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Testing the minimal S_4 model of neutrinos with the Dirac and Majorana phases

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ABSTRACT: We propose two new simple lepton flavor models in the framework of the S_4 flavor symmetry. The neutrino mass matrices, which are given by two complex parameters, lead to the inverted mass hierarchy. The charged lepton mass matrix has the 1-2 lepton flavor mixing, which gives the non-vanishing reactor angle θ_{13} . These models predict the Dirac phase and the Majorana phases, which are testable in the future experiments. The predicted magnitudes of the effective neutrino mass for the neutrino-less double beta decay are in the regions as $32 \text{ meV} \lesssim |m_{ee}| \lesssim 49 \text{ meV}$ and $34 \text{ meV} \lesssim |m_{ee}| \lesssim 59 \text{ meV}$, respectively. These values are close to the expected reaches of the coming experiments. The total sum of the neutrino masses are predicted in both models as $0.0952 \text{ eV} \lesssim \sum m_i \lesssim 0.101 \text{ eV}$ and $0.150 \text{ eV} \lesssim \sum m_i \lesssim 0.160 \text{ eV}$, respectively.

KEYWORDS: Neutrino Physics, CP violation, Discrete and Finite Symmetries

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1 Introduction

The neutrino experiments are going on the new step to reveal the CP violations in the lepton sector. If the neutrinos are the Majorana particles, there are one Dirac phase and two Majorana phases [1–3], which are sources of the CP violation. The T2K experiment, which has already confirmed the neutrino oscillation in the $\nu_\mu \rightarrow \nu_e$ appearance events [4], provides us the new information of the CP violating Dirac phase by combining the data of reactor experiments [5]–[8]. On the other hand, the neutrino-less double beta decay ($0\nu\beta\beta$) experiments lower gradually the upper-bound of the effective neutrino mass m_{ee} around $\mathcal{O}(100)$ meV. If the neutrino mass hierarchy is the inverted one, one can expect to observe the $0\nu\beta\beta$ in the near future. Its magnitude depends on the CP violating Majorana phases. Thus, the three CP violating phases can be observed in progressive neutrino experiments. Those CP violating phases provide us the important information to test the flavor model of neutrinos. There appear many works to test models with the CP violating Dirac and the Majorana phases of neutrinos in addition to the mixing angle sum rules [9]–[32].

The recent experimental data of neutrinos give us a big hint of the flavor symmetry. Actually, the lepton flavor structure has been discussed in the framework of the flavor symmetry. Before the reactor experiments reported the non-zero value of θ_{13} , there appears a paradigm of “tri-bimaximal mixing” (TBM) [33, 34], which is a simple mixing pattern for leptons and can be easily derived from flavor symmetries. Some authors succeeded to obtain the TBM in the A_4 models [35]–[39]. After those successes, the non-Abelian discrete groups are center of attention at the flavor symmetry [40]–[43]. The other groups were also examined to give the TBM [44]–[55]. The deviations from the TBM were estimated in many works [12, 56]–[74]. The observation of the non-vanishing θ_{13} enables the detail studies of flavor models in the context of the sum rules of mixing angles [43, 75]–[83]. The neutrino CP violation is also discussed in the context of the generalized CP symmetry [15, 84, 85]. The non-Abelian discrete groups with the CP symmetry have predicted the magnitude of the CP violating phases [86]–[105]. Furthermore, the neutrino mixing angles have been predicted linking with the quark mixing matrix by using some GUT models [106]–[121]. Thus, the flavor models in the lepton sector confront the neutrino experimental data of CP violating phases.

However, the flavor models do not always give the unique predictions since they have many parameters. Therefore, it is desirable to build a model with the small number of parameters for testing it. This attempt was proposed with Occam’s Razor [122], where the inverted mass hierarchy is predicted. In this paper, we build two neutrino flavor models with the small number of parameters as much as possible in the framework of the indirect approach with the S_4 flavor symmetry. The neutrino mass matrix is given by two complex parameters. The charged lepton mass matrix gives the non-vanishing reactor angle θ_{13} . The models lead to the inverted mass hierarchy, and predict the Dirac phase and the Majorana phases. Our predicted effective neutrino mass of the $0\nu\beta\beta$ is correlated with the Dirac phase.

In section 2, we propose two simple S_4 models, and discuss their implications. In section 3, we present the numerical analyses of the CP violating phases as well as the mixing angles, and then predict the effective neutrino mass for the $0\nu\beta\beta$. The section 4 is devoted to discussions and summary. In appendices, we present the multiplication rules of the S_4 group, and the neutrino mass matrix in the relevant basis of neutrinos.

2 S_4 flavor models

Let us build simple lepton flavor models with the S_4 group by the indirect approach of the flavor symmetry. We obtain the lepton mass matrices by introducing the relevant flavons and assuming the alignment of their vacuum expectation values (VEVs). The advantage of the S_4 group includes both the doublet and the triplets as the fundamental representations. Therefore, the S_4 flavor symmetry was easily extended to the quark sector [107, 108]. We obtain two models with the inverted neutrino mass hierarchy, which give two cases of the lightest neutrino mass $m_3 = 0$ and $m_3 \neq 0$, respectively. The particle assignments are same in both cases, while the vacuum alignments of flavon for the neutrino sector are different in each model.

	$\bar{L}$	l_R	τ_R	N_R^c	$N_{\tau R}^c$	H	ϕ_ν	ϕ'_ν	ϕ_ℓ	ϕ'_ℓ	ϕ''_ℓ	Θ
SU(2)	2	1	1	1	1	2	1	1	1	1	1	1
S_4	3	2	1	2	1	1	3	3	3	3'	3	1
Z_4	1	i	$-i$	-1	1	1	-1	1	$-i$	$-i$	i	1
$U(1)_{FN}$	0	$\ell + 2$	0	0	0	0	0	0	$-\ell$	$-\ell$	0	-1

Table 1. The assignments of leptons, Higgs and flavons.

2.1 S_4 flavor model for $m_3 = 0$

We present a model with the S_4 flavor symmetry in the case of the vanishing lightest neutrino mass. The particle assignments are shown in table 1. These assignments are similar to the model in refs. [107, 108] except for flavon fields. Under the S_4 group, the left-handed lepton doublet $\bar{L} = (\bar{L}_e, \bar{L}_\mu, \bar{L}_\tau)$ are assumed to transform as the triplet, while the right-handed charged lepton are assigned to the doublet $\ell_R = (e_R, \mu_R)$ and the singlet τ_R . The right-handed neutrinos are also assigned to the doublet $N_R^c = (N_{eR}^c, N_{\mu R}^c)$ and the singlet $N_{\tau R}^c$. The Z_4 charge is assigned relevantly to the leptons. In order to get the natural hierarchy between the muon and tauon masses, the Froggatt-Nielsen mechanism [123] is introduced as an additional $U(1)_{FN}$ flavor symmetry, where Θ denotes the Froggatt-Nielsen flavon. On the other hand, the Higgs doublet H is assigned to the S_4 singlet. The gauge singlet flavons are assigned to the triplet or triplet-prime, which have different Z_4 charges as seen in table 1.

We can now write down the $S_4 \times Z_4 \times U(1)_{FN}$ invariant Lagrangian for the Yukawa interaction in terms of the S_4 cutoff scale Λ and the $U(1)_{FN}$ cutoff scale $\bar{\Lambda}$ as follows:

$$\begin{aligned}
 \mathcal{L}_Y = & y_\ell \bar{L} l_R H \phi_\ell \Theta^2 / (\Lambda \bar{\Lambda}^2) + y'_\ell \bar{L} l_R H \phi'_\ell \Theta^2 / (\Lambda \bar{\Lambda}^2) + y''_\ell \bar{L} \tau_R H \phi''_\ell / \Lambda \\
 & + y_D \bar{L} N_R^c \tilde{H} \phi_\nu / \Lambda + y'_D \bar{L} N_{\tau R}^c \tilde{H} \phi'_\nu / \Lambda \\
 & + M_1 N_R^c N_R^c + M_2 N_{\tau R}^c N_{\tau R}^c,
 \end{aligned} \tag{2.1}$$

where y 's are Yukawa couplings, M_1 and M_2 are the Majorana masses, and $\tilde{H} = i\tau_2 H^*$. In this setup, we discuss the lepton mass matrices.

Let us begin with discussing the neutrino sector. In order to desirable mass matrices, the relevant VEV alignments are required. We will show the potential analysis to derive the VEV alignments in subsection 2.3.

We take the VEVs of the relevant flavons and VEV alignments as:

$$\langle H \rangle = v, \quad \langle \phi_\nu \rangle = v_\nu(1, 0, 0), \quad \langle \phi'_\nu \rangle = v'_\nu(1, 1, 1). \tag{2.2}$$

By using the multiplication rules in appendix A, we obtain the Dirac neutrino mass matrix M_D as

$$M_D = \begin{pmatrix} 0 & -\frac{2}{\sqrt{6}} y_D v_\nu & y'_D v'_\nu \\ 0 & 0 & y'_D v'_\nu \\ 0 & 0 & y'_D v'_\nu \end{pmatrix} \frac{v}{\bar{\Lambda}}, \tag{2.3}$$

and the right-handed Majorana neutrino mass matrix M_N as

$$M_N = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_1 & 0 \\ 0 & 0 & M_2 \end{pmatrix}. \quad (2.4)$$

By using the seesaw mechanism, the left-handed Majorana neutrino mass matrix M_ν is given as

$$M_\nu = \begin{pmatrix} a+b & b & b \\ b & b & b \\ b & b & b \end{pmatrix}, \quad a = \frac{2(y_D v_\nu v)^2}{3M_1 \Lambda^2}, \quad b = \frac{(y'_D v'_\nu v)^2}{M_2 \Lambda^2}, \quad (2.5)$$

where a and b are complex parameters. In order to compare with the TBM, we move to the TBM base. Then, we can easily seen the family structure of the neutrino mass matrix. After rotating M_ν with the TBM matrix V_{TBM} :

$$V_{\text{TBM}} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad (2.6)$$

the neutrino mass matrix turns to

$$M_\nu = V_{\text{TBM}} \begin{pmatrix} \frac{2}{3}a & \frac{\sqrt{2}}{3}a & 0 \\ \frac{\sqrt{2}}{3}a & \frac{1}{3}a + 3b & 0 \\ 0 & 0 & 0 \end{pmatrix} V_{\text{TBM}}^T. \quad (2.7)$$

This mass matrix gives us the inverted mass hierarchy with the one vanishing mass. The flavor mixing is deviated from the TBM only in the rotation of the (1-2) axis.

