship with his research on centrally-planned churches, while examining it in the context of contemporary developments and architects.

It is hard to deny that Leonardo, among many other inventions, should also be credited as the inventor of a new technique of representation, especially adapted to his research on centrally-planned churches, which combines, as a system, the building plan and a bird's-eye perspective of the whole, as in MS 2307, fol. 5v. (fig. 1).

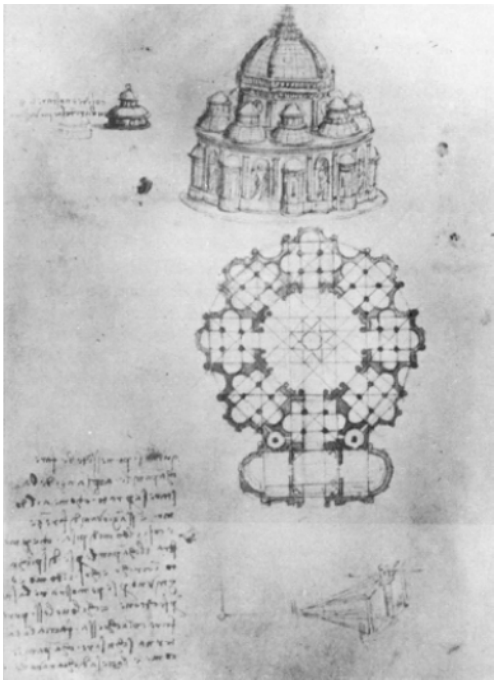

Fig. 1. Leonardo da Vinci, plan and bird's-eye view of a centrally-planned church (MS 2307, fol. 5v)

1590-5896/08/010077-24 DOI 10.1007/ s00004-007-0057-7

This was already pointed out by Murray, who wrote:

> …in the great majority of the drawings in the architectural treatise Leonardo presents a plan and a bird's-eye view, so arranged that the maximum amount of information about a building, both externally and internally, is given in two drawings (…) [Murray 1978: 62].

Previously, Lotz had connected the simultaneous use of these two kinds of drawings with the representation of centrally planned temples [Lotz 1997: 94].

For both Murray and Lotz, the great innovation is the bird's eye-view.

Murray relates this type of perspective with Leonardo's anatomical drawings, asserting that the use of the same technique in architecture "opened a whole field of new possibilities" and completes this statement with this challenging commentary:

> … it is probable that Bramante inspired him to invent this new form of draughtsmanship in connection with architecture and in turn, Bramante's whole cast of thought would have influenced by the new technique of visualization [Murray 1978: 62].

Lotz maintains that the bird's-eye view contains the elements of a perspective construction that will be known later as "cavalier perspective", a representational method developed in the last quarter of 1400, according to him.

In spite of these respectable opinions, I believe Leonardo's genuine and productive innovation was actually the systematic combination of a bird's-eye view and a plan in architectural representation. It is also possible to find the single use of the first kind of drawing in Francesco di Giorgio's *Trattati di Architettura Ingeneria e Arte Militare* (c. 1485), although not with the same coherence and quality of Leonardo's sketches, who was undoubtedly a gifted draftsman.

We should specify that we are not dealing with just any kind of bird's eye-view, which could be simply defined as a kind of perspective produced when the object the observer is looking at is above the horizon line, HL.[1] In fact, we are dealing with central perspective, which means that if we take a cube as reference, we have two faces parallel to the picture plane that are, consequently, in true form. But it is also a fact that in these drawings this supposed cube is laterally placed in relation to the central visual plane, which gives us the possibility to see another of its vertical faces, this one with the correspondent homological transformation. The high position of the observer allows the view of the top face, also transformed by perspective. As we see, the conjugation of all these factors leads to the most usual form of cavalier perspective, and we might think that this relationship would grow proportionally as the distance of observation increases. But here is the problem: the existence of a point of view placed at a finite distance from another one placed at the infinite is crucial, as it is actually the difference between linear and parallel perspective, or in other words, the difference between perspective and axonometry.

Massimo Scolari, in an interesting essay about the origins of axonometry [1984], contextualizing its inherent qualities in philosophic terms, talks about a voyage that begun in "Plotinus' interior eye" and arrives to the "eye of the Sun", to use Pietro Accolti's language,[2] becoming a few years later the "point at infinity" in Desargues's projective geometry. In any case, to Scolari, the development of axonometry coexisted with that of perspective. This means we had a combination, from the first attempts to represent spatial depth in a plane until the sedimentation of both representational systems in scientific terms, begun in the fifteenth century, of two ways to view or represent the world, or two ways of facing or thinking it: this interior eye vision found in axonometry and the perspective "body's eye" a former *mimesis* instrument to reproduce nature. And, as Alan Colquhoun explains,

> [This] debate between Euclid's and Democritus's theories of vision, with their rival aesthetic implications was resolved, in the course of 16^{th} and 17^{th} centuries by dividing the field of representation with two discrete parts. Mimesis in general became absorbed by perspective, while parallel projection and axonometry were preferred wherever the criterion of value was descriptive accuracy [Colquhoun 1992: 17].

As far as I know, the first rigorous cavalier perspectives appeared in Piero Della Francesca's manuscript *Libellus de Quinque Corporibus Regularibus*. It is not surprising that Luca Pacioli also utilized this kind of perspective in his own drawings for *Divina Proportione*. An example could be Piero's drawing of an icosahedron inscribed in a cube, which Pacioli repeats in an engraving suitable for printing (figs. 2 and 3). Curiously, Leonardo's drawings for Pacioli's treatise are not cavalier but bird's-eye perspectives (fig. 4). I will return to this further on because I believe this is rather significant.

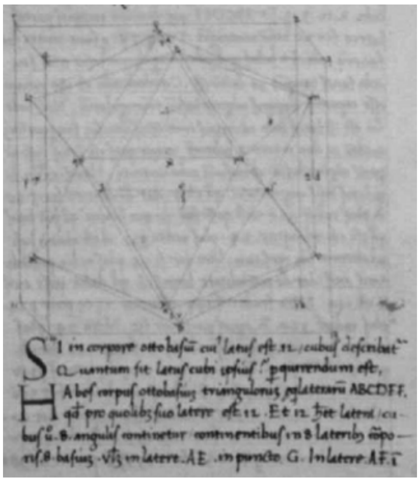

Fig. 2. Piero della Francesca, icosahedron in a cube. From *Libellus De Quinque Corporibus Regularibus*, fol. 40v

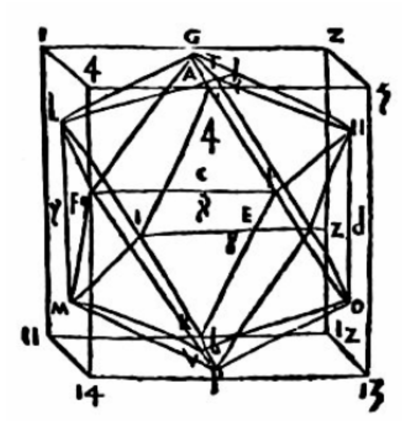

Fig. 3. Luca Pacioli, icosahedron in a cube, *Divina proportione*, Libellus, fol. 15r

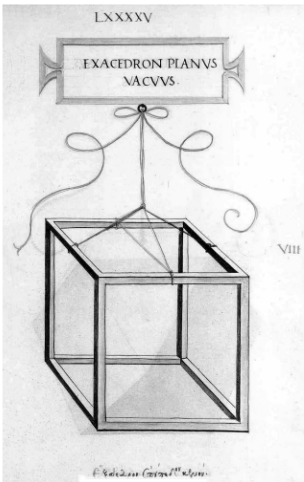

Fig. 4. Leonardo da Vinci, Hexahedron (cube), from Luca Pacioli's *Divina proporzione* (1509)

So, polyhedral representation or, more widely, the representation of spatial geometry will come to be preferred to cavalier perspective drawings. It seems, though, that when the objects of pure mathematics become objects of a scientific approach in the field of representation we tend more to axonometry than to perspective in spite of all the astonishing perspectives of polyhedrons presented in perspective treatises through the sixteenth and seventeenth centuries.

