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The Effect of Integrating Design Problems on Learning Mathematics in an Architecture College

A number of universities and colleges have developed mathematics courses based on the relationship between architecture and mathematics. Igor Verner and Sarah Maor report on a study of learning mathematics in professional context in one of the architecture colleges in Israel, with a focus on assessment and educational research. This paper considers in detail applied contents and learning activities in the course and our way forward in order to discuss them with the NEXUS community.

Introduction

The past decade has been a period of increasing research and debate on the interaction between architecture and mathematics. The mutual contributions of architectural and mathematical thinking [Salingaros 1999; Aroni 1999] have been emphasized and illustrated by numerous examples in:

- Geometrical analysis of architectural objects, e.g., calculating dimensions, proportions, surfaces, and volumes as well as positioning and performing spatial transformations [Kappraff 1991; March 1998];
- Formal description and interpretation of architectural concepts and symbols such as incidence, infinity, symmetry and multi-scaling [Williams 1998; Bovill 1996];
- Mathematical background concerning science and engineering aspects of architectural design and construction [Salvadori 1968; Rajchman 1998].

A number of universities and colleges have developed mathematics courses based on the relationship between subjects such as the course described by Jay Kappraff [1991]. However, only minimal information is available on the educational aspects of these courses [Banerjee and De Graaf 1996]. There is a need for a comprehensive survey and empirical studies of mathematics curricula for architecture education. This would help to reduce a gap between supporters and detractors of mathematics education in architecture.

This paper reports a study of learning mathematics in professional context in one of the architecture colleges in Israel. The effect of integrating architectural and structure design problems in the calculus curriculum on students' achievements and attitudes was examined. A previous paper of ours [2001] presented results of the study with focus on assessment and educational research. Here we will consider in more detail applied contents and learning activities in the course, and our way forward in order to discuss them with the NEXUS community.

Case Study Framework

Architecture education in Israel is offered at university and college levels. The Ministry of Labour coordinates the architecture programme in colleges and certifies the graduates as practical architects for designing certain types of buildings. As part of this programme, the mathematics curriculum comprises basic concepts in algebra, geometry and trigonometry.

The shortcomings of the curriculum were found through the long-term experience of teaching it in one of the architecture colleges. First, the curriculum provided only partial knowledge of that required for other disciplines and design activities. In particular, the lack of mathematical analysis skills hampered the students' achievements in mechanics and structural design. Furthermore, it did not provide motivation for learning mathematics and recognizing its importance in architecture.

A revised mathematics curriculum was developed in the college in order to eliminate these deficiencies and the study of calculus was added. Initially, when the course was taught in a conventional way, the students had difficulties in applying mathematical concepts to other disciplines. Subsequently, the need for interdisciplinary connections and applications was recognized.

The new curriculum have been implemented in the college since 1999 accompanied by an active research. The research goal was to examine the effect of integrating architectural and structure problems in the mathematics course on the students' achievements and attitudes towards the role of mathematics in design. The study consisted of four stages: pilot study, curriculum development, course implementation and assessment of outcomes.

Pilot Study

The goal of the 1997 pilot study was to assess the ability of students to apply mathematics to design problems and examine their attitude towards the subject. At that stage, architecture majors at one college and one university took an achievement test and filled in a questionnaire. Answers were received from 91 students, who had already completed their mathematics and statics studies. Also, practising architects were interviewed and asked to suggest appropriate subjects for an integrated curriculum. The achievement test included five mathematical problems in architectural design:

1. Object 1 is composed of equal cubic blocks and its incidence matrix. Define by means of an incidence matrix a figure consisting of block elements that should be added to Object 1 to make Object 2 (a cube).

ing or block elements that should be added to Object 1 to make

Incidence matrix of Object 1

111	101	1
110	11	10
100	0	0

Object 1

Incidence matrix of a cube

111	111	111
111	111	111
111	111	111

Object 2

2. Find the lengths of two different circular segments AC & BC of given radii R and $2R$, which are parts of a smooth path (biarc) between two given parallel lines, AN and BM (see fig.). The distance between the parallel lines is $H=3R/2$. Choose a correct answer among (a) - (e).