In order to get the mass eigenvalues and the flavor mixing angles of neutrinos, we examine $M_\nu^\dagger M_\nu$ as

$$\begin{aligned} M_\nu^\dagger M_\nu &= V_{\text{TBM}} \begin{pmatrix} \frac{2}{3}|a|^2 & \frac{\sqrt{2}}{3}|a|^2 + \sqrt{2}a^*b & 0 \\ \frac{\sqrt{2}}{3}|a|^2 + \sqrt{2}ab^* & \frac{1}{3}|a|^2 + 9|b|^2 + ab^* + a^*b & 0 \\ 0 & 0 & 0 \end{pmatrix} V_{\text{TBM}}^T \\ &= V_{\text{TBM}} \begin{pmatrix} \frac{2}{3}|a|^2 & \frac{\sqrt{2}}{3}|a|^2 + \sqrt{2}|a||b|e^{-i\varphi} & 0 \\ \frac{\sqrt{2}}{3}|a|^2 + \sqrt{2}|a||b|e^{i\varphi} & \frac{1}{3}|a|^2 + 9|b|^2 + 2|a||b|\cos\varphi & 0 \\ 0 & 0 & 0 \end{pmatrix} V_{\text{TBM}}^T, \end{aligned} \quad (2.8)$$

where $a = |a|e^{i\varphi_a}$, $b = |b|e^{i\varphi_b}$, and $\varphi = \varphi_a - \varphi_b$. Hereafter, we define $|a| \equiv a$ and $|b| \equiv b$ for simplicity. Then, the neutrino mass eigenvalues are

$$\begin{aligned} m_1^2 &= \frac{1}{2} \left[a^2 + 9b^2 + 2ab \cos \varphi - \sqrt{(a^2 + 9b^2 + 2ab \cos \varphi)^2 - 16a^2b^2} \right], \\ m_2^2 &= \frac{1}{2} \left[a^2 + 9b^2 + 2ab \cos \varphi + \sqrt{(a^2 + 9b^2 + 2ab \cos \varphi)^2 - 16a^2b^2} \right], \\ m_3^2 &= 0. \end{aligned} \quad (2.9)$$

Therefore, the neutrino mass squared differences are given as

$$\begin{aligned}\Delta m_{\text{atm}}^2 &= m_2^2 - m_3^2 = \frac{1}{2} \left[a^2 + 9b^2 + 2ab \cos \varphi + \sqrt{(a^2 + 9b^2 + 2ab \cos \varphi)^2 - 16a^2b^2} \right], \\ \Delta m_{\text{sol}}^2 &= m_2^2 - m_1^2 = \sqrt{(a^2 + 9b^2 + 2ab \cos \varphi)^2 - 16a^2b^2},\end{aligned}\quad (2.10)$$

and neutrino mixing matrix U_ν is

$$U_\nu = V_{\text{TBM}} \begin{pmatrix} \cos \eta & e^{-i\psi} \sin \eta & 0 \\ -e^{i\psi} \sin \eta & \cos \eta & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (2.11)$$

where

$$\tan 2\eta = \frac{2\sqrt{2}a\sqrt{a^2 + 9b^2 + 6ab \cos \varphi}}{27b^2 - a^2 + 6ab \cos \varphi}, \quad \tan \psi = -\frac{3b \sin \varphi}{a + 3b \cos \varphi}. \quad (2.12)$$

Thus, the neutrino mass hierarchy is only inverted one and the neutrino mixing angles are determined by two neutrino mass squared differences and relative phase φ in our model.

Next, we consider the charged lepton sector. Taking VEV and VEV alignments as¹

$$\langle \Theta \rangle = \theta, \quad \langle \phi_\ell \rangle = (v_{\ell 1}, v_{\ell 2}, 0), \quad \langle \phi'_\ell \rangle = v'_\ell (1, 0, 0), \quad \langle \phi''_\ell \rangle = v''_\ell (0, 0, 1), \quad (2.13)$$

the charged lepton mass matrix M_ℓ is given as

$$M_\ell = \begin{pmatrix} \frac{2}{\sqrt{6}}c_\ell - \frac{2}{\sqrt{6}}a_\ell & 0 \\ \frac{1}{\sqrt{2}}b_\ell & \frac{1}{\sqrt{6}}b_\ell & 0 \\ 0 & 0 & d_\ell \end{pmatrix}, \quad (2.14)$$

where

$$a_\ell = y_\ell v_{\ell 1} v \frac{\theta^2}{\Lambda \bar{\Lambda}^2}, \quad b_\ell = y_\ell v_{\ell 2} v \frac{\theta^2}{\Lambda \bar{\Lambda}^2}, \quad c_\ell = y'_\ell v'_\ell v \frac{\theta^2}{\Lambda \bar{\Lambda}^2}, \quad d_\ell = y''_\ell v''_\ell v \frac{1}{\Lambda}. \quad (2.15)$$

In order to reproduce the relevant mass hierarchy between the muon and the tauon, we take $\theta/\bar{\Lambda}$ as the Cabibbo angle 0.22 approximately.

The left-handed mixing of charged leptons is given by investigating $M_\ell M_\ell^\dagger$:

$$\begin{aligned}M_\ell M_\ell^\dagger &= \begin{pmatrix} \frac{2}{3}(|a_\ell|^2 + |c_\ell|^2) & -\frac{1}{3}(a_\ell - \sqrt{3}c_\ell)b_\ell^* & 0 \\ -\frac{1}{3}(a_\ell^* - \sqrt{3}c_\ell^*)b_\ell & \frac{2}{3}|b_\ell|^2 & 0 \\ 0 & 0 & |d_\ell|^2 \end{pmatrix} \\ &= \begin{pmatrix} \frac{2}{3}(|a_\ell|^2 + |c_\ell|^2) & -\frac{1}{3}(|a_\ell||b_\ell|e^{i\varphi_{ab}} - \sqrt{3}|b_\ell||c_\ell|e^{-i\varphi_{bc}}) & 0 \\ -\frac{1}{3}(|a_\ell||b_\ell|e^{-i\varphi_{ab}} - \sqrt{3}|b_\ell||c_\ell|e^{i\varphi_{bc}}) & \frac{2}{3}|b_\ell|^2 & 0 \\ 0 & 0 & |d_\ell|^2 \end{pmatrix},\end{aligned}\quad (2.16)$$

¹We take the VEV of ϕ_ℓ as $(v_{\ell 1}, v_{\ell 2}, 0)$ for simplicity. As the alternative choice, we can introduce one more field $\tilde{\phi}_\ell$, which charge assignment of S_4 and Z_4 is same as ϕ_ℓ , and take VEV alignments as $\langle \phi_\ell \rangle = (v_{\ell 1}, 0, 0)$, $\langle \tilde{\phi}_\ell \rangle = (0, v_{\ell 2}, 0)$. By using these VEV alignments, we obtain the same mass matrix in eq. (2.14).

where $a_\ell = |a_\ell|e^{i\varphi_{a_\ell}}$, $b_\ell = |b_\ell|e^{i\varphi_{b_\ell}}$, $c_\ell = |c_\ell|e^{i\varphi_{c_\ell}}$, and $\varphi_{ab} = \varphi_{a_\ell} - \varphi_{b_\ell}$, $\varphi_{bc} = \varphi_{b_\ell} - \varphi_{c_\ell}$. Hereafter, we define $|a_\ell| \equiv a_\ell$, $|b_\ell| \equiv b_\ell$, and $|c_\ell| \equiv c_\ell$ for simplicity. Then, the charged lepton masses are given as

$$\begin{aligned} m_e^2 &= \frac{1}{3} \left[a_\ell^2 + b_\ell^2 + c_\ell^2 - \sqrt{(a_\ell^2 + b_\ell^2 + c_\ell^2)^2 - b_\ell^2 (3a_\ell^2 + c_\ell^2 + 2\sqrt{3}a_\ell c_\ell \cos \varphi_\ell)} \right], \\ m_\mu^2 &= \frac{1}{3} \left[a_\ell^2 + b_\ell^2 + c_\ell^2 + \sqrt{(a_\ell^2 + b_\ell^2 + c_\ell^2)^2 - b_\ell^2 (3a_\ell^2 + c_\ell^2 + 2\sqrt{3}a_\ell c_\ell \cos \varphi_\ell)} \right], \\ m_\tau^2 &= d_\ell^2, \end{aligned} \quad (2.17)$$

where $\varphi_\ell = \varphi_{ab} + \varphi_{bc}$.

The $U(1)_{FN}$ symmetry guarantees the mass hierarchy of the muon and the tauon. Now, we discuss the electron and muon masses. If we assume $b_\ell \gg a_\ell, c_\ell$ in eq. (2.14), the electron and muon masses in eq. (2.17) are approximately written as

$$\begin{aligned} m_e^2 &\simeq \frac{1}{3} \left(\frac{3}{2}a_\ell^2 + \frac{1}{2}c_\ell^2 + \sqrt{3}a_\ell c_\ell \cos \varphi_\ell \right), \\ m_\mu^2 &\simeq \frac{1}{3} \left(2b_\ell^2 + \frac{1}{2}a_\ell^2 + \frac{3}{2}c_\ell^2 - \sqrt{3}a_\ell c_\ell \cos \varphi_\ell \right). \end{aligned} \quad (2.18)$$

Then, the muon mass is $\sqrt{2/3}b_\ell$ and the electron mass is reproduced by tuning a_ℓ and c_ℓ .

On the other hand, the left-handed charged lepton mixing matrix U_ℓ is

$$U_\ell = \begin{pmatrix} \cos \lambda & e^{-i\psi_\ell} \sin \lambda & 0 \\ -e^{i\psi_\ell} \sin \lambda & \cos \lambda & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (2.19)$$

where

$$\tan 2\lambda = \frac{b_\ell \sqrt{a_\ell^2 + 3c_\ell^2 - 2\sqrt{3}a_\ell c_\ell \cos \varphi_\ell}}{a_\ell^2 - b_\ell^2 + c_\ell^2}, \quad \tan \psi_\ell = \frac{a_\ell \sin \varphi_{ab} + \sqrt{3}c_\ell \sin \varphi_{bc}}{a_\ell \cos \varphi_{ab} - \sqrt{3}c_\ell \cos \varphi_{bc}}. \quad (2.20)$$

In the charged lepton sector, there are four real parameters ($a_\ell, b_\ell, c_\ell, d_\ell$) and two phases. After inputting charged lepton masses, there remains two independent parameters, which are the mixing angle λ and the CP violating phase ψ_ℓ . Those free parameters are adjusted to the experimental data of the lepton mixing matrix, namely Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix U_{PMNS} [124, 125] as

$$U_{\text{PMNS}} = U_\ell^\dagger U_\nu = \begin{pmatrix} \cos \lambda & -e^{-i\psi_\ell} \sin \lambda & 0 \\ e^{i\psi_\ell} \sin \lambda & \cos \lambda & 0 \\ 0 & 0 & 1 \end{pmatrix} V_{\text{TBM}} \begin{pmatrix} \cos \eta & e^{-i\psi} \sin \eta & 0 \\ -e^{i\psi} \sin \eta & \cos \eta & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (2.21)$$

The mixing matrix elements are written as

$$\begin{aligned}
 U_{e1} &= \frac{\cos \eta (2 \cos \lambda + e^{-i\psi_\ell} \sin \lambda)}{\sqrt{6}} + \frac{e^{i\psi} \sin \eta (-\cos \lambda + e^{-i\psi_\ell} \sin \lambda)}{\sqrt{3}}, \\
 U_{e2} &= \frac{\cos \eta (\cos \lambda - e^{-i\psi_\ell} \sin \lambda)}{\sqrt{3}} + \frac{e^{-i(\psi+\psi_\ell)} \sin \eta (2e^{i\psi_\ell} \cos \lambda + \sin \lambda)}{\sqrt{6}}, \\
 U_{\mu 1} &= \frac{\cos \eta (-\cos \lambda + 2e^{i\psi_\ell} \sin \lambda)}{\sqrt{6}} - \frac{e^{i\psi} \sin \eta (\cos \lambda + e^{i\psi_\ell} \sin \lambda)}{\sqrt{3}}, \\
 U_{\mu 2} &= \frac{\cos \eta (\cos \lambda + e^{i\psi_\ell} \sin \lambda)}{\sqrt{3}} + \frac{e^{-i\psi} \sin \eta (-\cos \lambda + 2e^{i\psi_\ell} \sin \lambda)}{\sqrt{6}}, \\
 U_{\tau 1} &= -\frac{\cos \eta}{\sqrt{6}} - \frac{e^{i\psi} \sin \eta}{\sqrt{3}}, \quad U_{\tau 2} = \frac{\cos \eta}{\sqrt{3}} - \frac{e^{-i\psi} \sin \eta}{\sqrt{6}}, \\
 U_{e3} &= \frac{e^{-i\psi_\ell} \sin \lambda}{\sqrt{2}}, \quad U_{\mu 3} = -\frac{\cos \lambda}{\sqrt{2}}, \quad U_{\tau 3} = \frac{1}{\sqrt{2}}. \tag{2.22}
 \end{aligned}$$

Therefore, the mixing angles are expressed as follows:

$$\begin{aligned}
 \sin^2 \theta_{12} &= \frac{|U_{e2}|^2}{|U_{e1}|^2 + |U_{e2}|^2} \\
 &= \frac{1}{6(3 + \cos 2\lambda)} \left[9 + 3 \cos 2\lambda (1 + \sin^2 \eta) + \sin^2 \eta (1 + 8 \cos \psi_\ell \sin 2\lambda) \right. \\
 &\quad \left. - \cos^2 \eta (1 + 3 \cos 2\lambda + 8 \cos \psi_\ell \sin 2\lambda) \right. \\
 &\quad \left. + 2\sqrt{2} \sin 2\eta \left\{ \cos \psi (1 + 3 \cos 2\lambda - \cos \psi_\ell \sin 2\lambda) - 3 \sin \psi \sin \psi_\ell \sin 2\lambda \right\} \right], \\
 \sin^2 \theta_{23} &= \frac{|U_{\mu 3}|^2}{|U_{\mu 3}|^2 + |U_{\tau 3}|^2} = 1 - \frac{1}{2 - \sin^2 \lambda}, \quad \sin^2 \theta_{13} = |U_{e3}|^2 = \frac{\sin^2 \lambda}{2}. \tag{2.23}
 \end{aligned}$$

In order to compare our model with the TBM in the neutrino sector, it is useful to write down the mixing angles without the contribution from the charged lepton sector, that is ones in eq. (2.11),

$$\sin^2 \theta_{12}^\nu = \frac{1}{3} \left(1 + \sin^2 \eta + \sqrt{2} \sin 2\eta \cos \psi \right), \quad \sin^2 \theta_{23}^\nu = \frac{1}{2}, \quad \sin \theta_{13}^\nu = 0, \tag{2.24}$$

where θ_{ij}^ν denote the mixing angles only in the neutrino sector. Our model is different from the TBM only in θ_{12}^ν . We will see how large our θ_{12}^ν is deviated from the TBM $\sin^2 \theta_{12}^\nu = 1/3$ in the next section.