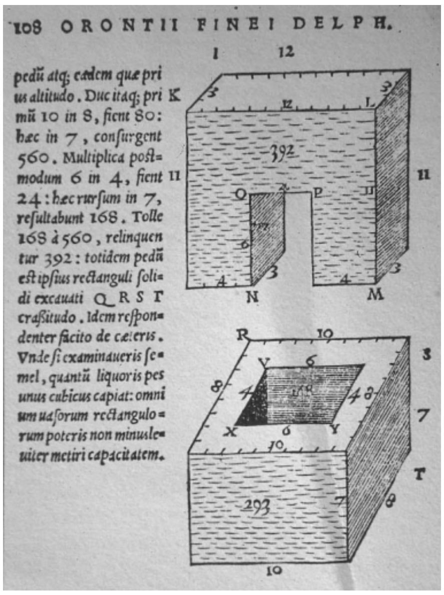

Fig. 5. From Oronce Finé, *Liber de Geometria Pratica*, 1544

But this, I believe, with a few exceptions, was more due to virtuosi and artistic purposes or pure pedagogical reasons, such as the use of such shapes that could facilitate the learning of perspective, than for a mathematical approach to geometry itself. However, it is interesting to note that Piero was responsible for the beginning of a tradition leading to our days, as cavalier perspective is still the main tool for visualization in mathematics literature. One of the reasons for this longevity could be its connection with an intuitive process of spatial representation, its simplicity and readiness of use and, of course, the possibility of obtaining true measurements. It was Oronce Finé (1544) who first scaled the solids edges legitimating the future designation (dated from the nineteenth century) of this representational system – axonometry (axis measurements) (fig. 5).

But the main reason, in this case, could be perhaps the consideration of the object as itself as axonometry gives us a key to "read" it without the subjectivity inherent in a personal eye. So, in axonometry, there are not eyes but a conventional one (actually, there is more than one), placed at infinity, which allows a close approach to the object, to its own characteristics and properties, without the veil that visual perception could impose in between.

These drawings, unequivocally rigorous and as thus perfect axonometric projections, achieved a long time before the theorization of this representational system (but this happened with all systems of representation), cohabit with the kind of bird's-eye views, used by Leonardo and Francesco di Giorgio, which are usually considered as pre-cavalier drawings, and thus pre-axonometric ones, as actually they are not far from geometric cavalier perspective. Of course, there is the lack of parallelism of the perpendicular lines to the picture plane, as they converge to the central vanishing point (the *punto centrico* as Alberti baptized it). There is also the impossibility of picking up true measures. Anyway, the considerable distance of the observer, placed at a point where birds usually are, in a middle stage between man's view in a standing position – that is, with feet on the floor – and the infinite distance of the eye of God, gives a complete three-dimensional view of the object and the possibility of describing it in a synthetic way, a quality typical of axonometry.

In the specific case of Leonardo, I think he deliberately proscribed parallel or cavalier perspective. He wanted to do exactly what he did: nothing other than bird's-eye perspectives!

What gives me permission to state this? Fundamentally, two things. First of all, Leonardo lived in the world of perspective, which he proclaimed several times as the unique instrument that makes it possible to get knowledge of nature and all the environment (including all the artifacts it is possibly to find in it or invent for it). Even in the field of perspective it is possible to verify that Leonardo never forgets that the point of view is actually the human eye, in contrast to Piero della Francesca's mathematical approach, where he considered it more a single geometric point (cf. [Cabezas 2002: 147]), although his preference for bird's-eye views could lie behind the image of the winged eye, shown in the verso of Matteo de Pasti's portrait medallion of Leon Battista Alberti, if we accept that this emblem, a symbol of perspective, inspired the kind of imaginative vision which came to encompass all his aesthetic ideas in its gaze, as noted by Gadol [1969: 69]. Secondly, as I remarked before, and I believe this is determinant, his drawings of polyhedra for Luca Pacioli were perspectives and not cavalier, as Luca Pacioli's own drawings were in the footsteps of Piero. So, he knew perfectly that kind of perspective, where lines he actually saw as convergent strangely remained parallel against the evidence of his senses. And this, for a man who had an absolute faith in experience, could never be tolerated!

Thus I believe that, with Leonardo, we are not dealing with a drawing production classifiable as pre-axonometric in the sense of a primitive stage of that kind of representational system. Such consideration is valid for the prior tradition as in Taccola's pre-cavalier perspectives of mechanisms (fig. 6), which both Francesco di Giorgio and Leonardo followed (fig. 7). But if in the engineering drawings made by Taccola and Francesco di Giorgio some detectable inconsistency can be attributed to their incapacity to control three-dimensional representation,[3] which does not compromise at all its complete efficacy for displaying their functional and constructive aspects, I think that with Leonardo things are completely different.

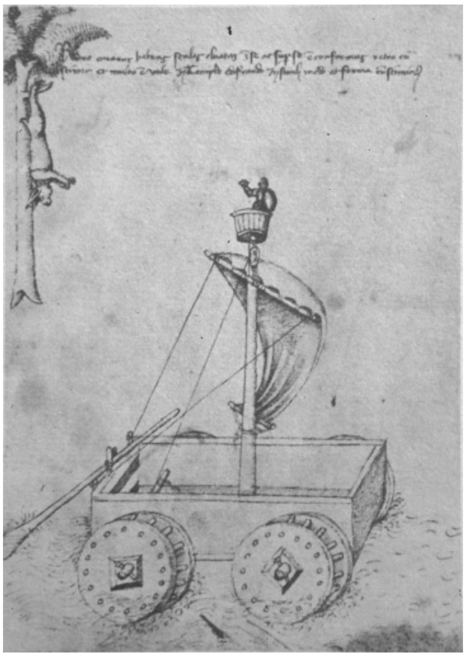

Fig. 6. Mariano di Jacopo (il Taccola), Sailing-car, *Liber Tertius de Ingenis ac Edifitiis non usitatis*, 1433

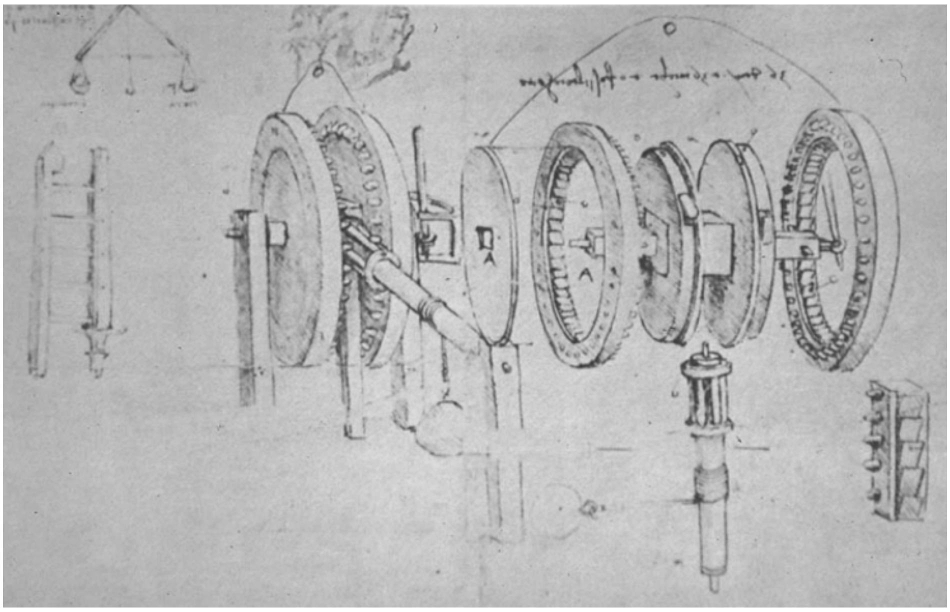

Fig. 7. Leonardo da Vinci, crane to lift heavy weights, *Codex Atlanticus* fols. 30v/8v-b

Once again, without leaving his beloved perspective in this form of bird's-eye views (a technique he mastered as no one ever before), he took advantage of another quality of axonometric representation: the capacity to construct/deconstruct an object into its component parts in order to clarify fitting and functioning, which was another main reason to keep axonometric drawings near to engineering production or architectural construction details.[4] If we look carefully at the bird's-eye perspectives he used to explore all the fields he was interested in, which were many, we can detect that the point of view he usually chose was not as high as it is now in conventional cavalier perspective.[5] As regards his architectural drawings, we verify no lack of realism, since the views of a single building or an entire city are obtained as if he were looking at them from the top of a hill, as in the first landscape we know of his (fig. 8).

Fig. 8. Leonardo da Vinci, landscape, 1473. Florence, Uffizi Gallery

Particularly, if we think of his centrally-planned churches crowned by a dome, we can easily perceive that he has drawn them as if he were viewing the dome of Santa Maria del Fiore from the neighboring campanile by Giotto, or the church of Santa Maria degli Angioli from the lantern of Santa Maria del Fiore, which were determinant referential models for his research on centrally-planned space. Carlo Pedretti goes even so far as to suggest that this view from a high point to three-quarters of the Tempio degli Angioli, similar to a view of an actual wooden model of the church placed on a table, must be related to Leonardo's use of this kind of perspective in architectural drawings [1981: 14].

