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Origins of an Obsession

Though many geometric shapes can be constructed from circles, this paper is about the geometric square. It will be demonstrated that while the square is not the easiest of the polygons to construct initially, it is both easily enlarged and easily subdivided. Step-by-step manipulations of the square provide an explanation for the architectural design of the Forum of Augustus.

Introduction

My fascination with the use of geometry in architecture began in 1978 during a month-long visit to Rome as a University of Kentucky architecture student. It was then that I “discovered” the geometrically rich Pantheon while in hot pursuit of cheap postcards.

This chance visit was my first experience of the sublime visceral architecture that is the Pantheon and the beginning of an ongoing fascination with the use of geometry in architecture. When I recently found myself back from Rome again in the summer of 2001 with a group of students from Indiana University–Purdue University in Fort Wayne, I began studying this use of geometry in depth.

But much to my surprise there was little information available on the step-by-step use of geometry in construction and even less information available to indicate the architectural reasons for any particular geometric arrangement in a design. I nonetheless continued to read and experiment with the analysis of published drawings by means of compass and straightedge while working to maintain, as much as possible, the perspective of a first-century architect, that is, the use of compass and straightedge for design and string and stakes for building and site layout. This paper is a result of that investigation.

One aspect of the investigation was my conclusion that, given ancient technology, it is likely that the tools and techniques used to design on paper are similar in purpose and technique to those used in laying out a building on its site. The tools and techniques used to apply the geometry in both cases would work in essentially the same way.

Fig. 1. One man, “stakes-string-chalk-tape”, twenty-two minutes

Confident that valuable insight would be gained from my personal use of both sets of tools, I decided to explore the use of stakes, chalk, tape, and string to mark on the ground what I had drawn in my office using compass, pencil, and straightedge. Fig. 1 shows a few of the photos that were taken to record this experiment. The layout of this rotated square within a square seen in fig. 1 was completed single-handedly by the author in twenty-two minutes with considerable accuracy.

This experiment reinforced my belief in the similarity of techniques used for “office” design and site layout in earlier times.

The Square derived

Though many geometric shapes can be constructed from circles, this paper is about the geometric square. It will be demonstrated that while the square is not the easiest of the polygons to construct initially, it is both easily enlarged and easily subdivided. Its derivation from circles is as follows:

A first circle is created by holding a point at one end of a string and rotating a marker attached to the strings other end all around the fixed point.
A second circle is added which has its center point at the left quadrant of the first circle. It is rotated around this point with its circumference passing through the first circles center point.
A third circles center point in located on the first circles right quadrant. This creates a horizontal line through all three circles center points dividing the original circle in half.
A perpendicular line to divide the original circle into quarters is created by the standard geometric technique of bisecting a line. A line drawn from the bisector point to the center point of the original circle divides the circle into four equal parts and locates the four points of a square inscribed in the circle.
This first and smaller square can now be shaded. With the original circles radius of 1.0 and an area of 3.14, the square has an area of 2.0 and a side length of 1.414
Now set similarly sized circles having center points at each end of the vertical bisector. The intersections of the two new circles with the previous two circles are the four points of a larger circumscribing square.
The larger shaded square is exactly twice the area of the smaller square and rotated 45°. This is commonly referred to as a rotating square.

Subdivisions and extensions of the square

The extensive and repeated use of squares in the built environment, though economic because identical shapes are used for wall and corner construction, can rapidly become a mind-numbing grid without distinction, uniqueness, or attraction. In order to maintain visually interesting surroundings, harmonious manipulations of the square are necessary. When seeking these geometric manipulations, I felt strongly that an effort should be made to harness the creative possibilities found in small sets of interrelated rules such as the seven Newtonian colors of the painter’s palette and the twelve harmonic tones that are the foundation of a Liszt concerto. It is my

belief that just such a set of synchronized geometric techniques, which I will now describe, would have an inherent potential to create any number of designs, each of which could be imbued with a sense of resonant wholeness.

Having derived the square from a circle and now embraced it as the basis for further development, some ancient geometer probably began experimenting with string and stakes to determine what other shapes could be derived from the square. Purity would naturally be associated with those shapes most easily generated and so, therefore, preference was given to those shapes defined with as few as two strokes.

Following are the results of this explication of the square with notes indicating commonly used designations. In this paper, however, I will not be using these designations, which I feel would carry some baggage and unnecessarily encumber a fresh look at the subject. I will be using designations such as “1.414 technique” that clearly indicates proportions and avoids highlighting any of them as more “sacred” than the others.

