Igor M. Verner
Sarah Maor

Department of Education
in Technology & Science
Technion – Israel Institute
of Technology
Haifa, 32000 ISRAEL
ttrigor@tx.technion.ac.il

Mathematical Mode of Thought in Architecture Design Education: A case study

Integrating mathematics and architecture design curricula has resulted in a positive change in students' abilities to apply mathematics to architectural design. The authors developed the first-year calculus-with-applications course based on the Realistic Mathematics Education approach. In order to encourage students to use mathematics in design projects, the integration of mathematics and architecture education was continued by developing and evaluating the second-year Mathematical Aspects in Architectural Design course based on the Mathematics as a Service Subject approach. The paper considers three directions of geometrical complexity studied in the course with a focus on the process of project-based learning of curved surfaces.

Introduction

Architectural design is treated as a purposeful reflection-in-action process of creating a structure which fits building, material and topographic standards, funding and time limitations, and requirements of culture, aesthetics, environment and proportions. The structure should also match optimization criteria with respect to shape, stability, energy and resources consumption [Lewis 1998, Gilbert 1999, Unwin 1997, Burt 1996]. To answer these professional requirements, in their design practice the architecture program graduates should develop conceptual ideas and implement them in material products, demonstrating deep understanding of cultural, social, technological and management aspects of the project [Gilbert 1999]. These professional skills lean on prerequisite knowledge in art, science and technology [Kappraff 1991].

The value of mathematical thinking in architecture has been emphasized in recent research, particularly in geometrical analyses, formal descriptions of architectural concepts and symbols, and engineering aspects of design [Luhur 1999, Williams 1998]. Architectural educators are calling for enhanced learning programs for mathematics in architectural education by revising the goals, content, and teaching/learning strategies [Salingaros 1999, Williams 1998, Luhur 1999, Consiglieri and Consiglieri 2003, Pedemonte 2001]. Educational research is currently required in order to accommodate didactical approaches to mathematics in architecture education.

The two main approaches to teaching mathematics in various contexts are "Realistic Mathematics Education" (RME) and "Mathematics as a Service Subject" (MSS). In RME, the mathematics curriculum integrates various context problems where the problem situation is experientially real to the student [Gravemeijer and Doormen 1999]. The MSS approach considers mathematics as part of professional education and focuses on mathematical competence required for professional practice [Pollak 1988]. This includes the capability to apply mathematics to design processes such as geometrical design in architecture [Houson et al. 1988, 8].

This paper considers an ongoing study which utilizes the RME and MSS approaches to developing an applications-motivated mathematics curriculum in one of the architecture colleges in Israel. At the first stage we developed a calculus course, based on the RME approach, as part of the first-year mathematics curriculum [Verner and Maor 2003]. The two-year follow-up indicated the positive effect of integrating applications on motivation, understanding, creativity and interest in mathematics. However, from the analysis of fifty-two graduate design projects of students who took the first year RME-based mathematics course, we found that the students did not apply to

their architecture design projects in any considerable way the mathematical knowledge acquired in the course.

This situation motivated us to continue the study and develop a “Mathematical Aspects in Architectural Design” (MAAD) course based on the MSS approach. This paper presents architectural design assignments and related mathematical models and activities in the course.

The MAAD course is given in the second college year. It relies on the first year mathematics course [Verner and Maor 2003] and offers mathematical learning as part of hands-on practice in architecture design studio. The course focuses on the analysis of geometrical forms through experiential learning activities.

Geometrical forms

In structuring the course we follow the principles of classification of geometrical forms in architecture education. Consigliere and Consigliere [2003] proposed to concentrate mathematics in the architecture curriculum on composing the variety of complex architectonic objects from elementary geometrical forms. Salingaros [2000] formulated the following eight principles of geometrical complexity of urban forms:

- Couplings – connecting elements on the same scale to form a module;
- Diversity – creating couples from different elements;
- Boundaries – connecting modules by their boundaries and not by internal elements;
- Forces – shaping an object by force loading;
- Organization – shaping a structure by superposition of forces;
- Hierarchy – assembling components of various scales from small to large;
- Interdependence – assembling depends on components’ properties but not vice versa;
- Decomposition – identifying and analyzing different types of units included in a form.