Even in his drawings of cities, Leonardo remained loyal to bird's-eye perspective (fig. 9), while Francesco di Giorgio was developing for his fortifications a kind of drawing where the front view and the plan tend to appear, simultaneously, in true form (fig. 10). Faced with the need to display clearly the geometry of his plans for fortified cities, Francesco di

Giorgio intuitively discovered a drawing method which proved to be perfect for representing military architecture and, later on, architecture in general. That is why the rigorous form of this kind of perspective, today called "planometric projection", was first known as "military perspective", or *prospettiva soldatesca* as Girolamo Maggi and Jacomo Castriotto called first it. According to Scolari, parallel perspective as an alternative to Renaissance perspective was clearly presented, for the first time, in their work *Della fortificazione delle città* (Venice, 1564).[6]

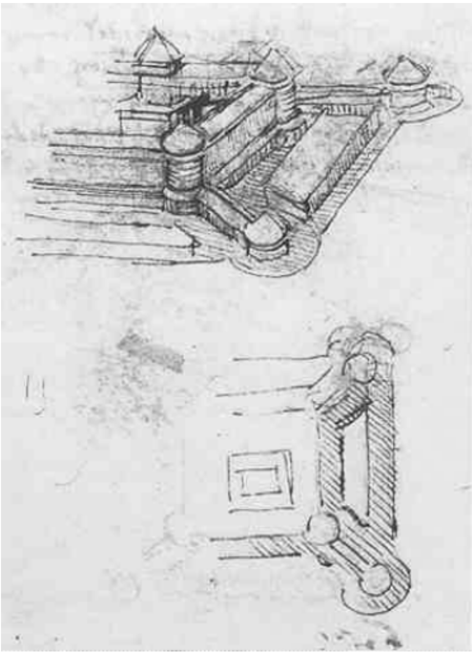

Fig. 9. Leonardo da Vinci, bird's-eye view of a fortress (c. 1504). Manuscript 8936, fol. 79r, Madrid, Biblioteca Nacional

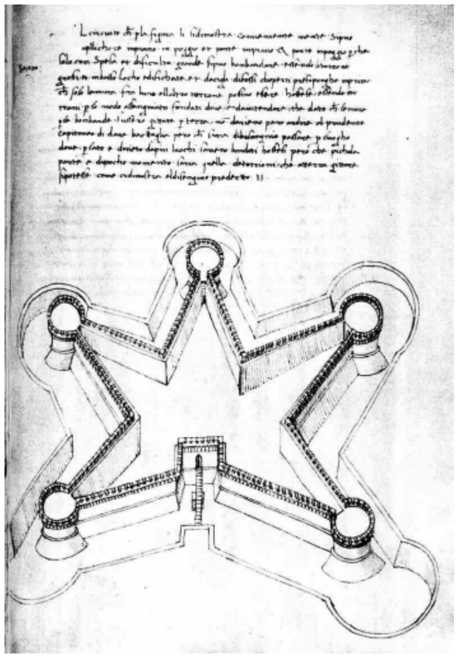

Fig. 10. Francesco di Giorgio, Pentagonal star fortress, *Trattati di Architettura Ingeneria e Arte Militare*, c. 1485

Returning to the new technique brought into architectural representation by Leonardo, we should now try to understand its novelty and special relationship with his research on centrally-planned churches, while examining its contextualization.

In a way we may consider Leonardo's system – plan plus bird's eye perspective – as a personal synthesis of Vitruvian ideas on architectural representation, since for him it was enough to use only two kinds of *dispositio*, as Vitruvius says: the so-called *ichonographia* (ground plan) and *scenographia* (perspective). It seems evident that the missing element of the triad, the *ortographia* (elevation), could be dispensed with, as it was implicitly included in the frontal bird's-eye views he used. In any case, as far as the position of the observer is concerned, Vitruvius's *scenographia* seems to be a bit different. There is no doubt that he is referring to central perspective, certainly in its pre-stage (there were no means, at that time, to establish correctly the diminution of distances in depth), when he says, "…perspective is the method of sketching a front with the sides withdrawing into the background, the lines all meeting in the centre of a circle" [Vitruvius 1970: I, ii, 15]. But as he only refers to the possibility of seeing the sides of the building and not the roof, we can infer that the observer was not at the high level where Leonardo always preferred to place him in order to control three-dimensionality in a single drawing. And, as we know, he was particularly interested in the design of the roof, since the shape, size and disposition of the domes, apart from their constructional aspects, were questions of main importance in ecclesiastic architecture. Further, according to Richter, it seems that these church drawings were expressly designed for a "Tratatto delle Cupole" that he envisioned writing [Richter 1970: 38].

I have already presented an ensemble of possibilities derived from such bird's-eye positioning in general: its three-dimensional ability to give a synthetic overview of the whole, the focusing on the object in itself, contributing to the possibility of underlining its geometric profile and showing its parts and the way they are spatially assembled and combined in order to compose the whole. And, concerning the measurements, we should also add, at least, the possibility of getting proportional measurements in the frontal views, since its form remains unaltered.

On the other hand, a plan drawing (*ichonographia*),[7] in addition to its main role in functional aspects of accessibility, circulation and interconnection, which are related to men's horizontal movements through space, is like an engraving on the ground of the building's geometrical matrix – a perfect mirror for its mathematics. And, actually, it is also from the ground up that it will grow.

In combining a plan – sometimes a mere diagram or just a scheme – with the 3D possibilities of bird's-eye perspective, Leonardo found an efficient instrument for sculpting a building in space, a way to control its growth through its final stage, ensuring its geometrical integrity from the floor to the roof. With these two drawings he could also give the maximum amount of information about it, as Murray underlined. But it is easy to prove that this is especially true in the case of centrally-planned temples, and even there we shall have opportunity to point to some indeterminations and unsolved questions.

It is evident that what makes this information "almost enough" is actually the central symmetry of the building.

Bird's-eye perspective, simultaneously presenting the front elevation and a lateral one, is implicitly telling us what the conformation of the other two sides not shown is. If any doubt remains, it is sufficient to look at the plan to confirm this. The elevation in frontal view also indicates the proportional relationship between horizontal and vertical measures of the façade, which is extensible to them all.

One of the façades, often the one in frontal view, is subtly different from the others because it is where Leonardo locates the entrance. Even so, it is difficult to speak about the existence of a main façade, since Leonardo's concern for emphasizing centrally-planned space forces him to avoid distinguishing any kind of direction. The door is there, but only to connect exterior and interior. It is not by chance that he sometimes even omits it. Further, sometimes we don't even know where it could possibly be placed, as in the plan for an octagonal church with round chapels in each side (fig. 11, upper right). To contradict this, there is also a more elaborated octagonal plan with Greek-cross chapels on each side (see fig. 1), with an exceptionally developed narthex. But here, as it does not appear in the bird's-eye view, it seems that it was added only in plan, as a possible solution for solving the difficulties inherent in the previous plan scheme. So, the prevailing feeling is that the narthex is, in this case, more a matter of concession than conviction and perhaps a result of looking at ancient examples. I believe that all these reasons could explain Leonardo's reluctance to use Alberti's system, which prescribed the use of drawing an elevation (*ortographia*) together with the plan, to which Rafael later added the use of the section, since he did not want to give preference to any of the façades of his centrally-planned temples.

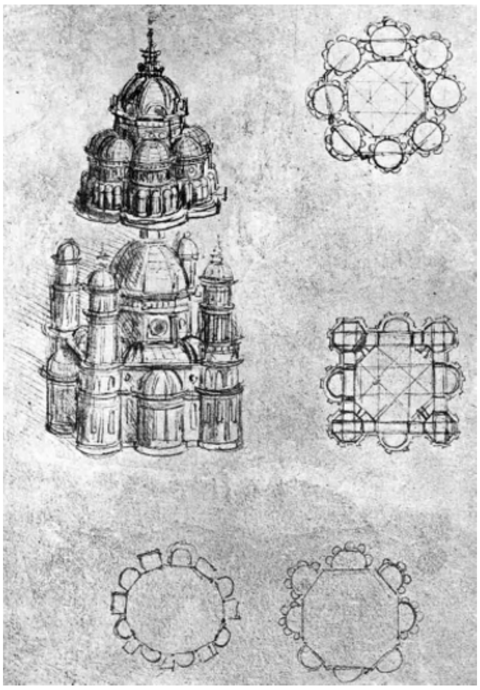

Fig. 11. Leonardo da Vinci, drawings of churches. Manuscript B, Institut de France, Paris

Due to central symmetry, the other thing a bird's-eye view makes evident is the whole configuration of the roof, where the main element is always a central dome. Its shape is generally octagonal or hemispherical. Sometimes there are smaller domes around, which can be intercalated by towers, and there can also be apses, sub-apses, niches and so on, with their corresponding domes. Whatever the building top is, even without seeing the whole, we are able to get all the information we need about it, as every element is symmetrically disposed in relation to a central vertical axis. This vertical axis, which runs from the center of the ground plan to the top of the lantern of the main dome, directs the connection between the ground and the roof plan and thus perspective displays three-dimensionally in the exterior the result of interior spatial shaping, particularly the balanced configuration of its different kinds of domes.