Fig. 2. The square's five subdivisions and five extensions

As can be seen from fig. 2, from the ten double-stroke derivatives of the square, many historically significant geometric ratios were anticipated. There are the 1:2, 2:3, 3:4, 3:5, and 4:5 small fraction ratios promulgated by Plato, Vitruvius, and Palladio for both music and architecture. Also shown are extensions such as the Sacred Cut ($1/\sqrt{2} = 0.707\dots$), and the Golden

Section ($\left(\frac{\sqrt{5}+1}{2}\right) = 1.618\dots$). Each of the five subdivisions was performed with two lines, and

each of the extensions was accomplished by means of one line and one arc. Clearly a remarkable variety of shapes was possible for a pre-numerical geometer working single-handedly using strings and stakes.

The test

Now we are ready to test the hypothesis that beautiful, visually resonating structures can be built with no greater a set of geometric techniques than what are being proposed in this paper. Am I thus claiming that any structure built with these techniques will be beautiful? Not a chance! What I am proposing is that a reasonably creative designer would have a better chance of designing a compositionally cohesive and visually attractive structure using this small interrelated set of geometric techniques than if that same designer were left to their own “inspired” devices. Exceptions exist, of course, but probably fewer than are thought.

Precision and accuracy

When trying to “prove” a reconstruction, comparison and agreement become two of the primary tools by which a reconstruction gains credibility. But even a credible comparison is not certain proof. There is always a possibility that your explanation may be apparently correct while actually wrong, as was Ptolemy’s explanation for the movement of the stars in his earth-centric explanation of the cosmos. However, predictive exactitude is still necessary even not sufficient.

In archaeological reconstruction the better the condition of the structure the more likely accurate dimensions can be established. Even so, as a practicing architect I know there is a considerable difference between an architect’s original idea and the constructed building. It begins with a pure idea that the architect struggles to get on paper. That paper is given to a contractor who struggles to build it accurately. The building then lives a life of shifting foundations and adjustments. An archaeologist sometime discovers the remains of the structure and posits a reconstruction. The reconstruction is published at a greatly reduced scale and finally someone like me photocopies it for study. The opportunity for error is vast when attempting to represent the original idea of the building, even when the process is rigorously performed. For this reason I expect only reasonably approximate correlations between my reconstructions and the published images of others’ proposals.

I have therefore adopted a process of collecting as many different representations of a given structure as possible and copying them to the same scale. Then using the tools and perspective of a first-century Roman architect, I study the building images using compass and straightedge. It is common in my experience for various reconstructions to have differing dimensions but also typical for them to share many aspects. It is at this point that I make a preliminary determination of the most likely shape and point of beginning for the reconstruction process. I then proceed, sometimes by chance and sometimes by plan, selecting those steps in the design that fit the criteria of being contiguous, continuous, and accurate.

This process has often begun with a one-unit square drawn using AutoCAD software and does so here. Using this software for drawing subdivisions and extensions gives me the capability of drawing with precision to eight decimal places. This readily allows the quick supporting or disproving of prospective steps in the process.

When overlaying existing published images with proposed step-by-step constructions, it is reassuring to know that my reconstruction is not exaggerated in a way as to support a predetermined outcome. The actual comparison process consists of proportionally enlarging the proposed reconstruction and overlaying it on an existing image for a visual analysis of its correspondence.

Forum Augustum

In order to rigorously test the preceding hypothesis it was decided to use a well documented and geometrically rich Roman imperial structure as a proof of concept. The Forum of Augustus was chosen as the example and a step-by-step examination of the design process was planned.

From the beginning of my study it was clear that Augustus's decision to locate his Forum alongside the Forum of Julius Caesar was not an accident. It is likely that the same political calculations¹ that placed a statue of Caesar inside Augustus's Temple of Mars, guided Augustus in placing his forum in close proximity to Forum Julium to maximize his own forum's "authority by association".

After much experimentation I decided that the beginning shape for generating Augustus's forum was a square located adjacent to Julius Caesar's Forum and shown as dark dashed lines in fig. 3.

Fig. 3. Forums Julium (lower left) and Augustus (upper right)

One factor contributing to this decision was that the square shown is approximately 194 Roman feet, compared to the 200 Roman foot initial square of a military camp [Polybus 1979]. From this it seems plausible that Augustus, the General, might depend on familiar military layout techniques, or technicians, when designing his forum. Somewhat supportive of this decision is the comment on military camps by Josephus "within are streets and tents, those of the commanders being in the middle, and in the midst of all is the generals tent, in the nature of a temple" [Josephus 93 A.D].

