## Microstate solutions from black hole deconstruction

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Abstract: We present a new family of asymptotic $A d S_{3} \times S^{2}$ solutions to eleven dimensional supergravity compactified on a Calabi-Yau threefold. They originate from the backreaction of $S^{2}$-wrapped M2-branes, which play a central role in the deconstruction proposal for the microscopic interpretation of the D4-D0 black hole entropy. We show that they are free of possible pathologies such as closed timelike curves and discuss their holographic interpretation.

Keywords: Black Holes in String Theory, AdS-CFT Correspondence, D-branes, M-Theory

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## 1 Introduction: the black hole deconstruction proposal

Starting with the seminal work of Strominger and Vafa [1], string theory has proven highly successful in giving microscopic accountings of the Bekenstein-Hawking entropy of certain supersymmetric black holes. Such accountings typically make optimal use of the protected nature of the entropy or index to do the computation in a regime where gravitational backreaction is absent and the relevant degrees of freedom are weakly coupled D-brane excitations. This approach leaves unanswered the question what the microstates evolve to in the regime where gravitational backreaction is significant. Furthermore, with the advent of AdS/CFT it became clear that the black hole microstates correspond to states in the Hilbert space of a CFT which captures the degrees of freedom in a near-horizon

AdS throat region. According to the standard AdS/CFT prescription, states in the CFT correspond semiclassically to turning on normalizeable fluctuations of the bulk fields near the boundary, and these are expected to lead to solutions of the full string/M theory on the AdS background.

Efforts to construct such solutions within the supergravity approximation to string/M theory can be grouped loosely under the fuzzball or microstate geometry program (see [2] and [3] for reviews and further references), although to which extent and under which circumstances the 2 -derivative low energy supergravity approximation is sufficient for this purpose is still a matter of debate. In this work we will make progress towards constructing supergravity solutions carrying the same charges as a large black hole in the context of the black hole deconstruction proposal [4]. In this proposal, it is argued that the leading contribution to the entropy of a 4D black hole arises from the large degeneracy of states carried by certain wrapped M2-branes, which so far were approximated as probes in the background of other rigid constituent branes. Our goal in this work is to construct the fully backreacted solutions. ${ }^{1}$ Our solutions contain brane sources near which the supergravity approximation breaks down, as might have been expected. Following the terminology of [3] we will refer to such solutions as microstate solutions as opposed to smooth microstate geometries.

Let us briefly review the main ingredients of the black hole deconstruction proposal. We start from the setup first introduced and studied by Maldacena, Strominger and Witten (MSW) [5]: consider M-theory on the background $\mathbb{R}^{1,3} \times S^{1} \times X$, with $X$ a Calabi-Yau threefold. When the radius of the circle is small in 11D Planck units, the type IIA string theory picture is appropriate. One can consider BPS states which are point-like in $\mathbb{R}^{1,3}$, arising from wrapped (D6, D4, D2, D0) branes ${ }^{2}$ and labelled by a charge vector $\Gamma=$ $\left(p^{0}, p^{A}, q_{A}, q_{0}\right)$. In the M-theory frame, these lift to (KK monopole, M5, M2, momentum) charges respectively, but we choose to use the IIA language throughout this paper. It is possible to construct a regular black hole carrying D4-D0 charges $\left(0, p^{A}, 0, q_{0}\right)$ which breaks half of the supersymmetry of the background ${ }^{3}$ and whose Bekenstein-Hawking entropy can be computed to be:

$$
\begin{equation*}
S=2 \pi \sqrt{q_{0} p^{3}} \tag{1.1}
\end{equation*}
$$

where $p^{3} \equiv D_{A B C} p^{A} p^{B} p^{C}$ where is triple self-intersection of the four-cycle in $X$ wrapped by the D 4 -brane.

We then proceed to take an M-theory decoupling limit

$$
\begin{equation*}
\frac{R}{l_{11}} \rightarrow \infty, \quad V_{\infty} \equiv \frac{V_{X}}{l_{11}^{6}} \quad \text { fixed }, \tag{1.2}
\end{equation*}
$$

where $R$ is the radius of $S^{1}$ and $l_{11}$ the 11D Planck length. For a more detailed discussion of this decoupling limit see [7]. Note that one can define a 't Hooft like coupling that is

[^0]invariant under this limit:
\[

$$
\begin{equation*}
\lambda \equiv \frac{p^{3}}{V_{\infty}} . \tag{1.3}
\end{equation*}
$$

\]

When this parameter is large, $\lambda \gg 1$, the bulk theory, M-theory in a (locally) $A d S_{3} \times$ $S^{2} \times X$ attractor throat geometry, is well described by its supergravity approximation as the curvature radius of $A d S_{3}$ and $S^{2}$ is $l=\lambda l_{11}$. To be precise the decoupled near horizon geometry originating from the 4 d black hole $/ 5 \mathrm{~d}$ black string is not global $A d S_{3}$ but rather a BTZ black hole, $\mathrm{BTZ} \times S^{2} \times X$.

When on the other hand $\lambda \ll 1$ the theory is more naturally described as the low energy limit of the M5-brane worldvolume theory dimensionally reduced over the CalabiYau directions to a $1+1$ dimensional sigma model $[5,8]$. This incompletely understood theory is referred to as the MSW CFT. It has, up to terms subleading in the $p^{A}$, central charges $c_{L}=c_{R}=p^{3}$, and possesses $(4,0)$ superconformal symmetry. In terms of the conformal generators, the D0-charge corresponds to $q_{0}=\bar{L}_{0}-L_{0}$, so that the BPS states which contribute to the black hole entropy take the form of Ramond ground states in the left-moving sector tensored with highly excited states on the right-moving side. They can be easily counted in the Cardy regime $\frac{\bar{L}_{0}}{c} \sim \frac{q_{0}}{p^{3}} \gg 1$, which is also the regime where the BTZ black hole has a large horizon, and their exponential degeneracy correctly reproduces the Bekenstein-Hawking entropy (1.1) of the original 4D black hole [5].