In some cases there is a lack of information about the interior space, which could only have been overcome with an orthographic section or, alternately, with a perspective of the interior.

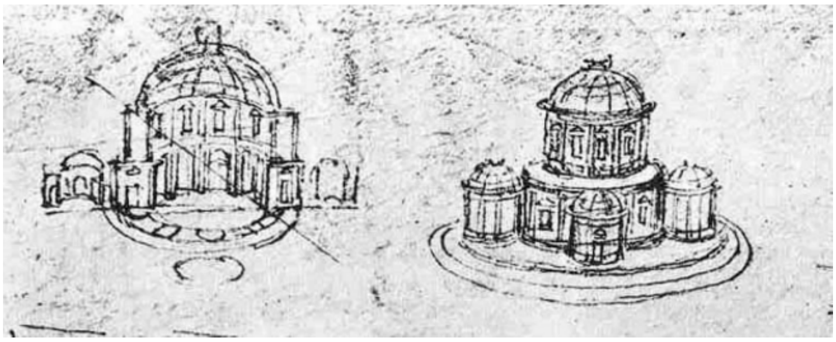

Fig. 12. Leonardo da Vinci, Circular temple with four circular chapels. *Codex Atlanticus*, fol 205 v

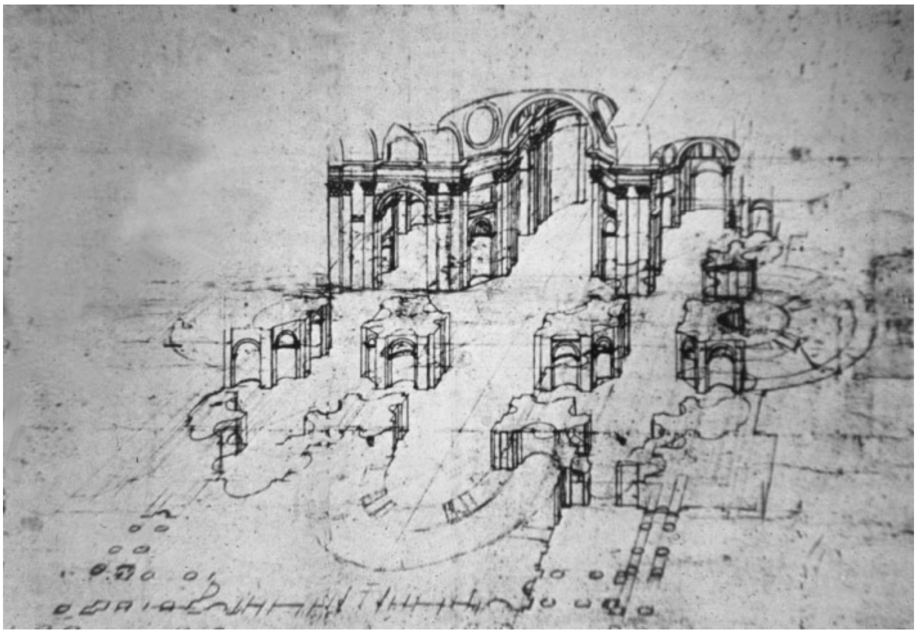

Fig. 13. Baldassarre Peruzzi, design for San Pietro di Roma. Firenze, U2A

While the resort to a section had been used since medieval times, the use of perspective in architectural representation, as pointed out by Lotz, made its appearance in Filarate's treatise, in the form of perspective-sections, and, in spite of Alberti's advice, the use of perspective, developed mainly in painting, became irresistible to architects as well. That is the case of Giuliano and Antonio da Sangallo, Francesco di Giorgio and Bramante.

Leonardo is an example of this, a very special one since he used perspective in a very peculiar way, as we have seen. But the fact is that he also used perspective sections and perspectives of the interior space. Sometimes the front views of his bird's-eye perspectives are actually sections, but there are also some situations where the observer is placed at a normal eye level, as if he were looking into the sectioned part of the building or into its interior.

One of the two drawings of a circular temple with four surrounding circular chapels is a bird's-eye perspective section (fig. 12). Here, it is undeniable that the section through one of its two symmetric vertical planes and the footprint of the half of the building not shown increases the information about it, even dispensing with the actual plan. This kind of strategy was announced Peruzzi's astonishing deconstructive bird's-eye perspective of his project for St. Peter's in Rome (fig. 13). But, evidently, axonometric drawing gradually appropriated this kind of approach for an architectural object, since it clearly departed from the field of naturalistic observation.

From the second kind of perspectives there are also some drawings regarding centrally-planned space in temples, even if the whole plan is classifiable as a basilica. All of them are related to the problem of covering a square space with apses on at least three sides, as the fourth one could be the basilica nave.

The drawing that has been related to an architectural idea for the cathedral of Pavia is a typical perspective section, according to tradition (fig. 14). The section plane, which acts as a picture plane, or an Albertian window, puts the observer aside the interior space in spite of the low point of view chosen. I believe that this kind of perspective is also in conformity with direct observation of wooden models, which were often sectioned along one of the vertical planes of symmetry, in order to show the interior.

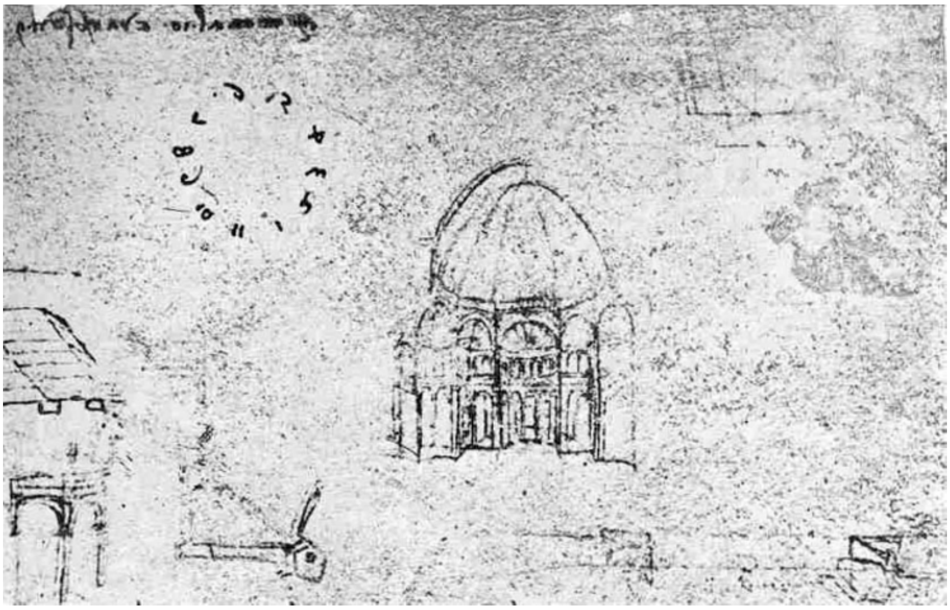

Fig. 14. Leonardo da Vinci, perspective-section of a church apse. *Codex Atlanticus*, fol. 7v-b

The other perspectives are quite different, because the observer is now immersed in the space. That is the case of a drawing for a proposal of a possible renovation of San Sepolcro in Milan (fig. 15).[8] The same attitude is found in a series of interior perspectives regarding a project (or projects) for a Greek-cross plan church (fig. 16). Such kind of perspectives, which Leonardo uses to control the articulation of different kind of elements that define space, are also in correspondence to a possible direct experience of space by the observer.

