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Mathland
The Role of Mathematics in Virtual Architecture

This paper is dedicated to some arguments that could be of interest both for students and practicing architects. A short adventure in the reign of mathematics and culture. The example that I have chosen is that of the idea of space, how this idea and the perception of space around us has changed up to the point where it has arrived to the form of virtual architecture.

Background

From my generation's student days—and by that I mean during the 1960s—our professors teaching mathematics courses at the university never missed an opportunity to underline the fact that pure mathematics was the only true mathematics, and that applications were of no interest at all to a "real" mathematician. Certainly this was not a new idea; it is sufficient to read some of G.H. Hardy's celebrated *A Mathematician's Apology*:

> The best mathematics is serious as well as beautiful, 'important' if you like, but the word is very ambiguous, and 'serious' expresses what I mean much better [Hardy 1940: 89].

Serious does not mean, as one might thing, full of practical applications. On the contrary, Hardy states that:

> … if a chess problem is, in the crude sense, 'useless', then that is equally true of most of the best mathematics; that very little of mathematics is useful practically, and that little is comparatively dull. The 'seriousness' of a mathematical theorem lies, not in its practical consequences…but in the significance of the mathematical ideas which it connects. We may say, roughly, that a mathematical idea is 'significant' if it can be connected, in a natural and illuminating way, with a large complex of other mathematical ideas [Hardy 1940: 89].

Naturally, we students developed with this idea in our heads. But once we had become mathematicians and then teachers in our turn, we found ourselves facing a situation that was completely different from what we expected, at least in Italy. First of all, after an increase in the number of university students during the 1960s and 1970s, including students of mathematics, the numbers evened out, and in particular the number of mathematics students began to decrease. This has meant that new teaching positions were obtained increasing often in faculties other than mathematics, a typical case being Architecture, where there has been an enormous and irrational increase in the number of students and teaching positions during the 1980s and 1990s. This phenomenon has continued, at least as far as the number of teaching positions goes, with the adoption of the 3 + 2 system, that is, three years for an initial or "brief" degree plus due years for a specialization. Further, during recent years the attitude of mathematicians with regard to the so-called "applied mathematics" has changed. Today we can confidently affirm that there are no more prejudices regarding the relationships between "pure" and "applied" mathematicians.

And yet, as recently as 2004 some mathematicians saw going to teach a mathematics course in a faculty of architecture as a kind of "punishment".

This attitude is brought on by at least two causes related to the little space that mathematics courses have in the architecture curriculum: there is an increasing tendency to reduce the number of classroom hours, and a parallel tendency to reduce drastically the arguments treated.

I believe that the ideal mathematics course in the architecture curriculum is, for the large majority of architects, a course in "recipes"—to paraphrase Robert Musil in *The Man without Qualities*, on the opinion of engineers regarding mathematics—which are to be applied without questioning why. The obvious corollary to this idea is that it would be better if the architects themselves taught these recipe courses without troubling the mathematicians to make some derivative or some integral. Although it is true that the mathematics courses serve as technical courses for architects, it is also true that the attitude of a great number of the students and professors of architecture is basically that in the end it is the engineers who have to deal with structures.

It is certainly very difficult to collaborate with other professors in non-mathematics courses for architecture given that the majority of these are ignorant (and prefer to be ignorant) of what could be done in a mathematics course. I recall the first year that I began to teach in architecture at *La Sapienza*, the University of Rome, in 1996. In presenting the courses the dean of the faculty praised the architect as creative and artistic, describing the courses as a kind of support for future architects for observing, gathering, feeling, almost sensing in the air the new tendencies in art and architecture. Architects as creators. How then can that arid discipline mathematics be of use?

By chance I taught for a year in 1992 at the IUAV in Venice before transferring to the University of Rome. After having taught for several years I posed myself the question, partly out of boredom of having always taught the same things in the same way, of how things could be changed radically. I left the architecture faculty and entered that of industrial design, hoping to find more imagination. In any case, I believed that the best thing I could do was not write yet another book on lessons and exercises in advanced calculus and analytical geometry (although obviously the great advantage of writing such books is that hundreds of students are "obliged" to buy them, to the great satisfaction of the authors) but instead to try to make comprehensible that mathematics has an enormous cultural value, that it can change our way of thinking and therefore the way that architects design in ways that they perhaps cannot even imagine.

The idea was born out of the project "Matematica ed arte" in 1976, and then in 1996 became the much more vast "Matematica e cultura" [Emmer 2002, 2003, 2004a etc]. Taking as a point of departure the ideas expressed in that dean's presentation of the courses, my ambition was to make it understood that among the many things to remember, observe, and understand there had to be mathematics as well. Not only because mathematics "is the essence of spirit", but because mathematics can be an inexhaustible font of ideas and suggestions, not only of "recipes". Besides, it can be an extraordinary "school of adaptation" for problems that have not yet been encountered.

