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The Geometer's Angle:
*An Introduction to the Art and Science of
Geometric Analysis*

Last summer, at the Nexus 2000 conference in Ferrara, Italy, a round table discussion was initiated by Kim Williams and chaired by Carol Watts where the issues of working out the inherent problems and possible procedures required to perform geometric analysis were initially opened and addressed [Watts 2001]. I would like to carry the theme forward in this column.

The role of geometry

Some geometers have believed that the two-dimensional space of Euclidean geometry is, in fact, closer to the philosophical and hermetic ideals of the absolute and the universal than the ten-dimensional universe of quantum mechanics being presented to us today. The farther we go into this higher-dimensional world, the less we know of the truths that geometry, its uses and meanings traditionally sought to present: form, structure, symbolism and relationships. It was on this two-dimensional plane that the plan, elevation and section of architecture, and the surface of traditional painting, drawing and printmaking were made. Also, many of us studied Euclidean geometry as a means to develop critical thinking. In these opposite roles, geometry as a design tool – the tool of choice from Babylon and Egypt, through the Renaissance, and down to the present day – has presented us with the difficulties of sorting out its dual nature, or at least its dual uses. As a result, our knowledge of the geometric systems employed by the ancients has oftentimes been lacking, confused and misunderstood. The duality is between the spiritual and scientific sides of the geometer's craft; the opposing sides present an incompatibility. In the past, the two could be understood as parts of a whole. Today, there is a great divide between them. In our age of specialization, we seem to have lost our abilities to see even divergent qualities as still being possibly parts of the same whole. What we were beginning to inquire about at the conference is if there may be an objective set of rules, in spite of this seeming dichotomy, that we can apply procedurally – objectively – in order to better understand the geometry we have inherited from our ancestors.

As stated, it is quite difficult to separate the hermetic side of geometric usage from its more practical role as a supreme building tool because of the fact that it *is* a supreme building tool. Geometric structure pervades the natural world, as well as architecture and design. This fact was not lost to our elders, and quite often, the user of the geometry we are planning to analyze saw the geometry, to borrow from Erwin Panofsky, as 'symbolic form'. Yet it is important to temper the analysis with a scientific approach, as the inheritors of the rationalist, scientific method would demand. In this column dedicated to the subject of analysis, I will be using some material I developed for the book on which I am currently at work, *The Art and Science of Geometric Analysis*.

Geometrical analysis and accuracy

The central issues of accurate geometric analysis must begin with a definition of "accurate", and I see no reason to ignore the concept of "percent deviation" in all measures. What that tolerance would be is best left in the hands of the mathematicians, architectural historians, artists,

geometers and scholars familiar with the subject. General acceptance of tolerance would be reasonable, I believe, to within standards possible at the time the design was done and the tools available in its making. With this comes the determination of tolerance based on workman's error and natural disasters from the time of original construction. This may require some sensitivity to intent by reason; that is, if it is found that a measure is 0.9995, the measure was, more than likely, intended to be 1.000. It is a long way from the drawing board to the laying of the cut stone, so some agreeable percent deviation should be set. Scholars then can make determinations on how close or far away a particular design is from true-to-ideal after the analysis is done. Some of the central considerations of geometric analysis in this determination should be:

- Geometric systems known and/or used prior to or concurrent with the design being studied;
- Uses of canons of measurement;
- Purpose and meaning of the geometric systems;
- Social, philosophical, symbolic and spiritual issues;
- Traceable historical developments vs. oral traditions;
- Theory vs. practice.

In my own experience, I have found that one of the greatest impediments to accuracy and understanding of the geometric system is in the attempt to convert measures. Critical to a more complete understanding of the approach to the analysis is to realize that a square will be a square regardless of the measuring system employed, and this carries over to the complete structure. Relationships of one length to another (ratios) remain, regardless of the measuring system. I have found more errors in conversion and inaccurate measurement than in any other areas. This is not to devalue the canons and measures used, because there are times when a more complete understanding of the geometer's intentions from a philosophical or spiritual point of view requires us to study the local units of measure (cubits, *braccia*, meters, feet, etc.). It just is not necessary during the analysis itself. The primary goal is to know the geometry, and many times more can be gleaned from a thorough understanding of the system than in knowing the length of a *braccio* (a particularly messy situation in Renaissance Italy, for example). It should also be considered that the measures used may have been truly arcane and known only to the few involved in the project and therefore completely unknown to us.

