Mark A.
Reynolds

Perspectiva Geometrica

Mark Reynolds presents some drawings and constructions for those whose interests are in systems of perspective, and the role that grid making may have played in the development of perspective in the early Renaissance.

Introduction

In this issue of the *Nexus Network Journal* dedicated to perspective, I would like to present some drawings and constructions for those whose interests are in systems of perspective, and the role that grid making may have played in the development of perspective in the early Renaissance. The information presented here may also be of assistance to those who are interested in learning some techniques for rendering perspective drawings, although this information assumes some preliminary knowledge of the elements, rules, and vocabulary of the science of perspective systems.

I recommend two books on the subject. *Principles of Perspective* [Walters and Bromham 1974] is a little known work on the subject of perspective. Written for those who wish to draw perspectively, this book offers one of the most accurate, understandable, and concise works on the subject to be found. There is not an abundance of text, and the illustrations are excellent, clear, and exact. Another book, probably known to many of our readers and that I highly recommend, is *Architectural Representation and the Perspective Hinge* [Perez-Gomez and Pelletier 1977]. Where *Principles of Perspective* is a precise and practical approach to the methods of perspective drawing, *Perspective Hinge* is a thoughtful discussion to the subject as it has related to architecture (although it is much more than that), and how perspective techniques used in architectural rendering can be found in the architecture itself, and the various ways the two have been intertwined over the centuries. It is excellent reading.

I have wondered, when developing a grid,¹ whether the same thoughts went through the masters' minds when they were making them; that is, there are aspects of the grid that suggest three-dimensional space, and a sufficiently developed grid contains the illusion of perspective space, complete with vanishing points and horizon lines. I have also considered that it is possible that if I've noticed it, early Renaissance perspectivists must surely have. The question must then be, did they "see" the similarities between certain geometric grids and the potential that these grids have for representing real space so as to make the connection? I was trained in how to do perspective and I learned how to see perspectively through the employment of geometric structures. But what helped painters of the thirteenth and fourteenth centuries to "see" perspectively through the use of geometric structures? In what ways does modern learning of perspective compare, if at all, to the way Giotto's and Duccio's observations were handed on to the Renaissance masters? Did these masters show how perspective space should look in a painting for Renaissance artists to learn from? Did Giotto's use of geometry in rendering architectural settings set the stage for how perspective space could look, and even more importantly: did his paintings and frescos present a situation that perhaps the time had come for

artists to begin to see space in a new and more natural way? These issues, then, are central to the development of linear perspective: how did the masters originally come to see specific geometric techniques as tools for the embodiment of "seeing" ideal, mathematical space?

In my own experiences and in discussions with others, notably with my colleague, Richard Talbot² (and who has an article in this issue), I have noted the idea of geometric grids as possibly generating the 'look' of perspective space, and that come as a product of the development of the grid. There are also certain techniques in grid construction that will provide proportioning systems for perspective drawings, as will be seen in my latter examples below. Whether the awareness of these qualities of the grid came first, that is, if geometric grids induced the founding fathers of this "mathematical space" to see in the grid itself a valid representation of space, or if the desire to create an ideal, yet practical, method for recording the illusion of space came first, I just cannot say with absolute certainty. In *The Science of Art* [1992], Martin Kemp states that perspective--as we think of it today--was the result of intellectual, scientific, and artistic inquiries into the subject of optics and seeing during the Renaissance [Kemp 1992, 1]. Specifically, Professor Kemp gives the credit for the invention of linear perspective to Filippo Brunelleschi in the early 1400s. I agree with this position, and yet I am also interested in Professor Kemp's statement in chapter one regarding "... questions of inexhaustible complexity, not only with respect to the historical circumstances which led to Brunelleschi's invention, but also in relation to the nature of what he actually invented" [Kemp 1992, 9]

It seems that Brunelleschi's experiences with measurements and surveying while in Rome, and his excellent education (which certainly must have included studies of Euclid's *Optics*, and medieval treatises on the same subject) put him in an ideal situation to conceive of the process of perspective drawing; but even with this in mind, I must also remain open to the possibilities that geometry (and, geometric construction), by its nature, was also a catalyst for this new way of thinking and seeing. Geometry was needed for most of what Filippo was doing. But what were all of the geometric systems he was utilizing and developing? Specifically, I am speaking of the nature of grids as generated from rectangles and the square, and his familiarity with them. Did Filippo create a system of geometry for rendering perspective directly and specifically with the intention of making perspective, or did he advance a technique based on a particular quality of geometric grid making that is present in rectangular armatures? It could be that both happened more or less simultaneously, but unfortunately, we do not now have any writings on the specifics. As an artist and geometer, I believe it is important to look at the process of grid making as possibly a key element in the development of perspective systems.