3. Find the centre of gravity coordinates for an object shown in the figure. The object is a circle of radius R centered in the origin O , with triangular and circular holes. Choose the correct answer among (a)-(d).

4. Determine dimensions of a circle, a rectangle and a triangle of maximal area inscribed in a given semicircle of radius R . Which of the three shapes has the maximal area?
5. Design a goblet of given volume $V=120 \text{ cm}^3$. Define its shape composed of spherical, cylindrical, conic or parabolic segments. Determine its dimensions and use formulas for segment volumes given in the figure. The two assessed factors in this task are shape complexity and proper volume of the goblet.

In addition to the test we conducted a questionnaire. It examined the student's attitudes towards:

- The competence in mathematical aspects of design required from an architect;
- The importance of mathematical thinking and capacity in architecture studies and particularly in structural design;
- The impact of design studies on understanding mathematical concepts;
- Personal experience in applying mathematical knowledge to design activities.

The results of the pilot study indicated that many college and university students could not cope with the test. On the other hand, the questionnaire results showed high student awareness of the importance of mathematics in design.

The interviews with architects reflected their view that mathematics belongs to the core of architecture education. However, in their own practice the architects tended to use graphical procedures instead of analytical methods because of the lack of mathematical knowledge.

In the interviews the architects also pointed out a number of actual mathematical problems in architectural design. Some of their ideas on applying the concept of function and calculus were implemented in the next stage of our research.

Application Oriented Curriculum

We developed a new calculus course to give students the mathematical skills required in mechanics and structural design courses and for solving actual mathematical problems in architectural practice.

Course Outline. The topics and learning hours of the calculus course were determined, as detailed in Table 1.

In the first variant of the course, the topics presented in Table 1 were taught by a disciplinary approach, closely following standard college mathematics texts. The ability of college students to apply their knowledge of calculus in the architectural context was assumed. However, teaching practice indicated the uncertainty of this assumption. Therefore, we developed an alternative variant of the course which integrated calculus concepts and their applications. In this integrated

curriculum the students were instructed to apply calculus to actual problems of architecture design.

In our study the two variants of the calculus course, one based on a disciplinary curriculum and another on an integrated curriculum, were taught in two parallel college groups by the same teacher. Learning achievements of the groups were compared.

Topics	Learning hours
1. Introduction	2
2. Functions	14
2.1 Basic properties of functions	
2.2 Basic functions and their graphs	
2.3 Basic analytic curves	
3. Differentiation	8
3.1 The derivative	
3.2 Techniques of differentiation	
4. Maxima and minima	8
5. Integration	13
5.1 The integral	
5.2 Techniques of integration	
5.3 Lengths, areas and volumes	
Total:	45

Table 1. Calculus course outline

Applied Contents. Here we considered the applied contents of the integrated curriculum and how it was delivered to students.

As the first step in designing the integrated curriculum, we formulated instructional objectives from the applications domain that should be added to objectives pertaining entirely to calculus. The following applied mathematical and attitudinal skills were included:

- Analysis of the steady state of structural elements;
- Geometrical design—analytical description of contours;
- Geometrical design—metrical analysis of structure dimensions;
- Geometrical design—optimization of lay-out plans;
- Motivation for applying calculus tools to architectural problems;

- Perception of functions as a means of interdisciplinary communication, part of professional language.

Next, applied contents corresponding to the instructional objectives were developed and fitted into the calculus topics of the curriculum. The applied problem solving activities were as follows.

Functions and graphs—Analytical description of structure contours. Pictures of various architectural buildings were presented to students. The students learned to identify contours of a building and the various types of basic functions to describe the contours analytically. Then they practised describing contours by means of those functions, when a parameter value of a function was identified approximately by means of templates. Each template presented a series of graphs of some basic function for several values of a parameter. Later, the students studied and practised finding analytically the value of a parameter.