The black hole deconstruction proposal [4] gives a tentative description of the typical microstates in the gravity regime $\lambda \gg 1$, as a particular bound state of low-entropy Dbrane centers. One starts with a two-center D6-anti-D6 configuration with worldvolume fluxes turned on, carrying the following charges:

$$
\begin{align*}
& \Gamma_{D 6}=\left(1, \frac{p^{A}}{2}, \frac{D_{A B C} p^{B} p^{C}}{8},-\frac{p^{3}}{48}\right) \\
& \Gamma_{\overline{D 6}}=\left(-1, \frac{p^{A}}{2},-\frac{D_{A B C} p^{B} p^{C}}{8},-\frac{p^{3}}{48}\right) . \tag{1.4}
\end{align*}
$$

The corresponding two-center supergravity solution can be constructed using the methods of [9], and upon taking the decoupling limit (1.2), one obtains the global $A d S_{3}$ geometry of the form $A d S_{3} \times{ }_{\text {rot }} S^{2} \times X$, where the subscript rot means the $S^{2}$ is nontrivially fibered. As we will review below, this solution represents, in a semiclassical sense, the Ramond ground state with maximum R-charge in the MSW CFT. To obtain a solution carrying the same charges as the 4 D black hole we have to add to the system an extra D0 charge $q_{0}+\frac{p^{3}}{24}$. One way to add this charge is in the form of many separate D0-brane centers, localized in the plane between the D6 and anti-D6. Such solutions can also be constructed using the methods of [9] and are illustrated in figure 1(a). However, despite there being a large moduli space of such solutions, the discrete set of states obtained upon quantization does not account for a sizeable fraction of the black hole entropy [10]. Another way to add the D0-brane charge is, in the spirit of the Myers effect [11], in the form of a D2-brane with worldvolume flux, which can supersymmetrically wrap an ellipsoid with the D6 and antiD6 branes at its centers, see figure 1(b). What makes these configurations relevant for the black hole entropy is that they couple to the D4-brane flux on the Calabi-Yau through the


Figure 1. Different ways of adding D0-brane charge (in red) to a two-centered D6-anti-D6 system (the blue dots). (a) In the form of separate D0-brane centers. (b) In the form of an ellipsoidal D2-brane with worldvolume flux. (c) Tadpole cancellation requires adding a fundamental string running between the D6 and anti-D6 branes, shown here as a green line.
worldvolume Wess-Zumino coupling $\int C_{3}$. Due to this coupling the D2-brane behaves as a particle in the magnetic fields threading the Calabi-Yau space, and has lowest Landau level degeneracy proportional to $p^{3}$. The combinatorics of distributing the total D0-charge over such D2-brane configurations then correctly accounts for the black hole entropy [12]. In the decoupling limit (1.2), this D2-brane configuration becomes an M2-brane which wraps the $S^{2}$ and is point-like in the $A d S_{3}$ part of the geometry. As we shall review below, this brane traces out a helical curve in $A d S_{3}$ whose radius is related to the D0-charge.

Although the configurations (a) have long been established as fully backreacted supergravity solutions, the configurations (b) were only constructed as M2 probes in a supergravity background. The goal of this work is to progress beyond the probe approximation for this wrapped M2-brane and construct the fully backreacted geometry. In doing so we will find that it is free of pathologies such as closed timelike curves, which plagued our earlier attempt in this direction [6], and has a standard asymptotically $A d S_{3}$ behaviour consistent with expectations from the MSW CFT.

We should also mention one complication that we will not address in this work, which arises from a worldvolume tadpole on the D6 branes of the type discussed in [13]. The D2 brane surrounding the D6-anti-D6 system produces a magnetic 6-form flux $F_{6}$ which induces a tadpole on the compact D6 worldvolume through the Wess-Zumino coupling $\int A \wedge F_{6}$. This tadpole can be cancelled by letting a fundamental string run between the D6 and anti-D6 branes, see figure 1(c). Furthermore, it can be argued that this string also carries an anti-D2 charge, so that the net D2-charge of the full configuration is zero. ${ }^{4}$ The M-theory decoupling limit of this configuration includes and additional anti-M2 brane at the center of AdS. Ignoring this tadpole does not lead to a direct inconsistency in the 5D supergravity picture we will use, as it is an effect in the internal Calabi-Yau directions. Nevertheless one might worry that not cancelling it leads to solutions which are ill-behaved

[^1]in some way. We will find that this is not the case, and that the main effect of ignoring it is, as far we can see, that the boundary theory is deformed by source terms proportional to the M2-charge, which otherwise would be absent.

This paper is organized as follows. In section 2 we review how the problem can be effectively reduced to a three dimensional description, a picture we will use in most of the article. We then discuss some physical properties of M2-brane probe particles, which originate from wrapping the internal $S^{2}$, in section 3 . The main new contributions of our work can be found in sections 4 and 5 . First we work out the details of the backreacted solution for an M2 at the center of $A d S_{3}$ and discuss at length various physical and holographic properties of this solution. In section 5 we present an additional family of solutions that tentatively describe the M2 particles spiralling at finite radius in $A d S_{3}$. We then connect back to the original 5 d setup in section 6 where we also discuss the supersymmetry properties of the solutions. We conclude in section 7 with a short outlook on possible future directions. For the convenience of the reader we also included the appendices $\mathrm{A}, \mathrm{B}, \mathrm{C}$ and D containing various technical details.

## 2 Effective three-dimensional description

As explained in the Introduction, the brane configurations we are interested in can be described as supersymmetric excitations of the long wavelength approximation to M-theory on the background $A d S_{3} \times$ rot $S^{2} \times X$, arising from wrapping an M2-brane on the $S^{2}$. We will make the approximation that the M2-brane charge is smeared on $X$, so that we can construct our solutions, after dimensional reduction on $X$, within 5D supergravity or, upon further reduction on $S^{2}$, in a three dimensional theory. We will use the simpler 3D point of view in most of the paper, and will comment on the geometric structure of our solutions from the 5 D point of view in section 6 .