In order to see an example of this quality of grid development, I first present a procedure for measured perspective, and then a grid that utilizes a geometric progression to create the illusion of depth. I also offer a drawing of mine in which the $\sqrt{\phi}$ is used for the perimeter ratio, and which demonstrates how space is created in various parts of the

A Rectangular Prism in Measured, Two-Point, Perspective

In Fig. 1 we begin with a circle, O, and the horizontal diameter. The diameter, AK, will function as the horizon line/eye level for the drawing. In this set up, the two points, A and K, will be the two vanishing points.

As we are in standard two point perspective, all the verticals will remain parallel, and have their vanishing points at infinity, above and below. In two point perspective, only two of the three dimensions are foreshortened; that is, the lines describing these two dimensions each have their own vanishing point. Hence, we have "two point perspective." The drawing also shows a vertical line, representing a vertical plane that cuts through the observer's eye and determines the specific point of intersection on the horizon line where the central visual ray (CVR) is directed. In this drawing, the CVR is labelled ϕ . I have placed the vertical line at the golden section of line AK. This is an

arbitrary decision, but it also has a direct influence on the position of the object as seen by the observer. This will be addressed in Step 2.

Here, in Fig. 2, a classical geometric construction is utilized in the perspective system. It is the Rule of Thales. The rule states that in a semicircle, any point on the semicircle, when connected by straight lines to the ends of the diameter, will always give a 90° angle. In measured perspective, these two connecting lines to the diameter, AP and KP, are sometimes called Van Pars, which stands for "vanishing parallels." These two lines represent the two dimensions that will be put into perspective, and the ends of the diameter become the two vanishing points that these two dimensions converge to. As the object we are rendering—a rectangular prism—has a 90° corner, the rule works perfectly. The 90° then can move along the vertical line, PR, and into perspective.

Fig. 2

The angles formed at the two vanishing points, A and K, determine the angle that we see the front edge, 90° angle of the object rotated to. Here, the angles are approximately 52° (PKA), and 38° (KAP). Because we are employing the rule of Thales, the two angles will always add up to 90° . Therefore, we can render the object in $30^\circ/60^\circ$, or $45^\circ/45^\circ$, or even $10^\circ/80^\circ$, any combination of angles that add up to 90° . As a result, the front 90° vertical edge will be rotated into those two angles as seen against the picture plane. This is because the Van Pars become the perspective front edges of the object as they are lifted from the 90° at the Observer (Obs) into the perspective space.

In Fig. 3, the lines AP and KP have been reflected into the lower half of the circle (still a semicircle), and P has become the point of the Observer. So,

- HL/EL = Horizon Line/Eye Level;
- Obs (SP) = Observer (Station Point);
- CVR = Central Visual Ray;
- CVP = Central Vertical Plane;
- VP1 and VP2 = The two Vanishing Points;
- Van Pars 1 and 2 = The two Vanishing Parallel Lines.

Fig. 3

In Fig. 4, measuring points, which are used in the determination of precise foreshortened measurements and are in conjunction with a calibrated scale of measurements, have been added.³ They are always generated by placing the compass point at a vanishing point, opening the lead to the observer, then rotating this measurement—the Vanishing Parallel Line (Van Par)—onto the horizon line. So,

$$VP1Obs = VP1MP1 \text{ and } VP2Obs = VP2Obs$$

Additionally, a base line (BL) and a height line (HL) have been introduced so that a scale of units can be used to determine sizes and lengths in the drawing. This scale is to be determined by the artist for the job at hand, and the units can represent any magnitude. As mentioned, the heights will not be foreshortened, but will only diminish by reduction of size. The crossing of BL and HL is indicated as "0", and the units to the left and right will correspond to those two directions of foreshortened lines; to the left, VP1, and to the right, VP2. The base line can be placed anywhere, and it will determine, at "0", where the front, lower corner and base of the rendered object will be.

