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Non-semisimple gauging of a magical $N = 4$ supergravity in three dimensions

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ABSTRACT: We construct a new $N = 4$ non-semisimple gauged supergravity in three dimensions with $E_{6(2)}/\text{SU}(6) \times \text{SU}(2)$ scalar manifold and $\text{SO}(4) \ltimes \mathbf{T}^6$ gauge group. Depending on the values of the gauge coupling constants, the theory admits both the maximally supersymmetric AdS_3 vacuum preserving $\text{SO}(4)$ gauge symmetry and half-supersymmetric domain walls with unbroken $\text{SO}(4)$ symmetry. We give all scalar masses at the supersymmetric AdS_3 critical point corresponding to an $N = (4, 0)$ superconformal field theory (SCFT) in two dimensions. The scalar potential also admits two flat directions corresponding to marginal deformations that preserve full supersymmetry and conformal symmetry. This $\text{SO}(4) \ltimes \mathbf{T}^6$ gauged supergravity is expected to arise from a dimensional reduction on a three-sphere of the minimal $N = (1, 0)$ supergravity in six dimensions coupled to three tensor and four vector multiplets.

KEYWORDS: Supergravity Models, Supersymmetric Effective Theories, Gauge-gravity correspondence, AdS-CFT Correspondence

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1 Introduction

Gauged supergravities in various dimensions play an important role in many aspects of string/M theory in particular the AdS/CFT correspondence [1]. Unlike higher dimensional analogues, gauged supergravity in three dimensions has a much richer structure due to the duality between vectors and scalars. Since three-dimensional supergravity fields are topological, the propagating bosonic degrees of freedom of the matter-coupled supergravity can be described entirely in terms of scalar fields. The resulting theory is a supersymmetric non-linear sigma model coupled to supergravity. All of these theories with different numbers (N) of supersymmetries have been classified in [2]. For $N > 4$, scalar fields must be described by a symmetric space of the form G/H in which G and H are global and local symmetries, respectively. The latter is the maximal compact subgroup of G and takes the form $\text{SO}(N) \times H'$ with $\text{SO}(N)$ being the R-symmetry group.

Gauged supergravity can be constructed by introducing gauge vector fields via Chern-Simons (CS) terms which are topological in nature. This CS formulation makes the connection to usual Yang-Mills (YM) gauged supergravities, in which gauge fields appear via YM kinetic terms, obscure. Furthermore, since conventional dimensional reductions result in YM gauged supergravities, the embedding of these CS gauged supergravities in higher dimensions is then a non-trivial task. However, it has been shown that CS gauged supergravity with a non-semisimple gauge group $G_0 \ltimes \mathbf{T}^{\dim G_0}$ is on-shell equivalent to YM

gauged supergravity with G_0 gauge group [3]. The $\mathbf{T}^{\dim G_0}$ factor is a nilpotent translational group transforming in the adjoint representation of a semisimple group G_0 . Therefore, at least some of these particular gauge groups might have higher dimensional origins in terms of known dimensional reductions. Since the embedding in higher dimensions is necessary for interpreting three-dimensional solutions in string/M-theory context, CS gauged supergravities with non-semisimple gauge groups are of particular interest in the AdS/CFT correspondence.

Until now, a number of works considering CS gauged supergravity with non-semisimple gauge groups and their application in the AdS/CFT correspondence have appeared [4–12]. All of these gauged supergravities are expected to arise from dimensional reductions of higher dimensional theories, and explicit reduction ansatze for obtaining three-dimensional gauged supergravities from a three-sphere reduction, both the full S^3 reduction and $SU(2)$ group manifold reduction, have been constructed in [13–16]. Furthermore, a number of $N = 2$ gauged supergravities with abelian gauge groups have also been obtained from wrapped branes in type IIB and M-theory [17–19].

In this paper, we construct new $N = 4$ gauged supergravity with $E_{6(2)}/SU(6) \times SU(2)$ scalar manifold and $SO(4) \ltimes \mathbf{T}^6$ gauge group. According to the result of [3], the resulting gauged supergravity is equivalent to $SO(4)$ YM gauged supergravity. A number of possible semisimple gaugings of this theory have already been classified in [8]. In the present work, we will study possible non-semisimple gauge groups and also classify supersymmetric vacua of the gauged supergravity both maximally supersymmetric and half-supersymmetric with the full $SO(4)$ symmetry unbroken. These would be useful in AdS_3/CFT_2 correspondence.

The theory considered here is one of the magical supergravities in three dimensions, see [20–22] for higher dimensional magical supergravities. In [8], another $SO(4) \ltimes \mathbf{T}^6$ gauged magical supergravity with $F_{4(4)}/USp(6) \times SU(2)$ scalar manifold has been studied. This theory is expected to arise from an S^3 reduction of $N = (1, 0)$ supergravity in six dimensions coupled to two vector and two tensor multiplets. In the present case, we expect the new $SO(4) \ltimes \mathbf{T}^6$ gauged theory with $E_{6(2)}/SU(6) \times SU(2)$ scalar manifold to arise from an S^3 reduction of this $N = (1, 0)$ supergravity coupled to four vector and three tensor multiplets. Both of these six-dimensional supergravities are also magical supergravities, and the possible gaugings have been systematically considered in [23] using the embedding tensor formalism.

The paper is organized as follow. In section 2, we review $N = 4$ three-dimensional gauged supergravity with symmetric scalar target spaces. We mainly focus on a specific case of exceptional coset $E_{6(2)}/SU(6) \times SU(2)$. In section 3, the embedding tensor of $SO(4) \ltimes \mathbf{T}^6$ is given. We then study the resulting scalar potential for the $SO(4)$ singlet scalars and identify some of their possible supersymmetric vacua. We end the paper by giving some conclusions and comments on a possible higher dimensional origin of the gauged supergravity constructed here in section 4. Two appendices with some useful formulae are also included.

2 $N = 4$ gauged supergravity in three dimensions with $E_{6(2)}/\text{SU}(6) \times \text{SU}(2)$ scalar manifold

Three dimensional matter-coupled supergravity is given by a nonlinear sigma model coupled to supergravity. Scalar fields will be denoted by ϕ^i , $i = 1, \dots, d$ with d being the dimension of the scalar target space. Throughout this paper, we will work in the $\text{SO}(N)$ covariant formulation of [24].

Coupling the non-linear sigma model to N extended supergravity requires $N - 1$ almost complex structures, denoted by f^{Pi}_j , $P = 2, \dots, N$, on the target space of the sigma model. The R-symmetry in three dimensions is given by $\text{SO}(N)$ under which scalars transform in a spinor representation. The tensor f^{IJ} , $I, J = 1, \dots, N$, generating $\text{SO}(N)$ R-symmetry in this spinor representation, can be constructed from f^P via

$$f^{1P} = -f^{P1} = f^P, \quad f^{PQ} = f^{[P} f^{Q]}. \quad (2.1)$$

The f^{IJ}_{ij} have a symmetry property $f^{IJ}_{ij} = -f^{JI}_{ji} = -f^{IJ}_{ji}$.

For $N = 4$ theory, the scalar target space must be a quaternionic manifold. Special to $N = 4$ supersymmetry, there exists a tensor $J = \frac{1}{6}\epsilon_{PQR}f^Pf^Qf^R$ that commutes with the almost complex structures and is covariantly constant. This implies the product structure of the target space. Therefore, a general $N = 4$ matter-coupled supergravity admits a scalar manifold of the form $\mathcal{M} = \mathcal{M}_- \times \mathcal{M}_+$ with a total dimension $d = d_+ + d_-$. The full $\text{SO}(4) \sim \text{SO}(3)_- \times \text{SO}(3)_+$ R-symmetry will split into each factor of the full target space. In this paper, we are however only interested in the so-called degenerate case with only one factor of $\mathcal{M}$ present. For definiteness, we will consider a non-vanishing $\mathcal{M}_+$ by setting $d_- = 0$. Furthermore, we will restrict ourselves to only a symmetric target space of the form G/H although this is not in general a requirement from $N = 4$ supersymmetry.