As we will see in more detail below, the M2-brane provides a source for the volume modulus of $X$, which we will call the dilaton $\tau_{2}$, as well as for an axion $\tau_{1}$ which is obtained from dualizing the M-theory three-form with all legs in the 5D part of the geometry. We will often combine these in a complex field $\tau=\tau_{1}+i \tau_{2}$, which we will refer to as the axiondilaton since it parametrizes the coset $\mathrm{SU}(1,1) / \mathrm{U}(1)$ just like the familiar axion-dilaton of type IIB supergravity/string theory. It was shown in [6], to which we refer for more details and conventions, that the consistent 11D reduction ansatz for our solutions is

$$
\begin{equation*}
d s_{11}^{2}=\tilde{\tau}_{2}^{-2 / 3}\left(d s_{3}^{2}+\frac{l^{2}}{4}\left(d \theta^{2}+\sin ^{2} \theta(d \phi-\mathcal{A})^{2}\right)\right)+l_{11}^{2} \tau_{2}^{1 / 3} d s_{X}^{2} \tag{2.1}
\end{equation*}
$$

Where $\tilde{\tau}_{2}=\frac{\tau_{2}}{V_{\infty}}$ denotes the fluctuating part of the dilaton field and the Calabi-Yau metric $d s_{X}^{2}$ is assumed to be normalized to have unit volume. The $\mathrm{U}(1)$ gauge field $\mathcal{A}$ incorporates the possibility of having a nontrivially fibered $S^{2}$.

The metric above (together with an appropriate 3-form) is a solution to 11D supergravity when the effective 3D fields parameterizing it extremize the action

$$
\begin{equation*}
S=\frac{1}{16 \pi G_{3}} \int_{\mathcal{M}}\left[d^{3} x \sqrt{-g}\left(R+\frac{2}{l^{2}}-\frac{\partial_{\mu} \tau \partial^{\mu} \bar{\tau}}{2 \tau_{2}^{2}}\right)+\frac{l}{2} \mathcal{A} \wedge d \mathcal{A}\right] \tag{2.2}
\end{equation*}
$$

where

$$
\begin{equation*}
16 \pi G_{3}=\frac{l_{11}^{3}}{2 \pi^{2} V_{\infty} l^{2}} \tag{2.3}
\end{equation*}
$$

This is equivalent to solving the equations of motion

$$
\begin{align*}
R_{\mu \nu}+\frac{2}{l^{2}} g_{\mu \nu}-\frac{\partial_{(\mu} \tau \partial_{\nu)} \bar{\tau}}{2 \tau_{2}^{2}} & =0  \tag{2.4}\\
\square \tau+i \frac{\partial_{\mu} \tau \partial^{\mu} \tau}{\tau_{2}} & =0  \tag{2.5}\\
d \mathcal{A} & =0 \tag{2.6}
\end{align*}
$$

We now describe our ansatz for the 3D fields describing the solutions of interest, which was proposed in [6] and further justified in [14]. We assume the metric to be stationary and written as a timelike fibration over a two-dimensional base, which we cover with a complex coordinate $z$. We will parameterize the metric as

$$
\begin{equation*}
d s^{2}=\frac{l^{2}}{4}\left[-(d t+\chi)^{2}+\tau_{2} e^{-2 \Phi} d z d \bar{z}\right] \tag{2.7}
\end{equation*}
$$

The real field $\Phi$, the one-form $\chi$ and the axion-dilaton $\tau$ are assumed to be time independent.

The equation (2.5) then reduces to

$$
\begin{equation*}
\partial_{z} \partial_{\bar{z}} \tau+i \frac{\partial_{z} \tau \partial_{\bar{z}} \tau}{\tau_{2}}=0 \tag{2.8}
\end{equation*}
$$

This equation allows for solutions where the axion-dilaton $\tau$ is holomorphic,

$$
\begin{equation*}
\tau=\tau(z) \tag{2.9}
\end{equation*}
$$

which motivated our choice of ansatz. Such holomorphic solutions are naturally expected to be supersymmetric, and we will show that this is indeed the case, though we defer a detailed discussion of the supersymmetry properties of our solutions to section 6. Our ansatz can be seen as a straightforward generalization of that in [15], which describes a codimension one BPS object in flat spacetime, to the case with a negative cosmological constant.

Choosing $\tau$ to be holomorphic and substituting this in the metric equation (2.4) leads to the following equations for $\chi$ and $\Phi$ :

$$
\begin{align*}
4 \partial_{z} \partial_{\bar{z}} \Phi+\tau_{2} e^{-2 \Phi} & =0  \tag{2.10}\\
d \chi+\frac{i}{2} \tau_{2} e^{-2 \Phi} d z \wedge d \bar{z} & =0 \tag{2.11}
\end{align*}
$$

The first equation is a deformation of the Liouville equation, to which it reduces when $\tau$ is constant. Given a solution to the first equation, the second equation can be solved uniquely up to the choice of a closed one-form $\Lambda$ :

$$
\begin{equation*}
\chi=2 \Im m \partial \Phi+\Lambda \tag{2.12}
\end{equation*}
$$

where $\partial$ is the standard holomorphic Dolbeault operator. Finally the solution is completed by choosing a flat gauge connection $\mathcal{A}$.

Note that our discussion was completely local so far. Below we will add susy-compatible source terms, which describe the M2-brane wrapped on $S^{2}$, completing the solution globally. Indeed, from the three dimensional point of view this brane looks like a charged point particle, and will create delta-function singularities in the fields which we will examine in section 4.

Before doing so, let us first discuss the solution which describes the background to which we want to add the M2-brane. As discussed in the Introduction, this background is the decoupling limit of the fluxed D6-anti-D6 configuration (1.4), which was worked out in $[4,7]$. It can be found as a solution of the 3D theory (2.2), with the AdS radius $l$ given in terms of the D4-charges as

$$
\begin{equation*}
l=\left(\frac{p^{3}}{6 V_{\infty}}\right)^{\frac{1}{3}} \frac{l_{11}}{2 \pi} . \tag{2.13}
\end{equation*}
$$

The fluxed D6-anti-D6 configuration is realized as a particular solution within the ansatz (2.7) with constant axion-dilaton:

$$
\begin{align*}
& \tau=i V_{\infty}  \tag{2.14}\\
& \Phi=\ln \frac{\sqrt{V_{\infty}}(1-z \bar{z})}{2}  \tag{2.15}\\
& \chi=2 \Im m \partial \Phi=-2 \frac{z \bar{z}}{1-z \bar{z}} d \arg z  \tag{2.16}\\
& \mathcal{A}=d t+d \arg z \tag{2.17}
\end{align*}
$$