Grounded in these principles, our study deals with three directions of complexity in geometrical objects for architectural design:

1. **Arranging regular shapes to cover the plane (tessellations).** Boles and Newman [1990] developed a curriculum that studied plane tessellations arranged by basic geometrical shapes with focus on proportions and symmetry. Applications from Fibonacci numbers and the golden section to designing tessellations were emphasized. Frederickson [1997] studied geometrical dissections of figures into pieces and their rearrangements to form other figures using two methods: examining a shape as element of the module, and examining a vertex as a connection of elements. Ranucci [1974] studied mathematical ideas and procedures of tessellation design implemented in Escher’s artworks.
2. **Bending bars and flat plates to form curve lines and surfaces (deformations).** Hanaor [1998] introduces a course “The Theory of Structures” which focuses on “the close link between form and structure, between geometry and the flow of forces in the structure”. He points out that distributed loads on straight bars and planar surfaces affect bending and shape (deformation) which can be described by different mathematical functions.

The curved surfaces defined by these functions are used to minimize deformation of structures under distributed loads and to express aesthetic principles.

The reciprocal connection between form and construction characterizes Gaudi's approach to architectural design [Alsina and Gomes-Serrano, 2002]. Gaudi systematically applied mechanical modelling to create geometrical forms and to examine their properties. His experimental tools included photographic workshops, plaster models, mirrors, bells, moveable ceilings and other models. He conducted experimental research in order to come up with geometrical solutions which were optimal from the construction viewpoint. He also created 3D surfaces such as paraboloids, helicoids and conoids by moving generator profiles "in a dynamic manner" [Alsina 2002, 89].

3. **Intersecting solids (constructions).** Burt [1996] examined integrating and subdividing space by different types of polyhedral elements. He emphasized that this design method can provide efficient architectural solutions.

Alsina [2002, 119-126] considered the design of complex three-dimensional forms by intersecting various geometrical forms. He showed how Gaudi used these forms in his creations in order to achieve functional purposes, such as the light-shining effects or symbolic expressions.

Case study framework

The Mathematical Aspects in Architectural Design (MAAD) course has been implemented in one of Israeli colleges as part of the architecture program that certifies graduates as practicing architects. This second-year course relied on the first-year mathematics course [Verner and Maor 2003] and offered mathematical learning as part of hands-on practice in architecture design studio.

Following Schoen [1988] we consider the design studio as an experiential learning environment that represents real world practice and involves students in learning by doing, knowing-in-action, and reflection-in-action experience. Using the studio as an authentic environment for architectural design education, the MAAD course offers design assignments which require self-directed mathematical learning. This includes inquiring mathematical aspects of architectural problems, studying new mathematical concepts on a need-to-know basis, and applying them to geometrical design.

The MAAD course consists of three parts corresponding to the three directions of geometrical complexity introduced in the previous section of this paper. Each part of the course includes the following components:

- Mathematical concepts and methods with connections to architecture;
- Practice in solving mathematical problems;
- Design projects.

The 56-hour MAAD course outline is presented in Table 1. The first column includes the three course subjects (tessellations, curved surfaces, and solids intersections). The second column details the above mentioned components for each of the course subjects. The third column contains instructional goals which directed mathematical learning in each of the subjects. The fourth column describes learning activities towards achieving the objectives.