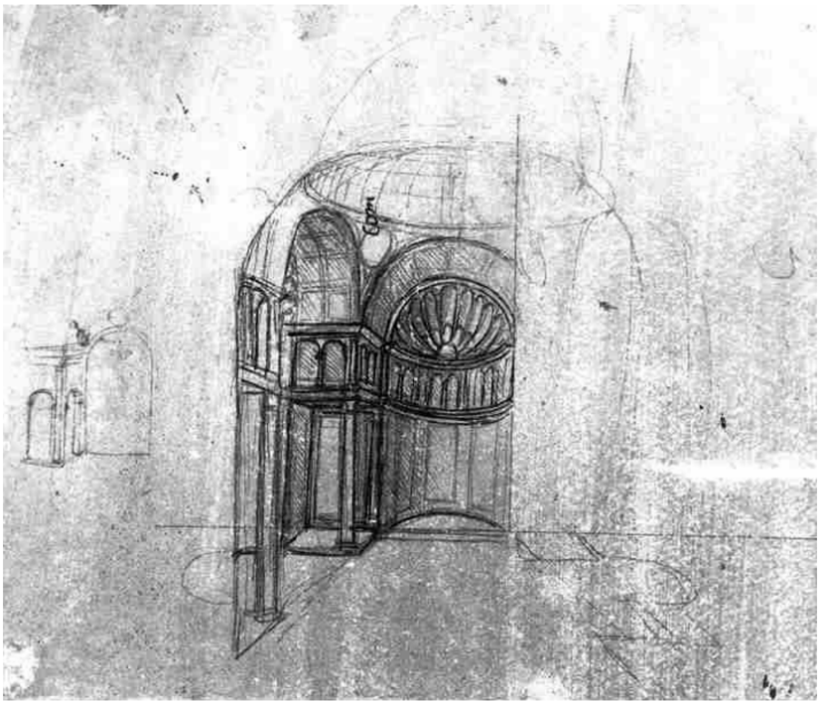

Fig. 15. Leonardo da Vinci, studies for a church apse, c. 1488. Windsor, n. 12609v

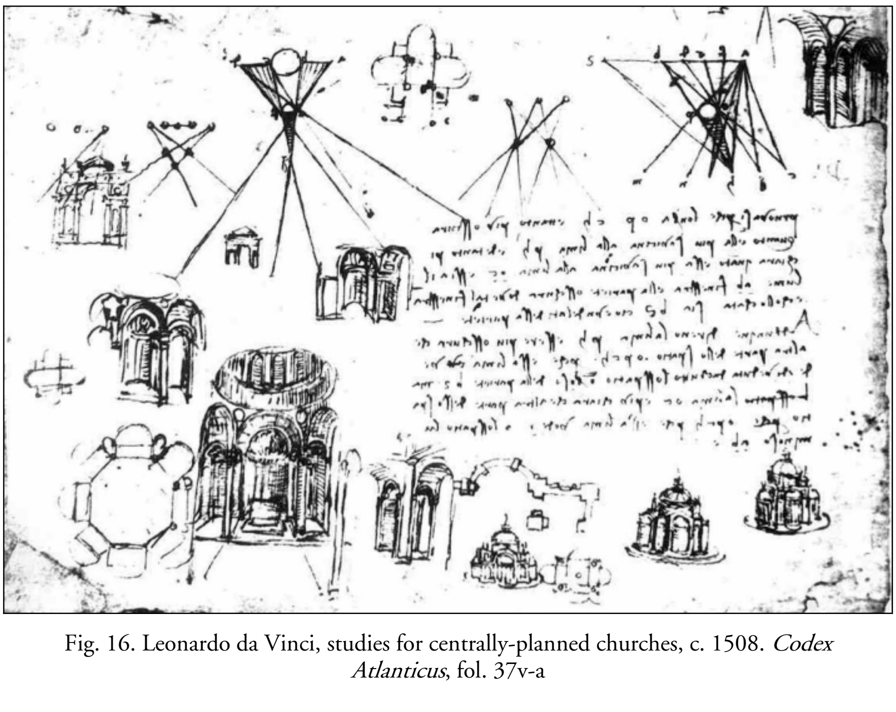

Fig. 16. Leonardo da Vinci, studies for centrally-planned churches, c. 1508. *Codex Atlanticus*, fol. 37v-a

I believe that this is the point where perspective really meets architecture, in the sense that we are allowed to state that space was shaped and thought of according to the way the spectator will see and feel it.

Elsewhere I have explored the close correspondence between the development of central perspective[9] and the research for centrally-planned space, in both urban and building planning, during the Renaissance [Xavier 1997]. The first known material evidence of such a tendency only came with the high Renaissance, as is the case of these perspectives of Leonardo's as well as the others attributed to Bramante, to which we should add the earlier panels of the *Ideal City* depicted by Urbino's circle (a presumed joint-venture between Luciano di Laurana, Francesco di Giorgio and Piero della Francesca) where Bramante, as a matter of fact, worked during his apprenticeship.

Returning to Leonardo's centrally-planned churches and his representational system, it can be seen that the information given is in fact sufficient in the majority of the situations, despite the exclusion of a section or a perspective of the interior.

First of all, we must take into account that we are working with sketches. So we have to deal with the hesitations, the superposition of different schemes or hypotheses, some imprecision in the drawing of lines or just their mere fading due to the passing of time. In any case, we should see that as a privilege, because it allows us to follow a mind at work through drawing, in this case seeking different and ingenious solutions and variations around the theme of centrally-planned space in temples. Leonardo himself proclaimed, *il disegno è una cosa mentale*, drawing is a mental thing, and in his case, this is absolutely true! So, we are not in the presence of drawings for execution nor was that their purpose. It appears that their main use was to be part of a theoretical treatise on architecture, which does not mean that these drawings could not reach a rigorous definition in order to produce a wooden model. Such models were still indispensable elements for preparing the project for construction.

Secondly, we have to consider that Leonardo was manipulating a stylistic grammar that was understandable to his fellows and followers. And, in fact, he uses a limited number of spatial cells, which were not unknown, but as he experiments with a wide range of possibilities and combinations, he arrives at some original results. However, it was not Leonardo but Bramante who was actually the first one who could amplify these achievements in executed buildings where special attention to proportion and the clarity of classical language became evident. This is also true for his designs, especially the one for San Pietro in Rome.

With the awareness of these limitations, it is possible to re-draw some of Leonardo's temples, which testify to the univocal sense of his representational system in such cases. There are even some models exhibited in museums, and it also possible to construct them virtually.

This is the case, for instance, of a temple with a regular octagonal plan surrounded by octagonal chapels on each side that are inscribed in a larger polygon with sixteen sides (fig. 17). There is no doubt that the main space is an octagonal prism crowned with an octagonal dome and we can be sure that the same occurs in the subsidiary chapels. There is also a similar plan where we find circular chapels on each side of the main octagon, but this time they are exterior, and show surrounding small apses that became niches in the interior side (see fig. 11, upper right). A final possibility is present in the very same plan, as there is a homothetic octagon whose sides cut these round chapels at the middle, transforming them into semi-circular apses. This is confirmed by a scheme present in the bottom of the same page. As we can see, the smaller apses remain. The corresponding bird's-eye perspectives confirm that these circular chapels are cylinders crowned with hemispherical domes and the apses half-cylinders covered with half-hemispheres, as shown more schematically on another page (fig. 18, above left).

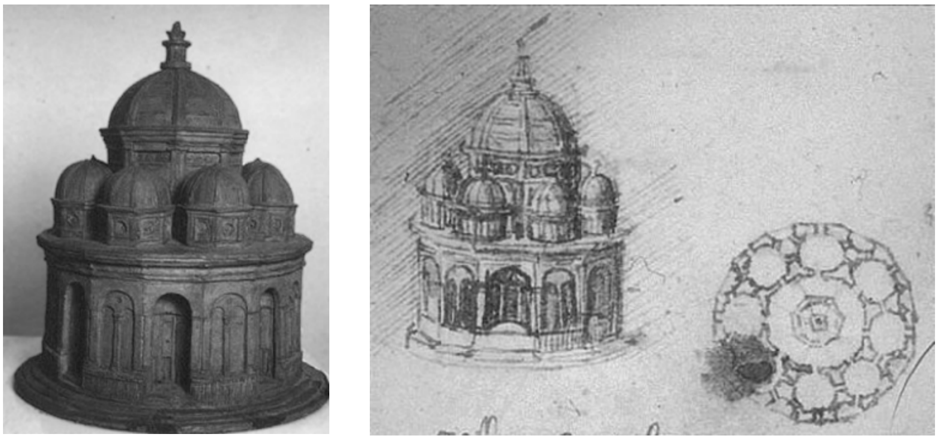

Fig. 17. Leonardo da Vinci, octagonal church with eight octagonal chapels around (Model - Museo Nazionale della Scienza e della Tecnologia "Leonardo da Vinci"; drawings - MS B, fol. 21 v)