Before closing this subsection, we present a simple sum rule between θ_{23} and θ_{13} in eq. (2.23):

$$\sin^2 \theta_{23} = 1 - \frac{1}{2 \cos^2 \theta_{13}}. \tag{2.25}$$

By using the experimental data [126] for 3σ range, the $\sin^2 \theta_{23}$ is predicted to be

$$0.487 \leq \sin^2 \theta_{23} \leq 0.490, \tag{2.26}$$

which is within the 3σ range for the experimental data [126] as $0.389 \leq \sin^2 \theta_{23} \leq 0.664$. We show the details of our numerical analyses in section 3.

2.2 S_4 flavor model for $m_3 \neq 0$

We can obtain another simple model with the non-vanishing lightest neutrino mass. The particle assignments are same as the model in table 1. Therefore, the $S_4 \times Z_4$ invariant Lagrangian for the Yukawa coupling is same as in eq. (2.1). The VEV alignments for the neutrino sector are different from eq. (2.2). Taking VEV alignments as²

$$\langle \phi_\nu \rangle = v_\nu(0, 1, -1), \quad \langle \phi'_\nu \rangle = v'_\nu(1, 1, 1), \quad (2.27)$$

we obtain the Dirac neutrino mass matrix M_D as

$$M_D = \begin{pmatrix} 0 & 0 & y'_D v'_\nu \\ \frac{1}{\sqrt{2}} y_D v_\nu & \frac{1}{\sqrt{6}} y_D v_\nu & y'_D v'_\nu \\ \frac{1}{\sqrt{2}} y_D v_\nu & -\frac{1}{\sqrt{6}} y_D v_\nu & y'_D v'_\nu \end{pmatrix} \frac{v}{\Lambda}. \quad (2.28)$$

The right-handed Majorana neutrino mass matrix M_N is same in eq. (2.4) as

$$M_N = \begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_1 & 0 \\ 0 & 0 & M_2 \end{pmatrix}. \quad (2.29)$$

By using the seesaw mechanism, the left-handed Majorana neutrino mass matrix M_ν is written as

$$M_\nu = \begin{pmatrix} b & b & b \\ b & 2a+b & a+b \\ b & a+b & 2a+b \end{pmatrix}, \quad a = \frac{(y_D v_\nu v)^2}{3M_1 \Lambda^2}, \quad b = \frac{(y'_D v'_\nu v)^2}{M_2 \Lambda^2}, \quad (2.30)$$

where a and b are complex parameters. Moving to the TBM base, the neutrino mass matrix is given as

$$M_\nu = V_{\text{TBM}} \begin{pmatrix} a & -\sqrt{2}a & 0 \\ -\sqrt{2}a & 2a+3b & 0 \\ 0 & 0 & a \end{pmatrix} V_{\text{TBM}}^T. \quad (2.31)$$

In order to get the mixing angles of the left-handed Majorana neutrino, we investigate $M_\nu^\dagger M_\nu$ as

$$M_\nu^\dagger M_\nu = V_{\text{TBM}} \begin{pmatrix} 3|a|^2 & -3\sqrt{2}(|a|^2 + |a||b|e^{-i\varphi}) & 0 \\ -3\sqrt{2}(|a|^2 + |a||b|e^{i\varphi}) & 3(2|a|^2 + 3|b|^2 + 4|a||b|\cos\varphi) & 0 \\ 0 & 0 & |a|^2 \end{pmatrix} V_{\text{TBM}}^T, \quad (2.32)$$

where $a = |a|e^{i\varphi_a}$, $b = |b|e^{i\varphi_b}$, and $\varphi_a - \varphi_b \equiv \varphi$. Hereafter, we define $|a| \equiv a$ and $|b| \equiv b$ for simplicity. Then, the neutrino mass eigenvalues are

$$\begin{aligned} m_1^2 &= \frac{3}{2} \left[3a^2 + 3b^2 + 4ab \cos \varphi - \sqrt{(3a^2 + 3b^2 + 4ab \cos \varphi)^2 - 4a^2 b^2} \right], \\ m_2^2 &= \frac{3}{2} \left[3a^2 + 3b^2 + 4ab \cos \varphi + \sqrt{(3a^2 + 3b^2 + 4ab \cos \varphi)^2 - 4a^2 b^2} \right], \\ m_3^2 &= a^2. \end{aligned} \quad (2.33)$$

²The alignments $\langle \phi_\nu \rangle / v_\nu = (0, 1, 1)$, $(0, -1, -1)$, $(0, -1, 1)$ also lead to the same M_ν in eq. (2.30).

	ϕ''_ν	ξ_0	ξ'_0	η_0	χ_0
SU(2)	1	1	1	1	1
S_4	3	1	1'	1	2
Z_4	-1	1	1	i	-1
$U(1)_{FN}$	$-z$	ℓ	ℓ	0	z
$U(1)_R$	0	2	2	2	2

Table 2. The assignments of the additional fields.

Therefore, the neutrino mass squared differences are

$$\begin{aligned}\Delta m_{\text{atm}}^2 &= m_2^2 - m_3^2 = \frac{3}{2} \left[\frac{7}{3} a^2 + 3b^2 + 4ab \cos \varphi + \sqrt{(3a^2 + 3b^2 + 4ab \cos \varphi)^2 - 4a^2 b^2} \right], \\ \Delta m_{\text{sol}}^2 &= m_2^2 - m_1^2 = 3\sqrt{(3a^2 + 3b^2 + 4ab \cos \varphi)^2 - 4a^2 b^2},\end{aligned}\quad (2.34)$$

and the neutrino mixing matrix U_ν is

$$U_\nu = V_{\text{TBM}} \begin{pmatrix} \cos \eta & e^{-i\psi} \sin \eta & 0 \\ -e^{i\psi} \sin \eta & \cos \eta & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (2.35)$$

where

$$\tan 2\eta = \frac{2a\sqrt{2(a^2 + b^2 + 2ab \cos \varphi)}}{a^2 + 3b^2 + 4ab \cos \varphi}, \quad \tan \psi = -\frac{b \sin \varphi}{a + b \cos \varphi}. \quad (2.36)$$

We can easily find the mass ordering with $m_2 > m_1 > m_3$ for any $\cos \varphi$. Therefore, the neutrino mass hierarchy is only inverted one and the neutrino mixing angles are determined by two neutrino mass squared differences and relative phase φ .

The charged lepton mass matrix is the same as the one in the previous subsection. Therefore, the PMNS mixing matrix elements are expressed in the same form in eq. (2.22). We have the same sum rule between θ_{23} and θ_{13} in eq. (2.25).

2.3 Potential analysis and VEV alignments

In this subsection, we present the potential analysis of flavons in the framework of the supersymmetry with the $U(1)_R$ symmetry. We can generate the vacuum alignment through F -terms by coupling flavons to driving fields, which carry the R charge $+2$ under $U(1)_R$ symmetry. We also assign R charge $+1$ to the lepton doublets, right-handed charged leptons, and right-handed Majorana neutrinos. In addition to driving fields with $R = 2$, we introduce a new flavon ϕ''_ν , which does not couple to leptons and right-handed Majorana neutrinos due to the $U(1)_{FN}$ charge. The charge assignments of the additional flavon and driving fields are summarized in table 2.

Let us write down the $S_4 \times Z_4 \times U(1)_{FN}$ invariant superpotential for the interaction of scalar fields as follow:

$$w_V = y_{\xi_0} \phi_\ell \phi''_\ell \xi_0 + y_{\xi'_0} \phi'_\ell \phi''_\ell \xi'_0 + y_{\eta_0} \phi_\nu \phi''_\ell \eta_0 + y_{\chi_0} \phi'_\nu \phi''_\nu \chi_0, \quad (2.37)$$

where y_{ξ_0} , $y_{\xi'_0}$, y_{η_0} and y_{χ_0} are arbitrary parameters. Then, the potential V is given as

$$V = \left| \frac{\partial w_V}{\partial X_0} \right|^2 = \left| y_{\xi_0} (\phi_{\ell 1} \phi''_{\ell 1} + \phi_{\ell 2} \phi''_{\ell 2} + \phi_{\ell 3} \phi''_{\ell 3}) \right|^2 + \left| y_{\xi'_0} (\phi'_{\ell 1} \phi''_{\ell 1} + \phi'_{\ell 2} \phi''_{\ell 2} + \phi'_{\ell 3} \phi''_{\ell 3}) \right|^2 \quad (2.38)$$

$$+ \left| y_{\eta_0} (\phi_{\nu 1} \phi''_{\ell 1} + \phi_{\nu 2} \phi''_{\ell 2} + \phi_{\nu 3} \phi''_{\ell 3}) \right|^2$$

$$+ \left| \frac{1}{\sqrt{2}} y_{\chi_0} (\phi'_{\nu 2} \phi''_{\nu 2} - \phi'_{\nu 3} \phi''_{\nu 3}) \right|^2 + \left| \frac{1}{\sqrt{6}} y_{\chi_0} (-2\phi'_{\nu 1} \phi''_{\nu 1} + \phi'_{\nu 2} \phi''_{\nu 2} + \phi'_{\nu 3} \phi''_{\nu 3}) \right|^2,$$

where $X_0 = \xi_0, \xi'_0, \eta_0, \chi_0$. Therefore, conditions to realize the potential minimum ($V = 0$) are given from eq. (2.38) as

$$\begin{aligned} y_{\xi_0} (\phi_{\ell 1} \phi''_{\ell 1} + \phi_{\ell 2} \phi''_{\ell 2} + \phi_{\ell 3} \phi''_{\ell 3}) &= 0, \\ y_{\xi'_0} (\phi'_{\ell 1} \phi''_{\ell 1} + \phi'_{\ell 2} \phi''_{\ell 2} + \phi'_{\ell 3} \phi''_{\ell 3}) &= 0, \\ y_{\eta_0} (\phi_{\nu 1} \phi''_{\ell 1} + \phi_{\nu 2} \phi''_{\ell 2} + \phi_{\nu 3} \phi''_{\ell 3}) &= 0, \\ \frac{1}{\sqrt{2}} y_{\chi_0} (\phi'_{\nu 2} \phi''_{\nu 2} - \phi'_{\nu 3} \phi''_{\nu 3}) &= 0, \\ \frac{1}{\sqrt{6}} y_{\chi_0} (-2\phi'_{\nu 1} \phi''_{\nu 1} + \phi'_{\nu 2} \phi''_{\nu 2} + \phi'_{\nu 3} \phi''_{\nu 3}) &= 0. \end{aligned} \quad (2.39)$$

One of the solutions which satisfies these conditions is given as

$$\begin{aligned} \langle \phi_{\ell 3} \rangle &= \langle \phi'_{\ell 2} \rangle = \langle \phi'_{\ell 3} \rangle = \langle \phi''_{\ell 1} \rangle = \langle \phi''_{\ell 2} \rangle = \langle \phi_{\nu 2} \rangle = \langle \phi_{\nu 3} \rangle = 0, \\ \langle \phi_{\nu 1} \rangle &= \langle \phi'_{\nu 2} \rangle = \langle \phi'_{\nu 3} \rangle, \quad \langle \phi''_{\nu 1} \rangle = \langle \phi''_{\nu 2} \rangle = \langle \phi''_{\nu 3} \rangle, \end{aligned} \quad (2.40)$$

then, the VEV alignments are

$$\begin{aligned} \langle \phi_{\ell} \rangle &= (v_{\ell 1}, v_{\ell 2}, 0), \quad \langle \phi'_{\ell} \rangle = v'_{\ell} (1, 0, 0), \quad \langle \phi''_{\ell} \rangle = v''_{\ell} (0, 0, 1), \\ \langle \phi_{\nu} \rangle &= v_{\nu} (1, 0, 0), \quad \langle \phi'_{\nu} \rangle = v'_{\nu} (1, 1, 1), \end{aligned} \quad (2.41)$$

which have been used in the subsection 2.1. We can also derive the VEV alignments of the subsection 2.2 in the similar discussion.