Course subjects	Components	Instructional goals	Learning activities
1. Tessellations	A. Mathematical concepts of tessellations design (4 hours)	Understanding the rules of harmonic dimensions and their use in art, architecture and music	Seminar presentations on golden section, Fibonacci sequence, logarithmic spiral, harmonic properties and applications.
	B. Practice in solving mathematical problems related to tessellations (4 hours)	Acquiring basic skills in analysis of proportions, symmetry, harmonic dimensions, and drawing tessellations	Drawing logarithmic spirals, analyzing basic geometrical figures and their possible combinations, and drawing tessellation fragments.
	C. Tessellation design project (8 hours)	Acquiring experience in tessellation design, making use of modularity, differentiation, proportion and harmony	Designing a flat tessellation of a given floor surface as a combination of modules inspired by a certain metaphorical subject
2. Curved surfaces	A. Algebraic surfaces used in constructions (4 hours)	Identifying types of mathematical surfaces utilized in roof design of existing gas stations and other constructions	Seminar presentations on roof surfaces of existing constructions such as the Sarger surface, hyper, and ellipsoid.
	B. Practice in drawing algebraic surfaces (6 hours)	Developing skills of drawing surfaces through calculating their parameters and coordinates	Exercises of drawing algebraic surfaces (plane, sphere, ellipsoid, cylinder, elliptic hyperboloid, hyperbolic paraboloid) and calculating their volumes and areas.
	C. Gas station design project (10 hours)	Application of calculus to defining complex roof shapes for large span solutions	Designing a gas station with a curved roof which answers the functionality, stability, and constructive efficiency criteria. Building a physical model which accurately represents the roof dimensions.
3. Solids / Intersections	A. Algebraic solids and intersections (4 hours)	Defining algebraic solids, their features, parameters and intersections, in application to structures design	Seminar presentations on solid intersections (cylinder with oblique cone and pyramid, sphere with ellipsoid and straight cone) in existing constructions.
	B. Practice in calculating parameters of solids and intersections (6 hours)	Developing skills of mathematical analysis of solids and intersections	Drawing algebraic solids, their sections and involutes (?). Calculating volumes, surface areas, and involutes
	C. Solids and intersections project (10 hours)	Acquiring the skill of analysis of composed structures using analytic geometry	Decomposing an existing architectural structure into solids and analyzing their intersections. Building a physical model of the structure

Table 1. The MAAD course outline

In this paper we will focus on the second course subject, related to surfaces of structures. In the study of curved surfaces, as in the two other subjects, the mathematics concepts were introduced in the form of student seminars. Students were guided with respect to seminar contents, references, and presentation procedures. Each of the seminars was given by a number of students and included definitions of mathematical concepts and their applications in architecture. Hands-on activities and discussions were encouraged. Seminars given by the students dealt with curved surfaces in public buildings. Two examples of the seminars are gas stations with curved roofs, and Sarger surface analysis in the “Le Marche de Royan” building.

The practice sections of the course focused on specific mathematical skills related to the subjects. In addition to exercises of drawing algebraic surfaces the students were required to calculate their parameters and coordinates.

The study of curved surfaces in the course was culminated by a design project assignment formulated as follows:

Design a plan and top covering of a gas station. Start from a zero level plan including access roads, parking, pumps, cars washing, coffee shop, and an office. Design a top covering for the pumps area, or the roof of the coffee shop and office building. Find a design solution that addresses the stability, functionality, constructive efficiency, complexity and aesthetics criteria.

The project stages are:

- Identifying the project data (place, design guidelines for gas stations, dimensions, and prototypes);
- Developing an architectural programme;
- Defining design factors relevant to the project;
- Generating alternative solutions;
- Analyzing alternatives and selecting the solution;
- Producing drawings, calculations, and a physical model.

The projects evaluation is based on the following criteria:

- design criteria: constructive efficiency (8%), aesthetics aspect (8%), architectural functionality (20%), program quality (8%);
- mathematics criteria: gas station dimensions calculation (8%), parametric analysis of surfaces (8%), roof model calculation (8%), building the physical model (8%), precision of model and calculation (8%), mathematical model analysis (8%), geometrical complexity of solutions (8%).

The projects were performed as individual assignments. The course meetings during the five-week period of the project were dedicated to guidance, studio discussions, and progress reports.

Methodology

The first step of the study was examining past senior projects (N=52) with regard to the mathematics applications. It revealed the following features:

1. All the students used proportions in designing spaces, areas and lines, but did not use the golden section or any other irrational ratio.
2. In all the projects, contours were described only by line and circular segments. Even if the students needed to use more complicated lines, they described them by circular segments. Only one of the students used a higher order curve, namely a logarithmic spiral, working under a mathematics teacher's guidance. A few of the students (9.6%) inscribed curved surfaces inside the structure but defined them graphically and not analytically. Students avoided trigonometric calculations of constructive parameters by using computer drawing operations. This method required them to draw additional sections of structure elements, which caused architectural mistakes in measuring distances and angles. Also, no experience was found in the analysis of special points, while conjugations of curves were performed graphically.
3. The final solution was not based on the mathematical optimization analysis. In fact, this analysis was not required by the assignment, which did not specify dimensional and cost limitations and criteria.
4. All design operations related to lengths, areas, and 3D geometry were performed by means of software tools. Lack of analytical evaluation of results caused mistakes in calculating geometrical parameters in the projects.