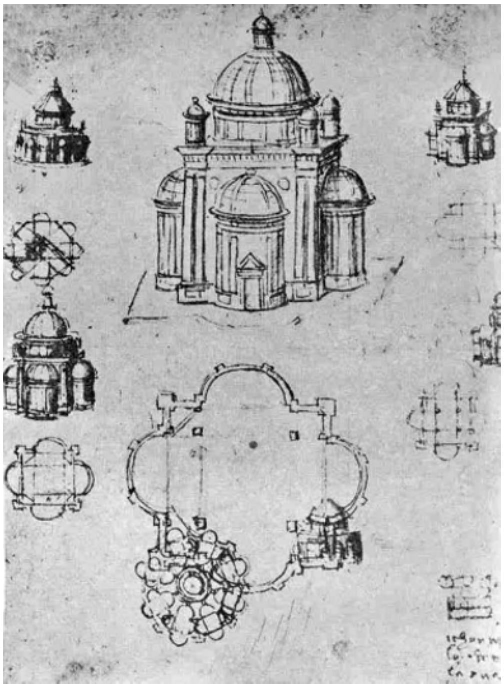

Fig. 18. Leonardo da Vinci, Drawing of churches MS BN. 2037, f. 3v. Paris, Institut de France

There is also another family of plans generated from a regular octagon to which correspond, once again, a prismatic space covered by an octagonal dome. The church presented in MS B, fol. 22 r is an example of this kind of plan. Here again there is no any possibility of misunderstanding (fig. 19). Considering the octagon as the result of the intersection of two concentric squares rotated 45º, we arrive to the whole plan with an *ad quadratum* growth of each square. Along the way, corresponding to the corners of each square, we will find four chapels and four apses. One of the exterior limiting squares, which cuts the apses at the middle, corresponds to the dominant quadrangular prismatic mass of the building from which the main dome and the four domes of the chapels are detached. In the example selected, the chapels are quadrangular prisms crowned with hemispherical domes perched on cylindrical drums; the half-detached side apses are cylinders with hemispherical domes.

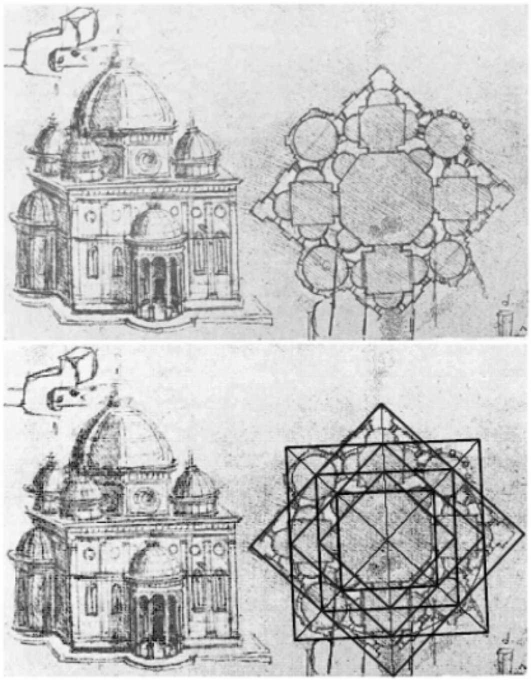

Fig. 19. Leonardo da Vinci, plan generated from a central octagon with an *ad quadratum* structure MS B, fol. 22 r., Paris, Institut de France. Geometric overlay by the author

But the problems arrive with the squared plan, which is actually related to the Greek-cross plan, to which corresponds in most cases a hemispherical dome with a cylindrical drum, although it can also be octagonal.[10] It is not by chance that all the interior perspectives we have are in relation to this plan, but even so, not everything can be disclosed.

It is sufficient to look at the several plans presented in MS 2037, fol. 3v to find the first difficulties (see fig. 18). The larger plan to which the larger bird's-eye view above presumably corresponds does not in fact match. Comparing the dimensions of the dome to the visible half-domes of the apses, we conclude that it should cover the whole space, which is incompatible with the presence of the piers in the plan. There is a more schematic plan at left, and another, even more schematic one at the right of the larger bird's-eye view, in which the piers do not appear, but we cannot be sure if this is a consequence of the simplicity of the sketches. What is certain is that the plan that matches the bird's-eye perspective cannot have piers. But there cannot be any kind of connection in the corners either, as is also shown in the plan, suggesting the definition of a semi-regular octagon inscribed in the square. Anyhow, there is also marked in the same plan (upper left corner) a short arc of a circle that could correspond to part of the projection of the dome shown in perspective. If, on the contrary, we stay with the four piers, the dome necessarily has to be shorter and should be inscribed in the square defined by these piers. But if it is the semi-regular octagon that remains, then the dome can be maximized again and here we have the interesting question posed by the spatial transition between a semi-regular octagon and a circle, a problem presented and perfectly solved in Bramante's project for San Pietro in Rome.

Considering all these possibilities, and others not yet mentioned, we begin to believe that from that plan it is possible to extract different plans, or more exactly, different variations of the same type of building, which is a clear indication that Leonardo was aware of such implications and was trying to find solutions for the problems posed by each. So, here again we see his mind in action.

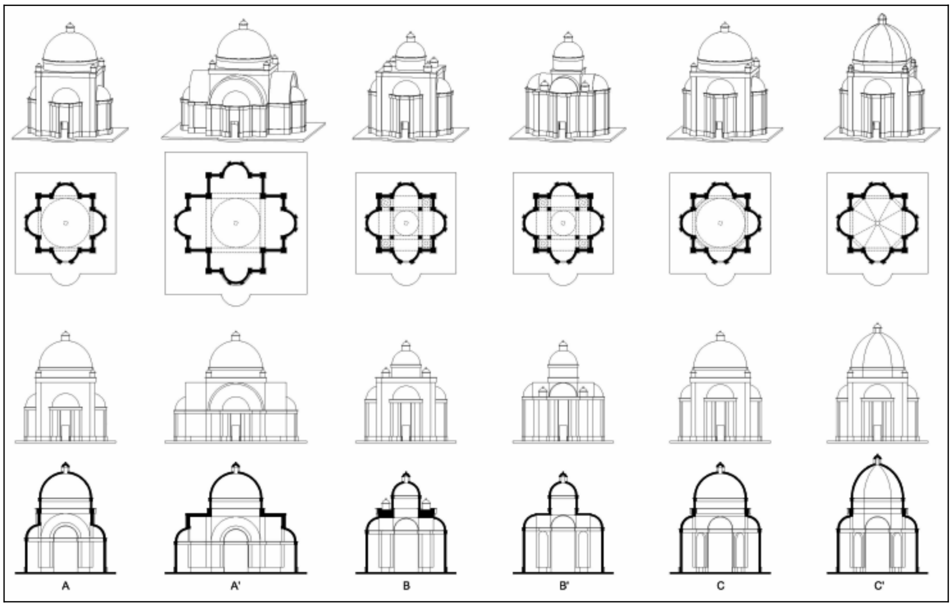

Table 1. Types of centrally-planned churches derived from MS BN.2037, fol 3v.
Paris, Institut de France

I have tried to reconstitute these possibilities diagrammatically, where each type (or sub-type) is presented in Leonardo's preferred system – plan plus bird's-eye perspective – to which I added a section and an elevation (Table 1). Because I could utilize CAD software to make these drawings, I also constructed virtual 3D models which we can (virtually) enter. With this instrument, the remaining doubts about the interior space become more evident.

All types take as their point of departure a squared plan. By subdividing each side into four units we get a net of sixteen squares, which underlies the dimensions of all the spatial elements present in all the types. It seemed to me that this grid fit, more or less, in the majority of the sketches, particularly the larger plan of MS 2307, fol. 3v (fig. 20).

