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Designing a Problem-Based Learning Course of Mathematics for Architects

In the past nine years, the teaching model of the Instituto Tecnológico y de Estudios Superiores de Monterrey has rapidly evolved, taking into account the development of abilities, attitudes and values without forgetting the development of knowledge. The mathematics for architecture course was redesigned, using problem-based learning and an intensive application of computer technology to overcoming those difficulties. Now, the main purpose is to develop a mathematical, physical and technological culture in students of architecture to allow them to analyze and solve complex problems related to mathematics in architecture and design. The course was planned and implemented for the first semester of the architecture program and is actually related (through curriculum integration) to future courses which require specific mathematical applications.

Introduction

In the past, the mathematics for the architecture course of the Instituto Tecnológico y de Estudios Superiores de Monterrey (ITESM) entailed difficulties for both teachers and students: poor knowledge retention, courses that were too centered in algebra instead broader mathematical concepts and their applications, the use of rules and algorithms rather than the practical applications in which students are usually interested. In addition, there were few connections with later courses in the architecture program.

In the past nine years, the teaching model of ITESM has rapidly evolved, taking into account the development of abilities, attitudes and values (AAV's) without forgetting the development of knowledge. The mathematics for architecture course was redesigned, using Problem-Based Learning (PBL) and an intensive application of computer technology to overcoming those difficulties. At present, the main purpose is to develop a mathematical, physical and technological culture in students of architecture to allow them to analyze and solve complex problems related to mathematics in architecture and design. The course was planned and implemented for the first semester of the architecture program and is actually related (through curriculum integration) to future courses which require specific mathematical applications and are now available online.

The course is centered around five mathematical themes which give the students a panoramic of mathematics in architecture, from design to building engineering:

- Functions and modeling;
- Derivatives and optimization;
- Applications of integrals in design, architecture and building engineering;
- Vectors and mathematics in physics;
- Three different types of geometry: Euclidean, spherical and fractal.

In this paper, we present a brief outline of the course, examples of student projects integrating architecture and mathematics, and experiences and statistical outcomes the course as regards the students.

Mathematics in the architecture curriculum at ITESM

The architecture undergraduate program, oriented towards design, was opened on our campus seven years ago (after than new teaching model of ITESM had been implemented). The curricula included a mathematics course entitled Mathematics for designers for the first-semester students in the program, which was centered around calculus, and including some themes of geometry.

Original implantation outcomes varied depending on the professor and the group of students, but in general it was observed that:

- The program was very ambitious and seldom fulfilled the learning requirements of students;
- The course was not very attractive because mathematics teachers were not knowledgeable about applications of use in architecture and design; the ones taken into consideration artificial and unrealistic;
- The course did not have a purposeful continuity with the rest of the courses in the architecture program.

Still, even with the implementation of the new educative model (based on technology and techniques of meaningful learning) and the efforts to bring this philosophy into this course, advances were slight and did not help to solve the existing situation.

Basic aspects in design

Through the experience and the contact that I have had with Nexus, I began in 1997 to delineate and design this course, taking as the central elements architecture and design, rather than mathematics itself. Additionally, elements of curricular design were considered which have been applied successful in the program Principia [Polanco, Calderón and Delgado 2000; Delgado, Santiago and Prado 2002; Delgado 2003] for learning based on problems and curricular integration for engineers in the Tecnológico de Monterrey. The course was oriented toward requirements for architects in Mexico and in the international environment (taking into consideration that the program is more oriented towards building design to engineering). Tens of programs of mathematics courses for architects were evaluated in universities in America, Europe and Asia, all of them different, and different even among courses offered by different institutions in the same country. The present paper doesn't intend to attempt to establish what an international course of mathematics for architecture must contain, but rather will describe the methodology of satisfying a local requirement as well as solving some of the problems mentioned before.

In this way, the following aims were established:

- To guarantee that each concept introduced addressed a later curriculum requirement of the students, specifically those of applied physics, resistance of materials, structural systems and passive systems;
- To include applications derived from actual concepts but with specific references to the areas of architecture and design;
- To involve the Architecture faculty in the design of the course;
- To include the use of computer technology for mathematics and architecture in order to arrive at the solutions of more complex problems related to both disciplines;

- To be consistent with the constructivist methodology followed in mathematics courses (normally using Problem-Based Learning, or PBL) [De Graaff and Bouhuijs 1993; De Graaf and Cowdroy 1997] and that for architecture courses (using Project Oriented Learning, POL), this course is centered around complex scenarios tied to both disciplines, in which there is a mathematical component.