The resulting metric is completely regular and corresponds to global $A d S_{3} ;$ presented as a timelike fibration over the Poincaré disc $|z|<1$. The following coordinate transformation takes us to standard global coordinates:

$$
\begin{align*}
|z| & =\tanh \rho  \tag{2.18}\\
t & =2 T  \tag{2.19}\\
\arg z & =\alpha-T \tag{2.20}
\end{align*}
$$

and we obtain

$$
\begin{equation*}
d s^{2}=l^{2}\left[-\cosh ^{2} \rho d T^{2}+d \rho^{2}+\sinh ^{2} \rho d \alpha^{2}\right] . \tag{2.21}
\end{equation*}
$$

The Wilson line for the $\mathrm{U}(1)$ gauge field $\mathcal{A}$ means that the $S^{2}$ is nontrivially fibered, so that the full 11-D geometry is of the form $A d S_{3} \times_{\text {rot }} S^{2} \times X$ as anticipated in the Introduction. Its first effect is to break the symmetry ${ }^{5}$ to $\mathrm{U}(1)_{L} \times \mathrm{SL}(2, \mathbb{R})_{R}$. Furthermore this Wilson line is singular at $\rho=0$ and has the effect of changing the periodicity of the fermions when encircling the center of $A d S_{3}$. As we will review below, the interpretation in the dual (4,0) theory is that it represents the Ramond ground state with maximal R-charge on the leftmoving side and the $s l(2)$ invariant vacuum on the right-moving side. In our conventions, this state has conformal dimensions $(h, \bar{h})=(0,-c / 24)$ and R-charge $j=c / 12$.

[^2]We can also consider the more general class of solutions obtained by shifting both $\chi$ and $\mathcal{A}$ by $(\mu-1) d \arg z$, leading to

$$
\begin{align*}
\tau & =i V_{\infty}  \tag{2.22}\\
\Phi & =\ln \frac{\sqrt{V_{\infty}}(1-z \bar{z})}{2}  \tag{2.23}\\
\chi & =\left(\mu-1-2 \frac{z \bar{z}}{1-z \bar{z}}\right) d \arg z  \tag{2.24}\\
\mathcal{A} & =d t+\mu d \arg z \tag{2.25}
\end{align*}
$$

The parameter $\mu$ is a constant whose physical values correspond to the range $0<\mu \leq 1$. For $\mu<1$, this introduces a Dirac string singularity in $\chi$ and we obtain a singular metric with the geometry of a spinning conical defect. These solutions represent Ramond ground states with lower than maximal R-charge on the left-moving side and the $s l(2)$ invariant vacuum on the right-moving side, with quantum numbers $(h, \bar{h})=(0,-c / 24)$ and $j=c \mu / 12$. They can be viewed as the result of backreacting a heavy BPS particle in the center of the $A d S_{3}$ solution (2.17) [16].

## 3 Probe approximation

We now turn to the issue of adding an M2-brane wrapped on the $S^{2}$ in the $A d S_{3} \times_{\text {rot }} S^{2} \times X$ background described by (2.17). From the 3D point of view this M2-brane is a charged point particle, which we will refer to as the 'M2-particle' in what follows. To start with we will review some results from treating the M 2 -particle as a probe $[4,12,17,18]$ i.e. ignoring its backreaction on the geometry.

By dimensionally reducing the M2-brane action over the $S^{2}$ one obtains the following 3 D action for an M2-particle of charge ${ }^{6} 2 \pi q_{\star}$

$$
\begin{equation*}
S_{\mathrm{M} 2}=\frac{1}{16 \pi G_{3}}\left[-2 \pi q_{\star} \int_{\mathcal{W}} d \xi \frac{\sqrt{-{ }^{*} g}}{\tau_{2}}\right]+2 \pi q_{\star} \int_{\mathcal{W}} A \tag{3.1}
\end{equation*}
$$

where $\mathcal{W}$ denotes worldline of the M2-particle and $A$ is the $\mathrm{U}(1)$ gauge field dual to the axion $\tau_{1}$.

The M2-particle action (3.1) in the $A d S_{3}$ background (2.21) reads, in a static gauge with respect to $T$,

$$
\begin{equation*}
S_{\mathrm{M} 2}=-\frac{q_{\star} l}{8 V_{\infty} G_{3}} \int d T \sqrt{\cosh ^{2} \rho-\dot{\rho}^{2}-\sinh ^{2} \rho \dot{\alpha}^{2}} \tag{3.2}
\end{equation*}
$$

It's easy to see that a solution is provided by having the particle rotate on a helical curve at fixed $\rho$ (see figure 2):

$$
\begin{equation*}
\rho=\rho_{0}, \quad \alpha=\alpha_{0}+T \tag{3.3}
\end{equation*}
$$

The constant $\alpha_{0}$ can be absorbed in $T$ and we will do so in what follows. Note that the

[^3]

Figure 2. Trajectories of M2-particles in global $\mathrm{AdS}_{3}$ depicted as a solid cylinder. (a): an M2particle in the 'center' $\rho_{0}=0$. (b): for $\rho_{0}>0$ the particle traces out a helical curve.

M2-particle is static with respect to the time coordinate $t$ in the coordinate system (2.20), in terms of which the metric is not manifestly static.

Let us comment on the symmetry properties of this class of solutions. It's useful to represent points in $A d S_{3}$ as $\mathrm{SL}(2, \mathbb{R})$ group elements

$$
g(T, \rho, \alpha)=\left(\begin{array}{cc}
X_{1}+X_{4}-X_{2}-X_{3}  \tag{3.4}\\
X_{2}-X_{3} & X_{1}-X_{4}
\end{array}\right) ; \quad X_{1}^{2}+X_{2}^{2}-X_{3}^{2}-X_{4}^{2}=1
$$

with

$$
\begin{array}{ll}
X_{1}=\cos T \cosh \rho & X_{3}=\cos \alpha \sinh \rho \\
X_{2}=\sin T \cosh \rho & X_{4}=\sin \alpha \sinh \rho . \tag{3.6}
\end{array}
$$

The worldlines of the solutions (3.3) are represented by

$$
\begin{equation*}
g_{w l}\left(\rho_{0} ; T\right)=g\left(T, \rho_{0}, T\right) . \tag{3.7}
\end{equation*}
$$