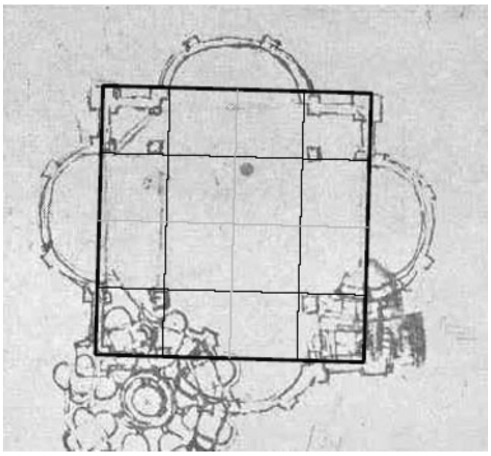

Fig. 20. Squared plan grid. Geometric overlay by the author

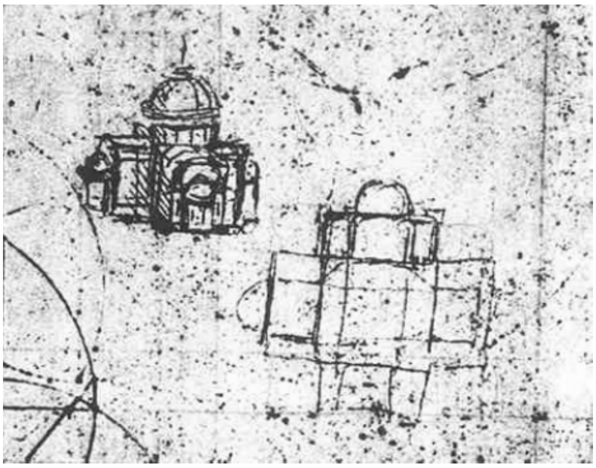

Fig. 21. Leonardo da Vinci, Greek-cross church. *Codex Atlanticus*, fol. 362r-b, v-b

Type A corresponds to a neat squared plan with one semicircular apse on each side. The main space is almost cubic and a hemispherical dome perched on a cylindrical drum covers it. The apses are cylindrical and crowned by half-hemispheres. In the exterior there are four pinnacles in the corners of the cubic prism, which are actually small *tempiettos*, as in the project for the Cathedral of Pavia. The transition between the square plan and the circular base of the cylindrical drum could be certainly solved with spherical triangles. The building that is most similar to this type of scheme is Bramante's Santa Maria delle Grazie in Milan. Both realities must be taken in consideration in order to explain Santa Maria della Consolazione in Todi. Here, however, the apses are wider, which brings the plan closer to a Greek cross.

Type A′ is configured as a clear Greek-cross plan, but the exterior appearance of the vaults of the detached arms of the cross is not convincing (fig. 21). Perhaps it could be covered with a saddle-roof with a pediment at the end, as is shown in a perspective in the *Codex Atlanticus*, fol. 37 v-a (second from right at the bottom of the page) (see fig. 16). The most similar examples in actual buildings are Giuliano da Sangallo's Santa Maria delle Carceri in Prato (1485-95), although apses are absent there, and San Biagio in Montepulciano (1518-1545) by Antonio da Sangallo the Elder, which presents two detached bell-towers in the corners and a lower semi-circular body, the sacristy, joined to the arm where the altar is.

Type B could be a perfect building *in se*, but we detect its presence more as a spatial unit that is part of a more complex composition, such as the plan for an octagonal church presented in MS 2307, fol. 5v (see fig. 1). It is also very similar to the church of San Sepolcro, on which Leonardo and Bramante probably worked, putting forth a proposal for its renovation (fig. 22).

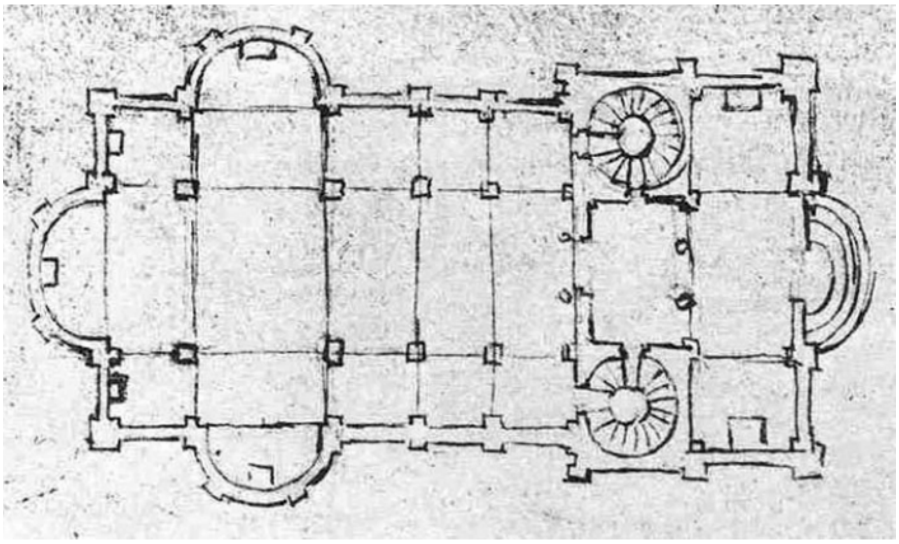

Fig. 22. Leonardo da Vinci, plan of San Sepolcro in Milan. Manuscript B, Institut de France, Paris

With the aid of an interior perspective drawing (see fig. 15), it is possible to make a sure interpretation of the configuration of this space. It is clear that the round dome is now circumscribed by the square defined by the four piers and its articulation is identical to type A. Over the smaller squares the situation is probably repeated, as we can see lanterns that appear on the roof. That is confirmed in the plans and interior perspectives of *Codex Atlanticus* fol. 37v-a (see fig. 16). But it is interesting to note the two solutions advanced by Leonardo to define the limits of these corners: with entablatures (the option for type B), and with arches.

Type B′ appears in the same sheet. Here the space corresponding to the squares in the corner is lower and thus the arched vault in between become apparent. Consequently, the articulation of the volumes underlines the subjacent Greek-cross structure of the plan.

Type C and C′ both have a semi-regular octagon inscribed in the square. Taking our grid as reference, its larger sides measures two units, the same as the diameter of the semi-circular apses, and the smaller sides $\sqrt{2}$. Once again, both types could be either independent buildings or spatial units, but we can detect their presence in the Cathedral of Pavia as well as in the Church of San Lorenzo in Milan. The importance of this Early Christian temple is well documented in Leonardo's research on centrally-planned churches. There is even a drawing for a church that is a clear development of San Lorenzo's plan that was executed at the Cathedral of Pavia (fig. 23).

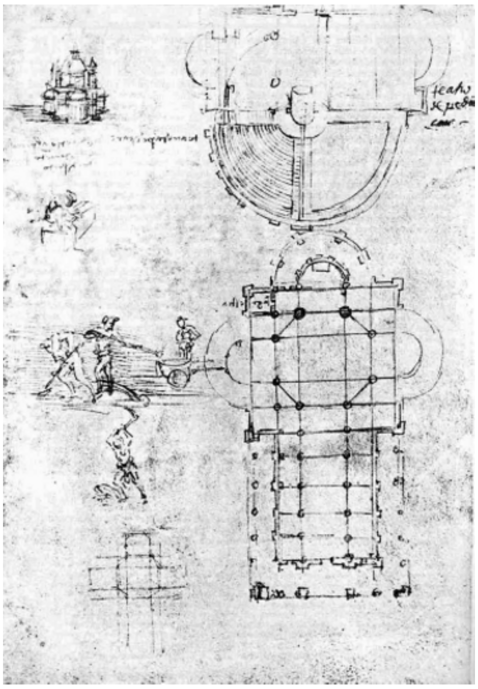

Fig. 23. Leonardo da Vinci, plan for a church based in San Lorenzo in Milan.
Manuscript B, Institut de France, Paris

Type C′ with an octagonal dome is the traditional solution, derived from the Gothic period, for covering a prismatic octagonal space, and this is the solution that was employed in Pavia (see fig. 14). Here we can verify that the height of the dome has been reduced in comparison to many of Leonardo's drawings, where the complete veneration of Brunelleschi and the influence of the dome for Santa Maria del Fiore are evident.

Type C, with a hemispherical dome, is the modern response. We know how Bramante solved the transition from the octagon to the circle where the cylindrical dome sits. It is in his project for San Pietro and it is clearly described in a drawing by one of his collaborators (fig. 24). A few years later, Raphael did the same thing on a much smaller scale in the Chigi

Chapel of Santa Maria del Popolo in Rome. We cannot be sure if Leonardo was the first to conceive this solution. His innovative representational system is not sufficient to allow us to be sure, or perhaps the problem is that insufficient information is given in the plan. In this particular case, since we do not have a wooden model, an orthographic section or an interior perspective would be welcome.