3 Numerical analyses

In this section, we show the numerical analyses in our S_4 models. We use the result of the global analyses of neutrino oscillation experiments [126–128]. The 3σ range of the experimental data [126] for the inverted neutrino mass hierarchy are

$$\begin{aligned} 7.02 \leq \frac{\Delta m_{\text{sol}}^2}{10^{-5} \text{ eV}^2} \leq 8.09, \quad 2.307 \leq \frac{|\Delta m_{\text{atm}}^2|}{10^{-3} \text{ eV}^2} \leq 2.590, \quad (3.1) \\ 0.270 \leq \sin^2 \theta_{12} \leq 0.344, \quad 0.389 \leq \sin^2 \theta_{23} \leq 0.664, \quad 0.0188 \leq \sin^2 \theta_{13} \leq 0.0251. \end{aligned}$$

Our numerical strategy is as follows. Fixing a random value for φ of eq. (2.10) in the region $-\pi \leq \varphi \leq \pi$, then, a and b are determined from the experimental data of Δm_{atm}^2 and Δm_{sol}^2 . Then, we can calculate the neutrino mixing matrix (η and ψ) from

eqs. (2.11) and (2.12). The magnitude of λ is determined in $\sin^2 \theta_{13}$ of eq. (2.23) by using the experimental data [126] for 3σ range. Finally, choosing a random value for ψ_ℓ in $-\pi \leq \psi_\ell \leq \pi$, we calculate the PMNS mixing matrix elements in eq. (2.21), which are adjusted to the experimental data of eq. (3.1).

In our numerical studies, we predict the Dirac phase and the Majorana phases. Our predicted PMNS matrix in the previous section should be compared with the following conventional parametrization [129] including the Majorana phases:

$$\begin{aligned} & \begin{pmatrix} e^{i\delta_e} & 0 & 0 \\ 0 & e^{i\delta_\mu} & 0 \\ 0 & 0 & e^{i\delta_\tau} \end{pmatrix} \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\text{CP}}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{\text{CP}}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{\text{CP}}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{\text{CP}}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{\text{CP}}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\alpha} & 0 & 0 \\ 0 & e^{i\beta} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} c_{12}c_{13}e^{i(\delta_e-\alpha)} & s_{12}c_{13}e^{i(\delta_e-\beta)} & s_{13}e^{i(\delta_e-\delta_{\text{CP}})} \\ (-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{\text{CP}}})e^{i(\delta_\mu-\alpha)} & (c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{\text{CP}}})e^{i(\delta_\mu-\beta)} & s_{23}c_{13}e^{i\delta_\mu} \\ (s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{\text{CP}}})e^{i(\delta_\tau-\alpha)} & (-c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{\text{CP}}})e^{i(\delta_\tau-\beta)} & c_{23}c_{13}e^{i\delta_\tau} \end{pmatrix}, \end{aligned} \quad (3.2)$$

where c_{ij} and s_{ij} denote $\cos \theta_{ij}$ and $\sin \theta_{ij}$, respectively. The phases δ_e , δ_μ , and δ_τ could be absorbed in the left-handed charged lepton fields, δ_{CP} is the Dirac phase, and α , β are the Majorana phases.

We can calculate the Dirac phase δ_{CP} as follows. The Jarlskog invariant [130], which is the measure describing the size of the CP violation, is given as

$$\begin{aligned} J_{\text{CP}} &= \text{Im} [U_{xi}U_{yj}U_{yi}^*U_{xj}^*] \\ &= \frac{\sin 2\lambda}{48} \left[3\sqrt{2} \cos \psi_\ell \sin \psi \sin 2\eta - \sin \psi_\ell \left(\sqrt{2} \cos \psi \sin 2\eta + 4 \cos 2\eta \right) \right], \end{aligned} \quad (3.3)$$

where $x, y = e, \mu, \tau$ and $i, j = 1-3$. Then, the CP violating Dirac phase is written in terms of the lepton mixing angles and J_{CP} as

$$\sin \delta_{\text{CP}} = \frac{J_{\text{CP}}}{s_{23}c_{23}s_{12}c_{12}s_{13}c_{13}^2}. \quad (3.4)$$

In our model, the one of the PMNS mixing matrix elements $U_{\tau 1}$ is written as

$$|U_{\tau 1}| = \left| -\frac{\cos \eta}{\sqrt{6}} - \frac{e^{i\psi} \sin \eta}{\sqrt{3}} \right| = \left| s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{\text{CP}}} \right|, \quad (3.5)$$

then, the CP violating Dirac phase is also written as

$$\cos \delta_{\text{CP}} = \frac{s_{12}^2 s_{23}^2 + c_{12}^2 c_{23}^2 s_{13}^2 - \frac{1}{6} \cos^2 \eta - \frac{1}{3} \sin^2 \eta - \frac{1}{3\sqrt{2}} \sin 2\eta \cos \psi}{2s_{12}s_{23}c_{12}c_{23}s_{13}}. \quad (3.6)$$

Thus, δ_{CP} is determined up to the quadrant.

Next, let us calculate the Majorana phases. In our model, $U_{\mu 3}$ and $U_{\tau 3}$ have no phases as seen in eq. (2.22), so we find $\delta_\mu = \delta_\tau = 0$ in eq. (3.2). For convenience, expressing the phases of the mixing matrix of eq. (2.22) as

$$U_{e1} = |U_{e1}|e^{i\delta_1}, \quad U_{e2} = |U_{e2}|e^{i\delta_2}, \quad U_{e3} = |U_{e3}|e^{-i\psi_\ell}, \quad (3.7)$$

Figure 1. The allowed region on ψ - $\sin^2 \theta_{12}^\nu$ plane. The red line corresponds to the TBM.

we derive the following relations by comparing with eq. (3.2):

$$\delta_1 = \delta_e - \alpha, \quad \delta_2 = \delta_e - \beta, \quad \psi_\ell = \delta_{\text{CP}} - \delta_e. \quad (3.8)$$

Eliminating δ_e in these relations, we obtain

$$\alpha = \delta_{\text{CP}} - \psi_\ell - \delta_1, \quad \beta = \delta_{\text{CP}} - \psi_\ell - \delta_2, \quad \alpha - \beta = \delta_2 - \delta_1, \quad (3.9)$$

from which we can calculate the Majorana phases numerically.

3.1 S_4 flavor model for $m_3 = 0$

In this subsection, we show the numerical results in the case of the lightest neutrino mass $m_3 = 0$. In order to reproduce the relevant mass squared differences, Δm_{atm}^2 and Δm_{sol}^2 for the inverted mass hierarchy, $a \simeq 3b$ and $\varphi \simeq \pm\pi$ should be satisfied. Then, the ψ of eq. (2.12) is restricted in $45^\circ \lesssim |\psi| \lesssim 90^\circ$. On the other hand, $\sin \eta$ in eq. (2.12) is restricted in $-0.70 \lesssim \sin \eta \lesssim -0.38$. In figure 1, we show the allowed region on ψ - $\sin^2 \theta_{12}^\nu$ plane, where θ_{12}^ν is the mixing angle of the neutrino sector without the charged lepton contribution as seen in eq. (2.24). As seen in this figure, our model is possibly deviated from the TBM considerably. Then, the mixing angle and the phase of the charged lepton sector become important to adjust the observed PMNS mixing angles.

Now we can calculate the Dirac phase and the Majorana phases. In figures 2 and 3, we show the Dirac phase δ_{CP} versus the Majorana phases α and β , respectively. The both positive and negative signs are allowed for α and β . The allowed regions of the Majorana phases are $110^\circ \lesssim |\alpha| \lesssim 150^\circ$ and $145^\circ \lesssim |\beta| \lesssim 170^\circ$. On the other hand, the Dirac phase is still allowed in the all region as $-\pi \lesssim \delta_{\text{CP}} \lesssim \pi$.

Since the difference between the Majorana phases α and β contributes to the $0\nu\beta\beta$, we show the Dirac phase versus $(\alpha - \beta)$ in figure 4. We obtain $10^\circ \lesssim |\alpha - \beta| \lesssim 50^\circ$. We also show the allowed region of the Majorana phases on the α - β plane in figure 5. The Majorana phases α and β are restricted in the two regions on this plane.

Figure 2. The predicted region on α - δ_{CP} plane.

Figure 3. The predicted region on β - δ_{CP} plane.

Figure 4. The predicted region on $(\alpha - \beta)$ - δ_{CP} plane.

Figure 5. The predicted region on α - β plane. The Majorana phases are restricted in the corner on this plane.

Based on the numerical results of the phases, we can estimate the effective neutrino mass for the $0\nu\beta\beta$. The effective neutrino mass is written as

$$m_{ee} \equiv \sum_{i=1}^3 m_i U_{ei}^2. \quad (3.10)$$

We show the effective neutrino mass of the $0\nu\beta\beta$ versus the difference of the Majorana phases and the Dirac phase for in figures 6 and 7, respectively. For the fixed effective neutrino mass for the $0\nu\beta\beta$, there are two fold degeneracy and four fold degeneracy in the Majorana phase $(\alpha - \beta)$ and the Dirac phase δ_{CP} , respectively. Therefore, the degeneracy will be solved if both δ_{CP} and m_{ee} are observed. At present, we predict $32 \text{ meV} \lesssim |m_{ee}| \lesssim 49 \text{ meV}$, which is close to the expected reaches of the coming experiments of the $0\nu\beta\beta$ [131].

Finally, we show the sum of the neutrino masses is predicted as

$$0.0952 \text{ eV} \lesssim \sum_i m_i \lesssim 0.101 \text{ eV}. \quad (3.11)$$

This total neutrino mass is within the reaches of the future cosmological and astrophysical measurements, such as the CMB spectrum, the galaxy distributions, and the red-shifted 21cm line in the future [132].

Figure 6. The predicted effective neutrino mass for the $0\nu\beta\beta$ versus $(\alpha - \beta)$.

Figure 7. The predicted effective neutrino mass for the $0\nu\beta\beta$ versus δ_{CP} .

Figure 8. The allowed region on ψ - $\sin^2 \theta_{12}^\nu$ plane. The red line corresponds to the TBM.