Fig. 5 shows a square of 4 units drawn beneath the base line. This has been done in order to construct a golden section rectangle to be drawn in perspective. Any plan may be measured or drawn using the units chosen by the artist. (Here, as the golden section length would not fall conveniently and neatly on a base line measurement, the construction method was used.) The measurements from the base line and the construction can now be transferred onto the orthogonal lines.

Four units are measured to the left, and a straight edge to the measure point is then used to transfer these four units onto the orthogonal going to VP1 at q1. Next, the same procedure is applied to the right, and the straight edge is then drawn to MP2. These four units are marked on the orthogonal going to VP2 at q2. Lastly, the golden section measurement, F, is drawn onto the orthogonal to VP1 at q3. Now, the front half of the rectangle in perspective plan is drawn to the VPs and the back half can now be added, as seen in Fig. 6, which shows the golden section rectangle, drawn in plan in Fig. 5, now in two-point perspective.

Fig. 6

In Fig. 7, an elevation of a golden section rectangle is drawn on the base line, and its heights are then transferred over to the height line, and the square is four units in height. A golden section rectangle has been drawn for the height of the prism. Measurements of height can be moved around by drawing orthogonals from the top and bottom of all the vertical measurements to the appropriate VPs. This is how the other three vertical edges of the prism were made in Fig. 8.

Fig. 7

Fig. 8

The rectangular prism with guide lines:

Fig. 9

The finished perspective drawing of the rectangular prism:

Fig. 10

I am including an interior space to demonstrate that measured perspective can also render space. The height line, base line, and the two vanishing points work in concert to move dimensions through the perspective space. Note the 45° measure point for the diagonals of the square grid. As units of magnitude, these squares can be scaled to any size. For example, in this drawing, if we assign a size of three feet to be equal to one square, our eye level is two units from the ground, or six feet (the height of the eye level/horizon line from the base line, 0), and we are then looking at the back walls as being about 21 feet back, and the ceiling 15 feet in height.

The Grid within the Golden Section Rectangle

In the next series, Figs. 11 through 13, I present a grid within the golden section rectangle that creates a lovely progression demonstrating diminution. (The ϕ rectangle was the ratio I originally used when I made this discovery. The grid becomes a perspective space, with the diminution starting at the bottom of the rectangle and proceeding to the top. The top of the rectangle acts as the horizon line. This technique can be employed in any rectangle, but the results can vary greatly.⁴ This particular progression is based on $\sqrt{\phi}$ within the ϕ ratio, as noted on the right.

Fig. 11

Fig. 12

In Fig. 11, we have golden section rectangle APRZ. The rectangle contains two squares, AKMZ and SPRT. O is the center of the rectangle. XY is the horizontal midline of the rectangle. ZQR is a semicircle, with center Y and radius ZY. $ZY = YQ = YR$.

Fig. 12 shows the vertical line NG generated by the intersections of the semicircle, ZQR, with the sides KM and ST, of the squares AKMZ and SPRT, at points E and C. This vertical line becomes the key instrument in the creation of the progression.

The procedure is as follows:

1. Draw horizontal line JH through F where the diagonal AR intersects GN at F.
2. Draw line JR from the generated point J.
3. Draw horizontal line ST through E where JR intersects NG at point E.
4. Draw line SR from the generated point S.
5. Continue this same procedure up the rectangle toward the top of the rectangle PR creating horizontal lines LY, KM, UV, WX, etc.

Fig. 13

The very tall rectangle on the right side of the illustration, APGN, has been taken from the master rectangle, APRZ. It has been isolated so that the diminution of spaces can be clearly seen, and also so that each horizontal band becomes just a rectangle. The first rectangle at the bottom, labelled, is a ϕ rectangle. The next one above it, labelled, is a $\sqrt{\phi}$ rectangle. The next is a square, and so on. Each rectangle, from bottom to top, decreases by $\sqrt{\phi}$, or, divided by 1.272...

The Grid with the $\sqrt{\phi}$ Rectangle

A variation of the above techniques involves the $\sqrt{\phi}$ rectangle.