The insufficient application of mathematical methods in the senior projects indicated the need of an additional course which teaches the applications of mathematics to architectural design. The new MAAD course was implemented in the college in the 2003-2004 academic year by one of the present authors (Sarah Maor) and attended by twenty-six second-year students. The goal of the course's follow-up was to examine mathematical learning in the architectural design studio. The study focused on the following questions:

- What are the features of mathematical learning in the studio environment?
- What is the effect of the proposed environment on learning mathematical concepts and methods?

The study used qualitative and quantitative methods, which analyzed architecture design experience and observed learning behaviour within the context of design studio [Schoen 1988]. Different tools were used for answering the research questions. Data on the features of mathematical learning were gathered from:

- Interviews with experts. Two experienced practicing architects considered their professional activities and relevant mathematical concepts throughout the design stages. From these considerations we derived ideas of the design activities in the three course projects, of the design education features, and of mathematical needs in design.
- Architectural design and mathematics education literature. Relevant teaching methods using context, visualization, heuristic and intuitive reasoning, algorithmic analysis, and reflection were selected. Through integrating them with the ideas given by the architects we developed the concepts of learning activities in the course.

Data on the effect of the environment were collected by:

- Design project portfolios. Portfolio evaluation included content analysis of the project activities and assessment of design solutions and mathematics applications. The design

assessment criteria were based on the existing practice of studio evaluation and referred to the three following aspects: concept, planning/detailing, and representation/ expression. The mathematics assessment criteria were: perception of mathematical problems, solving applied problems, precision in drawing geometrical objects, accuracy of calculations and parametric solutions. Frequencies and correlations of grades in design vs. mathematics evaluation grades were examined.

- Attitude questionnaire and interview. The post-course questionnaire asked students to list the mathematical concepts studied in the course, give their opinion about its importance, and evaluate the learning subjects and methods. The in-depth interview with one of the students in the end of the course focused on his experience of applying mathematics in design before and in the course.

Analysis of results

Interviews and literature.

Design stages. In the first stage of the study, two experienced practicing architects were interviewed. They described their considerations in professional activities and relevant mathematical concepts, throughout the design stages: concept design, data collection and analysis, design alternatives development, design criteria formulation, design solution selection, models and drawings producing and presentation, solution examining and revising. This sequence of stages is similar to that proposed in [Hanaor 1998].

Design criteria and related mathematical concepts. The criteria and mathematical concepts identified in the study include the following: aesthetics, geometrical form, space division, proportions, functionality, culture, environment, symbolism, climate, geology, topography, construction rules and processes, people flow, energy, materials, stability, durability, building limitations, efficiency, modularity and accuracy.

Geometrical forms in architectural design. As noted by the architects, when developing structural forms they rely on the above mentioned criteria and answer questions such as: “How does the structure work? How does it fit in the entire project and in the environment? Is it a heavy/concrete or light/steel construction? Does it fit the concept?” They apply mathematical concepts such as reflection, symmetry, function, fractals, topological features, chaos, proportion, equality, identity, scale, algebraic surfaces, surface area, dimensions, volume and polyhedra. The literature sources mentioned in the geometrical forms section of the paper present numerous applications of these mathematical concepts in architecture design.

Mathematical aspects and course contents. The architects recommended teaching the mathematics concepts through architecture design projects that deal with curved surfaces, transformations, large structures and spans in airports, stadiums, etc. They emphasized the importance of mathematical methods in obtaining accurate design solutions. They suggested introducing the mathematics activities in the architecture design studio and then focusing on inquiry and learning discovery, design experiments and critical discussions. The architects proposed learning about geometrical objects in the order of their geometrical complexity – from point to line, to plane, to surface, and finally to volume. In addition to these recommendations, relevant teaching methods using context, visualization, heuristic and intuitive reasoning, algorithmic analysis and reflection, were selected from educational literature.

Grounded in the characteristics extracted from interviews and literature, we developed the concepts of learning activities in the course, defined a hierarchy of architectural and developed the MAAD course curriculum.