Derivatives and integral—Analysis of distributed loads. Analysis of shear forces and bending moments in beams is a subject of the mechanics of structures course, studied during the same semester as the calculus course. The integrated calculus curriculum referred to the dependence between shear forces and bending moments in a beam and described it in terms of the derivative and the integral. Students applied methods of mechanics to find the distribution of shear forces and the distribution of bending moments in a loaded beam. Then the students defined functions that described the distribution of forces and moments in the beam under various loads. Through practical examination they checked the mathematical dependence of the two functions, so that the function of the forces was the derivative of the moments function, and the moments function is the indefinite integral of the forces function. The students practised determining the distribution of shear forces from the distribution of bending moments, and vice versa by means of mathematical operations of differentiation and integration.

Lengths, areas and volumes—Dimensions of structures and elements. In this section of the curriculum the students dealt with architectural plans of public and residential buildings. They learned to perceive the elements of a plan as geometrical figures. Then the students practised calculating and estimating geometrical parameters (lengths, areas and volumes) of elements from the plan.

Maxima and minima—Optimal design. The students were engaged in the process of designing a location and the dimensions of a building on a plot from an architectural plan. This is a typical problem of house building design, in which a solution of maximal actual living space is required. In this context, the students practised finding variants of regular geometrical locations of a building on a plot in accordance with its shape and design constraints. Then the students determined the optimal solution and calculated dimensions of a building of maximal possible area and its position on the plot.

The last step in designing the integrated curriculum was making decisions on how to combine applied topics with the calculus topics and deliver them to students. An approach to integration of mathematical and applied contents proposed by Niss [1989] and Alsina [1998] was accepted. Accordingly, in the integrated curriculum, calculus topics were considered as dominant and were studied in their conventional order, as presented in Table 1. The applications were studied through learning practice and concerned with constructive interpretation of the mathematical concepts.

Learning Sessions. The applied contents of the curriculum were delivered through a problem-oriented learning activities. Applied problems were solved through a common scheme: perception

of a real object and defining the mathematical problem, solving the problem and applying the solution.

Each type of applied problems mentioned in the previous section was studied through a tutorial, a workshop and a homework assignment sessions. In the tutorial session the teacher presented a sample problem and discussed its solution. Then, in the workshop session the students solved applied problems in class (working in pairs). Finally, they solved and submitted problems included in the individual homework assignment.

For example, we considered a problem related to the analytical description of structure contours. One of architectural buildings shown to the students is given in Fig. 5a. The students were asked to verify that the roof edge contour has the form of a hyperbola (Fig. 5b), and to describe it analytically.

Fig. 5. Analytical description of a structure contour: a) the building example; b) the hyperbola which describes the roof edge contour

To solve the problem, the students were given the following:

1. Copy the structure contour and the bottom edge of the glass wall lines to a squared paper sheet.
2. Define the position of the Y axis along the bottom edge of the glass wall.

3. Make sure that the contour line includes a segment which is symmetric with respect to some vertical axis. Define the position of the X axis to coincide with the symmetry axis of the contour.
4. Put a number of points (from five to seven) on the contour line and determine their coordinates.
5. Select a suitable hyperbola equation to describe the contour line.
6. Calculate the values of the hyperbola parameters a and b using two of the points.
7. Check that the other points satisfy the equation of the hyperbola with the parameters values found in 6.
8. Determine the domain of a contour line.
9. Calculate the coordinates of the hyperbola's foci.
10. Draw the second (missing) branch of the hyperbola. Make sure that the characteristic of the parabola as a locus of points actually exists.

The solution of the problem is presented in Fig. 5b. The students determined the coordinates of five points on the contour $M_1(6, 6)$, $M_2(0, 5)$, $M_3(-6, 6)$, $M_4(-12, 8.3)$, $M_5(-16, 10.2)$. They described the contour by means of an equation:

$$\frac{Y^2}{b^2} - \frac{X^2}{a^2} = 1$$

and calculated the values of parameters $a = 9$, $b = 5$ using points M_1 and M_2 . Then they checked that the coordinates of points M_3 , M_4 and M_5 satisfied the equation. The educational value of integrating such applied problems in the calculus curriculum on students' achievements and attitudes is examined in the next sections of the paper.