Of course, any consideration of a true analysis must consider measurements *in situ*. For a variety of reasons, geometric analysis is sometimes done on photographs, or from measures taken from others without regard to the measurers' tools and techniques, or the conditions, prejudices and the times in which the measurements were taken. Analyses of photographs are so problematic that a separate column could be devoted to them. Photographic sources can be used to supplement, demonstrate or reinforce the analysis, but even in these instances, the photographic information should be done either by the geometer or under the geometer's direct supervision. There are rare occasions where all we have are photographs to work from, and a good geometer can get a ballpark approximation of the system, but it must be recognized as educated guesswork or a close approximation at best. This same position must be taken with existent drawings. I have found assumptions by otherwise competent surveyors and measurers where a square was drawn when in fact the space was correctly in a ratio of 1.118...:1, or $\sqrt{5}/2$, presenting a totally different system than one based on a master square. If there is a situation in which photographs and prior

renderings are the only available sources, then as many sources as possible should be obtained to see if there may exist an average with a small percent deviation that indicates a particular geometric usage; these instances should be well-documented and footnoted.

Modern technology has afforded us with new laser-based measurement capabilities, and scanners that can scan every 0.5 mm. Even so, the results can be misleading without on-site drawings and measurements. (When I was in Cairo last year, we were allowed to use a digital camera to photograph a sculpture of a scribe in the Cairo Museum. When the information was put into the computer for analysis, we found that the scribe showed unusually large legs in his cross legged, seated pose. This happened because the legs were closer to the camera lens than were the torso and head.) Additionally, it is crucial to the analysis that true vertical through precise 90° horizontal axes be observed so that there is absolutely no foreshortening or parallax. The slightest deviation can result in a distorted ratio and an inaccurate analysis. Perhaps special software programs can be written to assist in geometric analysis, but as things stand today, there isn't anything available that functions better than the educated eye and mind of a skilled geometer.

As we develop methods and techniques for a good analysis, the mathematician and the artist will have to find some common ground where differences can be put aside in order to better understand the maker's intentions when using various geometric systems. Sometimes we make the mistake of taking modern thought and opinion and projecting it back into antiquity with the hope of getting at the truth, and it doesn't work. It is true that Euclid stated that a line has no breadth, and that is the mathematician's position. The designer seeks a more flexible interpretation of line, for without it, the drawing would not physically be seen. From the issue of line, for example, it may be more advisable to discuss the width of the line – the line weight – and how it can drastically alter an analysis: when the line refers to the thickness of a wall, to which side – inside or outside – do we take our measures? There needs to be some flexibility in these determinations, and some clarity when presenting the analysis itself.

I would also like to address the issue of the golden section, ϕ , mainly because of the frequency and intensity with which it comes up in discussion and debate. There is much attached to its use or lack thereof wherever one goes in the world of geometry and the arts. There is no doubt that it has unique mathematical properties and the ratio has significant representation in approximations to its ideal ratio of 1.61803... : 1 in nature, including the human body. There is also no doubt that it has some strong qualities as a design tool. (A future column will be devoted to these issues.) First, it is important to note that the golden section is not the only geometric system with wonderful qualities. Second, whether it is or is not employed in a particular work is not a basis for accepting or denying the overall qualities of a work of art, design or building, and assigning a passing or failing aesthetic grade to that work. I believe that the almost spiritual status it seems the golden section has achieved over the centuries has also made it a target for undue negative criticism as well as a device used by some contemporary pseudo-spiritualists to demonstrate the other worldly qualities of number mysticism. It is an extraordinary subject to study, and I recommend that we keep our wits about us when doing so. I personally love the ratio and its related magnitudes, $\sqrt{\phi}$ and $\phi + 1$, and I am fairly well versed in its uses in antiquity. I sense that above all else, the subject is misunderstood and its truths distorted. The subject will come up in the column, and all I ask of the reader is to keep an open mind when presented with information and documentation regarding its use.

In the next sections, I will give the steps I have found that can yield a competent analysis. I welcome any feedback, suggestions and questions regarding geometric analysis, especially from my colleagues in our conference discussion.

Beginning a geometric analysis

Here I would like provide a step-by-step guide for those who would like to try their skills at this “Art and Science of Geometric Analysis”. For our discussion, the suggested procedures are for paintings, architecture and most works of design. Sculpture presents a special set of problems that deserve special consideration.