The symmetric target space of the form G/H has a global symmetry G and a local symmetry H given by its maximal compact subgroup. In general, the local symmetry H contains the $\text{SO}(N)$ R-symmetry and takes the form $H = \text{SO}(N) \times H'$. But, for the $N = 4$ theory, we have the compact group $H_{\pm} = \text{SO}(3)_{\pm} \times H'_{\pm} \sim \text{SU}(2)_{\pm} \times H'_{\pm}$ for each subspace $\mathcal{M}_{\pm}$. In the present case, the scalar target space is given by $E_{6(2)}/\text{SU}(6) \times \text{SU}(2)$ of dimension 40.

We now decompose the $E_{6(2)}$ generators $t^{\mathcal{M}} = (T_+^{IJ}, X^{\alpha}, Y^A)$ into those of the compact $\text{SO}(3)_+ \times \text{SU}(6)$ and the non-compact generators Y^A , $A = 1, \dots, 40$. The $\text{SO}(3)_+$ generators denoted by T_+^{IJ} are given by the self-dual part of the full $\text{SO}(4)$ R-symmetry generators T^{IJ} :

$$T_+^{IJ} = T^{IJ} + \frac{1}{2}\epsilon_{IJKL}T^{KL}. \quad (2.2)$$

X^{α} , $\alpha = 1, \dots, 35$ are $\text{SU}(6)$ generators.

The $E_{6(2)}/\text{SU}(6) \times \text{SU}(2)$ manifold can be described by the coset representative L transforming under $E_{6(2)}$ and $\text{SU}(6) \times \text{SU}(2)$ by left and right multiplications, respectively. In particular, L can be used to find the $\text{SU}(6) \times \text{SU}(2)$ composite connections, Q_{+i}^{IJ} and Q_i^{α} , and the vielbein on $E_{6(2)}/\text{SU}(6) \times \text{SU}(2)$, e_i^A , by the relation

$$L^{-1}\partial_i L = \frac{1}{2}Q_{+i}^{IJ}T_+^{IJ} + Q_i^{\alpha}X^{\alpha} + e_i^A Y^A. \quad (2.3)$$

The metric on the target space g_{ij} can be computed from the vielbein e_i^A by the usual relation $g_{ij} = e_i^A e_j^B \delta_{AB}$. Here, indices $A, B = 1, \dots, 40$ can be considered as “flat” target space indices.

Gaugings are implemented by a symmetric and gauge invariant tensor called the embedding tensor $\Theta_{\mathcal{MN}}$ [25, 26]. A viable gauging consistent with supersymmetry is characterized by the embedding tensor that satisfies two consistency conditions. The first condition, called the quadratic constraint, requires that the gauge group is a proper subgroup of the global symmetry G . This constraint is explicitly given by

$$\Theta_{\mathcal{PL}} f^{\mathcal{KL}}_{(\mathcal{M}} \Theta_{\mathcal{N})\mathcal{K}} = 0 \quad (2.4)$$

where $f^{\mathcal{KL}}_{\mathcal{M}}$ are the G -structure constants. Furthermore, supersymmetry requires that the T-tensor defined by the image of the embedding tensor under a map $\mathcal{V}$

$$T_{AB} = \mathcal{V}^{\mathcal{M}}_{\mathcal{A}} \Theta_{\mathcal{MN}} \mathcal{V}^{\mathcal{N}}_{\mathcal{B}} \quad (2.5)$$

satisfies the constraint

$$T^{IJ, KL} = T^{[IJ, KL]} - \frac{4}{N-2} \delta^{[I[K} T^{L]M, MJ]} - \frac{2}{(N-1)(N-2)} \delta^{I[K} \delta^{L]J} T^{MN, MN}. \quad (2.6)$$

The T-tensor transforms under the local H symmetry with the index $\mathcal{A} = \{IJ, \alpha, A\}$. The above constraint implies that the $\boxplus$ representation of $\text{SO}(N)$ is absent from the $T^{IJ, KL}$ component of the T-tensor. Therefore, we can equivalently write this constraint as

$$\mathbb{P}_{\boxplus} T^{IJ, KL} = 0. \quad (2.7)$$

In the case of symmetric target spaces, the condition (2.7) can be expressed as a consistency condition on the embedding tensor which lives in a symmetric product of the adjoint representation of G . It turns out that under the decomposition of this product into irreducible representations of G , there is a unique representation of G , called R_0 , which gives rise to the $\text{SO}(N)$ representation $\boxplus$ under the branching $G \rightarrow \text{SO}(N)$. Therefore, in this case, the constraint (2.7) can be written in a G -covariant way by

$$\mathbb{P}_{R_0} \Theta_{\mathcal{MN}} = 0. \quad (2.8)$$

In the case of $G = E_{6(2)}$, the representation R_0 is given by **2430** [24]. In addition, for symmetric spaces in the form of a coset space, the map $\mathcal{V}$ will be an isomorphism, and its components are given by the relation

$$L^{-1} t^{\mathcal{M}} L = \frac{1}{2} \mathcal{V}^{\mathcal{M}}_{IJ} T^{IJ}_+ + \mathcal{V}^{\mathcal{M}}_{\alpha} X^{\alpha} + \mathcal{V}^{\mathcal{M}}_A Y^A. \quad (2.9)$$

Various components of the T-tensor can be straightforwardly computed from the embedding tensor by using the definition (2.5) and the map $\mathcal{V}$ from (2.9). Combinations of these T-tensor components are used to construct the A_1 , A_2 and A_3 tensors which will appear as fermion mass-like terms and the scalar potential in the gauged Lagrangian. They

are defined by

$$A_1^{IJ} = -\frac{4}{N-2}T^{IM,JM} + \frac{2}{(N-1)(N-2)}\delta^{IJ}T^{MN,MN}, \quad (2.10)$$

$$A_{2j}^{IJ} = \frac{2}{N}T^{IJ}_j + \frac{4}{N(N-2)}f_j^{M(IM}T^{J)M}_m + \frac{2}{N(N-1)(N-2)}\delta^{IJ}f_j^{KL}T^{KL}_m, \quad (2.11)$$

$$A_{3ij}^{IJ} = \frac{1}{N^2} \left[-2D_{(i}D_{j)}A_1^{IJ} + g_{ij}A_1^{IJ} + A_1^{K[I}f_{ij}^{J]K} + 2T_{ij}\delta^{IJ} - 4D_{[i}T_{j]}^{IJ} - 2T_{k[i}f^{IJk}_{j]} \right] \quad (2.12)$$

where D_i is the covariant derivative with respect to ϕ_i . In terms of these tensors, the scalar potential can be written as

$$V = -\frac{4}{N} \left(A_1^{IJ}A_1^{IJ} - \frac{1}{2}Ng^{ij}A_{2i}^{IJ}A_{2j}^{IJ} \right). \quad (2.13)$$

As a final ingredient, we give the supersymmetry transformations of the gravitini ψ_μ^I and the spinor fields χ^{iI} , involving only bosonic fields,

$$\delta\psi_\mu^I = \mathcal{D}_\mu\epsilon^I + gA_1^{IJ}\gamma_\mu\epsilon^J, \quad (2.14)$$

$$\delta\chi^{iI} = \frac{1}{2}(\delta^{IJ}\mathbf{1} - f^{IJ})^i_j \mathcal{D}\phi^j\epsilon^J - gNA_2^{JIi}\epsilon^J \quad (2.15)$$

where the covariant derivative of ϵ^I is given by

$$\mathcal{D}_\mu\epsilon^I = \partial_\mu\epsilon^I + \frac{1}{4}\omega_\mu^{ab}\gamma_{ab}\epsilon^I + \partial_\mu\phi^iQ_i^{IJ}\epsilon^J + g\Theta_{\mathcal{MN}}A_\mu^{\mathcal{M}}\mathcal{V}^{\mathcal{N}IJ}\epsilon^J. \quad (2.16)$$

The covariant derivative on scalars ϕ^i is defined by

$$\mathcal{D}_\mu\phi^i = \partial_\mu\phi^i + \Theta_{\mathcal{MN}}A_\mu^{\mathcal{M}}X^{\mathcal{N}i} \quad (2.17)$$

with $X^{\mathcal{N}i}$ being the Killing vectors associated to the isometries of G/H .