Fig. 14

Starting with $\sqrt{\phi}$ rectangle AKMZ, horizontal line PG is drawn at a golden section of the height, so that $PK/AP = AK/PK = F/1$. Points C and R are the golden sections of the width. Here any and all of the lines, PG, PR, and PM (and although there are more) act as line NG did in the last construction. In this construction, I particularly like the rectangular shapes within triangle PRM. Near P, the shape appears to be square, but as it proceeds to the top of the rectangle KM, which is the horizon line/eye level here, the shape is distorted into longer, and larger, rectangles, until, at PM, only the nearest edge of the last rectangle can be seen, converging with the horizon line, KM!

Fig. 15 is a graphite drawing of the dense grid developed using this technique. In this drawing, one can see diminution occurring in several places, including at the top, to the far left, and at the horizontal golden section of the rectangle.

Conclusion

I hope that these demonstrations motivate some of you to try drawing with these techniques. Perhaps even more, I hope that they may provide a point of discussion on the issue of the nature of geometric grids in creating perspective space as a natural consequence of the geometric armature. Although we may never be certain which

approach came first-the conscious decision to develop a working system of geometry to help explain the way we see, and therefore understand, the world, or the awareness of the ability of geometry to naturally demonstrate some of the ways in which we see-perhaps the drawings will keep both possibilities open whenever discussions about the history of perspective and geometric systems arise.

Fig. 15

Notes

1. The reader who may be interested in the elements of rectangular grids might wish to see the Geometer's Angle No. 2 and no. 3 in the *NNJ*.
2. Richard is a very intelligent and gifted artist. I would encourage anyone interested in seeing some amazing and beautiful perspective drawings to look at his work. (Please note that these drawings are done by hand, not computer.) Richard convincingly shows that there can always be a new approach to an ancient method with exquisite and aesthetically pleasing results. Some of us may have come to believe that much about perspective rendering is sterile and mechanical, but Richard has taken the art to a new, refreshing, and welcome level; and, his drawings are drawn freehand.
3. See [Walters and Bromham 1976] for the geometric proof of why the measure points work.

4. The reader is referred to the Nexus Journal/Geometer's Angle column, Volume 3, Number 4, where the subject of progressions and perspective are addressed.

Bibliography

- EUCLID. 1998. *The Arabic Version of Euclid's Optics: Edited and Translated with Historical Introduction and Commentary*. Elahieh Kheirandish, trans. Berlin: Springer-Verlag.
- ALBERTO PEREZ-GOMEZ and LOUISE PELLETIER. 1997. *Architectural Representation and the Perspective Hinge*. Cambridge MA: MIT Press.
- NIGEL V. WALTERS and JOHN BROMHAM. 1976. *Principles of Perspective*. New York: Watson-Guptil.
- MARTIN KEMP. 1992. *The Science of Art*. New Haven CT: Yale University Press.

About the author

Marcus the Marinite (aka Mark A. Reynolds) is contributing editor for the Geometer's Angle column of the *Nexus Network Journal*. He is a visual artist who works primarily in drawing, printmaking and mixed media. He received his Bachelor's and Master's Degrees in Art and Art Education at Towson University in Maryland. He was also awarded the Andelot Fellowship to do post-graduate work in drawing and printmaking at the University of Delaware. He is also an educator who teaches sacred geometry, linear perspective, drawing, and printmaking to both graduate and undergraduate students at the Academy of Art College in San Francisco, California. He was voted Outstanding Educator of the Year by the students in 1992. Additionally, he is a geometer, and his specialties in this field include doing geometric analyses of architecture, paintings, and design. For the past decade, Mr. Reynolds has been at work on an extensive body of drawings, paintings and prints that incorporate and explore the ancient science of sacred, or contemplative, geometry. He is widely exhibited, showing his work in group competitions and one person shows, especially in California. His work is in corporate, public, and private collections, and he is represented by the Mill & Short Gallery in San Francisco. He is a member of the California Society of Printmakers, the Los Angeles Printmaking Society, and the Marin Arts Council. He published "A Comparative Geometric Analysis of the Heights and Bases of the Great Pyramid of Khufu and the Pyramid of the Sun at Teotihuacan" in the *NNJ* vol.1 (1999).