Each of these M2-particle worldlines preserves a specific $\mathrm{U}(1)_{L} \times \mathrm{U}(1)_{R}$ subgroup of the $\mathrm{SL}(2, \mathbb{R})_{L} \times \mathrm{SL}(2, \mathbb{R})_{R}$ isometry group of the $A d S_{3}$ metric. This is easy to see for the worldline with $\rho_{0}=0$, which is invariant under translations of both $T$ and $\alpha$. The M2particle worldlines with $\rho_{0}>0$ are simply related to this one by the action of the broken generators in $\operatorname{SL}(2, \mathbb{R})_{R}$ :

$$
\begin{align*}
g_{w l}\left(\rho_{0} ; T\right) & =g_{w l}(0 ; T) \cdot R\left(\rho_{0}\right) ;  \tag{3.8}\\
R\left(\rho_{0}\right) & =\left(\begin{array}{cc}
\cosh \rho_{0} & \sinh \rho_{0} \\
\sinh \rho_{0} & \cosh \rho_{0}
\end{array}\right) \tag{3.9}
\end{align*}
$$

Each of the worldlines (3.3) therefore preserves a $\mathrm{U}(1)_{L} \times \mathrm{U}(1)_{R}$ whose embedding in $\mathrm{SL}(2, \mathbb{R})_{L} \times \mathrm{SL}(2, \mathbb{R})_{R}$ depends on $\rho_{0}$.

Another way to state this is that we can generate the M2-particle at finite radius $\rho_{0}$ from the one at $\rho_{0}$ through the coordinate transformation determined by

$$
\begin{equation*}
g(\tilde{T}, \tilde{\rho}, \tilde{\alpha})=g(T, \rho, \alpha) \cdot R\left(\rho_{0}\right) \tag{3.10}
\end{equation*}
$$

More explicitly it is given by

$$
\begin{align*}
\cosh ^{2} \tilde{\rho} & =\cosh ^{2}\left(\rho+\rho_{0}\right)-\sin ^{2} \frac{\psi}{2} \sinh 2 \rho \sinh 2 \rho_{0}  \tag{3.11}\\
\tilde{x}_{+} & =x_{+}-x_{-}+\arg \left[\left(1+e^{i x_{-}} \operatorname{coth} \rho_{0} \tanh \rho\right)\left(1+e^{i x_{-}} \tanh \rho_{0} \tanh \rho\right)\right]  \tag{3.12}\\
\tilde{x}_{-} & =\arg \left[\left(1+e^{i x_{-}} \operatorname{coth} \rho_{0} \tanh \rho\right)\left(1+e^{-i x_{-}} \tanh \rho_{0} \tanh \rho\right)\right] \tag{3.13}
\end{align*}
$$

Near the AdS boundary $\rho \rightarrow \infty$ this transformation takes the form

$$
\begin{align*}
\tilde{x}_{+} & =x_{+}+\mathcal{O}\left(e^{-2 \rho}\right)  \tag{3.14}\\
e^{i \tilde{x}_{-}} & =\frac{\cosh \rho_{0} e^{i x_{-}}+\sinh \rho_{0}}{\sinh \rho_{0} e^{i x_{-}}+\cosh \rho_{0}}+\mathcal{O}\left(e^{-2 \rho}\right) \tag{3.15}
\end{align*}
$$

We see that this reduces on the boundary to a purely right-moving conformal transformation in the $\mathrm{SL}(2, \mathbb{R})$ subgroup of the conformal group, which in terms of Virasoro generators can be written as

$$
\begin{equation*}
e^{\rho_{0}\left(\tilde{L}_{-1}-\tilde{L}_{1}\right)} . \tag{3.16}
\end{equation*}
$$

As was shown in $[4,6,17]$ from a worldvolume $\kappa$-symmetry analysis, the M2-particle solutions (3.3) are BPS, preserving half of the supersymmetry of the background. Furthermore, the solutions with different values of $\rho_{0}$ are mutually supersymmetric. This can also be understood from the point of view of the asymptotic superalgebra, since the solutions with different $\rho_{0}$ are related by a purely right-moving conformal transformation on the boundary, which does not affect the asymptotic supercharges which reside in the leftmoving sector. These observations will prove useful to obtain a proposal for the backreacted solutions with $\rho_{0}>0$ from the one with $\rho_{0}=0$, as we shall see in section 5 .

We now turn to the determination of some of the worldvolume Noether charges. One easily computes the energy $H_{T}$ with respect to $\partial_{T}$ and the $\alpha$-angular momentum $P_{\alpha}$ of of the solutions (3.3):

$$
\begin{align*}
H_{T} & =\frac{q_{\star}}{V_{\infty}} \frac{c}{12} \cosh ^{2} \rho_{0}  \tag{3.17}\\
P_{\alpha} & =\frac{q_{\star}}{V_{\infty}} \frac{c}{12} \sinh ^{2} \rho_{0}, \tag{3.18}
\end{align*}
$$

where we have introduced the Brown-Henneaux central charge

$$
\begin{equation*}
c=\frac{3 l}{2 G_{3}}=p^{3} . \tag{3.19}
\end{equation*}
$$

These suggest that the addition of the M2-particle changes the scaling dimensions in the dual CFT as follows

$$
\begin{align*}
\Delta h^{\text {probe }} & =\frac{1}{2}\left(H_{T}-P_{\alpha}\right)=\frac{q_{\star}}{V_{\infty}} \frac{c}{24}  \tag{3.20}\\
\Delta \bar{h}^{\text {probe }} & =\frac{1}{2}\left(H_{T}+P_{\alpha}\right)=\frac{q_{\star}}{V_{\infty}} \frac{c}{24} \cosh 2 \rho_{0} . \tag{3.21}
\end{align*}
$$

We note that the difference of the conformal dimensions $P_{\alpha}$ becomes the D0-charge $\Delta q_{0}$ after dimensional reduction on the $\arg z$ circle. Hence the greater the radius $\rho_{0}$, the greater
the D0-charge. Furthermore, viewing the M2-probe probe solution from the 5D point of view, one finds that it carries no $\phi$ angular momentum $J_{\phi}$ on the $S^{2}$, which translates into a statement on the R-charge in the CFT:

$$
\begin{equation*}
\Delta j^{\text {probe }}=J_{\phi}=0 \tag{3.22}
\end{equation*}
$$

Similarly one verifies that adding an M2-particle probe static with respect to $t$ to the more general backgrounds (2.22)-(2.25) leads to the same changes in the quantum numbers (3.20), (3.21), (3.22).