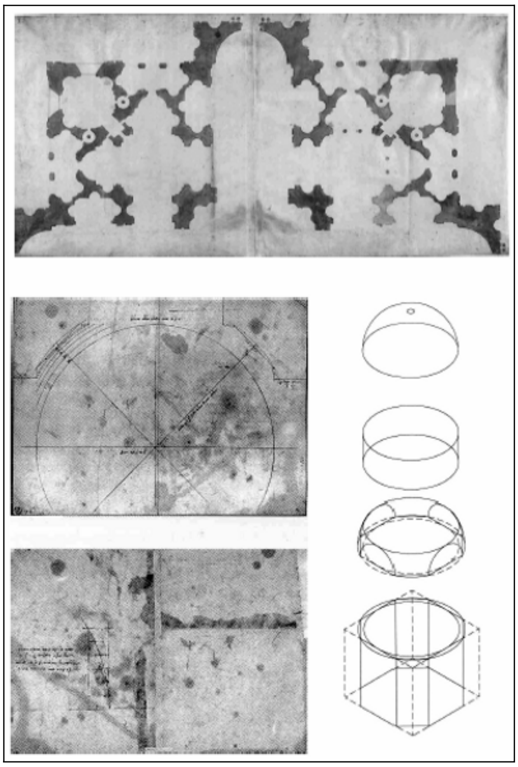

Fig. 24. Design for San Pietro in Rome (1505). Plan by Bramante. Plan and section of the project for the spherical penditives by Antonio di Pellegrino (for Bramante). Axonometry of the system by the author

Peruzzi solved the problem through an original synthesis of both procedures (see fig. 13). But he could not know that his approach was leading him away from the world of perspective and opening the doors for the definitive entrance of axonometry in the field of architectural representation.

Acknowledgment

The author wishes to thank Architect Domingos Tavares, History of Modern Architecture Lecturer and Professor at FAUP, for his advice on this paper, Architect Ana Ferreira, who constructed the virtual 3D models, and Architect Kim Williams, who so carefully revised the text. I am also grateful to Architect Sylvie Duvernoy, guest editor of this issue of the *Nexus Network Journal*.

Notes

1. Perhaps we should add "considerable" above HL presuming an observation from a high point of view.
2. Pietro Accolti, *Lo inganno degli occhi* (1625) relates parallel projection with the shadow projected by the Sun. Directing his attention to practicing painters he states:

   > *insegnandoci il testimonio del senso visivo (al quale unicamente sottoposta la pittura) manda l'ombre sue, parallele sul piano… con la infinita distanza del luminoso degli opachi… così restiamo capaci potersi all'occhio nostro, in disegnar far rappresentazione di quella precisa veduta di qualsivoglia dato corpo, esposto all'occhio (per così dire) del Sole quale ad esso Sole gli si rappresenta in veduta: onde si come speculando intendiamo il Sole non vedere giammai alcuna ombra degl'opachi, e superficie, ch'egli rimiri e illustri, cosi tutte quelle, che vengono in sua veduta, intendiamo restare lumeggiate e per contrario tutte le altre a lui ascose restare ombreggiate... Così intendiamo dover essere il suddetto disegno, per rappresentazione di veduta del Sole, terminato con linee, e lati paralleli, non occorrenti a punto alcuno di Prospettiva* [Scolari 1984: 46].

3. It wouldn't be fair not to point to Francesco di Giorgio's very intelligent way of drawing. In fact, his perspectives have a great flexibility in order to show at once what has to be shown. Thus he has no problem with intentionally distorting them on behalf of immediate legibility.
4. Among the drawings of *Codice Coner* it is possible to find some pre-cavalier perspectives, especially some architectural details seen from a low point of view (the so-called worm's-eye views, in contrast to the high position of the observer in the bird's-eye view).
5. In the most usual form of cavalier perspective the vanishing angle for the y-axis is 45º; the reduction coefficient along the y-axis is ½.
6. *Non pensi alcuno in queste mie opera vedere mode o regole di prospettiva, l' una per non essere professione di soldato non le saprei fare; l'altra perché li scorci che vi andrebbono, l'huomo leverebbe troppo dalle piante; però in esse piante, e profili consisterà il tutto di queste opera e questa si dirà prospettiva soldatesca* [Maggi and Castriotto 1564], cited in [Scolari 1984: 43].
7. *A groundplan is made by the proper successive use of compasses and rule, through which we get outlines for the plane surfaces of buildings* [Vitruvius 1960: I, ii, 15].
8. Carlo Pedretti speaks about a renewal program of possible Bramantesque inspiration. Cf. [Pedretti 1981: 24].
9. I am referring to perspective where the central vanishing point acts as the unique vanishing point. So, the y-axis is perpendicular to the picture plane.
10. The Latin-cross plan does not raise any new question, since in this situation the nave works as an antechamber to a centrally-planned temple.

References

BENEVOLO, Leonardo. 1993. *Storia dell' Architettura del Rinascimento* (1 ed. 1968). Bari: Editori Laterza.

BOIS, Yves-Alain. 1981. Metamorphosis of Axonometry. *Daidalos* 1: 41-58.

CABEZAS, Lino. 2002. Las máquinas de dibujar. Entre el mito de la visión objetiva y la ciencia de la representación. Pp. 83-348 in *Máquinas y herramientas de Dibujo*. Juan José Gómez Molina, ed. Madrid: Ediciones Cátedra.

COLQUHOUN, Alan. 1992. Assonometria: primitivi e moderni. Pp. 12-23 in, *Alberto Sartoris. Novanta gioielli*. Eds. Abriani, Alberti/Gubler, Jacques. Milan: Mazzotta.

GADOL, Joan. 1969. *Leon Battista Alberti: Universal Man of the Early Renaissance*. Chicago: University of Chicago Press.

LOTZ, Wolfgang. 1997. La rappresentazione degli interni nei disegni architettonici del Rinascimento. In: *L'Architettura del Rinascimento*. Milan: Electra Editrice.

MAGGI, G. and J. CASTRIOTTO. 1564. *Della fortificazione delle città*. Venice.

MILLON, Henry and Vittorio M. LAMPUGNANI. 1994. *Rinascimento da Brunelleschi a Michelangelo. La rappresentazione dell' Architecttura*. Milan: Bompiani.
MURRAY, Peter. 1978. *Renaissance Architecture*. Milan: Electa Editrice.
MURTINHO, Vitor M. Bairrada. 2001. La Piú Grassa Minerva'. A Representação do Lugar. Ph.D. dissertation, University of Coimbra.
MUSEO NAZIONALE DELLA SCIENZA E DELLA TECNOLOGIA "LEONARDO DA VINCI". http://www.museoscienza.it/leonardo/.
PEDRETTI, Carlo. 1981. *Leonardo Architetto*. Milan: Gruppo Editoriale Electa.
RICHTER, Jean Paul. 1970. *The Notebooks of Leonardo da Vinci*. New York: Dover Publications.
SAINZ, Jorge. 1990. *El dibujo de arquitectura. Teoría e historia de un lenguaje gráfico*. Madrid: Editorial Nerea, S.A.
SCOLARI, Massimo. 1984. Elementi per una storia dell' assonometri. *Casabella* 500 (March 1984): 42-48.
VELTMAN, Kim H. 1986. *Studies on Leonardo da Vinci. I – Linear Perspective and the Visual Dimensions of Science and Art*. Munich: Deutschen Kunstverlag.
VITRUVIUS, Marco Pollio. 1960. *The Ten Books on Architecture*. Trans. Morris Hicky Morgan. New York: Dover Publications.
WITTKOWER, Rudolf. 1971. *Architectural Principles of in the Age of Humanism*. New York: W.W. Norton.
XAVIER, João Pedro. 1997. *Perspectiva, perspectiva acelerada e contraperspectiva*. Porto: FAUP Publicações.

About the author

João Pedro Xavier is an architect and geometry teacher. He received his degree in Architecture from the Faculty of Architecture of the University of Porto (FAUP), a Ph.D. in Architecture in 2005, and has been licensed as an architect at the College of Architects in Porto since 1986. He has won academic prizes, including the Prémio Florêncio de Carvalho and Prémio Engº António de Almeida. He worked in Álvaro Siza's office from 1986 to 1999. At the same time, he established his own practice as an architect. He has been teaching geometry since 1985 at the Architecture School of Cooperativa Árvore in Porto, the Fine Arts School of Porto and at FAUP from 1991 to the present. He is the author of *Perspectiva, perspectiva acelerada e contraperspectiva* (FAUP Publicações, 1997) and *Sobre as origens da perspectiva em Portugal* (FAUP Publicações, 2006). Xavier has always been interested in the relationship between architecture and mathematics, especially geometry. He published several works and papers on the subject, presented conferences and lectures and taught courses to high school teachers. He presented "António Rodrigues, a Portuguese architect with a scientific inclination" at the Nexus 2002 conference in Óbidos, Portugal.