Note that there are only d physical χ^i fields in consistent with d scalar fields ϕ^i as required by supersymmetry. In order to work with the $\text{SO}(N)$ covariant formulation of [24], in which the explicit dependence on f^P is absent, the χ^i fields have been written in terms of the constrained fields χ^{iI} satisfying

$$\chi^{iI} = \frac{1}{N}(\delta^{IJ}\delta_j^i - f^{IJi}_j)\chi^{jJ}. \quad (2.18)$$

We finally note here that, for maximally symmetric vacua, the unbroken supersymmetry corresponds to Killing spinors ϵ^I satisfying the relation

$$A_1^{IK}A_1^{KJ}\epsilon^J = -\frac{V_0}{4}\epsilon^I. \quad (2.19)$$

This relation can be derived by solving the BPS conditions $\delta\psi_\mu^I = 0$ and $\delta\chi^{iI} = 0$ at constant scalars, see the relevant detail in [24].

3 $N = 4$, $\text{SO}(4) \ltimes \mathbf{T}^6$ gauged supergravity and some supersymmetric vacua

We first give an explicit construction of the $E_{6(2)}/\text{SU}(6) \times \text{SU}(2)$ coset space. Generators of the compact $H = \text{SU}(6) \times \text{SU}(2)$ subgroup and the 40 non-compact generators are given in appendix A.

We first describe the embedding of $\text{SO}(4) \ltimes \mathbf{T}^6$ gauge group in the global symmetry group $E_{6(2)}$. In order to do this, we will decompose $E_{6(2)}$ into its maximal subgroup $\text{SO}(6, 4) \times \text{U}(1)$ generated by $\tilde{X}^{\hat{i}\hat{j}}$ and $\hat{X}$ given explicitly in appendix A. The full gauge group $\text{SO}(4) \ltimes \mathbf{T}^6$ can be embedded in $\text{SO}(6, 4)$ as follow.

The semisimple part $\text{SO}(4)$ is given by a diagonal subgroup of $\text{SO}(4) \times \text{SO}(4) \subset \text{SO}(6) \times \text{SO}(4)$ which is in turn the maximal compact subgroup of $\text{SO}(6, 4)$. This $\text{SO}(4)$ is accordingly generated by

$$J^{ab} = \tilde{X}^{ab} + \tilde{X}^{a+6, b+6}, \quad a, b = 1, \dots, 4. \quad (3.1)$$

The other combination $\tilde{X}^{ab} - \tilde{X}^{a+6, b+6}$ together with a suitable set of non-compact generators will give rise to the translational generators $\mathbf{T}^6$ transforming as an adjoint representation of the above mentioned $\text{SO}(4)$. All of the $\mathbf{T}^6$ generators also commute with each other.

In order to identify the appropriate non-compact generators constituting $\mathbf{T}^6$, we decompose the Y^A generators under the $\text{SO}(4)$ part of the gauge group. Under $\text{SU}(6) \times \text{SU}(2)$, the 40 generators Y^A transform as $(\mathbf{20}, \mathbf{2})$. Under $\text{SU}(6) \times \text{SU}(2) \rightarrow \text{SU}(4) \times \text{SU}(2) \times \text{U}(1) \times \text{SU}(2)$, we find

$$(\mathbf{20}, \mathbf{2}) \rightarrow (\mathbf{4}_3, \mathbf{1}_3, \mathbf{2}) + (\bar{\mathbf{4}}_{-3}, \mathbf{1}_{-3}, \mathbf{2}) + (\mathbf{6}_0, \mathbf{2}_0, \mathbf{2}). \quad (3.2)$$

From now on, we will neglect all the $\text{U}(1)$ charges since they will not play any important role in the following analysis. We now decompose $\text{SU}(4) \sim \text{SO}(6) \rightarrow \text{SO}(4) \times \text{SO}(2)$ by the embedding $\mathbf{6} \rightarrow \mathbf{4} + \mathbf{1} + \mathbf{1}$. With the $\text{SO}(2) \sim \text{U}(1)$ charges neglected, further decompositions to $\text{SO}(4) \times \text{SO}(4)$ and finally to $\text{SO}(4)_{\text{diag}}$ respectively give

$$\begin{aligned} \text{SO}(4) \times \text{SO}(4) : & 2 \times (\mathbf{1}, \mathbf{2}; \mathbf{1}, \mathbf{2}) + 2 \times (\mathbf{2}, \mathbf{1}; \mathbf{1}, \mathbf{2}) + (\mathbf{2}, \mathbf{2}; \mathbf{2}, \mathbf{2}) + (\mathbf{1}, \mathbf{1}; \mathbf{2}, \mathbf{2}) \\ \text{SO}(4)_{\text{diag}} : & 3 \times (\mathbf{1}, \mathbf{1}) + 2 \times (\mathbf{1}, \mathbf{3}) + 4 \times (\mathbf{2}, \mathbf{2}) + (\mathbf{3}, \mathbf{3}) \\ & + (\mathbf{1}, \mathbf{3}) + (\mathbf{3}, \mathbf{1}) \end{aligned} \quad (3.3)$$

where we have denoted the $\text{SO}(4) \sim \text{SU}(2) \times \text{SU}(2)$ representations by $(\mathbf{2j}_1 + \mathbf{1}, \mathbf{2j}_2 + \mathbf{1})$ with j_1, j_2 corresponding to the spins of the two $\text{SU}(2)$'s.

The last two representations in (3.3) are the adjoint representation of $\text{SO}(4)_{\text{diag}}$. These will be part of the gauge generators $\mathbf{T}^6$. Explicitly, we find that these generators are given by

$$t^{ab} = \tilde{X}^{ab} - \tilde{X}^{a+6, b+6} + \tilde{X}^{a, b+6} + \tilde{X}^{a+6, b}, \quad a, b = 1, \dots, 4. \quad (3.4)$$

Note that $\tilde{X}^{a, b+6}$ and $\tilde{X}^{a+6, b}$ are non-compact generators of $E_{6(2)}$. It can be verified that (J^{ab}, t^{ab}) generators satisfy the $\text{SO}(4) \ltimes \mathbf{T}^6$ algebra

$$[J^{ab}, J^{cd}] = -4\delta^{[a[c} J^{d]b]}, \quad [J^{ab}, t^{cd}] = -4\delta^{[a[c} t^{d]b]}, \quad [t^{ab}, t^{cd}] = 0. \quad (3.5)$$

The non-vanishing components of the embedding tensor corresponding to the full gauge group are denoted by $\Theta_{\mathbf{ab}}$ and $\Theta_{\mathbf{bb}}$ with $\mathbf{a}$ and $\mathbf{b}$ associated to the J^{ab} and t^{ab} parts, respectively [3]. It turns out that the embedding tensor satisfying the linear and quadratic constraints given in (2.4) and (2.8) is given by

$$\Theta = g_1 \Theta_{\mathbf{ab}} + g_2 \Theta_{\mathbf{bb}} \quad (3.6)$$

where both $\Theta_{ab,cd}^{\mathbf{ab}}$ and $\Theta_{ab,cd}^{\mathbf{bb}}$ are given by ϵ_{abcd} . This is much similar to the $N = 8$ and $N = 4$ theories studied in [6] and [8]. It should be noted that supersymmetry allows for two independent coupling constants.