3.2 S_4 flavor model for $m_3 \neq 0$

In this subsection, we show the numerical results in the case of the non-vanishing lightest neutrino mass $m_3 \neq 0$. In order to reproduce the relevant mass squared differences, Δm_{atm}^2 and Δm_{sol}^2 for the inverted mass hierarchy, $a \simeq b$ and $\varphi \simeq \pm\pi$ are satisfied. Then, the ψ of eq. (2.36) is restricted in $50^\circ \lesssim |\psi| \lesssim 90^\circ$ numerically. On the other hand, $\sin \eta$ in eq. (2.36) is restricted in $-0.70 \lesssim \sin \eta \lesssim -0.54$.

In figure 8, we show the allowed region on ψ - $\sin^2 \theta_{12}^\nu$ plane, where θ_{12}^ν is the mixing angle of the neutrino sector without the charged lepton contribution as seen in eq. (2.24). This result is very similar to the case of $m_3 = 0$ of figure 1 in the previous subsection.

At first, in figures 9 and 10, we show the Dirac phase δ_{CP} versus the Majorana phases α and β , respectively. The both positive and negative signs are allowed for α and β . The allowed regions of the Majorana phases are $105^\circ \lesssim |\alpha| \lesssim 135^\circ$ and $145^\circ \lesssim |\beta| \lesssim 170^\circ$. The Dirac phase is still allowed in the all region as $-\pi \lesssim \delta_{CP} \lesssim \pi$. We also show the Dirac phase δ_{CP} versus $(\alpha - \beta)$ in figure 11. We obtain $10^\circ \lesssim |\alpha - \beta| \lesssim 60^\circ$. In figure 12, we show the allowed region of the Majorana phases α and β . The Majorana phases are restricted in the two regions on this plane as well as the case of the previous subsection.

Next, we also show the effective neutrino mass m_{ee} for the $0\nu\beta\beta$ versus the Dirac phase in figure 13. For the fixed effective neutrino mass for the $0\nu\beta\beta$, there are four fold

Figure 9. The predicted region on α - δ_{CP} plane.

Figure 10. The predicted region on β - δ_{CP} plane.

Figure 11. The predicted region on $(\alpha - \beta)$ - δ_{CP} plane.

Figure 12. The predicted region on α - β plane. The Majorana phases are restricted in the corner of this plane.

degeneracy in the Dirac phase δ_{CP} as well as the case of the previous subsection. This degeneracy will be solved if both δ_{CP} and m_{ee} are observed. The predicted magnitude of m_{ee} is $34 \text{ meV} \lesssim |m_{ee}| \lesssim 59 \text{ meV}$, which is close to the expected reaches of the coming experiments of the $0\nu\beta\beta$ [131].

Finally, we show the allowed region of neutrino masses m_2 and m_3 in figure 14. The neutrino mass m_2 is $58.7 \text{ meV} \lesssim m_2 \lesssim 62.2 \text{ meV}$. The lightest neutrino mass m_3 is $31.0 \text{ meV} \lesssim m_3 \lesssim 35.8 \text{ meV}$. The sum of the neutrino masses is predicted as

$$0.150 \text{ eV} \lesssim \sum_i m_i \lesssim 0.160 \text{ eV}. \quad (3.12)$$

This value is also within the reaches of the future cosmological and astrophysical measurements.

4 Discussions and summary

We have presented the minimal S_4 flavor models of leptons. By using the alignments of the VEV's of the flavons obtained in subsection 2.3, the models lead to the two different neutrino mass spectra with the inverted mass hierarchy. The neutrino mass matrix is

Figure 13. The prediction on $\delta_{\text{CP}}-|m_{ee}|$ plane. The allowed region of effective neutrino mass for the $0\nu\beta\beta$ is $34\text{ meV} \lesssim |m_{ee}| \lesssim 59\text{ meV}$.

Figure 14. The allowed region on m_3-m_2 plane. The lightest neutrino mass m_3 is proportional to m_2 .

given by two complex parameters in eqs. (2.5) and (2.30). The first case has one vanishing neutrino mass $m_3 = 0$, and the second case has three non-vanishing neutrino masses. The charged leptons are not flavor diagonal. There remains the first and second family mixing, which give the non-vanishing reactor angle θ_{13} . We have moved to the TBM basis of neutrinos in order to clarify the deviation from the TBM.

It is not necessary to go to the TBM basis to calculate the mixing angles, but this is merely a convenient basis to do so. We present alternative basis, the bi-maximal base or the μ - τ symmetric one, where it is also convenient to study the mass matrices in appendices B and C, respectively.

Inputting the experimental data of two neutrino mass squared differences and the mixing angles θ_{12} , θ_{23} and θ_{13} , we can predict the Dirac phase and the Majorana phases. Furthermore, we predict the magnitude of the effective neutrino mass of the $0\nu\beta\beta$, which is correlated with the Dirac phases. For the fixed effective neutrino mass for the $0\nu\beta\beta$, there are two fold degeneracy and four fold degeneracy in the Majorana phase difference $(\alpha - \beta)$ and the Dirac phase δ_{CP} , respectively. Therefore, if both δ_{CP} and m_{ee} are observed, the Majorana phases are determined finally. At present, the predicted magnitudes of m_{ee} are in the regions as $32\text{ meV} \lesssim |m_{ee}| \lesssim 49\text{ meV}$ and $34\text{ meV} \lesssim |m_{ee}| \lesssim 59\text{ meV}$ for the cases of $m_3 = 0$ and $m_3 \neq 0$, respectively. These values are close to the expected reaches of the coming experiments of the $0\nu\beta\beta$.

The sum of three neutrino masses is also predicted in both models as $0.0952\text{ eV} \lesssim \sum m_i \lesssim 0.101\text{ eV}$ and $0.150\text{ eV} \lesssim \sum m_i \lesssim 0.160\text{ eV}$, respectively. Those total masses are also within the reaches of the future cosmological and astrophysical measurements.

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A Multiplication rules of the S_4 group

We show the multiplication rules of S_4 :

$$\begin{aligned}
 \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}_2 \otimes \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}_2 &= (a_1 b_1 + a_2 b_2)_1 \oplus (-a_1 b_2 + a_2 b_1)_{1'} \oplus \begin{pmatrix} a_1 b_2 + a_2 b_1 \\ a_1 b_1 - a_2 b_2 \end{pmatrix}_2, \\
 \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}_2 \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_3 &= \begin{pmatrix} a_2 b_1 \\ -\frac{1}{2}(\sqrt{3}a_1 b_2 + a_2 b_2) \\ \frac{1}{2}(\sqrt{3}a_1 b_3 - a_2 b_3) \end{pmatrix}_3 \oplus \begin{pmatrix} a_1 b_1 \\ \frac{1}{2}(\sqrt{3}a_2 b_2 - a_1 b_2) \\ -\frac{1}{2}(\sqrt{3}a_2 b_3 + a_1 b_3) \end{pmatrix}_{3'}, \\
 \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}_2 \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{3'} &= \begin{pmatrix} a_1 b_1 \\ \frac{1}{2}(\sqrt{3}a_2 b_2 - a_1 b_2) \\ -\frac{1}{2}(\sqrt{3}a_2 b_3 + a_1 b_3) \end{pmatrix}_3 \oplus \begin{pmatrix} a_2 b_1 \\ -\frac{1}{2}(\sqrt{3}a_1 b_2 + a_2 b_2) \\ \frac{1}{2}(\sqrt{3}a_1 b_3 - a_2 b_3) \end{pmatrix}_{3'}, \\
 \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}_3 \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_3 &= (a_1 b_1 + a_2 b_2 + a_3 b_3)_1 \oplus \begin{pmatrix} \frac{1}{\sqrt{2}}(a_2 b_2 - a_3 b_3) \\ \frac{1}{\sqrt{6}}(-2a_1 b_1 + a_2 b_2 + a_3 b_3) \end{pmatrix}_2 \\
 &\quad \oplus \begin{pmatrix} a_2 b_3 + a_3 b_2 \\ a_1 b_3 + a_3 b_1 \\ a_1 b_2 + a_2 b_1 \end{pmatrix}_3 \oplus \begin{pmatrix} a_3 b_2 - a_2 b_3 \\ a_1 b_3 - a_3 b_1 \\ a_2 b_1 - a_1 b_2 \end{pmatrix}_{3'}, \\
 \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}_{3'} \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{3'} &= (a_1 b_1 + a_2 b_2 + a_3 b_3)_1 \oplus \begin{pmatrix} \frac{1}{\sqrt{2}}(a_2 b_2 - a_3 b_3) \\ \frac{1}{\sqrt{6}}(-2a_1 b_1 + a_2 b_2 + a_3 b_3) \end{pmatrix}_2 \\
 &\quad \oplus \begin{pmatrix} a_2 b_3 + a_3 b_2 \\ a_1 b_3 + a_3 b_1 \\ a_1 b_2 + a_2 b_1 \end{pmatrix}_3 \oplus \begin{pmatrix} a_3 b_2 - a_2 b_3 \\ a_1 b_3 - a_3 b_1 \\ a_2 b_1 - a_1 b_2 \end{pmatrix}_{3'}, \\
 \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}_3 \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{3'} &= (a_1 b_1 + a_2 b_2 + a_3 b_3)_{1'} \oplus \begin{pmatrix} \frac{1}{\sqrt{6}}(2a_1 b_1 - a_2 b_2 - a_3 b_3) \\ \frac{1}{\sqrt{2}}(a_2 b_2 - a_3 b_3) \end{pmatrix}_2 \\
 &\quad \oplus \begin{pmatrix} a_3 b_2 - a_2 b_3 \\ a_1 b_3 - a_3 b_1 \\ a_2 b_1 - a_1 b_2 \end{pmatrix}_3 \oplus \begin{pmatrix} a_2 b_3 + a_3 b_2 \\ a_1 b_3 + a_3 b_1 \\ a_1 b_2 + a_2 b_1 \end{pmatrix}_{3'}.
 \end{aligned} \tag{A.1}$$

More details are shown in the review [41, 42].

B Neutrino mass matrix in the bimaximal base

B.1 $m_3 = 0$ case

The left-handed Majorana neutrino mass matrix M_ν is

$$M_\nu = \begin{pmatrix} a + b & b & b \\ b & b & b \\ b & b & b \end{pmatrix}. \tag{B.1}$$

After rotating M_ν with the bimaximal mixing matrix V_{BM} as

$$V_{\text{BM}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad (\text{B.2})$$

the left-handed Majorana neutrino mass matrix is rewritten as

$$M_\nu = V_{\text{BM}} \begin{pmatrix} \frac{1}{2}(a + (3 - 2\sqrt{2})b) & \frac{1}{2}(a - b) & 0 \\ \frac{1}{2}(a - b) & \frac{1}{2}(a + (3 + 2\sqrt{2})b) & 0 \\ 0 & 0 & 0 \end{pmatrix} V_{\text{BM}}^T. \quad (\text{B.3})$$

B.2 $m_3 \neq 0$ case

The left-handed Majorana neutrino mass matrix M_ν is

$$M_\nu = \begin{pmatrix} b & b & b \\ b & 2a + b & a + b \\ b & a + b & 2a + b \end{pmatrix}. \quad (\text{B.4})$$

After rotating M_ν with the bimaximal mixing matrix V_{BM} , the left-handed Majorana neutrino mass matrix is rewritten as

$$M_\nu = V_{\text{BM}} \begin{pmatrix} \frac{1}{2}(3a + (3 - 2\sqrt{2})b) & -\frac{1}{2}(3a + b) & 0 \\ -\frac{1}{2}(3a + b) & \frac{1}{2}(3a + (3 + 2\sqrt{2})b) & 0 \\ 0 & 0 & a \end{pmatrix} V_{\text{BM}}^T. \quad (\text{B.5})$$

C Neutrino mass matrix in the μ - τ maximal base

C.1 $m_3 = 0$ case

The left-handed Majorana neutrino mass matrix M_ν is

$$M_\nu = \begin{pmatrix} a + b & b & b \\ b & b & b \\ b & b & b \end{pmatrix}. \quad (\text{C.1})$$