Educational Study Framework

The new calculus curriculum has been implemented in the college since 1999. The assessment focused on the following questions:

1. What is the impact of integrating applied problems in the calculus course on the students' achievements?
2. What are the changes in the students' attitudes towards the role of mathematics in design as a result of the course?

The research sample included two groups of the architecture college first year students. The control group ($N=30$) studied calculus based on a disciplinary approach while the experimental group ($N=30$) studied an interdisciplinary course described in the previous section. Both groups studied the same mathematical concepts and methods, taught by the same teacher. However, practice in applying mathematical knowledge to design was offered only to the experimental group.

The instruments that were used for the purposes of this study were application skills tests and attitude questionnaires (pre-course and post-course), and student interviews at the end of the course, as detailed below.

The achievement pre-test examined the mathematical knowledge of both groups before the course. It was decided to use the same achievement test as that used in the pilot study in order to validate its relevancy to the research population.

The achievement post-test was also given to the control and experimental groups and consisted of five open mathematical problems:

1. A metal strip of 25 m length and 1.5 m width is given to produce a drainage channel. For each of the four possible channel profiles shown in Fig. 6, calculate the value of a parameter X which provides a maximum area section. Which one of the four profiles has the largest water supply?

Fig. 6. Drainage channel profiles: a) quadratic; b) rectangular; c) semicircular; d) stepped

2. A semi-circle plot of land $R = 20\text{m}$ is given to build a T-shape structure CDEFGH as shown on the plan (Fig. 7). Distances of the structure contour from the plot perimeter, required by a municipality are as follows: 5 m from the arc, and 4 m from the diameter of the semi-circle. The structure includes four apartments, three of them are represented on the plan by rectangles $X \times Y$, and for the fourth one only one dimension X is given.

- Describe the total area of the three equal apartments as a function of $S(X)$;
- Calculate the X -value which provides maximum of S ;
- For the X -value found in (b) calculate the total area of the structure;
- What would be the total area of the structure in case that all four its apartments are equal squares.

Summary of Results

Learning achievement. Detailed statistical analysis of results of our study was presented in a previous article of ours [2001]. Here we summarize the research findings related to the abovementioned instruments, and research sample (experimental and control groups).

Initially, backgrounds of the experimental and control groups were compared by five independent variables: student gender, age, level of mathematics studied in high school, mathematics grade in the matriculation certificate, and Psychometric test grade. A salient conclusion was that the differences between the groups before the course were non-significant. The psychometric grades of both groups were higher than average for architecture students in the college. Mathematical pre-test achievements were low. In contrast to the pre-test, the groups' post-test results were significantly different. The post-test mean grade of the experimental group 73.2% was significantly higher than that of the control group 61.0%. Further more, the percentage of post-course failures in the experimental group (10%) was much lower than in the control group (43%).

The dependence of the post-test grade on the factors represented by the five variables and the teaching method factor was analyzed by a stepwise regression. Only two of these factors, namely the level of mathematics studied at school and the teaching method were found to be significant predictors. Through analysis of variances of the two factors we found that the proposed teaching method contributed to students with heterogeneous mathematical backgrounds.

In the pre- and post-test assessment of each problem we related to the student's ability to deal with the following stages: defining the mathematical problem, solving it and applying the solution. While for the pre-course test applied problems 3 and 4 only 36% of students from both groups succeeded in defining the mathematical problems, after the course both groups performed better in solving similar applied problems. However, the definition results in the experimental group (83%) were significantly higher than in the control group (65%). The experimental group also obtained better results in the applied interpretation of mathematical solutions.