I did not want, however, to look at the questions "in abstract" (abstraction is one of the great defects attributed to mathematicians by those who do not understand that this is instead one of their great merits) [Osserman 1995]. Therefore I wanted to start with a concrete example in which the relationship between mathematics and culture had profoundly changed our way to looking at the world around us, and therefore also the architect's way of thinking and acting. The theme was that of space, of the mutation of our idea of space, using as the perfect guide the extraordinary book *Flatland* by E. Abbot, a book published in 1884 but timely today if only one will penetrate the surface and not consider it merely as a somewhat trite criticism of Victorian society. Having made an animated film by the same title [Emmer 1994], I had had the experience of "designing" the space that the book talks about, as well as the characters, the city, and the universe described by the hero, the Square. This is the reason by the book that I wrote on this theme is entitled *Mathland* [Emmer 2004b].

In the present paper I would like the refer to some of the arguments that this book discusses that I believe are of interest to both student and practicing architects. This is a brief reading of man's adventure of thought in the reign of relationships between mathematics and culture. The example that I have chosen is that of the idea of space, how this idea and the perception of the space around us has changed up to the point where it has arrived to the form of virtual architecture.

Premise

In summer 2002 the *Biennale di Architettura* took place in Venice. Among the many designs and many ideas exhibited—some were of genuine interest, while others were merely more or less extravagant—was the project for a museum of the Hellenic world by the group of architects called *Anamorphosis Architects*, made up of Nikos Georgiadis, Tota Mamalaki, Kostas Kakoyiannis, and Vaios Zitounolis (fig. 1).

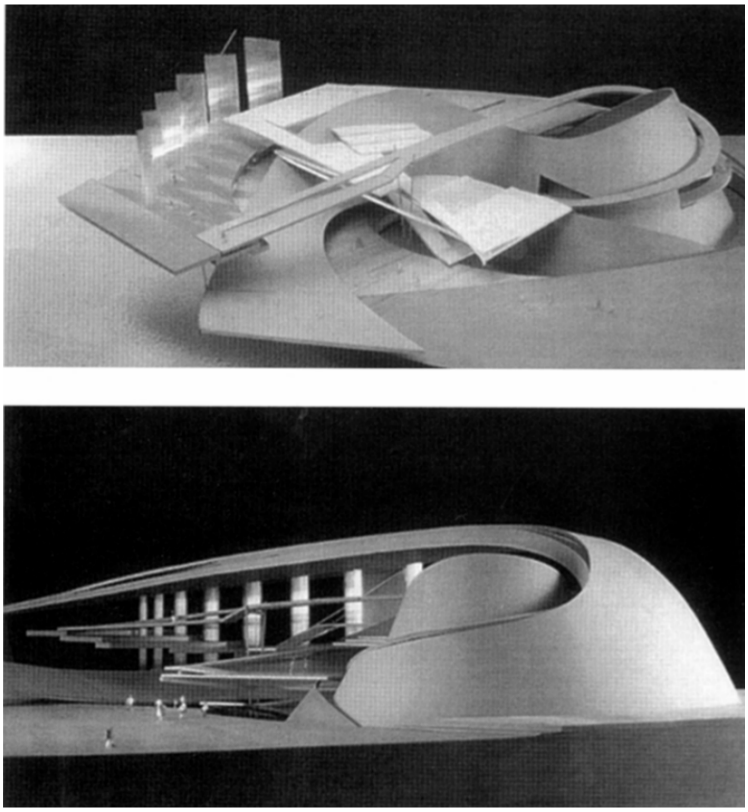

Fig. 1. Anamorphosis Architects, Athens, Greece, "Project for the Museum of the Hellenic Wolrd"(2002) © Anamorphosis architects

There was a great emphasis in this project on the spatiality of the construction, a large continuous space in transformation with curved lines that wrap into a spiral, with at its center the exhibition of the Classical period of Greek civilization. This building was in some sense the beginning and the (temporary) end of a dialogue that began with Euclidean geometry thousands of years ago, a geometry that was the basis, together with Greek philosophy, of the formation of Western civilization as we know it today. It shouldn't be forgotten that the influence of many other civilizations, first of all the Islamic, permitted Europe to rediscover the forgotten Greek civilization.

There are some questions to investigate in order to understand at least in part how philosophical, artistic, scientific elements—in a word, culture—contributed over the course of centuries to the synthesis of a project such as that for the museum of Greek civilization. It is a sort of voyage into Western civilization of the past 2000 years and more, with an emphasis on cultural aspects related to geometry, mathematics, and architecture.

Space is mathematics

> I feel I am perceiving a decline in the belief that in philosophy it is necessary to lean on the opinions of some celebrated author; as if our mind should remain completely sterile and infertile when not mixed with another's discourse; and perhaps it should believe philosophy is a book of fantasy of man, such as the Iliad or Orlando Furioso, books in which the least important thing is that what is written is true. But things are not thus. Philosophy is written in this great book that is continuously open in front of our eyes (I mean the universe), but it can not be understood without first learning to understand its language and characters, in which is written. It is written in a mathematical language and the characters are triangles, circles and other geometric figures; without these, it is impossible to humanly understand a word; without these it is a mere wandering in vain around a dark maze [Emmer 2004b, translation by Stephen Jackson].

These are the words of Galileo Galilei, written in *Il Saggiatore*, published in Rome in 1623. Without mathematical structures it is not possible to comprehend nature. Mathematics is the language of nature.

