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keywords: arch construction,
voussoir, Interior Angle Sum
Conjecture, Exterior Angle Sum
Conjecture, triangles, trapezoids

Research

Solving Ertha Diggs's Ancient Stone Arch Mystery

Abstract. According to legend, when the Romans made an arch, they would make the architect stand under it while the wooden support was removed. That was one way to be sure that architects carefully designed arches that wouldn't fall! Educator Michael Serra led AAAS symposium participants in a surprising and fun hands-on arch construction project using familiar objects—Chinese take-out cartons—in an unfamiliar way: “these are stone voussoirs from an ancient miniature bridge uncovered by my friend, archaeologist Ertha Diggs. She has asked us to determine the number of stones in the original bridge.” This makes it possible to understand both arch mechanics and the mathematics behind the arch through actually constructing them.

Introduction

These lessons took place in a geometry class in which students work in cooperative groups of four to discover the properties of geometry. They are familiar with the basic tools of geometry: compass and straightedge, patty papers, and The Geometer's Sketchpad. They have recently discovered the Interior Angle Sum Conjecture, the Exterior Angle Sum Conjecture, and properties of isosceles triangles and trapezoids (fig. 1).

Fig. 1.

Lesson: Part 1

I began the lesson by sharing photos and transparencies of Roman and Chinese arches while discussing their history and development. Whether true or not, my students especially enjoyed the tale about the Roman architect for an arch. During class discussion we arrived at the

geometric characteristics of an arch. We agreed that the arch is half of a regular polygon. We conjectured that half the number of sides of the regular polygon must be an odd number (in order to have a keystone). For example, half of a regular 18-gon gives us an arch with 9 stones, 8 voussoirs and a keystone (fig. 2).

Fig. 3. If the arch is made of 9 isosceles trapezoidal stones then the interior angles are 80° , 80° , 100° , and 100° .

Fig. 4. If the arch is made of 15 isosceles trapezoidal stones then the interior angles are 84° , 84° , 96° , and 96° .

Then each group of four students was to design and build a two-dimensional arch of isosceles trapezoids (with more than 9 stones). I required the angle measures of the isosceles trapezoids to be positive integers. The students then discussed, planned, designed, and constructed their two-dimensional arches with compass, straightedge, and protractor. Next they wrote a description of what they did, describing the mathematics used.

But that was just the warm-up. As the period came to a close I pulled out a box of Chinese take-out cartons, that is, the characteristically-shaped paper

boxes used by Chinese restaurants (fig. 5). I told them, in mock scientific seriousness, that "these are stone voussoirs from an ancient miniature bridge uncovered by my friend, archaeologist Ertha Diggs. She has asked us to determine the number of stones in the original bridge." I gave one to each group, the bell rang and class was dismissed.

Lesson: Part 2

When the students came in to class the next day they began discussing the problem posed yesterday in their groups. I interrupted the group discussions to announce that when a group determines how many pieces in the original (Chinese take-out carton) bridge they are to write up an explanation and then call me over. When they call me over and explain their reasoning, I then give them the additional cartons they think they need to build a replica of the bridge. By the end of the period, when each group has built their arch, we bring them all together and assemble them into a vault! Of course these teenagers cannot resist crawling through the vault. The objective was for students to review and apply the properties of isosceles triangles, trapezoids, regular polygons, and of interior and exterior angle sums. They were to practice communicating mathematically and modeling in two and three dimensions. It is a fun two-day activity of hands-on mathematics and problem solving.

Follow-up

During the previous summer there was a lot of remodeling at our school site. So we had a large number of cardboard boxes stacked in piles all over the school building. When I see a free resource lying around I am compelled to make use of it in some way. So I created a lesson on nets for solids. On Day one, I asked my students to explore all the possible non-congruent nets for a particular isosceles trapezoidal prism (fig. 6).

Fig. 6. Three isosceles trapezoidal prisms are shown above. Can you find the others?

When completed we selected the one to best fit on one of the folded flat cardboard boxes. Once we had the largest possible net designed on the cardboard we passed it around and each group traced it onto their cardboard. The next day they cut and assembled each net into the isosceles trapezoidal stone ready for assembly into an arch. Would the arch be large enough for us to walk under? Having recently completed the trigonometry chapter I was determined to get them to apply their new trig skills. The task of each group was to measure their trapezoidal stone and calculate the span and rise of the arch that was going to be created by these voussoirs (fig. 7).

Fig. 7.

If a (length of large base) = 26 cm
and b (length of small base) = 22 cm,
then
Span = $2r = b/\sin 6^\circ = 22/\sin 6^\circ \approx 210.5$ cm
Rise = $h = .5b/\tan 6^\circ = 11/\tan 6^\circ \approx 104.7$ cm

By the second and third geometry classes of the day we had enough voussoirs to complete the arch. It was agreed that I was the chief architect of the arch so I was required to remain beneath the arch as all supporting hands were removed. I managed to survive the last stage of construction. Once again Geometry and Architecture blended beautifully into a few days of fun applications for my geometry students and myself. At the St. Louis Arch symposium (during the AAAS conference) in February of 2006 I had the pleasure of watching a group of professional architects and professors of architecture get on their hands and knees and play with my Chinese take-out carton arches. Whereas high school students and teachers that I have worked with in the past are fascinated with the building of arches and vaults, my new students, these architectural professionals, seemed much more interested in seeing how they would collapse! Whether building or forcing collapse, it was agreed that a fun time was had by all (figs 8-12).

Fig. 8. AAAS symposium participants studying the “relics” found by Ertha Diggs

Figs. 9 and 10. Studying the constructed arch

Fig. 11. Manipulation of the arch to see the various deformations that lead to collapse

Fig. 12. AAAS symposium speakers Matthys Levy, Santiago Huerta (holding arch), Kim Williams, Paul Calter (behind arch), John Ochsendorf and Michael Serra

About the author

Since the 1990 publication of the first edition of *Discovering Geometry: An Investigative Approach (DG)* (San Francisco: Key Curriculum Press), Michael Serra has been a popular classroom teacher, speaker, and teacher trainer at Geometry Institutes. When not teaching, he is either writing new material or traveling the country giving workshops to districts that have already adopted or are thinking of adopting *DG*. He also gives presentations at a number of National Council of Teachers of Mathematics (NCTM) regional conferences, or state mathematics conference around the country. Internationally, Michael has consulted for mathematics teachers in Costa Rica, South Africa, Australia, and international schools in China. In 2002 the third edition, *Discovering Geometry: An Investigative Approach* was released. The fourth edition of *Discovering Geometry: An Investigative Approach* is scheduled for release in January of 2007. Other publications include the very popular supplementary geometry book, *Patty Paper Geometry*, and the set of five classroom starter workbooks *Mathercise*. Key Curriculum Press has just published *What's Wrong With This Picture? Critical Thinking Exercises in Geometry*. His next project is a book of Mathematical Games and Puzzles.