Attitudes. The answers to the pre-course questionnaire given by the experimental and control groups reinforced the pilot study results. In particular, 43% of the students believed that the architect should be involved in the mathematical aspects of design. After the course, the number of students who held this opinion rose to 80% in both groups, due to the study of mathematics. Before the course, 37% of students considered mathematics a necessary tool for architecture studies. The post-course questionnaire indicated that this opinion was shared by 47% of the control group students vs. 80% in the experimental group. The Wilcoxon test comparison of pre- and post-course results showed that the attitudes changed significantly only in the experimental group.

The difference between the groups in the post-course questionnaire was also significant for the effect of mathematics studies on abilities to calculate dimensions of structural elements, and to understand computer-aided design operations. Practice in contour design contributed to better understanding of the mathematical function concept.

Section B of the post-course questionnaire examined attitudes of the experimental group towards the proposed integrated curriculum. It was found that 67% of the students recognized the importance and relevance of studying applied problems for their architecture studies. The students also mentioned that this approach stimulated their motivation for learning mathematics (60%), interest in this subject (70%) and creativity (30%). For 40% of the students, the

integrated approach reduced learning difficulties. In addition, 87% of the respondents became interested in continuing mathematics studies by this approach.

Student interviews. The students interviewed from the experimental group pointed out that they were quite impressed by the new approach of the course. They especially mentioned the value of practice in mathematical analysis of real architectural artifacts presented visually in the course. The students independently defined and solved applied problems in their structural design project, which was parallel to the course. They stated that, thanks to the course, they developed a better understanding of the statics concepts and recognized the role of mathematics as a design tool.

The Way Forward

The research presented above focused on the mathematics course at the first year of studies in the college. The two-years follow-up indicated that our integrated calculus course has become a part of the core mathematics curriculum. The retention results show that the course maintains its values which were identified in our study. The course graduates demonstrated capabilities to apply mathematical knowledge in their architecture design projects.

With regard to an architecture education program in general, it is required to continue studying mathematics after the course towards developing mathematical thinking in the context of creative design and professional communication in architecture. To answer the need, we started a follow-up research considering mathematical aspects of the architectural design education. The goal of our new research is to develop a studio environment for design projects, which inspires students to think mathematically, and examine its value for learning mathematics and architecture.

This learning environment will provide experience with a hierarchy of architectural objects from drawing basic geometrical figures to analytic design and building physical models of composed 3D structures. Project assignments will require students to rely on the following design factors: dimensions, symbolism and expression, efficiency and functionality, steadiness, aesthetics, optimal planning, and diversity of shapes.

The learning population includes second-year students in the college, who studied mathematics in the first year of their studies, following the integrated curriculum. Each of the students will perform three different project assignments. The first project will deal with designing a combination of basic geometrical figures of harmonic proportions, to face a given surface. In the second project the student will design structure contours using mathematical functions, and build their physical models. The third project will focus on spatial morphology, dimensions and physical modelling of complex structures.

Our study will apply ethnographic research methods, including interviews with designers and observations of their work in the architectural studios, as well as interviews with students and observations of their design experiences. The focus will be on employing mathematical concepts and skills.

Conclusions

In our case study, the two variants of the calculus course were taught in two parallel college groups, one based on a disciplinary curriculum and another on an integrated curriculum. As indicated by the study, the integration of design problems in the mathematics course can help students majoring in architecture to:

- Better understand abstract mathematical concepts. Through practice in applications the students developed the ability to synthesize and evaluate functions for better understanding the calculus concepts than in the conventional course;
- Develop applied problem solving skills. The students gained experience in applied problem solving including defining a mathematical problem, finding the solution and its interpretation;
- Study technological disciplines;
- Because of the course, the students had a better understanding of concepts of statics and recognized the role of mathematics as a design tool;
- Acquire computer-designing skills. Knowledge of mathematical methods in contour design is an important prerequisite for CAD studies.

This approach can also increase the motivation and confidence of the students in studying mathematics and encourage them to use mathematical methods in their personal design activities.

The proposed calculus curriculum can serve as a module for a new mathematics programme for architecture colleges.

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