Let us jump forward several centuries. In 1904 a famous painter wrote to Emile Bernard,

> *Traiter la nature par le cylindre, la sphère, le cône, le tous mis en perspective, soit que chaque côté d'un objet, d'un plan, se dirige vers un point central. Les lignes parallèles à l'horizon donnent l'étendue, soit une section de la nature….Les lignes perpendiculaires à cet horizon donnent le profondeur. Or, la nature, pour nous hommes, est plus en profondeur qu'en surface, d'où la nécessité d'introduire dans nos vibrations de lumière, représentée par les rouges et le jaunes, une somme suffisante de bleutés, pour faire sentir l'air [Venturi 1970: 269].*

Art historian Lionello Venturi commented that he didn't see any cylinders, spheres and cones in the work of Cézanne (as this the painter we are talking about), and therefore his sentences expressed the ideal aspirations of an organization of forms that transcended nature, nothing more.

In the same years in which Cézanne painted, or rather some years earlier, the panorama of geometry had changed since Galileo's time. In the course of the second half of the nineteenth century geometry had changed profoundly. Between 1830 and 1850 Lobachevskij and Bolyai constructed the first examples of non-Euclidean geometry, in which Euclid's famous fifth postulate on parallel lines was no longer valid. Not without doubts and opposition, Lobacevskij would call his geometry (which today is called non-Euclidean hyperbolic geometry) "imaginary geometry", in as much as it opposed the common meaning of the term. Non-Euclidean geometry would still remain for some years marginal with respect to the rest of geometry, a sort of curiosity, until it was incorporated into mathematics as an integral part by means of the general concepts of G.F.B. Riemann (1826-1866). In 1854 Riemann gave his famous lecture to the University of Gottingen entitled *Über die Hypothesen welche der Geometrie zur Grunde liegen* ("On the Hypotheses which Lie at the Foundations of Geometry"), which was not published until 1867. In his presentation Riemann set forth a global vision of geometry as a study of the variety of any number of dimensions in any kind of space. According to Riemann's ideas, geometry did not even necessarily deal with points or space in the ordinary sense, but with groups of n ordinates. In 1872 Felix Klein (1849-1925) became professor at Erlangen and gave his inaugural address, known as the Erlangen Program, in which he described geometry as the study of the properties of figures having characters that were invariant with respect to particular groups of transformations. As a consequence every classification of groups of transformation became a codification of various geometries. For example, Euclidean plane geometry is the study of properties of the figures that remain invariant with respect to groups of rigid transformations of the plane made up of translations and rotations. Jules Henri PoInkaré affirmed that:

> … geometric axioms are neither a priori synthetic judgments nor experimental data. They are conventions; our choice among all the possible conventions is guided by experimental data, but remains free and is not limited by the necessity of avoiding all contradiction. Thus the postulates remain rigorously true, even if the experimental laws that determined their adoption are only approximate.
>
> In other words, geometric axioms are only definitions in other guises.
>
> In the meantime, what shall we make of the question, "Is Euclidean geometry true?" This makes no sense, just as it makes no sense to ask oneself if the metric system is true and old systems of measurement false, or if Cartesian coordinates are true and polar coordinates false. One geometry cannot be truer than another; it can only be more convenient. Euclidean geometry is, and will remain, the most convenient [PoInkaré 1968: 75-76].

Also due to PoInkaré is the official birth of that sector of mathematics that today is called *topology*, with his book *Analysis Sitûs*, a Latin translation of a Greek name published in 1895: "As far as I am concerned, all the various research in which I have been involved have led me to Analysis Sitûs (literally, Analysis of position)." PoInkaré defined topology as the science by which we know the qualitative properties of geometric figures not only in ordinary space but in space with more than three dimensions as well.

If to all this we add the geometry of complex systems, fractal geometry, chaos theory and all the "mathematical" images discovered (or invented) by mathematicians in the last thirty years using computer graphics, it is easy to understand how mathematics has contributed in an essential way to changing over and over our idea of space, both the space in which we live as well as the idea of space itself.

That mathematics is not merely a "kitchen recipe", but has contributed, when it has not actually determined, the way we have of conceiving space on earth and in the universe. What is lacking is the awareness of mathematics as an essential cultural instrument. This explains the great delay in comprehension and therefore in coming up with new ideas that mathematicians have experienced for decades.

This is so in particular with regards to topology, the science of transformation and invariance. One example is the design of Frank O. Gehry for the new Guggenheim Museum in New York, a design that is even more stimulating, even more *topological* than the Guggenheim in Bilbao (fig. 2).

Certainly, the cultural leap is noteworthy; the construct using techniques and materials that permit the realization of an almost constant transformation, a sort of contradiction between the finished construction and its deformation. It is an interesting sign that one begins studying contemporary architecture using even such instruments as mathematics and science make available, instruments that are cultural as well as technical.

It is worth underlining how the discovery (or invention) of non-Euclidean geometries and of higher dimensions, beginning with the fourth, is one of the most interesting in terms of the profound repercussions that many ideas of mathematicians will have on humanistic culture and art. Every good voyage requires an itinerary, one with the elements that will be utilized in order to give a sense of Space.