Development

With regards to these criteria, it was determined that original course, centered in Calculus, although it was very extensive, was only partially oriented towards later courses, because it was deficient in the following areas:

- Vectors and their applications to physics and mechanics;
- Mathematical modeling (vector functions and multivariable functions);
- Optimization;
- Applications of integral (inertia momentum and mass centers);
- Introduction to differential equations;
- Emphasis on geometry.

On the other hand, some elements were identified in the original course which were excessively covered but lacked applications, such as:

- Exhaustive study of functions;
- Limits and continuity;
- Treatment of derivatives and integrals centered in algebra;
- Integration methods;
- Plotting functions.

With this perspective, the new course replaced or simplified some subjects to include others that were more important. Five thematic units were constituted:

- Functions and modeling;
- Derivatives and optimization;
- Applications of integrals in design, architecture and building engineering;
- Vectors and mathematics in physics;
- Three different types of geometry: Euclidean, spherical and fractal.

In addition, five complex (but generic) mathematical scenarios in architecture or design were determined for each unit. These scenarios provide a perspective for these disciplines.

PBL Scenarios

For this course, five real scenarios in the sense of PBL methodology [De Graaff and Bouhuijs 1993] were identified and selected for integrate contents of each thematic unit:

- Parametric propagation of form in structural architecture;
- Optimal design of packages and containers;
- Ergonomics and design;
- Tensegrity;
- Fractal architecture.

All of them share the following characteristics:

- They are centered on knowledge of the actual thematic unit;
- They use some of the concepts reviewed in previous units;
- They emphasize the necessity of new knowledge which will be learned in later units or later courses;
- They show the relationships between mathematics and architecture or design.

As an example, we will describe briefly the scenario corresponding to “parametric propagation of form” [Szalabaj 2001] in which, given a space to construct for a railway station, the student must design mathematically the form of the dome for it. Basically the activity is centered in designing a parametric function:

$$x(\alpha, \theta) = (r(\theta) + \alpha) \cos(\theta)$$

$$y(\alpha, \theta) = (r(\theta) + \alpha) \sin(\theta)$$

$$z(\alpha, \theta) = f(\alpha)$$

where $f(\alpha)$ describes the form of dome and $r(\theta)$ the way in which this form will propagate. Fig. 1 shows a proposal of one group of students for one specific situation. Some parts of resulting work is finally evaluated not only by the teacher (a mathematician), but by teachers of architecture faculty (who don't teach the course), integrating some elements of design that the students are learning in the same semester.

Fig. 1. Parametric propagation of form

It is important to note that previous to this complex activity, the lectures as well as the exercises of this unit included analysis, modeling and construction of simple domes and cupolas, based on modeling with multivariable functions and vector functions. Another of the scenarios used in the course is integrated into the design of an electronic device, using geometry and calculus. In this activity, technical requirements such as capacity and size are combined with aesthetic requirements (fig. 2).

Fig. 2. Ergonomics and design

In order to arrive at these scenarios successfully, the course attempts to be a constructivist guide for developing skills. So, the student had previously been required to solve diverse problematic situations involved in the design of Kingdom Centre, the design of the Al Faisaliah tower, or with self-sustainable structures. These previous activities endow to the student with useful abilities which will be necessary in the more complex scenarios of PBL.

Outcomes

The new *Mathematics for Designers* course in its final version has been applied continuously since summer 2003 by a group of four professors (each one having different group of students), who in some cases have proposed alternative applications to those considered in the basic course, but in accordance with the same philosophy. Fig. 3 shows the average results in basic exams for the Mathematics department (there are three partial exams; the final grade of course final depends on the final exam).

Fig. 3. Averages for semester in department exams and partial evaluations

It is clear that dispersion has been reduced (in some sense this is to the general advantage of students in the whole architecture course), since the introduction of this course. Nevertheless, the finals grade for the course aren't significantly different. Another positive result is that students appreciate the professors who have shown a commitment to the course.

Future evolution

The lines of development for this course are:

- the involvement of more professors who enrich the course with their own contributions of cases of analyses that can be used as scenarios;
- the deliberate use of architecture tools (ArchiCAD by example) must be considered, since its use predominates the other types of technologies not so able for a student of architecture as far as visualization 3D.

A pending aspect in the present version of the course is a revision of the contents. As was mentioned at the beginning, there is no consensus on the mathematical contents that must be learned by the architecture student, either on the national level, nor on the international level (still within the different profiles and directions that can be given to the formation of an architect). I believe without a doubt that in any one of the contexts defined, there will always exist many applications that can be useful as learning scenarios.

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