3.1 Maximally supersymmetric vacua with $\text{SO}(4)$ symmetry

We now consider some vacua of the $N = 4$ gauged supergravity constructed previously. From equation (3.3), we see that there are three scalars which are singlets under $\text{SO}(4)$. These singlets correspond to the following non-compact generators

$$\begin{aligned} \hat{Y}_1 &= Y_1 - Y_4 + Y_5 - Y_7 - Y_9 - Y_{12} + Y_{13} + Y_{15}, \\ \hat{Y}_2 &= Y_2 + Y_3 - Y_6 + Y_8 + Y_{10} - Y_{11} - Y_{14} - Y_{16}, \\ \hat{Y}_3 &= Y_{17} + Y_{19} + Y_{24} + Y_{26} + Y_{29} - Y_{31} + Y_{36} - Y_{38}. \end{aligned} \quad (3.7)$$

The coset representative L can be parametrized by

$$L = e^{\Phi_1 \hat{Y}_1} e^{\Phi_2 \hat{Y}_2} e^{\Phi_3 \hat{Y}_3}. \quad (3.8)$$

By using the formulae in section 2 and appendix B, we obtain the scalar potential for (Φ_1, Φ_2, Φ_3) given by

$$\begin{aligned} V &= 32g_1 [4 \cosh(2\Phi_1) \cosh(2\Phi_2) \cosh(2\Phi_3) - 4 \sinh(2\Phi_3)]^2 \times \\ &\quad \times [4g_2 \cosh(2\Phi_1) \cosh(2\Phi_2) \cosh(2\Phi_3) - 4g_2 \sinh(2\Phi_3) - 6g_1]. \end{aligned} \quad (3.9)$$

From this potential, it can be readily verified that the $\text{SO}(4) \ltimes \mathbf{T}^6$ gauged supergravity admits a maximally supersymmetric AdS_3 critical point. By shifting the values of scalar fields at the vacuum, we can bring the critical point to $L = \mathbf{I}_{27 \times 27}$ at which $\Phi_1 = \Phi_2 = \Phi_3 = 0$. This can also be achieved by setting $g_2 = g_1 = g$. The cosmological constant at the critical point is given by $V_0 = -1024g^2$. In our convention, the AdS_3 radius is given by

$$L = \sqrt{-\frac{1}{V_0}} = \frac{1}{32g} \quad (3.10)$$

where we have taken $g > 0$ for definiteness.

It can be checked by using the supersymmetry transformations (2.14) and (2.15) or the relation (2.19) that this critical point preserves the full eight supercharges corresponding to $N = (4, 0)$ superconformal symmetry in the dual two-dimensional SCFT. All of the scalar masses at this critical point are given in table 1. In the table, we have also given the dimensions of the dual operators in the dual SCFT according to the relation $m^2 L^2 =$

SO(4) representations	$m^2 L^2$	Δ
$(\mathbf{1}, \mathbf{1})$	$0_{(\times 2)}, 3$	$2, 3$
$(\mathbf{1}, \mathbf{3}) + (\mathbf{3}, \mathbf{1})$	$0_{(\times 6)}$	2
$(\mathbf{1}, \mathbf{3})$	$-\frac{8}{9}_{(\times 6)}$	$\frac{2}{3}, \frac{4}{3}$
$(\mathbf{2}, \mathbf{2})$	$-\frac{3}{4}_{(\times 16)}$	$\frac{1}{2}, \frac{3}{2}$
$(\mathbf{3}, \mathbf{3})$	$-1_{(\times 9)}$	1

Table 1. Scalar masses at the $N = 4$ supersymmetric AdS_3 critical point and the corresponding dimensions of the dual operators

$\Delta(\Delta - 2)$. All of the masses agree with the behavior of the corresponding scalar fields near the AdS_3 critical points.

From the table, we see the presence of six massless scalars in the adjoint representations of SO(4), $(\mathbf{1}, \mathbf{3}) + (\mathbf{3}, \mathbf{1})$. These are Goldstone bosons corresponding to the symmetry breaking $SO(4) \ltimes \mathbf{T}^6 \rightarrow SO(4)$ at the vacuum. Furthermore, there are additional massless scalars which are singlets under SO(4). These are expected to correspond to marginal deformations in the dual SCFT. The deformations preserve all supersymmetry as well as the full SO(4) symmetry. These deformations can be given explicitly by the relation

$$\cosh(2\Phi_1) \cosh(2\Phi_2) \cosh(2\Phi_3) = 1 + \sinh(2\Phi_3). \quad (3.11)$$

When $\Phi_1 = \Phi_2 = 0$, there is only one solution $\Phi_3 = 0$. Non-vanishing values of Φ_1 and Φ_2 give rise to the same value of V_0 , unbroken SO(4) symmetry and the same number of supersymmetry. Therefore, Φ_1 and Φ_2 correspond to flat directions of the potential.

There is another class of vacua given by the relation

$$\cosh(2\Phi_1) \cosh(2\Phi_2) = \tanh(2\Phi_3). \quad (3.12)$$

This gives supersymmetric Minkowski vacua in three dimensions with $V_0 = 0$.

3.2 Half-supersymmetric domain walls

We now move to half-supersymmetric vacuum solutions. To find these solutions, we set up the corresponding BPS equations from the supersymmetry transformations (2.14) and (2.15). The three-dimensional metric is taken to be the standard domain wall ansatz

$$ds^2 = e^{A(r)} dx_{1,1}^2 + dr^2. \quad (3.13)$$

With the projection $\gamma_r \epsilon^I = -\epsilon^I$ corresponding to $N = (4, 0)$ supersymmetry in two dimensions, equations $\delta\chi^{iI} = 0$ and $\delta\psi_\mu^I = 0$ for $\mu = 0, 1$ give

$$\Phi'_1 = \frac{16 \sinh(2\Phi_1)}{\cosh(2\Phi_2) \cosh(2\Phi_3)} [g_1 - g_2 \cosh(2\Phi_1) \cosh(2\Phi_2) \cosh(2\Phi_3) + g_2 \sinh(2\Phi_3)], \quad (3.14)$$

$$\Phi'_2 = \frac{16 \sinh(2\Phi_2) \cosh(2\Phi_1)}{\cosh(2\Phi_3)} [g_1 - g_2 \cosh(2\Phi_1) \cosh(2\Phi_2) \cosh(2\Phi_3) + g_2 \sinh(2\Phi_3)], \quad (3.15)$$

$$\begin{aligned}\Phi_3' &= 16 [g_1 - g_2 \cosh(2\Phi_1) \cosh(2\Phi_2) \cosh(2\Phi_3) + g_2 \sinh(2\Phi_3)] \times \\ &\quad \times [\cosh(2\Phi_1) \cosh(2\Phi_2) \sinh(2\Phi_3) - \cosh(2\Phi_3)] ,\end{aligned}\quad (3.16)$$

$$\begin{aligned}A' &= 32 [2g_1 - g_2 \cosh(2\Phi_1) \cosh(2\Phi_2) \cosh(2\Phi_3) + g_2 \sinh(2\Phi_3)] \times \\ &\quad \times [\cosh(2\Phi_1) \cosh(2\Phi_2) \cosh(2\Phi_3) - \sinh(2\Phi_3)] ,\end{aligned}\quad (3.17)$$

where $'$ denotes the r -derivative. All of these equations satisfy the second-order field equations. Some details of this verification is given in appendix C.