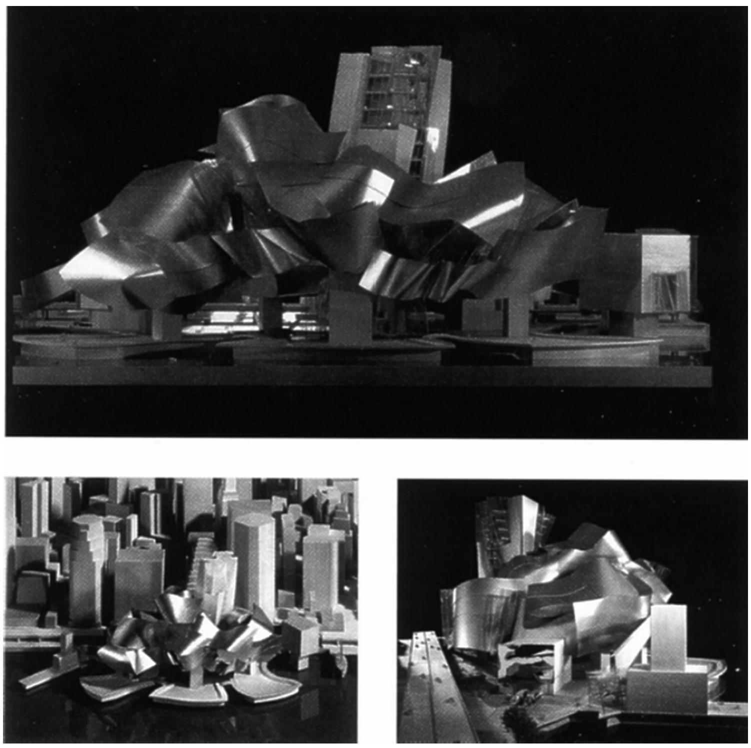

Fig. 2. Frank O. Ghery, “Project for the New Guggenheim Museum in Manhattan”, courtesy of © Keith Mendenhall for the Gehry Partners Studio

The first element is without a doubt the space that Euclid delineated, with the definitions, axioms, and properties of objects that must find a place in this space, a space that is perfection, Platonic space. Man as the genesis and measure of the universe is an idea that has come down through the centuries. Mathematics and geometry must explain everything, even the form of the human being. *The Curves of Life* was the title of the famous book of 1900 of Cook, who certainly never imagined how true it could be that all forms are found in nature, even those that give rise to life, some mathematic curves. From D’Arcy Thompson’s book *On Growth and Form* of 1914 to catastrophe theory of René Thom, to complexity and the Lorentz effect and dynamic non-linear systems.

The second element was freedom: mathematics and geometry seem to be an arid reign. One who has never dealt with mathematics, has never studied mathematics in school with interest, cannot begin to comprehend the deep emotion that mathematics can stimulate. Nor can he conceive that mathematics is an activity that is highly creative, nor that it is the domain of liberty where it is not only possible to invent (or discover) new objects, new theories, new fields of research activity, but that it is even possible to invent problems. And since mathematics does not always require huge economic resources, it can rightly be said that it is the reign of freedom and fantasy. And certainly of rigor. Of correct reasoning.

The third element to reflect on is how all these ideas are transmitted and assimilated, perhaps not completely understood and only vaguely listened to by various sectors of society. Architect Alicia Imperiale has written in a chapter entitled "Digital Technologies and New Surfaces" in the book *New Bidimensionalities* [2001], "Architects freely appropriate specific methodologies from other disciplines. This can be attributed to the fact that ample cultural changes are being verified more quickly in other contexts than in architecture." She adds,

> Architects freely appropriate specific methodologies from other disciplines. This can be attributed to the fact that broad cultural changes are verified quicker in other contexts than in architecture. Architecture reflects the changes that occur in culture, albeit many feel at a painfully slow pace...In constantly seeking to occupy an avant-garde role, architects think the information borrowed from other disciplines can be rapidly assimilated into architectural design. Nevertheless, this 'translatability', the transfer from one language to another, remains a problem...Architects more and more frequently look to other disciplines and industrial processes for inspiration, and make ever greater use of computer design and industrial production software originally developed for other sectors." [2001: 38].

Further, Imperiale says,

> It is interesting to note that in the information age, disciplines that were at one time distinct are now tied to each other by a universal language: the digital binary code.

Does the computer resolve all problems?

The fourth element is the computer, the graphic computer, the logical and geometrical machine *par excellence*. The idea realized by the intelligent machine that is capable of facing problems that are very different if only it is possible to make them comprehend the language being used. This was the general idea of one mathematician, Alan Turing [see Hodges 1991], which was carried to term under the stimulation of a war. A machine constructed by man, in which has been inserted a logic, that too constructed by man, conceived by man. A very sophisticated machine, irreplaceable, not only for architecture. In short, an instrument.