In the fully backreacted solution one expects these probe predictions to be corrected due to energy and angular momentum stored in the interactions of fields sourced by the M2particle. Naively one might expect these corrections to appear at second and higher orders in a perturbative expansion in the M2-brane charge $q_{\star}$. This is indeed what we will find for the right-moving scaling dimension $\Delta \bar{h}$. On the left-moving side, where supersymmetry resides, things will turn out to be more subtle because of the existence of the spectral flow isomorphism of the $N=4$ superconformal algebra [19], which in the bulk corresponds to a coordinate redefinition which doesn't vanish near the boundary [20]. It will turn out that the backreaction produces such a redefinition, which will modify the relations (3.20), (3.22) already at linear order in $q_{\star}$. We will find in particular that in the backreacted configuration $\Delta h=0$ which is characteristic for a Ramond ground state in the dual CFT.

## 4 Backreacted M2-particle in the center of $\boldsymbol{A d S} S_{3}$

In the next sections we will describe the backreaction on the 3D supergravity fields of an M2-particle moving on one of the BPS trajectories (3.3). For simplicity, we will first consider the backreaction of a probe in the 'center' of AdS, at $\rho_{0}=0$ as in figure 2(a), and discuss in detail its physical properties and holographic interpretation. Having obtained this solution we will describe in section 5 how to act with broken symmetry generators in order to obtain solutions that thus tentatively describe the backreacted M2-particle moving on a helical curve with finite radius.

### 4.1 Setting up the equations

We will first set up the equations following from the action (2.2) in the presence of the source terms (3.1) produced by an M2-particle with charge $2 \pi q_{\star}$ placed at the 'center' $\rho_{0}=0$ of $A d S_{3}$. Reverting to the coordinates $t, z, \bar{z}$ of the ansatz (2.7), the M2-particle is located at $z=0$. Varying the combined action (2.2), (3.1) with respect to $\tau$, one finds that (2.5) gets modified to

$$
\begin{equation*}
\partial_{z} \partial_{\bar{z}} \tau+i \frac{\partial_{z} \tau \partial_{\bar{z}} \tau}{\tau_{2}}=-\pi i q_{\star} \delta^{2}(z, \bar{z}) \tag{4.1}
\end{equation*}
$$

It's straightforward to check that the imaginary part of this equation follows from the $\tau_{2}$ variation of the combined action (2.2), (3.1); the real part requires more work as the source is written in terms of the $\mathrm{U}(1)$ field $A$ dual to the axion. However, the real part of (4.1)
is guaranteed to work out by supersymmetry, which requires $\tau$ to be holomorphic. The equation (4.1) is solved by

$$
\begin{equation*}
\tau=-i q_{\star} \ln z+i V_{\infty} \tag{4.2}
\end{equation*}
$$

Note that, as expected, the M2-charge induces a monodromy of $\tau$ when encircling the M2-particle:

$$
\begin{equation*}
\tau \rightarrow \tau+2 \pi q_{\star} \quad \text { under } \psi \rightarrow \psi+2 \pi \tag{4.3}
\end{equation*}
$$

In principle, we could have added to (4.2) an arbitrary holomorphic function regular in $z=0$; however since our configuration must preserve rotational symmetry in the plane transverse to the M2-worldline (which is the diagonal subgroup of the $\mathrm{U}(1)_{L} \times \mathrm{U}(1)_{R}$ symmetry referred to in the previous section), such additional terms are forbidden. ${ }^{7}$

Our notation $V_{\infty}$ for the constant term in (4.2) requires some explanation: as we will see below, the backreacted solution has a conformal boundary at some radius $\left|z_{0}\right|$. By rescaling $z$ we can assume the boundary to be at $|z|=1$ without loss of generality, and $V_{\infty}$ then represents the boundary value of dilaton, which has the meaning of the size of the Calabi-Yau space in 11D Planck units. In order for the supergravity approximation to be reliable, we will require

$$
\begin{equation*}
V_{\infty} \gg 1 \tag{4.4}
\end{equation*}
$$

Now we turn to the source terms coming from varying the combined action (2.2), (3.1) with respect to the metric. Our metric ansatz (2.7) explicitly involves $\tau$, and careful examination shows that, if $\tau$ satisfies the sourced equation (4.1), the equations (2.10), (2.11) for $\chi$ and $\Phi$ do not receive any delta function terms. In particular, $\chi$ should remain free of Dirac string singularities, so from (2.11) the expansion of $\Phi$ near the origin should not include a logarithmic term: ${ }^{8}$

$$
\begin{equation*}
\lim _{|z| \rightarrow 0} \frac{\Phi}{\ln |z|}=0 \tag{4.5}
\end{equation*}
$$

The requirement of rotational invariance furthermore implies that we can choose $\Phi$ to be a function of $r=|z|$ alone. To complete the solution, we also have to specify the flat connection $\mathcal{A}$. Since it is not sourced by the M2-particle nor coupled to any of the fields sourced by it, we take $\mathcal{A}$ to be the same as for the D6-anti-D6 solution, namely (2.17).

To simplify the equations somewhat, it will be useful to map the coordinate $z$ on the disc to a coordinate $w$ on the semi-infinite cylinder

$$
\begin{equation*}
z=e^{w} \tag{4.6}
\end{equation*}
$$

Furthermore, we set

$$
\begin{equation*}
w=x+i \psi \tag{4.7}
\end{equation*}
$$

[^4]so the the M2-particle source is at $x \rightarrow-\infty$ and the conformal boundary at $x \rightarrow 0$. In order for our ansatz (2.7) to be invariant under conformal transformations, $e^{-2 \Phi}$ must transform not as a scalar but as a density. Denoting the field in the $w$-frame by $\Phi^{\text {cyl }}$, we have
\[

$$
\begin{equation*}
\Phi^{\mathrm{cyl}}(w)=\Phi(w)-\frac{1}{2}(w+\bar{w}) . \tag{4.8}
\end{equation*}
$$

\]