As previously mentioned, the ideal situation is to have measurements that you have personally carried out, or whose accuracy you do not question. Barring this situation, it is important to have measures and visual references that are as precise as possible. This means that the visual representations be as close to the original drawings as possible. The more generations there are (that is, the more removed the reproduction is from the original), the greater the risk of distortion and inaccuracy. Quite often, photographs are suspect except as a check on measures taken *in situ*. There are too many types of lenses, too great a variety of films and film speeds, too many possibilities for distortion, and too many chances that shadows and highlights may obscure the desired, calculated accuracy. Is there parallax? Was a tripod used? Is the Center Line of Sight a true perpendicular to that which is to be analyzed? Unless you are there to supervise the use of a large format camera in the hands of a skilled and mature photographer, don't consider photographs your ally. If you are using the camera as supporting documentation, be sure to keep a diary of each shot, preferably with an accompanying sketch to augment each picture. Finally, the curse that befalls most analyses is that the image/reproduction has been cropped, sometimes severely, and sometimes on more than only one or two sides. There is no way to know for sure without comparison to the original.

There is no need to convert from one measuring system to another. Rather, find one that is suitable, practical and comfortable for your needs. Use it for all your measures, and do not bother to set up a scale, unless you would like to. For the purposes of pure geometric analysis and the geometric systems and constructions we are looking for, specific scales and units of measurement are not relevant. It is only necessary to have a simple system that you can keep track of and that will help you in your drawing. For some, this is a hard concept to grasp. (Yet don't confuse this issue with the interest scholars have in the study of the importance of the many canons of measurement to be found throughout history; it's a different aspect of analysis that is outside our current task.) Quite often, I have found that an analysis has been incorrectly concluded or the final interpretation of data is overly problematic because of the erroneous need to convert number measurements from one system to another; the most frequent being the conversion between the metric and the English systems, and the further confusion they both are subjected to when trying to fit, for example, the Italian Renaissance system of *braccia* into the equations. (In keeping with this thinking are the myriad of errors in conversions of measures to be found in art and architectural history books, especially between centimeters and inches. It is truly incredible.) What is most important at this initial stage is in the establishment of the unit One, or 1. A simple example will help to clarify this concept.

Take the measurements of a rectangular ground plan to establish the ratio of the lengths of the short and long sides. Divide the short side measure into the long side measure. In so doing, the short side is established as 1. Now this 1 can change over the course of an analysis, but it is helpful to the beginning analyst to establish the possibility of the presence of specific geometric

systems employed by the artist or designer. This simple calculation can help to determine whether whole numbers or irrationals like the square roots or the golden section systems were used. This has the great advantage in alerting the analyst to be on the lookout for specific devices that are part of that particular system.

After getting a few analyses completed, it may be of interest to see if a specific canon, or unit, of measure was employed to see if number symbolism or number games may have been used. Some research is advisable to see if certain numbers were preferred over others, especially in architecture. (I am reminded of an analysis I was doing with a graduate student on the plan of the Mormon Tabernacle Church in Salt Lake City. There was a problem in determining the diameters of the turret-like structures in the four corners in relation to the length of the walls. We were finally successful when we determined that the diameter was one-tenth the length of the short side wall. The student remarked that the faithful are asked to tithe ten percent of their earnings to the church annually, and he found it amazing that this concept had been placed at the four “pillars” of the building.)

It is recommended that this procedure of establishing ratios by the method of dividing the smaller number into the larger be maintained throughout the analysis for the sake of clarity and understanding. Additionally, the number systems being used all have the understanding: when compared to 1. It would be impossible to understand, “mean and extreme ratio” if we first didn’t assign 1 to either the entire line length, or to one or the other segments. What is the meaning of the $\sqrt{2}$, 1.4142136... if we first don’t state that the side of the square is 1? Stating that the golden section is either 0.618... or 1.618... is meaningless, unless we also add, “when compared to 1”.

A basic step-by-step guide to the procedures of geometric analysis

- 1) If at all possible, obtain the original dimensions of the work to be analyzed. Often, if the ratio of the work corresponds to one of the classical ratios of sacred geometry, it will be much easier to ascertain specifically what qualities and methods were being “exploited”, or were being used, by the artist/designer.
- 2) Draw a “frame” around the work that is tangent to, or touches, the work to be done. Usually, this frame is rectangular. Once this is done, a ratio can be calculated. It is important to note that any measuring system will work. Whether in inches, centimeters, *braccia*, cubits or kens, the ratio always comes out the same. Simply divide the lesser measure to the greater. As mentioned, this is the best way to establish the lesser measure as UNITY, or 1.
- 3) Draw the two diagonals, which place the center.
- 4) From this center, bisect the four angles of the two diagonals. In a square, these angles are 45° but in many other rectangles, these four angles can be any angles, depending on the ratio of the rectangle. These bisected lines will give the vertical and horizontal midlines of the rectangle, parallel to the sides.
- 5) Draw the eight half-diagonals. These lines work with the diagonals so that thirds, fourths, and other fractional parts of the rectangular surface can be found.
- 6) If desired, do the constructions for the other fractional parts of the rectangle. These will include fifths, sixths, eighths, tenths, etc. For example, if fifths are found, three of them will yield $3/5 = 60\%$. This number is very close to the golden section, which is .618033... = 61.8033% of the height or width.