The fifth element is progress, the word progress. If we consider non-Euclidean geometry, new dimensions, topology, the explosion of geometry and of mathematics in the twentieth century, can we speak of progress? Of knowledge, without a doubt, but not in the sense that new results cancel old ones. Mathematicians used to say that "Mathematics is like pork, nothing should be thrown away, and sooner or later even the things that appear to be most abstract and even senseless will become useful". Alicia Imperiale writes that topology is effectively an integral part of the system of Euclidean geometry. What escaped the author was what concerns the meaning of the word space in geometry. That is, words. Where instead changing geometry serves to confront problems that are different because the structure of space is different. Space is in its properties, not the objects contained therein. Words.

The sixth element are words. One of the great capacities of humanity is to give a name to things. Many times in "naming" words are used that are already in current use. This habit sometimes creates problems because one gets the impression when hearing these words of having understood or at least have listened to the things being spoken of. In mathematics this has happened often in recent years with words such as fractals, catastrophe, complexity, and hyperspace. Symbolic words, metaphors. Even topology, dimensionality and sequentiality are by now part of the everyday vocabulary, or at least that of architects.

To sum up, the voyage is undertaken through words, axioms, transformations, liberty. One word that will be of great importance during this voyage into the idea of space: topology. For the other remaining aspects I shall turn to the book *Mathland: From Flatland to Hypersurfaces* [Emmer 2004].

From topology to virtual architecture

> Towards the mid-nineteenth century geometry underwent a completely new development that was soon destined to become one of the great forces of modern mathematics [Courant and Robbins 1941: 353].

These are words of Courant and Robbins in the famous book *What is Mathematics?*

> The new argument, called analysis situs or topology, has as its object the study of the properties of geometric figures that persist even when they are subjected to such profound transformations so as to lose all their metric and projective characteristics.

PoInkaré defined topology as "the science that permits us to know the qualitative properties of geometric figures not only in ordinary space but in space of more than three dimensions." Topology therefore has as its object the study of geometric figures that when subjected to profound transformations so that they lose all of their metric and projective properties, as for example form and dimension, nevertheless remain invariant, that is, geometric figures that maintain their qualitative properties. Figures constructed at will of deformable materials come to mind, which cannot be lacerated or welded; there are properties that are conserved even when a figure like this is deformed in any way possible.

In 1858 the German mathematician and astronomer August Ferdinand Möbius (1790-1868) described for the first time in a paper presented to the Parisian Academy of Science a new surface in three-dimensional space, which is today known by the name "Möbius strip". This new surface has interesting properties. One consists in the fact that if one follows its longest axis with a finger, one eventually returns to the point of departure without ever crossing over the edge of the strip; the Möbius strip has only one side, not two, one external and the other internal as is the case, for example, of a cylinder. While in the case of the cylindrical surface, one can follow with one's finger the upper edge of the cylinder and never arrive at the border of the lower edge, in the case of the Möbius strip, one can follow the whole thing and return to the point of departure, that is, it has only one edge. All of this has important consequences from a topological point of view; among other things, the Möbius strip is the first example of a surface on which it is not possible to fix an orientation, that is, a direction of travel.

Courant and Robbins wrote further:

> From the first, the novelty of the methods used in the new field did not give a way to mathematicians to present their results in the traditional deductive form of elementary geometry; instead, the pioneers such as PoInkaré were forced to base themselves largely on geometric intuition. Even today (n.b. the book of Courant and Robbins dates from 1941) a scholar of topology will find that by insisting too much on formal rigor in the exposition it is easy to lose sight of the essential geometric content of a quantity of formal details [Courant and Robbins 1941: 355].

The key phrase is "geometric intuition". Obviously mathematicians over the years have seen that topology has been brought into the context of more rigorous mathematics, but the aspect of intuition has remained. It is indeed these two aspects, that of deformation that yet preserves the properties of geometric figures, and that of intuition, which play a central role in the idea of space and form that, beginning in the nineteenth century, has come down to us today. Some of the ideas of topology would be *intuited* through the decades, first by the artists, and then much later by the architects. It is worthwhile to tell the story of the discovery of a topological form by one of the great artists of the twentieth century, a form that, though discovered by the artist, already existed in the world of mathematical ideas. The artist is Max Bill, a great artist and architect, who passed away in 1994 (Fig. 3).

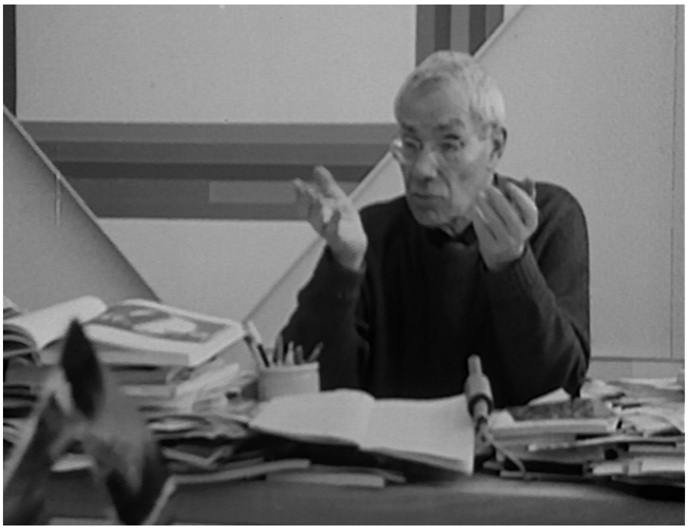

Fig. 3. Max Bill in his Zürich studio (1981) . From the film "The Moebius Band", ©M. Emmer