It will be convenient to to make a further shift

$$
\begin{equation*}
\tilde{\Phi}=\Phi^{\mathrm{cyl}}-\frac{1}{2} \ln V_{\infty} \tag{4.9}
\end{equation*}
$$

which makes manifest the property that the 3D metric actually only depends on the combination

$$
\begin{equation*}
\epsilon \equiv \frac{q_{\star}}{V_{\infty}} \tag{4.10}
\end{equation*}
$$

which is a small parameter in the regime of interest.
The field $\tilde{\Phi}$ depends on $x$ alone due to rotational invariance, and must satisfy the nonlinear ODE

$$
\begin{equation*}
\tilde{\Phi}^{\prime \prime}+(1-\epsilon x) e^{-2 \tilde{\Phi}}=0 \tag{4.11}
\end{equation*}
$$

Our task will be to solve this equation subject to (4.5), which in the new variables becomes the asymptotic condition

$$
\begin{equation*}
\tilde{\Phi} \xrightarrow{x \rightarrow-\infty}-x+\mathcal{O}(1) . \tag{4.12}
\end{equation*}
$$

From this behaviour we see that the solution for $\chi$ which is free of Dirac string singularities as $x \rightarrow-\infty$ is

$$
\begin{equation*}
\chi=\left(\tilde{\Phi}^{\prime}+1\right) d \psi . \tag{4.13}
\end{equation*}
$$

Furthermore, in order to assure that the solution describes the M2-particle backreacted in the background (2.17), we will look for solutions that reduce to (2.17) in the limit that the M2-particle charge $q_{\star}$ is taken to zero:

$$
\begin{equation*}
\lim _{\epsilon \rightarrow 0} \tilde{\Phi}=\ln \sinh (-x) . \tag{4.14}
\end{equation*}
$$

We will see that, under these conditions, we are led to a solution of (4.11) which is asymptotically $A d S_{3}$, which in terms of $\tilde{\Phi}$ means that

$$
\begin{equation*}
\tilde{\Phi} \xrightarrow{x \rightarrow 0_{-}} \ln (-x)+\mathcal{O}(1) \tag{4.15}
\end{equation*}
$$

As explained above, we have chosen the coordinate $x$ such that the conformal boundary is at $x \rightarrow 0_{-}$.

To recapitulate, we have reduced the problem to determining a single function $\tilde{\Phi}(x)$, which has to solve (4.11) under the conditions (4.12) and (4.14). The fields of our solution are then given by

$$
\begin{align*}
\tau & =q_{\star} \psi+i\left(V_{\infty}-q_{\star} x\right)  \tag{4.16}\\
d s_{3}^{2} & =\frac{l^{2}}{4}\left[-\left(d t+\left(\tilde{\Phi}^{\prime}+1\right) d \psi\right)^{2}+(1-\epsilon x) e^{-2 \tilde{\Phi}}\left(d x^{2}+d \psi^{2}\right)\right]  \tag{4.17}\\
\mathcal{A} & =d t+d \psi \tag{4.18}
\end{align*}
$$

### 4.2 Perturbative solution

We now turn to the solution of (4.11) under the conditions (4.12), (4.14). As we will explain in more detail in section 4.7 , (4.11) is equivalent to a first order Abel equation which does not belong to any subclass that has currently been solved. It turns out however that when considering the problem as a perturbative expansion in $\epsilon=\frac{q_{\star}}{V_{\infty}} \ll 1$ one can find an iterative solution, explicit up to quadrature, to all orders. To start we make a power series ansatz for $\tilde{\Phi}$ :

$$
\begin{equation*}
\tilde{\Phi}=\tilde{\Phi}_{0}+\sum_{\epsilon=1}^{\infty} \tilde{\Phi}_{n} \epsilon^{n} \tag{4.19}
\end{equation*}
$$

The subsidiary condition (4.14) fixes $\tilde{\Phi}_{0}$ to be

$$
\begin{equation*}
\tilde{\Phi}_{0}=\ln \sinh (-x) \tag{4.20}
\end{equation*}
$$

The non-linear equation (4.11) then decomposes order by order in $\epsilon$ into the following linear equations

$$
\begin{equation*}
\tilde{\Phi}_{n}^{\prime \prime}-\frac{2}{\sinh ^{2} x} \tilde{\Phi}_{n}=S_{n}(x) \tag{4.21}
\end{equation*}
$$

where

$$
\begin{equation*}
S_{n}(x)=\frac{1}{\sinh ^{2} x}\left(x \sum_{\vec{p} \in \mathcal{P}_{n-1}} \prod_{l=1}^{n-1} \frac{\left(-2 \tilde{\Phi}_{l}\right)^{p_{l}}}{p_{l}!}-\sum_{\vec{p} \in \mathcal{P}_{n}^{\prime}} \prod_{l=1}^{n-1} \frac{\left(-2 \tilde{\Phi}_{l}\right)^{p_{l}}}{p_{l}!}\right) \tag{4.22}
\end{equation*}
$$

With $\mathcal{P}_{n}$ we denote the set of integer partitions of $n$, namely all positive integer vectors $\vec{p}$ such that $\sum_{l} l p_{l}=n$, and $\mathcal{P}_{n}^{\prime}$ is that set minus the 'trivial' partition of $n$, i.e. $n$ itself, which corresponds to $\vec{p}=(0,0, \ldots, 1)$.

It is important to note that $S_{n}$ is fully determined, and actually algebraic, in terms of the $\tilde{\Phi}_{i}$ with $i<n$, so that (4.21) can be solved iteratively. It is interesting that the homogeneous part of (4.21) is identical at each order, it has the two linearly independent solutions

$$
\begin{equation*}
a(x)=\operatorname{coth} x \quad b(x)=1-x \operatorname{coth} x \tag{4.23}
\end{equation*}
$$

One can then solve (4.21) including the source term by the method of variation of parameters. Using an argument by induction the unique solution satisfying the boundary conditions (4.12), (4.15) can be found to be

$$
\begin{equation*}
\tilde{\Phi}_{n}(x)=(x \operatorname{coth} x-1) \int_{-\infty}^{x} S_{n}(u) \operatorname{coth} u d u+\operatorname{coth} x \int_{x}^{0} S_{n}(u)(u \operatorname{coth} u-1) d u \tag{4.24}
\end{equation*}
$$

The first few orders can be integrated explicitly to yield

$$
\begin{align*}
\tilde{\Phi}_{1}= & \frac{1}{2}(-x-x \operatorname{coth} x+1)  \tag{4.25}\\
\tilde{\Phi}_{2}= & \frac{1}{24}\left(-6 \operatorname{Li}_{2}\left(e^{2 x}\right) \operatorname{coth} x-3\left(2 x^{2}+x^{2} \operatorname{csch}^{2} x+4 \log (-2 \sinh x)-1\right)\right. \\
& \left.+\left(\pi^{2}-6(x-2) x\right) \operatorname{coth} x\right) \tag{4.26}
\end{align*}
$$