Design Project Portfolios

Each of the students in the MAAD course performed three projects and reported them in project portfolios. In this section we will consider the curved surfaces project, which is described in the “Case Study Framework” section of this paper.

Curved surfaces design project – grades. Our focus in evaluating student projects was on the correlation between students’ achievements in design and in mathematics. Tables 3A and 3B present results of project assessment – average grades and standard deviations (S.D.) – following the design and mathematics criteria mentioned in the tables.

	Constructive efficiency	Aesthetics aspect	Architectural functionality	Program quality	Design grade
Average	76.9	85.6	73.5	87.5	78.9
S.D.	13.6	14.3	11.0	12.1	9.1

Table 3A. Curved surfaces design grades (%)

	Gas station dimensions calculation	Parametric analysis of surfaces	Roof model calculation	Building the physical model	Precision of model and calculation	Mathematical model analysis	Geometrical complexity of solutions	Math grade
Average	76.9	68.8	81.3	76.3	81.9	81.9	85.0	78.9
S.D.	13.6	13.8	10.4	9.9	13.8	12.5	11.1	6.0

Table 3B. Mathematics grades (%)

Tables 3A and 3B reveal the following features:

1. In the design assessment the students achieved high average grades for the program quality (87.5) and aesthetics (85.6) – the skills that they already acquired in the architecture courses. The grades for functionality (73.5) and efficiency (76.9) are lower because their insufficient experience in mechanics and advanced design.
2. The highest average grade among the mathematics evaluation factors was achieved in the geometrical complexity of solutions (85.0%). In spite of the relatively low credit given for this factor in the project assignment (8%), the students developed complex solutions since they were internally motivated to acquire experience in designing complex shapes. The average grades for the model related criteria (81-82) are higher than that for drawing calculations criteria (69-77). The possible reason is the educational advantage of creating real physical models which is emphasized in the educational literature [Oxman, 1999].
3. Close correlation between the individual design and mathematics grades was found, $\rho = 0.698$. This result indicates the tight integration of design and mathematical aspects of the course project.

A

B

C

Sarger segment equation:

$$Z = d \cdot \left(1 - \frac{y^2}{L^2} \right) + f \cdot \frac{y^2}{L^2} \cdot \cos \left(\frac{\pi \cdot L \cdot x}{l \cdot y} \right)$$

Sarger roof segment equation:

$$Z = 11 \cdot \left(1 - \frac{y^2}{24^2} \right) + 6.5 \cdot \frac{y^2}{24^2} \cdot \cos \left(\frac{\pi \cdot 24 \cdot x}{15 \cdot y} \right)$$

Hypar roof equation:

$$\frac{x^2}{16^2} - \frac{y^2}{5} = \frac{2 \cdot z}{3}$$

E

D

Figure 1. A. Gas station plan 0:00; B. Sarger roof physical model for the pumps area; C. Sarger segment; D. Hypar roof physical model for the office area; E. Equations

Curved surfaces design project – example. Each of the project portfolios included the concept explanation, a module drawing, the module allocation plan, and a description of design and mathematical solutions. Here we present an example of project work produced by two students. The example includes a gas station plan drawing, performed by the students together, two roof models which they created individually, and equations (see fig. 1).

Our comments with regard to the design process description given by the students are as follows:

- The student applied knowledge of algebraic surfaces in order to design roof surfaces of desired curve shape and implement the precisely in the physical models.
- The drawings, models and mathematical descriptions given in the project report indicate that the students understand how to define and analyze surfaces, and their sections by coordinates. The students also demonstrate an ability to synthesize analytically surfaces which have desired properties and answer given constraints, such as Sarger and hyper surfaces.
- When designing the gas station roofs, the students revealed the limitations of the CAD software in precise drawing of curved surfaces. To solve this problem they learned how to draw mathematical surfaces using the MatLab software and used it to design the Sarger and hyper surfaces which are then implemented in the physical models.
- When building the physical models, the students acquired experience of dealing with efficiency and construction stability factors in design solutions.
- The solutions presented in fig. 1 were designed by the students through iterations and selection of alternative variants.