This is how Bill, in an article entitled "How I began to make surfaces with single sides" tells the story of how he *discovered* the Möbius strip (Bill call his sculptures in the form of Möbius strips "endless ribbons"):

> Marcel Breuer, my old friend from the Bauhaus, was responsible for my one-sided sculptures. Here is how it happened. In 1935 in Zurich, along with Emile and Alfred Roth, I was constructing the Doldertal houses that had quite a following at that time. One day Marcel told me he had received the assignment to build a model house for an exhibition in London where everything, even a small fireplace, had to be electric. It was clear to us that an electric fireplace that shines but has no fire was not the most attractive object. Marcel asked me if I would like to make a sculpture to put over it. I began to search for a solution, a structure that

> could be hung over the fireplace and maybe spin in the ascending current of air and, thanks to its form and movement, would act as a substitute for the flames. Art instead of fire! After long experiments, I found a solution that seemed reasonable to me [Quintavalle 1977: 23-25].

The interesting thing to note is that Bill believed that he had discovered a completely new form. Still more intriguing is that he discovered (invented?) it while playing with a strip of paper, in the same way that Möbius had discovered it many years before!

We could say that as in the case of the fourth dimension where the object that made the greatest impression on the imagination was the hypercube, or cube in four dimensions, in the case of topology this role was played by the Möbius strip. These forms that had interested Bill in the 1940s could not help but raise the same interest in architects, even if somewhat later; it was necessary to arrive to the wide-spread application of computer graphics that permitted the visualization of mathematical objects and consequently sustain intuition which is otherwise difficult to manipulate for the non-mathematician.

This is what Alicia Imperiale writes in the chapter entitled "Topological Surfaces":

> Architects Ben Van Berkel and Caroline Bos of UN Studio discuss the impact of new scientific discoveries on architecture. Scientific discoveries have radically changed the definition of the term "space", attributing to it a topological form. Rather than a static model of constitutive elements, space is perceived as something malleable, changeable, and its organization, its appropriations, become elastic [Imperiale 2001: 34-35].

This is the role of topology, as seen by an architect:

> Topology is the study of the behaviour of a structure of surfaces subjected to deformation. The surfaces register the changes of the shifting space-time differences in a continuous deformation. This causes ulterior potentialities for architectural deformation. Continuous deformation of a surface can lead to the intersections of exterior and interior planes in a continuous morphological mutation, exactly as in the Möbius strip. Architects use this topological form in the design for a house, inserting different fields of space and time into a structure that is otherwise static [Imperiale 2001: 34-35].

Naturally some of the words and ideas are deformed as well as they pass from a strictly scientific context into one that is artistic and architectonic, when seen with a different viewpoint. But this is not actually a problem, nor is it meant to be a criticism. There are ideas that circulate freely and everyone interprets them in his own way, trying to gather, as topology, the essence. In all this the role of computer graphics is essential, as this permits the insertion of that variable of deformation-time that would be Inkapable of being conceived let alone constructed.

With regards to Möbius, Imperiale continues:

> The house by Van Berkel inspired by the Möbius strip (Möbius House) is conceived as a structure that is programmatically continuous, that integrates the continuous change of sliding dialectic pairs that flow one into the other, from the interior to the exterior, from activities of work to those of play, from the load-bearing structure to that which is non-load-bearing [Imperiale 2001: 34-35].

The Klein bottle, another famous topological form, according to van Berkel, "can be translated into a system of canalization that incorporates all the elements it meets and makes them fall into a new kind of internally connected integral organization"; of particular note are the terms "integral" and "internally connected", which have precise meanings in mathematics. But this is not a problem because "the diagrams of these topological surfaces are not used architecturally in a way that is rigorously mathematical, but constitute abstract diagrams, three-dimensional models that permit architects to incorporate into architecture differentiated ideas of space and time."

Max Bill had written something analogous in 1949 regarding the links between art, form and mathematics: "By a mathematical approach to art, it is hardly necessary to say that I do not mean any fanciful ideas for turning out art by some ingenious system of ready reckoning with the aid of mathematician's formulas" [Bill 1993: 5].

In modern art as well artists have made use of regulating methods based on calculation, since these elements, along with those of a more personal and emotional nature, have formed *equilibrium* and *harmony* to every plastic work. Such methods, however, became increasing superficial, according to Bill, since, aside from the exception of the theory of perspective, the repertoire of methods used by artists had stopped growing by the time of ancient Egypt. The new fact arrived with the beginning of the twentieth century:

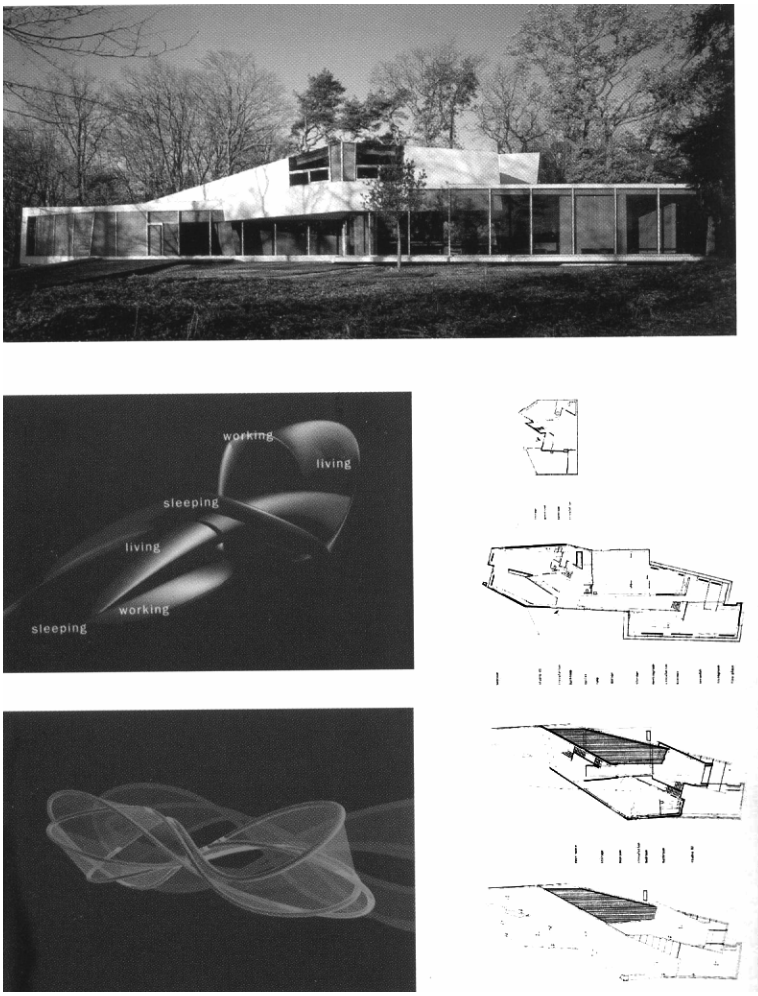

Fig. 4. Möbius House by © Ben van Berkel (UN Studio/van Berkel & Bos), 1993-1997

> It was probably Kandinsky who gave the immediate impulse towards an entirely fresh conception of art. As early as 1912, in his book on The Spiritual Harmony in Art, Kandinsky had indicated the possibility of a new direction which, if followed to its logical conclusion, would lead to the substitution of a mathematical approach for improvisations of the artist's imagination [Bill 1993: 5].

It was then Mondrian who more than anyone else distanced himself from the traditional concept of art. Mondrian wrote,

> Neoplasticism has its roots in cubism. It can also be called abstract-real painting because the abstract (as in the mathematical sciences but without reaching their absolute) can be expressed by a plastic reality in the painting. That is a composition of coloured rectangular planes that expresses a more profound reality, which comes across by means of plastic expression of relationships and not by means of the natural appearance. … The new plasticity poses its problems in aesthetic equilibrium and thus expresses the new harmony [Mondrian 1921: 18-19].

It is the opinion of Bill that Mondrian exhausted the remaining possibilities of painting: "I am convinced it is possible to evolve a new form of art in which the artist's work could be founded to quite a substantial degree on a mathematical line of approach to its content" [Bill 1993: 5].

Further, these mathematical representations, these restricted cases in which mathematics is plastically manifest undoubtedly have an aesthetic effect, adds Bill. And here is the definition of what a mathematical concept of art has to be:

> It must not be supposed that an art based on the principles of mathematics, such as I have just adumbrated, is in any sense the same thing as a plastic or pictorial interpretation of the latter. Indeed, it employs virtually none of the resource implicit in the term "pure mathematics". The art in question can, perhaps, best be defined as the building up of significant patterns from the everchanging relations, rhythms and proportions of abstract forms, each one of which, having its own causality, is tantamount to a law unto itself [Bill 1978: 112].

In order to be convincing, Bill must provide examples that are interesting from his artistic point of view, that is, examples that recall the mystery of mathematical problematics such as the "ineffability of space, the moving away from or coming closer to the infinite, the surprise of a space that begins in one part and ends in another, that is at the same time the same, the delimitation without exact limits, the parallels that intersect, and the infinite that returns to itself". In other words, the Möbius strip.

As we have said, architects as well, if with some delay, also became aware of the new scientific discoveries in the field of topology, and more than design and construct, they began to reflect. In a 1999 doctoral thesis, Giuseppa Di Cristina writes:

> The final conquest of architecture is space: this is generated by means of a positional logic of elements, that is, by means of the arrangement that generates spatial relationships; the formal value comes to be substituted by the spatial value of the configuration. And thus in topological geometry, deprived of "measure" and proper to non-rigid figures, is not something purely abstract that is first found in architecture, but is the trace left by that modality of action in the spatial concretization of architecture [1999; 2001].