After rotating M_ν with the μ - τ maximal mixing matrix V_{23} as

$$V_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad (\text{C.2})$$

the left-handed Majorana neutrino mass matrix is rewritten as

$$M_\nu = V_{23} \begin{pmatrix} a + b & \sqrt{2}b & 0 \\ \sqrt{2}b & 2b & 0 \\ 0 & 0 & 0 \end{pmatrix} V_{23}^T. \quad (\text{C.3})$$

C.2 $m_3 \neq 0$ case

The left-handed Majorana neutrino mass matrix M_ν is

$$M_\nu = \begin{pmatrix} b & b & b \\ b & 2a+b & a+b \\ b & a+b & 2a+b \end{pmatrix}. \quad (\text{C.4})$$

After rotating M_ν with the μ - τ maximal mixing matrix V_{23} , the left-handed Majorana neutrino mass matrix is rewritten as

$$M_\nu = V_{23} \begin{pmatrix} b & \sqrt{2}b & 0 \\ \sqrt{2}b & 3a+2b & 0 \\ 0 & 0 & a \end{pmatrix} V_{23}^T. \quad (\text{C.5})$$

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References

- [1] S.M. Bilenky, J. Hosek and S.T. Petcov, *On Oscillations of Neutrinos with Dirac and Majorana Masses*, *Phys. Lett. B* **94** (1980) 495 [[INSPIRE](#)].
- [2] J. Schechter and J.W.F. Valle, *Neutrino Masses in $SU(2) \times U(1)$ Theories*, *Phys. Rev. D* **22** (1980) 2227 [[INSPIRE](#)].
- [3] M. Doi, T. Kotani, H. Nishiura, K. Okuda and E. Takasugi, *CP Violation in Majorana Neutrinos*, *Phys. Lett. B* **102** (1981) 323 [[INSPIRE](#)].
- [4] T2K collaboration, K. Abe et al., *Observation of Electron Neutrino Appearance in a Muon Neutrino Beam*, *Phys. Rev. Lett.* **112** (2014) 061802 [[arXiv:1311.4750](#)] [[INSPIRE](#)].
- [5] DAYA BAY collaboration, F.P. An et al., *Observation of electron-antineutrino disappearance at Daya Bay*, *Phys. Rev. Lett.* **108** (2012) 171803 [[arXiv:1203.1669](#)] [[INSPIRE](#)].
- [6] RENO collaboration, J.K. Ahn et al., *Observation of Reactor Electron Antineutrino Disappearance in the RENO Experiment*, *Phys. Rev. Lett.* **108** (2012) 191802 [[arXiv:1204.0626](#)] [[INSPIRE](#)].
- [7] MINOS collaboration, P. Adamson et al., *Measurement of Neutrino and Antineutrino Oscillations Using Beam and Atmospheric Data in MINOS*, *Phys. Rev. Lett.* **110** (2013) 251801 [[arXiv:1304.6335](#)] [[INSPIRE](#)].
- [8] DOUBLE CHOOZ collaboration, Y. Abe et al., *Background-independent measurement of θ_{13} in Double CHOOZ*, *Phys. Lett. B* **735** (2014) 51 [[arXiv:1401.5981](#)] [[INSPIRE](#)].
- [9] S.K. Kang, C.S. Kim and J.D. Kim, *Neutrino masses and leptonic CP-violation*, *Phys. Rev. D* **62** (2000) 073011 [[hep-ph/0004020](#)] [[INSPIRE](#)].
- [10] M. Fukugita and M. Tanimoto, *Lepton flavor mixing matrix and CP-violation from neutrino oscillation experiments*, *Phys. Lett. B* **515** (2001) 30 [[hep-ph/0107082](#)] [[INSPIRE](#)].
- [11] C. Giunti and M. Tanimoto, *CP violation in bilarge lepton mixing*, *Phys. Rev. D* **66** (2002) 113006 [[hep-ph/0209169](#)] [[INSPIRE](#)].

- [12] Z.-z. Xing, *Nearly tri bimaximal neutrino mixing and CP-violation*, *Phys. Lett. B* **533** (2002) 85 [[hep-ph/0204049](#)] [[INSPIRE](#)].
- [13] B. Adhikary and A. Ghosal, *Constraining CP -violation in a softly broken A_4 symmetric Model*, *Phys. Rev. D* **75** (2007) 073020 [[hep-ph/0609193](#)] [[INSPIRE](#)].
- [14] B. Adhikary and A. Ghosal, *Nonzero U_{e3} , CP -violation and leptogenesis in a see-saw type softly broken A_4 symmetric model*, *Phys. Rev. D* **78** (2008) 073007 [[arXiv:0803.3582](#)] [[INSPIRE](#)].
- [15] G.C. Branco, R.G. Felipe and F.R. Joaquim, *Leptonic CP -violation*, *Rev. Mod. Phys.* **84** (2012) 515 [[arXiv:1111.5332](#)] [[INSPIRE](#)].
- [16] G.C. Branco, R. Gonzalez Felipe, F.R. Joaquim and H. Serodio, *Spontaneous leptonic CP -violation and nonzero θ_{13}* , *Phys. Rev. D* **86** (2012) 076008 [[arXiv:1203.2646](#)] [[INSPIRE](#)].
- [17] Y.H. Ahn and S.K. Kang, *Non-zero θ_{13} and CP -violation in a model with A_4 flavor symmetry*, *Phys. Rev. D* **86** (2012) 093003 [[arXiv:1203.4185](#)] [[INSPIRE](#)].
- [18] H. Ishimori, S. Khalil and E. Ma, *CP Phases of Neutrino Mixing in a Supersymmetric B - L Gauge Model with T_7 Lepton Flavor Symmetry*, *Phys. Rev. D* **86** (2012) 013008 [[arXiv:1204.2705](#)] [[INSPIRE](#)].
- [19] H. Ishimori and E. Ma, *New Simple A_4 Neutrino Model for Nonzero θ_{13} and Large δ_{CP}* , *Phys. Rev. D* **86** (2012) 045030 [[arXiv:1205.0075](#)] [[INSPIRE](#)].
- [20] W. Rodejohann and H. Zhang, *Simple two Parameter Description of Lepton Mixing*, *Phys. Rev. D* **86** (2012) 093008 [[arXiv:1207.1225](#)] [[INSPIRE](#)].
- [21] D. Marzocca, S.T. Petcov, A. Romanino and M.C. Sevilla, *Nonzero $|U_{e3}|$ from Charged Lepton Corrections and the Atmospheric Neutrino Mixing Angle*, *JHEP* **05** (2013) 073 [[arXiv:1302.0423](#)] [[INSPIRE](#)].
- [22] P. Ballett, S.F. King, C. Luhn, S. Pascoli and M.A. Schmidt, *Testing atmospheric mixing sum rules at precision neutrino facilities*, *Phys. Rev. D* **89** (2014) 016016 [[arXiv:1308.4314](#)] [[INSPIRE](#)].
- [23] P. Ballett, S.F. King, C. Luhn, S. Pascoli and M.A. Schmidt, *Testing solar lepton mixing sum rules in neutrino oscillation experiments*, *JHEP* **12** (2014) 122 [[arXiv:1410.7573](#)] [[INSPIRE](#)].
- [24] Z.-z. Xing and S. Zhou, *A partial μ - τ symmetry and its prediction for leptonic CP -violation*, *Phys. Lett. B* **737** (2014) 196 [[arXiv:1404.7021](#)] [[INSPIRE](#)].
- [25] G.C. Branco, M.N. Rebelo, J.I. Silva-Marcos and D. Wegman, *Quasidegeneracy of Majorana Neutrinos and the Origin of Large Leptonic Mixing*, *Phys. Rev. D* **91** (2015) 013001 [[arXiv:1405.5120](#)] [[INSPIRE](#)].
- [26] S.T. Petcov, *Predicting the values of the leptonic CP -violation phases in theories with discrete flavour symmetries*, *Nucl. Phys. B* **892** (2015) 400 [[arXiv:1405.6006](#)] [[INSPIRE](#)].
- [27] S.K. Kang and C.S. Kim, *Prediction of leptonic CP phase from perturbatively modified tribimaximal (or bimaximal) mixing*, *Phys. Rev. D* **90** (2014) 077301 [[arXiv:1406.5014](#)] [[INSPIRE](#)].

- [28] Y. Shimizu, M. Tanimoto and K. Yamamoto, *Predicting CP-violation in Deviation from Tri-bimaximal mixing of Neutrinos*, *Mod. Phys. Lett. A* **30** (2015) 1550002 [[arXiv:1405.1521](#)] [[INSPIRE](#)].
- [29] I. Girardi, S.T. Petcov and A.V. Titov, *Determining the Dirac CP-violation Phase in the Neutrino Mixing Matrix from Sum Rules*, *Nucl. Phys. B* **894** (2015) 733 [[arXiv:1410.8056](#)] [[INSPIRE](#)].
- [30] I. Girardi, S.T. Petcov and A.V. Titov, *Predictions for the Dirac CP-violation Phase in the Neutrino Mixing Matrix*, *Int. J. Mod. Phys. A* **30** (2015) 1530035 [[arXiv:1504.02402](#)] [[INSPIRE](#)].
- [31] I. Girardi, S.T. Petcov and A.V. Titov, *Predictions for the Leptonic Dirac CP-violation Phase: a Systematic Phenomenological Analysis*, *Eur. Phys. J. C* **75** (2015) 345 [[arXiv:1504.00658](#)] [[INSPIRE](#)].
- [32] S.K. Kang and M. Tanimoto, *Prediction of Leptonic CP Phase in A_4 symmetric model*, *Phys. Rev. D* **91** (2015) 073010 [[arXiv:1501.07428](#)] [[INSPIRE](#)].
- [33] P.F. Harrison, D.H. Perkins and W.G. Scott, *Tri-bimaximal mixing and the neutrino oscillation data*, *Phys. Lett. B* **530** (2002) 167 [[hep-ph/0202074](#)] [[INSPIRE](#)].
- [34] P.F. Harrison and W.G. Scott, *Symmetries and generalizations of tri-bimaximal neutrino mixing*, *Phys. Lett. B* **535** (2002) 163 [[hep-ph/0203209](#)] [[INSPIRE](#)].
- [35] E. Ma and G. Rajasekaran, *Softly broken A_4 symmetry for nearly degenerate neutrino masses*, *Phys. Rev. D* **64** (2001) 113012 [[hep-ph/0106291](#)] [[INSPIRE](#)].
- [36] K.S. Babu, E. Ma and J.W.F. Valle, *Underlying A_4 symmetry for the neutrino mass matrix and the quark mixing matrix*, *Phys. Lett. B* **552** (2003) 207 [[hep-ph/0206292](#)] [[INSPIRE](#)].
- [37] G. Altarelli and F. Feruglio, *Tri-bimaximal neutrino mixing from discrete symmetry in extra dimensions*, *Nucl. Phys. B* **720** (2005) 64 [[hep-ph/0504165](#)] [[INSPIRE](#)].
- [38] G. Altarelli and F. Feruglio, *Tri-bimaximal neutrino mixing, A_4 and the modular symmetry*, *Nucl. Phys. B* **741** (2006) 215 [[hep-ph/0512103](#)] [[INSPIRE](#)].
- [39] I. de Medeiros Varzielas, S.F. King and G.G. Ross, *Tri-bimaximal neutrino mixing from discrete subgroups of SU(3) and SO(3) family symmetry*, *Phys. Lett. B* **644** (2007) 153 [[hep-ph/0512313](#)] [[INSPIRE](#)].
- [40] G. Altarelli and F. Feruglio, *Discrete Flavor Symmetries and Models of Neutrino Mixing*, *Rev. Mod. Phys.* **82** (2010) 2701 [[arXiv:1002.0211](#)] [[INSPIRE](#)].
- [41] H. Ishimori, T. Kobayashi, H. Ohki, Y. Shimizu, H. Okada and M. Tanimoto, *Non-Abelian Discrete Symmetries in Particle Physics*, *Prog. Theor. Phys. Suppl.* **183** (2010) 1 [[arXiv:1003.3552](#)] [[INSPIRE](#)].
- [42] H. Ishimori, T. Kobayashi, H. Ohki, H. Okada, Y. Shimizu and M. Tanimoto, *An introduction to non-Abelian discrete symmetries for particle physicists*, *Lect. Notes Phys.* **858** (2012) 1 [[INSPIRE](#)].
- [43] S.F. King, A. Merle, S. Morisi, Y. Shimizu and M. Tanimoto, *Neutrino Mass and Mixing: from Theory to Experiment*, *New J. Phys.* **16** (2014) 045018 [[arXiv:1402.4271](#)] [[INSPIRE](#)].
- [44] R.N. Mohapatra, S. Nasri and H.-B. Yu, *S_3 symmetry and tri-bimaximal mixing*, *Phys. Lett. B* **639** (2006) 318 [[hep-ph/0605020](#)] [[INSPIRE](#)].