The comments given above refer mainly to the design process. In the study we also examined the mathematical learning process as it was reported in the student's description of mathematical solutions. This analysis revealed that principal features of mathematical learning [Tirosh 1999], such as algorithmic, formal, intuitive, logical, and affective processes, appeared in the curved surfaces design context.

Attitude Questionnaire

The post-course questionnaire included four open questions. The examination of students' responses is based on categorizing the answers through context analysis and calculating their frequencies.

The first question evaluated students' opinion about the importance of the three course subjects (design of tessellations, curved surfaces, and the intersections of solids) for their architecture studies. The first column of Table 4 cites typical statements given by the students which specify aspects of the course contribution. The second column presents attitude categories related to these contributions. The frequencies of the contribution categories are given in the third column.

The absolute majority of the students (93%) acknowledged the great importance of the three course subjects. They identified the seven main categories of the course contribution which are presented in Table 4. The frequencies shown in the table indicate the percentage of students who mentioned certain categories as especially important for their cases. The majority of students emphasized the course contribution to deeper thinking on the subject of tessellation (category 3),

understanding mathematical concepts applied in architecture (category 1), designing geometrical forms (category 4) and connecting mathematics and architecture (category 5).

The second question of the attitude questionnaire asked every student to list mathematics concepts that he/she learned in the course. The answers are given in Table 5.

Attitude statements (aspects of the course contribution)	Categories of contributions	Frequency of categories (%)
1. <i>The course contributed to deeper understanding of golden section, Sarger surface, hyper and other concepts that were introduced in the Art and Architecture History courses.</i>	Understanding mathematical concepts applied in architecture	76
2. <i>After studying the course we are able to make a deeper analysis of structures with regard to additional aspects.</i>	Skills acquisition for analyzing architecture works	36
3. <i>Tessellation design is an important part of an architect's professional activities. This experience facilitated deeper thinking on the subject including module design, proportions, geometrical forms, symmetry, and harmony.</i>	Deeper thinking on the tessellation subject	84
4. <i>Mathematics is necessary for architectural design. How to make it precise? How to diversify geometrical forms? How to define structure contours without mathematics?</i>	Designing geometrical forms	64
5. <i>The unexpected discovery from the course was the connection between mathematics and harmony, aesthetics, and efficiency in various areas such as biology, music, anatomy, art and architecture.</i>	Discovery of universal mathematics formulations for harmony, aesthetics, and efficiency	88
6. <i>The course contributed to the general background knowledge, the connections between mathematics and different subjects such as botany and music were very interesting.</i>	General background enhancement	44
7. <i>In design we need less specific calculation and more structured thinking and systematic consideration. In this project I acquired the ability of step-by-step design.</i>	Structured thinking and systematic consideration of the project	24

Table 4. Students evaluation of the course

Mathematics concepts	Frequency (%)
Proportions, sequences, logarithmic spirals, polygons, symmetry, harmonic division, algebraic surfaces and line intersections (polyhedral, cylindrical, spherical, elliptic, and conic)	90-100
Cartesian and polar coordinates, circles and arcs, exponential and logarithmic functions	70-89
Similarity of triangles, irrational numbers, geometrical dimensions	50-69
Fractals, trigonometric functions, derivatives	30-49
Parabolas, limits, radians, tangents, equations and inequalities, differentiability, vectors and matrices	Less than 30

Table 5. Mathematical concepts learned in the course

Table 5 shows that the students in the course were exposed to a variety of mathematics concepts learned in class or on a need-to-know basis.

The third question related to the impact of the course on students' attitudes toward mathematics. The answers are summarized in Table 6 (which is similar to Table 4).

Attitude statements (aspects of change)	Categories of attitude change	Frequency of categories (%)
1. <i>I always had fears of mathematics, also after the first year course even though it differed from the school subject. In the course, mathematics has become so friendly, relevant and interesting that I succeeded in applying it.</i>	Finding interest in mathematics and its relevance	68
2. <i>The atmosphere of projects competition in the course motivated me to apply diverse geometrical forms, and this caused me to study functions deeper than I expected from myself.</i>	Recognizing the challenge of mathematics application	36
3. <i>When studying Calculus I asked myself: why do I need it, as I will never use it. And to my great surprise, I opened my calculus note-book looking for formulas that could help me in the project.</i>	Self-directed mathematical learning	48

Table 6. Attitudes toward mathematics

The majority of the students (72%) noted that the course changed their attitude towards mathematics. Almost all these students (68%) affirmed that the course aroused their interest in mathematics and demonstrated its relevance to architecture. Some of the students recognized the challenge and even looked for new applications of mathematics in architecture by their own.