In "The Topological Tendency in Architecture", the Preface to a volume on the theme of architecture and science, Giuseppa Di Cristina explains,

> The articles in this volume concern, directly or indirectly, the topological approach in architecture which has developed progressively through the last decade. The articles that are included here bear witness to the interweaving of this architectural avant-garde with scientific mathematical thought, in particular topological thought: although no proper theory of topological architecture has yet been formulated, one could nevertheless speak of a topological tendency in architecture at both the theoretical and operative levels...In particular the developments in modern geometry or mathematics, perceptual psychology and computer graphics have an influence on the present formal renewal of architecture and on the evolution of architectural thought. What most interests architects who theorise about the logic of curvilinearity and pliancy is the meaning of 'event', 'evolution' and 'process', that is, of the dynamism that is innate in the fluid and flexible configurations of what is now called topological architecture. Architectural topology means the dynamic variation of form facilitated by computer-based technologies, computer-assisted design and animation software. The topologising of architectural form according to dynamic and complex configurations leads architectural design to a renewed and often spectacular plasticity, in the wake of the Baroque and of organic Expressionism [Di Cristina 2001: 7-8].

This is what Stephen Perrella, of the most interesting "virtual" architects today, has to say about Architectural Topology:

> Architectural topology is the mutation of form, structure, context and programme into interwoven patterns and complex dynamics. Over the past several years, a design sensibility has unfolded whereby architectural surfaces and the topologising of form are being systematically explored and unfolded into various architectural programmes. Influenced by the inherent temporalities of animation software, augmented reality, computer-aided manufacture and informatics in general, topological 'space' differs from Cartesian space in that it imbricates temporal events-within form. Space then, is no longer a vacuum within which subjects and objects are contained, space is instead transformed into an interconnected, dense web of particularities and singularities better understood as substance or filled space. This nexus also entails more specifically the pervasive deployment of teletechnology within praxis, leading to a usurping of the real and an unintentional dependency on simulation [Perrella 2001: 143].

In these observations ideas on geometry flow together with those of topology, computer graphics, space-time. The cultural nexus in the course of the years has functioned: new words, new meanings, new relationships.

Concluding observations

I have attempted to relate some of the important moments that have led to a change in our conception and perception of space, trying to gather, in addition to the technical and formal aspects which are certainly essential to mathematics, the cultural aspect, speaking of the idea of space in relationship to some aspects of contemporary architecture. I would especially like to recall two magic words of great importance: fantasy and freedom. There are perhaps the two magic words that have permitted contemporary architecture to enrich our design patrimony. Fantasy and liberty, derived from the flowing together of many elements over the course of the years: computer logic, new geometries, topology, computer graphics. Because even if only a few realize it, mathematics is, or can be, I repeat, the domain of fantasy and liberty.

Without all this, the design of a museum of the Hellenic world would be inconceivable. A culture that arose in that place thousands of years ago is celebrated in that same place with a highly symbolic building of the story of the culture of the Mediterranean.

Fig. 5. S. Perrella and R. Carpenter, "The Möbius House Study", © Perrella, Carpenter 1997-1998

It would be nonsense to fail to relate this fundamental aspect of the link between mathematics, culture and architecture to students of architecture, to those future architects who will be responsible for the space in which the generations of tomorrow will live.

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Michele Emmer, born in Milan September 15, 1945, is full professor of mathematics at the University of Rome. He was previously professor at the University of Ferrara, Trento, Viterbo, L'Aquila, Sassari, Venice and visiting professor, among others, at Princeton, Paris Orsay, Campinas, Barcelona and in several Japanese Universities. His area of activity were PDE and minimal surfaces, computer graphics, mathematics and arts, mathematics and culture, films and videos. He received in 1998 the "Galileo" award from the Italian Math Association for best popularization of Mathematics. In 2004 he received the "Pitagora" award. He was president for three years of the Italian associations for scientific media, part of the European association "Media in Science". Member of the American Mathematical Society, of the American Ass. For Aesthetics, of the European Math ass., ISAMA (art and Math ass), ISAST, etc. President of the electronic scientific journal *Galileo* (http://www.galileo.webzone.it); collaborator in the last twenty years of the cultural and scientific pages of the newspaper *L'Unità* and other magazines; *Diario*, *Telema*, *Sapere*, *Le Scienze* (Italian edition of *Scientific American*), *Alliage* (in French), *FMR*. Member of the board of the journal *Leonardo: Art, Science and Technology*, MIT Press. Filmmaker, almost all his movies in the series "Art and Math" have been broadcasted by the State Italian television and other television; all the videos are distributed in many countries in the various version (English, French, Spanish, Italian, Japanese). He has organized several exhibitions and conferences on the topic of "Art and Mathematics" including the annual conference on "Mathematics and Culture" at the University of Venice (http://www.mat.uniroma1.it/venezia2005); the exhibitions and conferences on M.C. Escher (1985 and 1998) at the University of Rome; the section on Space at the Biennale of Venice (1986), the 1989 travelling exhibition "The Eye of Horus" (Rome, Bologna, Milan, Parma). He was responsible for the exhibition and congress on "Math & Art", Bologna, 2000. He is the editor of the book series "Mathematics and Culture" published by Springer-verlag; the series "The Visual Mind" published by MIT press; video series "Video math" published by Springer-verlag. He has been responsible for the math section for the Science Center in Naples and for many other travelling exhibitions on math.