- [45] I. de Medeiros Varzielas, S.F. King and G.G. Ross, *Neutrino tri-bi-maximal mixing from a non-Abelian discrete family symmetry*, *Phys. Lett. B* **648** (2007) 201 [[hep-ph/0607045](#)] [[INSPIRE](#)].
- [46] P.D. Carr and P.H. Frampton, *Group Theoretic Bases for Tribimaximal Mixing*, [hep-ph/0701034](#) [[INSPIRE](#)].
- [47] F. Feruglio, C. Hagedorn, Y. Lin and L. Merlo, *Tri-bimaximal Neutrino Mixing and Quark Masses from a Discrete Flavour Symmetry*, *Nucl. Phys. B* **775** (2007) 120 [Erratum *ibid.* **836** (2010) 127] [[hep-ph/0702194](#)] [[INSPIRE](#)].
- [48] C.S. Lam, *Determining Horizontal Symmetry from Neutrino Mixing*, *Phys. Rev. Lett.* **101** (2008) 121602 [[arXiv:0804.2622](#)] [[INSPIRE](#)].
- [49] F. Bazzocchi and S. Morisi, *S_4 as a natural flavor symmetry for lepton mixing*, *Phys. Rev. D* **80** (2009) 096005 [[arXiv:0811.0345](#)] [[INSPIRE](#)].
- [50] W. Grimus and L. Lavoura, *A Model for trimaximal lepton mixing*, *JHEP* **09** (2008) 106 [[arXiv:0809.0226](#)] [[INSPIRE](#)].
- [51] W. Grimus and L. Lavoura, *Tri-bimaximal lepton mixing from symmetry only*, *JHEP* **04** (2009) 013 [[arXiv:0811.4766](#)] [[INSPIRE](#)].
- [52] H. Ishimori, T. Kobayashi, H. Okada, Y. Shimizu and M. Tanimoto, *Lepton Flavor Model from $\Delta(54)$ Symmetry*, *JHEP* **04** (2009) 011 [[arXiv:0811.4683](#)] [[INSPIRE](#)].
- [53] F. Bazzocchi, L. Merlo and S. Morisi, *Fermion Masses and Mixings in a $S(4)$ -based Model*, *Nucl. Phys. B* **816** (2009) 204 [[arXiv:0901.2086](#)] [[INSPIRE](#)].
- [54] F. Bazzocchi, L. Merlo and S. Morisi, *Phenomenological Consequences of See-Saw in S_4 Based Models*, *Phys. Rev. D* **80** (2009) 053003 [[arXiv:0902.2849](#)] [[INSPIRE](#)].
- [55] G. Altarelli, F. Feruglio and L. Merlo, *Revisiting Bimaximal Neutrino Mixing in a Model with S_4 Discrete Symmetry*, *JHEP* **05** (2009) 020 [[arXiv:0903.1940](#)] [[INSPIRE](#)].
- [56] Z.-z. Xing and S. Zhou, *Tri-bimaximal Neutrino Mixing and Flavor-dependent Resonant Leptogenesis*, *Phys. Lett. B* **653** (2007) 278 [[hep-ph/0607302](#)] [[INSPIRE](#)].
- [57] S.F. King, *Parametrizing the lepton mixing matrix in terms of deviations from tri-bimaximal mixing*, *Phys. Lett. B* **659** (2008) 244 [[arXiv:0710.0530](#)] [[INSPIRE](#)].
- [58] M. Honda and M. Tanimoto, *Deviation from tri-bimaximal neutrino mixing in A_4 flavor symmetry*, *Prog. Theor. Phys.* **119** (2008) 583 [[arXiv:0801.0181](#)] [[INSPIRE](#)].
- [59] M. Hirsch, S. Morisi and J.W.F. Valle, *Modelling tri-bimaximal neutrino mixing*, *Phys. Rev. D* **79** (2009) 016001 [[arXiv:0810.0121](#)] [[INSPIRE](#)].
- [60] S. Morisi, *Tri-Bimaximal lepton mixing with $A_4 \times (Z_2)^3$* , *Phys. Rev. D* **79** (2009) 033008 [[arXiv:0901.1080](#)] [[INSPIRE](#)].
- [61] A. Hayakawa, H. Ishimori, Y. Shimizu and M. Tanimoto, *Deviation from tri-bimaximal mixing and flavor symmetry breaking in a seesaw type A_4 model*, *Phys. Lett. B* **680** (2009) 334 [[arXiv:0904.3820](#)] [[INSPIRE](#)].
- [62] J. Barry and W. Rodejohann, *Deviations from tribimaximal mixing due to the vacuum expectation value misalignment in A_4 models*, *Phys. Rev. D* **81** (2010) 093002 [Erratum *ibid.* **81** (2010) 119901] [[arXiv:1003.2385](#)] [[INSPIRE](#)].
- [63] C.H. Albright, A. Dueck and W. Rodejohann, *Possible Alternatives to Tri-bimaximal Mixing*, *Eur. Phys. J. C* **70** (2010) 1099 [[arXiv:1004.2798](#)] [[INSPIRE](#)].

- [64] H. Ishimori, Y. Shimizu, M. Tanimoto and A. Watanabe, *Neutrino masses and mixing from S_4 flavor twisting*, *Phys. Rev. D* **83** (2011) 033004 [[arXiv:1010.3805](#)] [[INSPIRE](#)].
- [65] I. de Medeiros Varzielas and L. Merlo, *Ultraviolet Completion of Flavour Models*, *JHEP* **02** (2011) 062 [[arXiv:1011.6662](#)] [[INSPIRE](#)].
- [66] S.F. King and C. Luhn, *Trimaximal neutrino mixing from vacuum alignment in A_4 and S_4 models*, *JHEP* **09** (2011) 042 [[arXiv:1107.5332](#)] [[INSPIRE](#)].
- [67] S.F. King and C. Luhn, *A_4 models of tri-bimaximal-reactor mixing*, *JHEP* **03** (2012) 036 [[arXiv:1112.1959](#)] [[INSPIRE](#)].
- [68] Y. Shimizu, M. Tanimoto and A. Watanabe, *Breaking Tri-bimaximal Mixing and Large θ_{13}* , *Prog. Theor. Phys.* **126** (2011) 81 [[arXiv:1105.2929](#)] [[INSPIRE](#)].
- [69] E. Ma and D. Wegman, *Nonzero $\theta(13)$ for neutrino mixing in the context of A_4 symmetry*, *Phys. Rev. Lett.* **107** (2011) 061803 [[arXiv:1106.4269](#)] [[INSPIRE](#)].
- [70] H. Ishimori and T. Kobayashi, *Towards Minimal S_4 Lepton Flavor Model*, *JHEP* **10** (2011) 082 [[arXiv:1106.3604](#)] [[INSPIRE](#)].
- [71] S. Antusch and V. Maurer, *Large neutrino mixing angle θ_{13}^{MNS} and quark-lepton mass ratios in unified flavour models*, *Phys. Rev. D* **84** (2011) 117301 [[arXiv:1107.3728](#)] [[INSPIRE](#)].
- [72] P.S. Bhupal Dev, R.N. Mohapatra and M. Severson, *Neutrino Mixings in $SO(10)$ with Type II Seesaw and θ_{13}* , *Phys. Rev. D* **84** (2011) 053005 [[arXiv:1107.2378](#)] [[INSPIRE](#)].
- [73] P.S. Bhupal Dev, B. Dutta, R.N. Mohapatra and M. Severson, *θ_{13} and Proton Decay in a Minimal $SO(10) \times S_4$ model of Flavor*, *Phys. Rev. D* **86** (2012) 035002 [[arXiv:1202.4012](#)] [[INSPIRE](#)].
- [74] F. Bazzocchi and L. Merlo, *Neutrino Mixings and the S_4 Discrete Flavour Symmetry*, *Fortsch. Phys.* **61** (2013) 571 [[arXiv:1205.5135](#)] [[INSPIRE](#)].
- [75] J.A. Acosta, A. Aranda, M.A. Buen-Abad and A.D. Rojas, *Non-diagonal charged lepton mass matrix and non-zero θ_{13}* , *Phys. Lett. B* **718** (2013) 1413 [[arXiv:1207.6093](#)] [[INSPIRE](#)].
- [76] C. Hagedorn, S.F. King and C. Luhn, *$SUSY S_4 \times SU(5)$ revisited*, *Phys. Lett. B* **717** (2012) 207 [[arXiv:1205.3114](#)] [[INSPIRE](#)].
- [77] G. Altarelli, F. Feruglio, L. Merlo and E. Stamou, *Discrete Flavour Groups, θ_{13} and Lepton Flavour Violation*, *JHEP* **08** (2012) 021 [[arXiv:1205.4670](#)] [[INSPIRE](#)].
- [78] G. Altarelli, F. Feruglio and L. Merlo, *Tri-Bimaximal Neutrino Mixing and Discrete Flavour Symmetries*, *Fortsch. Phys.* **61** (2013) 507 [[arXiv:1205.5133](#)] [[INSPIRE](#)].
- [79] S.F. King, *Tri-bimaximal-Cabibbo Mixing*, *Phys. Lett. B* **718** (2013) 136 [[arXiv:1205.0506](#)] [[INSPIRE](#)].
- [80] S.F. King, T. Neder and A.J. Stuart, *Lepton mixing predictions from $\Delta(6n^2)$ family Symmetry*, *Phys. Lett. B* **726** (2013) 312 [[arXiv:1305.3200](#)] [[INSPIRE](#)].
- [81] Y. Shimizu, R. Takahashi and M. Tanimoto, *Minimal Neutrino Texture with Neutrino Mass Ratio and Cabibbo Angle*, *PTEP* **2013** (2013) 063B02 [[arXiv:1212.5913](#)] [[INSPIRE](#)].
- [82] S.F. King, *Minimal predictive see-saw model with normal neutrino mass hierarchy*, *JHEP* **07** (2013) 137 [[arXiv:1304.6264](#)] [[INSPIRE](#)].
- [83] S.F. King, *Minimal see-saw model predicting best fit lepton mixing angles*, *Phys. Lett. B* **724** (2013) 92 [[arXiv:1305.4846](#)] [[INSPIRE](#)].