We first consider the case $\Phi_1 = \Phi_2 = 0$. It can be readily seen that the first two equations are identically satisfied. We are left with two equations

$$\Phi_3' = 16e^{-4\Phi_3}(g_2 - g_1e^{2\Phi_3}), \quad (3.18)$$

$$A' = 32e^{-4\Phi_3}(2g_1e^{2\Phi_3} - g_2). \quad (3.19)$$

For $g_2 \neq 0$, the Φ_3' equation has a critical point at $\Phi_3 = \frac{1}{2} \ln \left[\frac{g_2}{g_1} \right]$ while the A' equation gives $A = \frac{32g_1^2}{g_2}r + C$. This is the maximally supersymmetric AdS_3 critical point identified previously.

Equations (3.18) and (3.19) can be solved explicitly with the corresponding solution

$$\begin{aligned}A &= -2\Phi_3 - \ln(e^{2\Phi_3}g_1 - g_2) + C_1, \\ 32g_1^2r &= -g_1e^{2\Phi_3} - g_2 \ln(e^{2\Phi_3}g_1 - g_2) + C_2.\end{aligned}\quad (3.20)$$

It should be noted that the integration constants C_2 and C_1 can be removed by shifting the coordinate r and rescaling the x^0 and x^1 coordinates, respectively. The solution interpolates between $N = 4AdS_3$ critical point at $r \rightarrow \infty$ to a half-supersymmetric domain wall at a finite value of r . At large $|\Phi_3|$, we find

$$\Phi_3 \sim \frac{1}{2} \ln(C - 32g_1r), \quad A \sim -\ln(C - 32g_1r) \quad (3.21)$$

with C being a constant. We have set $g_2 = g_1$ for simplicity. The metric at $r \sim \frac{C}{32g_1}$ becomes

$$ds^2 = (C - 32g_1r)^{-2} dx_{1,1}^2 + dr^2 \quad (3.22)$$

which takes the form of a domain wall. However, the scalar potential becomes unbounded when $\Phi_3 \rightarrow -\infty$. The singularity of the above metric is then unphysical by the criterion of [27]. Therefore, the holographic interpretation of the solution as an RG flow between an $N = (4, 0)$ SCFT and an $N = (4, 0)$ non-conformal field theory in two dimensions cannot be given at least in the three-dimensional framework. It would be interesting to further investigate the singularity in the context of higher dimensions in which this solution is embedded. Note also that since the operator dual to Φ_3 is irrelevant, we would expect the $N = (4, 0)$ SCFT to appear in the IR.

There is another simple exact solution to equations (3.14), (3.15), (3.16) and (3.17) namely

$$\Phi_3 = 0, \quad g_2 \cosh(2\Phi_1) \cosh(2\Phi_2) = g_1, \quad A = \frac{32g_1^2}{g_2}r + C. \quad (3.23)$$

This solution corresponds to a marginal deformation of the supersymmetric AdS_3 critical point.

We now move to another type of domain walls which is a half-supersymmetric vacuum of the theory without any limit with enhanced supersymmetry. Recall that supersymmetry allows for two independent gauge couplings g_1 and g_2 , by setting $g_2 = 0$, we also have a consistent gauged supergravity. In this case, the resulting gauged supergravity possesses a half-supersymmetric domain wall vacuum. We will present a simple solution of this type. With $\Phi_1 = \Phi_2 = g_2 = 0$, the BPS equations become

$$\begin{aligned}\Phi_3' &= -16g_1e^{-2\Phi_3}, \\ A' &= 64g_1e^{-2\Phi_3}\end{aligned}\tag{3.24}$$

which admit a solution

$$\Phi_3 = \frac{1}{2} \ln(C' - 32g_1r), \quad A = -2 \ln(C' - 32g_1r)\tag{3.25}$$

with an integration constant C' . This solution gives a domain wall metric

$$ds^2 = (C' - 32g_1r)^{-4} dx_{1,1}^2 + dr^2.\tag{3.26}$$

For $\Phi_2 = 0$ which satisfies equation (3.15), a more general solution can be found by treating Φ_1 as an independent variable. After combining equations (3.16) and (3.17) with (3.14), we can solve for Φ_3 and A as functions of Φ_1 . The result is then substituted in equation (3.14) to find the solution for $\Phi_1(r)$. The resulting solution is given by

$$\begin{aligned}\Phi_3 &= \frac{1}{4} \ln \left[\frac{C_1 - \coth \Phi_1}{\tanh \Phi_1 - C_1} \right], \\ A &= \ln(1 - e^{4\Phi_1}) - \frac{1}{2} \ln \left[(1 + 4C_1)^2 - (1 - 4C_1)^2 e^{4\Phi_1} \right] \\ &\quad - \frac{1}{2} \ln \left[\left[(1 - 4C_1)^2 e^{4\Phi_1} - (1 + 4C_1)^2 \right] g_1^2 - 16C_1^2 (e^{4\Phi_1} - 1) g_2^2 \right. \\ &\quad \left. - 16C_1 g_1 g_2 e^{2\Phi_1} \sqrt{\sinh(2\Phi_1) [8C_1 \cosh(2\Phi_1) - (1 + 16C_1^2) \sinh(2\Phi_1)]} \right], \\ 128C_1 g_1^2 r &= g_1 \sqrt{8C_1 \coth(2\Phi_1) - 16C_1^2 - 1} + 2C_1 g_2 \ln \sinh(2\Phi_1) \\ &\quad - 2C_1 g_2 \ln \left[(g_1^2 + (g_1^2 + g_2^2) 16C_1^2) \sinh(2\Phi_1) - 8C_1 g_1^2 \cosh(2\Phi_1) \right] \\ &\quad - 4C_1 g_2 \tanh^{-1} \left[\frac{g_1 \sqrt{8C_1 \coth(2\Phi_1) - 16C_1^2 - 1}}{4C_1 g_2} \right].\end{aligned}\tag{3.27}$$

In the above equations, we have neglected additive integration constants in $\Phi_1(r)$ and $A(\Phi_1)$ solutions.

Apart from these solutions, we are not able to completely solve the BPS equations with all scalars non-vanishing in an analytic form. We will however give a partial result on this solution since it might be useful for further investigation. It turns out that combining equations (3.14) and (3.15) gives an equation for Φ_2 as a function of Φ_1 with a solution

$$\Phi_2 = \frac{1}{4} \ln \left[\frac{1 - e^C \sinh(2\Phi_1)}{1 + e^C \sinh(2\Phi_1)} \right].\tag{3.28}$$

If all of the integration constants are set to zero, a simple solution for Φ_3 can also be found

$$\Phi_3 = \cosh^{-1} \left[\frac{1}{2} \sqrt{2 - \operatorname{csch}(2\Phi_1) \sqrt{\cosh(4\Phi_1) - 3}} \right]. \quad (3.29)$$

With these relations, equation (3.17) would in principle give a solution for $A(\Phi_1)$ while equation (3.14) would give a solution for $\Phi_1(r)$. We have not succeeded in obtaining an analytic form for these solutions.

4 Conclusions

In this paper, we have constructed $N = 4$ gauged supergravity in three dimensions with $\mathrm{SO}(4) \ltimes \mathbf{T}^6$ gauge group and $E_{6(2)}/\mathrm{SU}(6) \times \mathrm{SU}(2)$ scalar manifold. We have studied some of the maximally supersymmetric and half-supersymmetric vacua of this gauged supergravity. The $N = 4 \mathrm{AdS}_3$ critical point with $\mathrm{SO}(4)$ symmetry corresponds to an $N = (4, 0)$ SCFT in two dimensions. We have given the full scalar mass spectrum at this critical point which might be useful for other holographic applications. In addition, some half-supersymmetric domain wall solutions have also been explicitly given in an analytic form. According to the DW/QFT correspondence [28], we expect these solutions to be dual to $N = (4, 0)$ non-conformal field theories in two dimensions.

