Michael Serra | *Ertha Diggs and the Ancient
Stone Arch Mystery*

Michael Serra describes a class project for constructing arches and examining their properties. The objective was for students to review and apply the properties of isosceles triangles, trapezoids, regular polygons, and of interior and exterior angle sums. They were to practice communicating mathematically and modeling in two and three dimensions. It is a fun two-day activity of hands-on mathematics and problem solving.

The Setting

A geometry class in which students work in cooperative groups of four to discover the properties of geometry. They are familiar with the basic tools of geometry: compass and straightedge, patty papers, and The Geometer's Sketchpad. They have recently discovered the Interior Angle Sum Conjecture, the Exterior Angle Sum Conjecture, and properties of trapezoids.

The Lesson (Day One)

I began the lesson by sharing photos and transparencies of Roman and Chinese arches while discussing their history and development. Whether true or not, my students especially enjoyed the tale about the Roman architect for an arch. (According to legend when the Romans made an arch, they would make the architect stand under it while the wooden support was removed. That was one way to be sure that architects carefully designed arches that wouldn't fall!) During class discussion we arrived at the geometric characteristics of an arch. We agreed that the arch is half of a regular polygon. We conjectured that half the number of sides of the regular polygon must be an odd number (in order to have a keystone). For example half of a regular 18-gon gives us an arch with 9 stones, 8 voussoirs and a keystone. Then each group of four students was to design and build a two-dimensional arch of isosceles trapezoids. I required the angle measures of the isosceles trapezoids to be positive integers. The students then discussed, planned, designed, and constructed their two-dimensional arches. Next they wrote a description of what they did, describing the mathematics used. But that was just the warm-up. As the period came to a close I pulled out a box of Chinese take-out cartons. I told them, in mock scientific seriousness, that "these are stone voussoirs from an ancient miniature bridge uncovered by my friend, archaeologist Ertha Diggs. She has asked us to determine the number of stones in the original bridge." I gave one to each group, the bell rang and class was dismissed (Figure 1).

Fig. 1. Students in Michael Serra's geometry class construct the "ancient stone bridge" made from Chinese take-out cartons

The Lesson (Day Two)

The lesson (day two). When the students came in to class the next day they began discussing the problem posed yesterday in their groups. I interrupted the group discussions to announce that when a group determines how many pieces in the original (Chinese take-out carton) bridge they are to write up an explanation and then call me over. When they call me over and explain their reasoning, I then give them the additional cartons they think they need to build a replica of the bridge. By the end of the period, when each group has built their arch, we bring them all together and assemble them into a vault! Of course these teenagers cannot resist crawling through the vault (Figure 2). The objective was for students to review and apply the properties of isosceles triangles, trapezoids, regular polygons, and of interior and exterior angle sums. They were to practice communicating mathematically and modeling in two and three dimensions. It is a fun two-day activity of hands-on mathematics and problem solving.

Fig. 2. Students in Michael Serra's geometry class test the stability of the "ancient stone bridge" made from Chinese take-out cartons

Two Days of Follow-Up Near the End of the School Year

During the previous summer there was a lot of remodeling at our school site. So we had a large number of cardboard boxes stacked in piles all over the school building. When I see a free resource lying around I am compelled to make use of it in some way. So I created a lesson on nets for solids. On Day one, I asked my students to explore all the possible non-congruent nets for a particular isosceles trapezoidal prism. When completed we selected the one to best fit on one of the folded flat cardboard boxes. Once we had the largest possible net designed on the cardboard we passed it around and each group traced it onto their cardboard. The next day they cut and assembled each net into the isosceles trapezoidal stone ready for assembly into an arch. Would the arch be large enough for us to walk under? Having recently completed the

trigonometry chapter I was determined to get them to apply their new trig skills. The task of each group was to measure their trapezoidal stone and calculate the span and rise of the arch that was going to be created by these voussoirs. By the second and third geometry classes of the day we had enough voussoirs to complete the arch. It was agreed that I was the chief architect of the arch so I was required to remain beneath the arch as all supporting hands were removed (Figure 3, left). I managed to survive the last stage of construction. Once again Geometry and Architecture blended beautifully into a few days of fun applications for my geometry students and myself.

First published in the NNJ online Autumn 2002

About the Author

Since the 1990 publication of the first edition of *Discovering Geometry: An Investigative Approach* (DG) (San Francisco: Key Curriculum Press), Michael Serra has been a popular classroom teacher, speaker, and teacher trainer at Geometry Institutes. When not teaching, he is either writing new material or traveling the country giving workshops to districts that have already adopted or are thinking of adopting DG. He also gives presentations at a number of National Council of Teachers of Mathematics (NCTM) regional conferences, or state mathematics conference around the country. In 2002 the third edition, *Discovering Geometry: An Investigative Approach* was released. Other publications include the very popular supplementary geometry book, *Patty Paper Geometry*, and the set of five classroom starter workbooks *Mathercise*. Key Curriculum Press has just published *What's Wrong With This Picture? Critical Thinking Exercises in Geometry*. His next project is a book of Mathematical Games and Puzzles.