The fourth question asked students to evaluate instruction in the course. As found, 64% of the students think that the studio-based instruction increased their motivation to learn mathematics, for 84% it stirred their interest, curiosity and was a challenge. The studio method enhanced students' creativity (60%) and opened a skylight to mathematics (68%).

Conclusions

Our longitudinal study shows the positive change of students' ability to apply mathematics to architectural design as a result of integrating the mathematics and architecture design curricula. The study started from developing the first year calculus-with-applications course based on the Realistic Mathematics Education approach. The course follow-up revealed significant improvement of learning achievements in mathematics and attitudes towards the subject. However, reviewing graduate architectural design projects performed at the college revealed that students scarcely used mathematical tools acquired in the first year mathematics course. In their design solutions the students avoided applying complex forms and surfaces in their design solutions.

In order to encourage students to use mathematics in design projects, we continued the integration of mathematics and architecture education by developing and evaluating the second-year MAAD course based on the MSS approach. This course offers mathematical learning as part of hands-on practice in an architecture design studio. It deals with three aspects of complexity in geometrical objects for architectural design: (1) arranging regular shapes to cover the plane (tessellations); (2) bending bars and flat plates to form curved lines and surfaces (deformations); (3) integrating and subdividing space by solids (constructions).

This paper focuses on the second aspect of geometrical complexity and considers the process of project-based learning of curved surfaces. The 20-hour curved surfaces design course consisted of three sections: mathematical concepts and methods with connections to architecture, practice in mathematical analysis of curved surfaces for architectural design, and a design project.

In the 2002-03 course follow-up study we used qualitative (ethnographic) methods, which observed learning behaviour within the context of the design studio using observations and interviews, attitude questionnaire, and project portfolios.

Our observations showed that the students approached the project experience with curiosity and motivation, and interest in deepening studies in mathematical subjects and their use. Assessment of students' activities in the projects indicated that the majority of them refreshed and practically applied their background mathematical knowledge. They also learned on a need-to-know basis and applied algebraic surfaces such as ellipsoid, elliptic hyperboloid, hyperbolic paraboloid, Sarger segment, etc. The correlation between design and mathematics grades showed the tight integration of the two subjects in the projects.

Analysis of attitudes questionnaires revealed students' high positive evaluation of the course. The majority of the students noted that the course aroused their interest in mathematics and demonstrated its relevance to architecture. The studio method encouraged students' creativity. Some of the students recognized the challenge and even looked for new applications of mathematics in architecture by their own.

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About the authors

Igor M. Verner received his M.S. degree in Mathematics from the Urals State University (1975) and a Ph.D. in computer-aided design systems in manufacturing from the Urals Polytechnical Institute (1981), Sverdlovsk, Russia. He is a certified teacher of mathematics and technology in Israel and a Senior Lecturer at the Department of Education in Technology and Science, Technion – Israel Institute of Technology. His research interests include experiential learning and E-learning, computational geometry, robotics, design, spatial vision development, mathematics in engineering and architecture education. He supervised the M.S. and Ph.D. studies of Sarah Maor, completed at the Technion in 2000 and 2005, in which architecture college courses "Calculus with applications" and "Mathematical Aspects of Architectural Design" were developed, implemented, and evaluated. Results of the studies were described in the papers published in the International Journal of Mathematics Education in Science and Technology (2001 and 2005), and in the NNJ (2003). The studies were presented at the International Conference on Applications and Modeling in Mathematics Education in London (2005). In Fall 2005 Dr. Verner worked as a visiting professor at the Tufts University and in January 2006 as a visiting scholar at the University of California, Berkeley. He gave talks on mathematics in architecture education at the Tufts MSTE Program seminar and at the Harvard Mathematics Education Seminar Series.

Sarah Maor received her Ph.D. in 2005 from the Department of Education in Technology and Science, Technion and is a Lecturer at Hadassa-Wizo College of Design, Haifa, Israel