We have also identified two flat directions of the scalar potential corresponding to exactly marginal deformations of the $N = (4, 0)$ SCFT that preserve all supersymmetries and $\mathrm{SO}(4)$ symmetry. Remarkably, these flat directions are not Goldstone bosons. This is in contrast to the four-dimensional $N = 4$ gauged supergravity studied in [29]. In that case, all flat directions correspond to Goldstone bosons. It would be interesting to identify the $N = (4, 0)$ SCFT and two-dimensional non-conformal field theories dual to the vacua identified here. Further investigations of the scalar potential in other scalar sectors invariant under smaller residual gauge symmetry could be useful for the study of other deformations of the dual $N = (4, 0)$ SCFT in particular relevant deformations given by scalars in $(\mathbf{1}, \mathbf{3})$, $(\mathbf{2}, \mathbf{2})$ and $(\mathbf{3}, \mathbf{3})$ representations.

Due to the equivalence between the gauged supergravity constructed here and the Yang-Mills gauged supergravity with $\mathrm{SO}(4)$ gauge group, it is possible that this theory might be obtained from higher dimensions. The ungauged $N = 4$ supergravity with $E_{6(2)}/\mathrm{SU}(6) \times \mathrm{SU}(2)$ scalar manifold can be obtained from a reduction on a 3-torus (T^3) of the minimal supergravity in six dimensions coupled to three tensor and four vector multiplets. The three tensor multiplets consist of three scalars parametrized by the $\mathrm{SO}(3, 1)/\mathrm{SO}(3)$ coset manifold. After dimensional reduction and dualization of the vector fields coming from the six-dimensional metric and the (anti) self-dual tensor fields, the resulting three-dimensional supergravity consists of 40 scalars as required by the dimension of $E_{6(2)}/\mathrm{SU}(6) \times \mathrm{SU}(2)$ coset manifold.

We then expect that the $\mathrm{SO}(4) \ltimes \mathbf{T}^6$ gauged supergravity constructed here should arise from a dimensional reduction of this six-dimensional supergravity on a 3-sphere (S^3). Along the line of this uplifting, it could be useful to compute vector and fermion masses and match with the $\mathrm{AdS}_3 \times S^3$ spectrum of $N = (1, 0)$ six-dimensional supergravity carried

out in [32]. It should also be remarked here that, when coupled to hypermultiplets with hyper-scalars described by $\mathcal{M}_-$ manifold, the six-dimensional supergravity could give rise to three-dimensional gauged supergravity with two scalar target manifolds $\mathcal{M}_+ \times \mathcal{M}_-$. It would be interesting to construct an explicit reduction ansatz similar to the recent result in [16]. The result will be very useful in uplifting the three-dimensional solutions to higher dimensions. We leave this issue and related ones for future works.

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A Generators of $E_{6(2)}$ and relevant subgroups

E_6 generators in the fundamental representation, used throughout this paper, have been constructed in [30] and [31]. All of these 78 generators are denoted by c_i , $i = 1, \dots, 78$ and are normalized according to

$$\text{Tr}(c_i c_j) = -6\delta_{ij}. \quad (\text{A.1})$$

In order to construct the non-compact form $E_{6(2)}$ from the compact E_6 , we first identify the maximal compact subgroup $H = \text{SU}(6) \times \text{SU}(2)$ with the $\text{SU}(2)$ factor corresponding to the $\text{SU}(2)_+$ subgroup of the full $\text{SO}(4)$ R-symmetry. We then apply the “Weyl unitarity trick” to the remaining 40 generators to make them non-compact.

A.1 $\text{SU}(6) \times \text{SU}(2)$ subgroup and non-compact generators

The R-Symmetry group $\text{SU}(2)_+ \sim \text{SO}(3)_+$ is generated by

$$T^{12} = -\frac{1}{2}(c_{51} + c_{78}), \quad T^{13} = \frac{1}{2}(c_{52} - c_{77}), \quad T^{23} = \frac{1}{2}(c_{36} + \tilde{c}_{53}). \quad (\text{A.2})$$

The generators of the group $H' = \text{SU}(6)$ are given by

$$\begin{aligned} X_i &= c_i, & i &= 1, \dots, 15, \\ X_{16} &= \frac{1}{\sqrt{2}}(c_{52} + c_{77}), & X_{17} &= \frac{1}{\sqrt{2}}(c_{51} - c_{78}), & X_{18} &= \frac{1}{\sqrt{2}}(\tilde{c}_{53} - c_{36}), \\ X_{19} &= \frac{1}{\sqrt{2}}(c_{22} + c_{60}), & X_{20} &= \frac{1}{\sqrt{2}}(c_{23} - c_{59}), & X_{21} &= \frac{1}{\sqrt{2}}(c_{24} + c_{61}), \\ X_{22} &= \frac{1}{\sqrt{2}}(c_{25} - c_{58}), & X_{23} &= \frac{1}{\sqrt{2}}(c_{26} + c_{57}), & X_{24} &= \frac{1}{\sqrt{2}}(c_{27} + c_{55}), \\ X_{25} &= \frac{1}{\sqrt{2}}(c_{28} - c_{54}), & X_{26} &= \frac{1}{\sqrt{2}}(c_{29} - c_{56}), & X_{27} &= \frac{1}{\sqrt{2}}(c_{37} + c_{64}), \\ X_{28} &= \frac{1}{\sqrt{2}}(c_{38} - c_{66}), & X_{29} &= \frac{1}{\sqrt{2}}(c_{39} - c_{62}), & X_{30} &= \frac{1}{\sqrt{2}}(c_{40} + c_{67}), \\ X_{31} &= \frac{1}{\sqrt{2}}(c_{41} + c_{63}), & X_{32} &= \frac{1}{\sqrt{2}}(c_{42} - c_{65}), & X_{33} &= \frac{1}{\sqrt{2}}(c_{43} - c_{69}), \\ X_{34} &= \frac{1}{\sqrt{2}}(c_{44} + c_{68}), & X_{35} &= \tilde{c}_{70} \end{aligned} \quad (\text{A.3})$$

where $\tilde{c}_{53}$ and $\tilde{c}_{70}$ generators are defined by

$$\tilde{c}_{53} = \frac{1}{2}c_{53} + \frac{\sqrt{3}}{2}c_{70}, \quad \tilde{c}_{70} = -\frac{\sqrt{3}}{2}c_{53} + \frac{1}{2}c_{70}. \quad (\text{A.4})$$

It is useful to note that the $\text{SU}(4) \times \text{SU}(2) \times \text{U}(1) \sim \text{SO}(6) \times \text{SO}(3) \times \text{U}(1)$ subgroup of $\text{SU}(6)$ is generated by X_i , $i = 1, \dots, 15$, (X_{16}, X_{17}, X_{18}) and X_{35} , respectively.