- 7) Do the “rebattment”, that is, the rotation of the short sides onto the long sides of the rectangular perimeter. This procedure will yield two squares, one at each end. Often, this technique can be found in classical paintings, and the diagonals of these two squares were also utilized in the placement of imagery. It is likely that the midlines of these squares were used as well. Occasionally, it can be a good idea to construct a square in the center of the rectangle. With this, it is also sometimes found that a circle has also been utilized in the center.
- 8) If desired, find the “occult center(s)”. The occult centers are a bit more arcane, but the masters know of them and how to use them. On occasion, vertical and horizontal lines can be drawn through an occult center. Occult centers are those intersections, or “eyes”, where: a) a diagonal and reciprocal cross; b) when that reciprocal is drawn from a corner of the rectangle, crossing the diagonal at 90°. This mechanism is an important concept in analysis.
- 9) Another suggestion in the analysis is to apply the golden section(s) to the work.

Table of frequently-used measures

- 1) 1:1 (Square).
- 2) π : radius (Circle).
- 3) Triangles (there are 6 types: Equilateral, Isosceles, Scalene, Right, Acute, and Obtuse).
- 4) 1. 366... : 1 ($\sqrt{3} - 1 \div 2 + 1$).
- 5) The Square Root Rectangle Series (there are an infinite number of square root rectangles; the ones given here are the most commonly used):
 - i) $\sqrt{2} = 1.4142... : 1$
 - ii) $\sqrt{3} = 1.732... : 1$
 - iii) $\sqrt{4} = 2$
 - iv) $\sqrt{5} = 2.236... : 1$
 - v) $\sqrt{9} = 3$
 - vi) $\sqrt{16} = 4$
- 6) The Golden Section, or Phi (ϕ), Group:
 - i) ϕ , or 1.618... , : 1
 - ii) $1/\phi$, or 0.618... , : 1 (reciprocal)
 - iii) 1.118... : 1 ($\sqrt{5} \div 2$)
 - iv) 1.272... : 1 ($\sqrt{\phi} : 1$)
 - v) 2.618... : 1 ($\phi + 1$)
- 7) The “Musical Ratios”, or the Whole Number Series (there are other musical ratios, but these other ratios are found and used only rarely):
 - i) 1 : 1 (Square)
 - ii) 1 : 2 (Double Square)
 - iii) 2 : 3 (Major Fifth)
 - iv) 3 : 4 (Major Fourth)
 - v) 3 : 5 (Major Sixth)

- vi) 5 : 8 (Minor Sixth)
- vii) 4 : 5 (Major Second)
- viii) 8 : 9 (The Tone)

8) 2.4142... : 1 (Theta, θ , or $\sqrt{2} + 1$)

Remember that this is not a complete list of all the ratios available to the artist or designer. A good working knowledge of grid structures and harmonic deconstruction procedures can supply the creative mind with compositional possibilities regardless of the ratios involved. The list given here contains a series of ratios that have particular, individual qualities that are unique for each ratio, a form of visual personality specific to only that ratio. This list gives the artist a double set of compositional elements to work from: the general, common procedures for all rectangles, and the qualities found in each specific ratio on the list.

Anatomy of a rectangle

The table that follows is a list of geometric elements and their descriptions.