The process

In describing the following process I am not attempting to establish the one and only process or ideal design of the Forum, but rather I am demonstrating just one of many harmonious designs possible with the set of techniques already noted. Many designs could have been adapted to this project's constraints during the step-by-step design process. But in the end, my decision to begin the design process adjacent to the Forum Julium was a more geometrically productive and politically significant point of beginning.

<p>Step One – Using AutoCAD software, a replica of the originating square in the central open space is represented by a square of 1.0 unit sides. The square is divided horizontally and a 1.618 technique is performed resulting in a rectangle of sides 1.0 and 1.618 respectively.</p>
<p>Step Two – Here the original square is quartered and the lower left quarter is divided in half vertically. A 1.618 technique is performed and establishes the outer edge of the Forums northern side aisle.</p> <p>The process is mirrored to establish the outer edge of the southern side aisle and results in defining a larger square of 1.618 unit sides.</p>
<p>Step Three – A line is established just inside the North portico edge. A 1.50 technique is performed on the lower left quadrant of the original square generating a vertical line. This line is extended eastward until it intersects the larger square previously created. This intersection will provide the center points for the placement of the Forums two exedras.</p>
<p>Step Four – The sizing of the exedrae is established by defining a circle's diameter from the northern portico edge to the midpoint of the original square's western side using the 1.618 technique of Step Two.</p> <p>The circles are bisected by the outer portico lines with the smaller outer segments remaining as the exedrae.</p>

Intervening Comment

This concludes the development of the Forum precinct and before I proceed to do the same with the temple I would like to talk about the issue of alternatives. In the process of writing this paper and having it reviewed by various peers, alternative solutions have been proposed. Some of these were improvements and some were not. In all cases, suggested changes need to fit precisely in my ongoing process of development or the critic was expected to propose a complete process of their own.

Even my own process necessitated different steps to accurately represent different published versions of the Forum of Augustus. There are essentially two different published versions of the Forum, the Gismondi-Bauer version as found in the *Lexicon Topographicon* [Steinby 1995], and the Rodico Reif version as found in Favro's *Urban Image* [Favro 1996]. The Gismondi-Bauer version lacks parts of the Forum precinct but it non-the-less preferred by J.H. Humpfrey, editor of the *Journal of Roman Archaeology*, as the best choice. Between the two images, the primary differences are the length of the Temple of Mars-Altor and the radius/center point of its apse. However, due to its complete representation of the entire precinct as well as the temple, I have decided to present here my step-by-step comparison to the Rodico-Reif representation.

<p>Step Seven – Within the new platform square another square rotated 45° is placed. This new square is rotated and placed adjacent to and centered on the under side of the upper edge of the temple platform.</p>
<p>Step Eight - As was the case in placing the exedrae, a slight shift is made in the placement of the perhaps to accommodate the Suburra edge, but more likely for aesthetic reasons.</p> <p>To establish this new back of the temple, the small square is divided horizontally and a 1.618 technique is applied vertically.</p>
<p>Step Nine – Using a 1.50 technique, the square is restored at a point slightly beyond the temple platform. It is my opinion that this offset, similar to the excedra offsets, is a sophisticated approach to geometric design whereby a perceptible but subtle tension is created between these not-so-obviously related parts of the Forum.</p>

<p>Step Ten – Next comes the establishment of the temple apse. I have decided to demonstrate competing techniques for deriving the apse's radius. Which of the two is chosen depends on whether you are trying to outline the front or back of the apse.</p> <p>The larger circle's diameter is determined by drawing yet another rotated square in the temple square and inscribing it with a circle. The other candidate is generated by dividing the temple square in half vertically and using a 1.618 technique. This circle's diameter is the increased dimension part as shown at the top of the image.</p>
<p>Step Eleven – The center point for the temple apse is located by dividing the temple square into fifths. The point is at the intersection of the temple square's vertical bisector and a horizontal line representing the upper fifth of the square.</p>
<p>Step Twelve – This stage goes back to the original square, bisects it horizontally, and applies a 1.50 technique in order to locate the front of the temple steps.</p>

<p>Step Thirteen – In order to finish the baselines for the Temple, we vertically divide the lower left quadrant of the original square and extend this line, and its mirror image, to the eastern end of the temple.</p> <p>From the temple apse, lines are extended downward toward the front of the cella and this completes our geometric reconstruction of the Forum Augustum.</p>
<p>This proposed line drawing appears to have promise and will now be overlaid on the published Rodico-Reif image of the Forum of Augustus (see fig. 4).</p>

Fig. 4. Comparison of proposed derivation and the Rodico Reif representation

Conclusion

While considerably accurate, there is room to contest this reconstruction at several points. However, I request those tempted to do so refer to the “intervening comments” earlier in this paper.

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