With the compact H generators defined above, the 40 non-compact generators are accordingly given by

$$\begin{aligned} Y_1 &= \frac{i}{2}(c_{22} - c_{60}), & Y_2 &= \frac{i}{2}(c_{23} + c_{59}), & Y_3 &= \frac{i}{2}(c_{24} - c_{61}), \\ Y_4 &= \frac{i}{2}(c_{25} + c_{58}), & Y_5 &= \frac{i}{2}(c_{26} - c_{57}), & Y_6 &= \frac{i}{2}(c_{27} - c_{55}), \\ Y_7 &= \frac{i}{2}(c_{28} + c_{54}), & Y_8 &= \frac{i}{2}(c_{29} + c_{56}), & Y_9 &= \frac{i}{2}(c_{37} - c_{64}), \\ Y_{10} &= \frac{i}{2}(c_{38} + c_{66}), & Y_{11} &= \frac{i}{2}(c_{39} + c_{62}), & Y_{12} &= \frac{i}{2}(c_{40} - c_{67}), \\ Y_{13} &= \frac{i}{2}(c_{41} - c_{63}), & Y_{14} &= \frac{i}{2}(c_{42} + c_{65}), & Y_{15} &= \frac{i}{2}(c_{43} + c_{69}), \\ Y_{16} &= \frac{i}{2}(c_{44} - c_{68}), & Y_{17} &= \frac{i}{2}(c_{16} + c_{45}), & Y_{18} &= -\frac{i}{2}(c_{17} + c_{46}), \\ Y_{19} &= \frac{i}{2}(c_{18} + c_{47}), & Y_{20} &= -\frac{i}{2}(c_{19} + c_{48}), & Y_{21} &= -\frac{i}{2}(c_{20} + c_{49}), \\ Y_{22} &= -\frac{i}{2}(c_{21} + c_{50}), & Y_{23} &= \frac{i}{2}(c_{30} - c_{71}), & Y_{24} &= \frac{i}{2}(c_{72} - c_{31}), \\ Y_{25} &= \frac{i}{2}(c_{32} - c_{73}), & Y_{26} &= \frac{i}{2}(c_{74} - c_{33}), & Y_{27} &= \frac{i}{2}(c_{75} - c_{34}), \\ Y_{28} &= \frac{i}{2}(c_{76} - c_{35}), & Y_{29} &= \frac{i}{2}(c_{16} - c_{45}), & Y_{30} &= \frac{i}{2}(c_{46} - c_{17}), \\ Y_{31} &= \frac{i}{2}(c_{18} - c_{47}), & Y_{32} &= \frac{i}{2}(c_{48} - c_{19}), & Y_{33} &= \frac{i}{2}(c_{49} - c_{20}), \\ Y_{34} &= \frac{i}{2}(c_{50} - c_{21}), & Y_{35} &= \frac{i}{2}(c_{30} + c_{71}), & Y_{36} &= -\frac{i}{2}(c_{31} + c_{72}), \\ Y_{37} &= \frac{i}{2}(c_{32} + c_{73}), & Y_{38} &= -\frac{i}{2}(c_{33} + c_{74}), & Y_{39} &= -\frac{i}{2}(c_{34} + c_{75}), \\ Y_{40} &= -\frac{i}{2}(c_{35} + c_{76}). \end{aligned} \quad (\text{A.5})$$

For advantages of future investigations, we give the non-compact generators corresponding to scalar fields that are singlets under various subgroups of the $\text{SO}(4)$ gauge symmetry. The following results can be obtained by further decompositions of the $\text{SO}(4)$ representations given in (3.3).

- $\text{SO}(3)_{\text{diag}} \subset \text{SO}(3) \times \text{SO}(3) \sim \text{SO}(4)$ singlets:

$$\begin{aligned} \hat{Y}_1 &= Y_{17} + Y_{19} + Y_{24} + Y_{29} - Y_{31} + Y_{36}, \\ \hat{Y}_2 &= Y_4 + Y_7 + Y_{12} - Y_{15}, & \hat{Y}_3 &= Y_{26} - Y_{38}, \\ \hat{Y}_4 &= Y_6 - Y_8 + Y_{14} + Y_{16}, & \hat{Y}_5 &= Y_{27} - Y_{39}, \end{aligned}$$

$$\begin{aligned}\hat{Y}_6 &= Y_2 + Y_3 + Y_{10} - Y_{11}, & \hat{Y}_7 &= Y_{28} - Y_{40}, \\ \hat{Y}_8 &= Y_1 + Y_5 - Y_9 + Y_{13}\end{aligned}\tag{A.6}$$

- $SU(2) \times SO(2)_s$ singlets:

$$\begin{aligned}\hat{Y}_1 &= Y_{17} + Y_{19} + Y_{24} + Y_{26} + Y_{29} - Y_{31} + Y_{36} - Y_{38}, \\ \hat{Y}_2 &= Y_{18} + Y_{20} - Y_{23} - Y_{25} + Y_{30} - Y_{32} - Y_{35} + Y_{37}, \\ \hat{Y}_3 &= Y_1 - Y_4 + Y_5 - Y_7, & \hat{Y}_4 &= Y_2 + Y_3 - Y_6 + Y_8, \\ \hat{Y}_5 &= Y_9 + Y_{12} - Y_{13} - Y_{15}, & \hat{Y}_6 &= Y_{10} - Y_{11} - Y_{14} - Y_{16}\end{aligned}\tag{A.7}$$

- $SU(2)_s$ singlets:

$$\begin{aligned}\hat{Y}_1 &= Y_1 - Y_4 + Y_5 - Y_7 - Y_9 - Y_{12} + Y_{13} + Y_{15}, \\ \hat{Y}_2 &= Y_2 + Y_3 - Y_6 + Y_8 + Y_{10} - Y_{11} - Y_{14} - Y_{16}, \\ \hat{Y}_3 &= Y_{17} + Y_{24} - Y_{31} - Y_{38}, & \hat{Y}_4 &= Y_{18} - Y_{23} + Y_{32} - Y_{37}, \\ \hat{Y}_5 &= Y_{19} + Y_{26} + Y_{29} + Y_{36}, & \hat{Y}_6 &= Y_{20} - Y_{25} - Y_{30} + Y_{35}\end{aligned}\tag{A.8}$$

The $SU(2)_s$ denotes the $SU(2)$ subgroup of $SO(4)$ generated by self-dual $SO(4)$ generators with $SO(2)_s \subset SU(2)_s$.

A.2 $SO(6, 4) \times U(1)$ subgroup

The $U(1)$ is generated by $\hat{X} = \tilde{c}_{70}$ while the $SO(6, 4)$ generators are given by

$$\begin{aligned}\tilde{X}^{12} &= c_1, & \tilde{X}^{13} &= -c_2, & \tilde{X}^{23} &= c_3, & \tilde{X}^{34} &= c_6, \\ \tilde{X}^{14} &= c_4, & \tilde{X}^{24} &= -c_5, & \tilde{X}^{15} &= c_7, & \tilde{X}^{25} &= -c_8, \\ \tilde{X}^{35} &= c_9, & \tilde{X}^{45} &= -c_{10}, & \tilde{X}^{56} &= -c_{15}, & \tilde{X}^{16} &= c_{11}, \\ \tilde{X}^{26} &= -c_{12}, & \tilde{X}^{46} &= -c_{14}, & \tilde{X}^{36} &= c_{13}, & \tilde{X}^{17} &= ic_{16}, \\ \tilde{X}^{27} &= -ic_{17}, & \tilde{X}^{47} &= -ic_{19}, & \tilde{X}^{37} &= ic_{18}, & \tilde{X}^{67} &= -ic_{21}, \\ \tilde{X}^{57} &= -ic_{20}, & \tilde{X}^{78} &= -c_{36}, & \tilde{X}^{18} &= ic_{30}, & \tilde{X}^{28} &= -ic_{31}, \\ \tilde{X}^{48} &= -ic_{33}, & \tilde{X}^{38} &= ic_{32}, & \tilde{X}^{68} &= -ic_{35}, & \tilde{X}^{58} &= -ic_{34}, \\ \tilde{X}^{29} &= -ic_{46}, & \tilde{X}^{19} &= ic_{45}, & \tilde{X}^{49} &= -ic_{48}, & \tilde{X}^{39} &= ic_{47}, \\ \tilde{X}^{69} &= -ic_{50}, & \tilde{X}^{59} &= -ic_{49}, & \tilde{X}^{89} &= -c_{52}, & \tilde{X}^{79} &= -c_{51}, \\ \tilde{X}^{1,10} &= -ic_{71}, & \tilde{X}^{2,10} &= ic_{72}, & \tilde{X}^{3,10} &= -ic_{73}, & \tilde{X}^{4,10} &= ic_{74}, \\ \tilde{X}^{5,10} &= ic_{75}, & \tilde{X}^{6,10} &= ic_{76}, & \tilde{X}^{7,10} &= c_{77}, & \tilde{X}^{8,10} &= c_{78}, \\ \tilde{X}^{9,10} &= -\tilde{c}_{53}.\end{aligned}\tag{A.9}$$