<i>Point</i>	A specific location defined by a unique segment of a line. Unlike the mathematical definition, the artist makes a point that can be seen and quantified.
<i>Perimeter</i>	The boundary around a specific amount of space (area) that defines that space by specific measurements and shapes; a border. For the purposes of analysis, the perimeter is in the 2-D plane. That 2-D plane is seen as 3-D space by plan (top), elevation (front) and section (end), each two-dimensional references.
<i>Angle</i>	A part of 360 degrees made by the intersection and specific relationships between two lines. An angle of 180° is a straight line. A standing person makes a 90° angle with the earth. In all rectangles, excluding the square, the two angles of a diagonal are unequal. In the square, the 90° angle is bisected, or cut into two equal 45° angles.
<i>Rectangle</i>	A geometric shape consisting of 4 straight lines, with 2 different pairs of equal and parallel lines. Each line as a 90° angle at one end and is attached to the next straight line to make a perimeter by the 90° change in the direction of each line. In a parallelogram, the pairs of lines are parallel but the angles are not 90°
<i>Diagonal</i>	A directional angle made by straight lines that connect opposite corners of a rectangle. The diagonals cross in the precise center equidistant from all 4 corners and 4 sides. This center is sometimes referred to as the “dead center”. The diagonal is critical to any rectangle by defining, by its angle, the ratio of the rectangle. It is also a powerful design component in the composition. It is called ‘the power of the diagonal’.
<i>Half-Diagonal</i>	A line segment that connects opposite sides of the rectangle through their mid-lines, and the lines are parallel to the sides of the rectangle. Also called horizontal and vertical bisectors.

<i>Mid-Line</i>	A line segment that connects opposite sides of the rectangle through their mid-lines, and the lines are parallel to the sides of the rectangle. Also called horizontal and vertical bisectors.
<i>Mid-Point</i>	Halfway on the side of a rectangle or other element.
<i>Reciprocal</i>	A smaller rectangle in the same ratio as the master rectangle, and can be constructed with a reciprocal line.
<i>Reciprocal Line</i>	A straight line that intersects a diagonal at 90° and creates a smaller rectangle that is in the same ratio as the “master” or primary, rectangle to which the diagonal belongs.
<i>Occult Center</i>	The intersection of the diagonal and reciprocal lines when, and only when the reciprocal line is drawn from one of the four corners. When all four reciprocal lines are drawn from the four corners, the “eyes” made will surround the dead center and become a frame for it. These four eyes, because they fall along the diagonals and are parallel to the four sides of the master rectangle, will make a third rectangle of the same ratio as the master and reciprocal rectangles. This eye is called ‘occult’ because it is ‘hidden from the eye’. The origin of this term is obscure. (The occult center divides the diagonal and reciprocal into four lengths of a geometric progression, i.e., when a number is multiplied or divided by a number that is “constant”, or, remains the same. This constant is the ratio that the rectangle’s sides are in. Any two lengths in sequence from short to long can make a new rectangle that is in the same ratio as the master rectangle. These lengths can also be used to construct a spiral.)
<i>Eye</i>	The point where two or more lines intersect, or cross, each other.
<i>Rebattment</i>	The 90° rotation of the short side of a rectangle onto the long side for the purpose of constructing a square within the plane of the rectangle.
<i>Field</i>	The area enclosed by the perimeter of the rectangle.
<i>Right Triangle</i>	A triangle that contains a 90° (right) angle. The side opposite the 90° angle is called the hypotenuse. In a rectangle the diagonal can be considered a hypotenuse.
<i>Bisection</i>	The division of an element into two equal parts.
<i>Perpendicular</i>	Placed at 90° to something.
<i>Half-circle or Semi-circle</i>	In geometric analysis, half of a circle that uses either the short side or the long side of a rectangle as its diameter. The half-circle can be used to locate an occult center or in the construction of the golden section.
<i>Caesura</i>	A vertical or horizontal line that is used to break up the movement of the eye across the surface of the composition. Usually the term is used in music and poetry as a tool that cuts or breaks the flow of sounds, but it can mean the same thing visually. Some painters of the Renaissance used this concept by applying musical ratios to the composition and placing

key parts of the imagery on these lines in the grid. The golden section is another example of how this concept can be used.

Arc

Part of a circle, curve or sinuous line used to easily move the eye from one place to another. An 'S-curve' is exactly what it says. Arcs can sometimes be made by rotation (rebatment) of a length from one place to another and then used in a composition. 'French curves' are more free-flowing than geometric ones. Logarithmic curves are more mathematically plotted.

Conclusion

Although we have just scratched the surface, so to speak, of this complex science, I hope that you at least know where and how to begin your own analyses. It is always advisable to have a variety of sources and images on the particular study under consideration...and remember the old carpenter's dictum: "Measure twice; cut once."

In coming issues, I would like to do an analysis that demonstrates the subject matter we have covered, and I would like to do a couple of constructions that show some of the uses of the golden section as a design tool. This will help our readers get a better picture of this controversial subject from a design point of view.

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About the geometer

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