- [84] W. Grimus and L. Lavoura, *A nonstandard CP transformation leading to maximal atmospheric neutrino mixing*, *Phys. Lett. B* **579** (2004) 113 [[hep-ph/0305309](#)] [[INSPIRE](#)].
- [85] M.-C. Chen and K.T. Mahanthappa, *Group Theoretical Origin of CP-violation*, *Phys. Lett. B* **681** (2009) 444 [[arXiv:0904.1721](#)] [[INSPIRE](#)].
- [86] F. Feruglio, C. Hagedorn and R. Ziegler, *A realistic pattern of lepton mixing and masses from S_4 and CP*, *Eur. Phys. J. C* **74** (2014) 2753 [[arXiv:1303.7178](#)] [[INSPIRE](#)].
- [87] G.-J. Ding, S.F. King and A.J. Stuart, *Generalised CP and A_4 Family Symmetry*, *JHEP* **12** (2013) 006 [[arXiv:1307.4212](#)] [[INSPIRE](#)].
- [88] P.M. Ferreira, W. Grimus, L. Lavoura and P.O. Ludl, *Maximal CP-violation in Lepton Mixing from a Model with Delta(27) flavour Symmetry*, *JHEP* **09** (2012) 128 [[arXiv:1206.7072](#)] [[INSPIRE](#)].
- [89] I. de Medeiros Varzielas and D. Emmanuel-Costa, *Geometrical CP-violation*, *Phys. Rev. D* **84** (2011) 117901 [[arXiv:1106.5477](#)] [[INSPIRE](#)].
- [90] S. Morisi, K.M. Patel and E. Peinado, *Model for T2K indication with maximal atmospheric angle and tri-maximal solar angle*, *Phys. Rev. D* **84** (2011) 053002 [[arXiv:1107.0696](#)] [[INSPIRE](#)].
- [91] S. Gupta, A.S. Joshipura and K.M. Patel, *Minimal extension of tri-bimaximal mixing and generalized $Z_2 \times Z_2$ symmetries*, *Phys. Rev. D* **85** (2012) 031903 [[arXiv:1112.6113](#)] [[INSPIRE](#)].
- [92] I. de Medeiros Varzielas, *Geometrical CP-violation in multi-Higgs models*, *JHEP* **08** (2012) 055 [[arXiv:1205.3780](#)] [[INSPIRE](#)].
- [93] I. de Medeiros Varzielas, D. Emmanuel-Costa and P. Leser, *Geometrical CP-violation from Non-Renormalisable Scalar Potentials*, *Phys. Lett. B* **716** (2012) 193 [[arXiv:1204.3633](#)] [[INSPIRE](#)].
- [94] M. Holthausen, M. Lindner and M.A. Schmidt, *CP and Discrete Flavour Symmetries*, *JHEP* **04** (2013) 122 [[arXiv:1211.6953](#)] [[INSPIRE](#)].
- [95] M.-C. Chen, M. Fallbacher, K.T. Mahanthappa, M. Ratz and A. Trautner, *CP Violation from Finite Groups*, *Nucl. Phys. B* **883** (2014) 267 [[arXiv:1402.0507](#)] [[INSPIRE](#)].
- [96] R.N. Mohapatra and C.C. Nishi, *S_4 Flavored CP Symmetry for Neutrinos*, *Phys. Rev. D* **86** (2012) 073007 [[arXiv:1208.2875](#)] [[INSPIRE](#)].
- [97] E. Ma, *Neutrino Mixing and Geometric CP-violation with $\Delta(27)$ Symmetry*, *Phys. Lett. B* **723** (2013) 161 [[arXiv:1304.1603](#)] [[INSPIRE](#)].
- [98] I. Girardi, A. Meroni, S.T. Petcov and M. Spinrath, *Generalised geometrical CP-violation in a T' lepton flavour model*, *JHEP* **02** (2014) 050 [[arXiv:1312.1966](#)] [[INSPIRE](#)].
- [99] G.-J. Ding, S.F. King, C. Luhn and A.J. Stuart, *Spontaneous CP-violation from vacuum alignment in S_4 models of leptons*, *JHEP* **05** (2013) 084 [[arXiv:1303.6180](#)] [[INSPIRE](#)].
- [100] A. Meroni, S.T. Petcov and M. Spinrath, *A SUSY $SU(5) \times T'$ Unified Model of Flavour with large θ_{13}* , *Phys. Rev. D* **86** (2012) 113003 [[arXiv:1205.5241](#)] [[INSPIRE](#)].
- [101] C.-C. Li and G.-J. Ding, *Generalised CP and trimaximal TM_1 lepton mixing in S_4 family symmetry*, *Nucl. Phys. B* **881** (2014) 206 [[arXiv:1312.4401](#)] [[INSPIRE](#)].
- [102] C. Luhn, *Trimaximal TM_1 neutrino mixing in S_4 with spontaneous CP-violation*, *Nucl. Phys. B* **875** (2013) 80 [[arXiv:1306.2358](#)] [[INSPIRE](#)].

- [103] G.-J. Ding and Y.-L. Zhou, *Predicting lepton flavor mixing from $\Delta(48)$ and generalized CP symmetries*, *Chin. Phys. C* **39** (2015) 021001 [[arXiv:1312.5222](#)] [[INSPIRE](#)].
- [104] P. Chen, C.-C. Li and G.-J. Ding, *Lepton Flavor Mixing and CP Symmetry*, *Phys. Rev. D* **91** (2015) 033003 [[arXiv:1412.8352](#)] [[INSPIRE](#)].
- [105] C.-C. Li and G.-J. Ding, *Lepton Mixing in A_5 Family Symmetry and Generalized CP*, *JHEP* **05** (2015) 100 [[arXiv:1503.03711](#)] [[INSPIRE](#)].
- [106] F. Bazzocchi, M. Frigerio and S. Morisi, *Fermion masses and mixing in models with $SO(10) \times A_4$ symmetry*, *Phys. Rev. D* **78** (2008) 116018 [[arXiv:0809.3573](#)] [[INSPIRE](#)].
- [107] H. Ishimori, Y. Shimizu and M. Tanimoto, *S_4 Flavor Symmetry of Quarks and Leptons in $SU(5)$ GUT*, *Prog. Theor. Phys.* **121** (2009) 769 [[arXiv:0812.5031](#)] [[INSPIRE](#)].
- [108] H. Ishimori, K. Saga, Y. Shimizu and M. Tanimoto, *Tri-bimaximal Mixing and Cabibbo Angle in S_4 Flavor Model with SUSY*, *Phys. Rev. D* **81** (2010) 115009 [[arXiv:1004.5004](#)] [[INSPIRE](#)].
- [109] R. de Adelhart Toorop, F. Bazzocchi and L. Merlo, *The Interplay Between GUT and Flavour Symmetries in a Pati-Salam $\times S_4$ Model*, *JHEP* **08** (2010) 001 [[arXiv:1003.4502](#)] [[INSPIRE](#)].
- [110] I.K. Cooper, S.F. King and C. Luhn, *SUSY $SU(5)$ with singlet plus adjoint matter and A_4 family symmetry*, *Phys. Lett. B* **690** (2010) 396 [[arXiv:1004.3243](#)] [[INSPIRE](#)].
- [111] I.K. Cooper, S.F. King and C. Luhn, *$A_4 \times SU(5)$ SUSY GUT of Flavour with Trimaximal Neutrino Mixing*, *JHEP* **06** (2012) 130 [[arXiv:1203.1324](#)] [[INSPIRE](#)].
- [112] S.F. King and C. Luhn, *A Supersymmetric Grand Unified Theory of Flavour with $PSL_2(7) \times SO(10)$* , *Nucl. Phys. B* **832** (2010) 414 [[arXiv:0912.1344](#)] [[INSPIRE](#)].
- [113] A.S. Joshipura, B.P. Kodrani and K.M. Patel, *Fermion Masses and Mixings in a μ -tau symmetric $SO(10)$* , *Phys. Rev. D* **79** (2009) 115017 [[arXiv:0903.2161](#)] [[INSPIRE](#)].
- [114] K.M. Patel, *An $SO(10) \times S_4$ Model of Quark-Lepton Complementarity*, *Phys. Lett. B* **695** (2011) 225 [[arXiv:1008.5061](#)] [[INSPIRE](#)].
- [115] A.S. Joshipura and K.M. Patel, *Fermion Masses in $SO(10)$ Models*, *Phys. Rev. D* **83** (2011) 095002 [[arXiv:1102.5148](#)] [[INSPIRE](#)].
- [116] C. Hagedorn, S.F. King and C. Luhn, *A SUSY GUT of Flavour with $S_4 \times SU(5)$ to NLO*, *JHEP* **06** (2010) 048 [[arXiv:1003.4249](#)] [[INSPIRE](#)].
- [117] S.F. King, C. Luhn and A.J. Stuart, *A Grand $\Delta(96) \times SU(5)$ Flavour Model*, *Nucl. Phys. B* **867** (2013) 203 [[arXiv:1207.5741](#)] [[INSPIRE](#)].
- [118] S. Antusch, C. Gross, V. Maurer and C. Sluka, *A flavour GUT model with $\theta_{13}^{PMNS} \simeq \theta_C/\sqrt{2}$* , *Nucl. Phys. B* **877** (2013) 772 [[arXiv:1305.6612](#)] [[INSPIRE](#)].
- [119] S.F. King, *A to Z of Flavour with Pati-Salam*, *JHEP* **08** (2014) 130 [[arXiv:1406.7005](#)] [[INSPIRE](#)].
- [120] M. Tanimoto and K. Yamamoto, *Linking Leptonic CP-violation to Quark Unitarity Triangle*, *JHEP* **04** (2015) 037 [[arXiv:1501.07717](#)] [[INSPIRE](#)].
- [121] C.-Y. Yao and G.-J. Ding, *Lepton and Quark Mixing Patterns from Finite Flavor Symmetries*, *Phys. Rev. D* **92** (2015) 096010 [[arXiv:1505.03798](#)] [[INSPIRE](#)].

- [122] K. Harigaya, M. Ibe and T.T. Yanagida, *Seesaw Mechanism with Occam's Razor*, *Phys. Rev. D* **86** (2012) 013002 [[arXiv:1205.2198](#)] [[INSPIRE](#)].
- [123] C.D. Froggatt and H.B. Nielsen, *Hierarchy of Quark Masses, Cabibbo Angles and CP-violation*, *Nucl. Phys. B* **147** (1979) 277 [[INSPIRE](#)].
- [124] Z. Maki, M. Nakagawa and S. Sakata, *Remarks on the unified model of elementary particles*, *Prog. Theor. Phys.* **28** (1962) 870 [[INSPIRE](#)].
- [125] B. Pontecorvo, *Neutrino Experiments and the Problem of Conservation of Leptonic Charge*, *Sov. Phys. JETP* **26** (1968) 984 [[INSPIRE](#)].
- [126] M.C. Gonzalez-Garcia, M. Maltoni and T. Schwetz, *Updated fit to three neutrino mixing: status of leptonic CP-violation*, *JHEP* **11** (2014) 052 [[arXiv:1409.5439](#)] [[INSPIRE](#)].
- [127] D.V. Forero, M. Tortola and J.W.F. Valle, *Global status of neutrino oscillation parameters after Neutrino-2012*, *Phys. Rev. D* **86** (2012) 073012 [[arXiv:1205.4018](#)] [[INSPIRE](#)].
- [128] G.L. Fogli, E. Lisi, A. Marrone, D. Montanino, A. Palazzo and A.M. Rotunno, *Global analysis of neutrino masses, mixings and phases: entering the era of leptonic CP-violation searches*, *Phys. Rev. D* **86** (2012) 013012 [[arXiv:1205.5254](#)] [[INSPIRE](#)].
- [129] PARTICLE DATA GROUP collaboration, K.A. Olive et al., *Review of Particle Physics*, *Chin. Phys. C* **38** (2014) 090001 [[INSPIRE](#)].
- [130] C. Jarlskog, *Commutator of the Quark Mass Matrices in the Standard Electroweak Model and a Measure of Maximal CP-violation*, *Phys. Rev. Lett.* **55** (1985) 1039 [[INSPIRE](#)].
- [131] J.J. Gómez-Cadenas and J. Martín-Albo, *Phenomenology of neutrinoless double beta decay*, *PoS(GSSI14)004* [[arXiv:1502.00581](#)] [[INSPIRE](#)].
- [132] TOPICAL CONVENERS: K.N. ABAZAJIAN, J.E. CARLSTROM, A.T. LEE collaboration, K.N. Abazajian et al., *Neutrino Physics from the Cosmic Microwave Background and Large Scale Structure*, *Astropart. Phys.* **63** (2015) 66 [[arXiv:1309.5383](#)] [[INSPIRE](#)].