All generators are labelled by $SO(6, 4)$ adjoint indices with $\tilde{X}^{\hat{i}\hat{j}} = -\tilde{X}^{\hat{j}\hat{i}}$, $\hat{i}, \hat{j} = 1, \dots, 10$. The compact subgroup $SO(6) \times SO(4)$ is generated by $\tilde{X}^{\hat{i}\hat{j}}$, $\hat{i}, \hat{j} = 1, \dots, 6$ and $\tilde{X}^{\hat{i}\hat{j}}$, $\hat{i}, \hat{j} = 7, \dots, 10$, respectively. This coincides with the $SO(6) \times SO(3) \subset SU(6)$ together with the $SO(3)$ R-symmetry. The 24 non-compact generators are identified with $\tilde{X}^{\hat{i}, \hat{j}+6}$ for $\hat{i} = 1, \dots, 6$ and $\hat{j} = 1, \dots, 4$.

B Essential formulae

For a general symmetric space of the form G/H with G and $H = \text{SO}(N) \times H'$ being the global and local symmetry groups, the G algebra is given by

$$\begin{aligned} [T^{IJ}, T^{KL}] &= -4\delta^{[I[K}T^{L]J]}, & [T^{IJ}, Y^A] &= -\frac{1}{2}f^{IJ,AB}Y_B, \\ [X^\alpha, X^\beta] &= f^{\alpha\beta}_\gamma X^\gamma, & [X^\alpha, Y^A] &= h^\alpha_B{}^A Y^B, \\ [Y^A, Y^B] &= \frac{1}{4}f_{IJ}^{AB}T^{IJ} + \frac{1}{8}C_{\alpha\beta}h^{\beta AB}X^\alpha. \end{aligned} \quad (\text{B.1})$$

$C_{\alpha\beta}$ is an invariant tensor of H' , and h_{AB}^α are antisymmetric tensors that commute with f_{AB}^{IJ} .

Using the above algebra, we find that components of f^{IJ} tensor written in flat coset space indices are given by

$$f_{AB}^{IJ} = -\frac{2}{3}\text{Tr}(Y^B [T^{IJ}, Y^A]). \quad (\text{B.2})$$

In term of the coset representative, various components of the $\mathcal{V}$ map can be computed by using the relations

$$\begin{aligned} \mathcal{V}_a{}^{ab,IJ} &= -\frac{1}{3}\text{Tr}(L^{-1}J^{ab}LT^{IJ}), & \mathcal{V}_b{}^{ab,IJ} &= -\frac{1}{3}\text{Tr}(L^{-1}t^{ab}LT^{IJ}), \\ \mathcal{V}_a{}^{ab,A} &= \frac{1}{3}\text{Tr}(L^{-1}J^{ab}LY^A), & \mathcal{V}_b{}^{ab,A} &= \frac{1}{3}\text{Tr}(L^{-1}t^{ab}LY^A). \end{aligned} \quad (\text{B.3})$$

The T-tensors are then obtained from

$$\begin{aligned} T^{IJ,KL} &= g_1 \left(\mathcal{V}_a{}^{ab,IJ} \mathcal{V}_b{}^{cd,KL} + \mathcal{V}_b{}^{ab,IJ} \mathcal{V}_a{}^{cd,KL} \right) \epsilon_{abcd} + g_2 \mathcal{V}_b{}^{ab,IJ} \mathcal{V}_b{}^{cd,KL} \epsilon_{abcd}, \\ T^{IJ,A} &= g_1 \left(\mathcal{V}_a{}^{ab,IJ} \mathcal{V}_b{}^{cd,A} + \mathcal{V}_b{}^{ab,IJ} \mathcal{V}_a{}^{cd,A} \right) \epsilon_{abcd} + g_2 \mathcal{V}_b{}^{ab,IJ} \mathcal{V}_b{}^{cd,A} \epsilon_{abcd}. \end{aligned} \quad (\text{B.4})$$

From these relations, the tensors A_1^{IJ} , A_{2i}^{IJ} and the scalar potential can be straightforwardly computed.

C Field equations for $\text{SO}(4)$ singlet scalars and the metric

In this appendix, we explicitly verify that the BPS equations given in (3.14), (3.15), (3.16), and (3.17) indeed satisfy the corresponding second-order field equations. With only scalar fields and the metric, the Lagrangian of the gauged supergravity read, in our convention,

$$\mathcal{L} = \sqrt{-g} \left[\frac{1}{2}R - \frac{1}{2}P_\mu^A P^{A\mu} - V \right] \quad (\text{C.1})$$

where the scalar potential is given in (3.9). The scalar kinetic term is written in term of the coset vielbein e_i^A as

$$P_\mu^A = \partial_\mu \phi^i e_i^A. \quad (\text{C.2})$$

It should be noted that, with the relation $g_{ij} = e_i^A e_j^A$, the scalar kinetic term is the same as that given in [24]

$$-\frac{1}{2}P_\mu^A P^{A\mu} = -\frac{1}{2}g_{ij}\partial_\mu \phi^i \partial^\mu \phi^j. \quad (\text{C.3})$$

In the present case, the coset vielbein can be computed from (2.3)

$$P_\mu^A = \frac{1}{3} \text{Tr}(L^{-1} \partial_\mu L Y^A). \quad (\text{C.4})$$

For completeness, we will explicitly give the result here

$$-\frac{1}{2} P_\mu^A P^{A\mu} = -4\Phi_3'^2 - 4 \cosh^2(2\Phi_3) [\cosh^2(2\Phi_2) \Phi_1'^2 + \Phi_2'^2]. \quad (\text{C.5})$$

Einstein's equation coming from the above Lagrangian is given by

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = P_\mu^A P_\nu^A - g_{\mu\nu} \left[\frac{1}{2} P_\rho^A P^{A\rho} + V \right] \quad (\text{C.6})$$

For the metric ansatz (3.13), non-vanishing components of the Ricci tensor and Ricci scalar are the following

$$\begin{aligned} R_{\mu\nu} &= -e^{2A} \eta_{\mu\nu} (A'' + 2A'^2), \\ R_{rr} &= -2(A'' + A'^2), \\ R &= -4A'' - 6A'^2 \end{aligned} \quad (\text{C.7})$$

for $\mu, \nu = 0, 1$. These together with the scalar potential (3.9) and equation (C.5) imply that all components of the Einstein's equation are satisfied.

For scalar field equations, it is more convenient to write the scalar Lagrangian as

$$\mathcal{L}_{\text{scalar}} = -e^{2A} \left[\frac{1}{2} P_r^A P^{Ar} + V \right]. \quad (\text{C.8})$$

From this, the scalar field equations are given by

$$\frac{d}{dr} \frac{\partial \mathcal{L}_{\text{scalar}}}{\partial \Phi_i'} - \frac{\partial \mathcal{L}_{\text{scalar}}}{\partial \Phi_i} = 0, \quad i = 1, 2, 3. \quad (\text{C.9})$$

The resulting equations are quite complicated, so we refrain from giving their explicit form here. It can however be verified that all of these equations are satisfied by the BPS equations (3.14), (3.15), (3.16), and (3.17).

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