Finally, as we will need this later, we also work out the leading terms at small $x$ :

$$
\begin{align*}
& \tilde{\Phi}_{1}(x)=-\frac{1}{2} x-\frac{1}{6} x^{2}+\mathcal{O}\left(x^{3}\right) \\
& \tilde{\Phi}_{2}(x)=\frac{1}{6} x^{2} \log (-x)+\frac{1}{6}\left(\log 2-\frac{19}{12}\right) x^{2}+\mathcal{O}\left(x^{3}\right)  \tag{4.27}\\
& \tilde{\Phi}_{n}(x)=\left(\frac{1}{3} \int_{-\infty}^{0} S_{n}(u) \operatorname{coth} u d u\right) x^{2}+\mathcal{O}\left(x^{3}\right) \quad(n \geq 3)
\end{align*}
$$

### 4.3 Asymptotics

The equation (4.11) also allows for a perturbative expansion near the boundary, i.e for small $|x| \ll 1$, which will be useful to determine the asymptotic charges of our solution. We make the following ansatz in terms of a 'transseries' in $x$ :

$$
\begin{equation*}
e^{\tilde{\Phi}}=\sum_{n=0}^{\infty} P_{n}(u) \frac{\epsilon^{n-1}(-x)^{n}}{n!}, \quad u \equiv \ln (-x)+\frac{C}{\epsilon^{2}} \tag{4.28}
\end{equation*}
$$

where $C$ is an integration constant ${ }^{9}$ on which we will comment below.
Substituting the ansatz (4.28) into (4.11) one finds that the condition that the AdS boundary is at $x \rightarrow 0_{-}$, see (4.15), fixes the solutions for $P_{0}$ and $P_{1}$ to be

$$
\begin{align*}
& P_{0}=0  \tag{4.31}\\
& P_{1}=1 . \tag{4.32}
\end{align*}
$$

For the equations determining the $P_{k}(u)$ for $k>1$ one finds the recursion relations

$$
\begin{array}{r}
\ddot{P}_{k}+(2 k-3) \dot{P}_{k}+k(k-3) P_{k}=-2 \delta_{k, 2}-\sum_{n=1}^{k-2}\binom{k}{n}\left[(2 n-k) P_{k-n} P_{n+1}-\dot{P}_{k-n} P_{n+1}\right. \\
\left.+\frac{3 n-k+1}{n+1} P_{k-n} \dot{P}_{n+1}-\frac{\dot{P}_{k-n} \dot{P}_{n+1}}{n+1}+\frac{P_{k-n} \ddot{P}_{n+1}}{n+1}\right] \tag{4.33}
\end{array}
$$

where a dot means differentiation with respect to $u$. These equations determine the $P_{k}(u), k>1$ to be polynomials of order $[k / 2]$ in $u$. The integration constant $C$ in (4.28) arises because the coefficients in (4.33) are $u$-independent, and one can verify that the system (4.33) does not give rise to further integration constants that cannot be absorbed in a redefinition of $C$. These recursive equations can be solved e.g. with Mathematica to

[^5]for a constant $c_{0}$.
rapidly obtain the $P_{n}$ up to high values of $n$. For our purposes we will only need $P_{2}$ and $P_{3}$ as we will see that only they enter in the asymptotic charges. They are given by
\[

$$
\begin{align*}
P_{2} & =1  \tag{4.34}\\
P_{3} & =u . \tag{4.35}
\end{align*}
$$
\]

The value of the integration constant $C$ in (4.28) appropriate for our solution is determined by matching onto the correct delta-function sources in the deep interior of the bulk where $x \rightarrow-\infty$, as expressed by the boundary condition (4.12). We will see below that $C$ enters in the expression for the asymptotic charge $\bar{h}$ which is not fixed by supersymmetry. The $\epsilon$ dependence of $C$ is determined by comparing (4.28) with our power series in $\epsilon$ (4.24), whose expansion to order $x^{2},(4.27)$, is sufficient for this purpose. One finds that

$$
\begin{equation*}
C=1-\epsilon+\left(\ln 2-\frac{5}{6}\right) \epsilon^{2}+2 \sum_{n=3}^{\infty} \epsilon^{n} \int_{0}^{\infty} S_{n}(u) \operatorname{coth} u d u \tag{4.36}
\end{equation*}
$$

It's interesting to note that there exists a similar perturbative expansion of the equation for large $|x| \gg 1$, i.e. near the M2-particle position, which we derive in appendix A. After imposing the near-brane behaviour (4.12), this expansion contains a single integration constant $D$ which we expect to be completely fixed by imposing the behaviour (4.15) near $x \rightarrow 0_{-}$. The first terms of this expansion lead to

$$
\begin{equation*}
\tilde{\Phi}=-x+\ln D-\frac{\epsilon+(1-\epsilon x)}{4 D^{2}} e^{2 x}+\mathcal{O}\left(e^{3 x}\right) \tag{4.37}
\end{equation*}
$$

For example, to order $\epsilon^{2}$ we find from comparison to our perturbative expansion (4.26):

$$
\begin{equation*}
D=\frac{1}{2}+\frac{\epsilon}{4}+\left(\frac{1}{8}-\frac{\pi^{2}}{48}\right) \epsilon^{2}+\mathcal{O}\left(\epsilon^{3}\right) \tag{4.38}
\end{equation*}
$$

### 4.4 Properties of the 3D geometry

Before turning to the holographic interpretation of our solution, we would like to discuss some of the properties of its three-dimensional Lorentzian geometry. As is to be expected, the scalar curvature diverges near the M2-particle trajectory $x \rightarrow-\infty$ due to the source term (4.1). The curvature of a generic solution of the system (2.4)-(2.6) with holomorphic $\tau$ is

$$
\begin{equation*}
l^{2} R=-6+\frac{4\left|\partial_{z} \tau\right|^{2} e^{2 \Phi}}{\tau_{2}^{3}} \tag{4.39}
\end{equation*}
$$

For our specific solution one finds, using the near-M2 expansion (4.37), the leading $x \rightarrow-\infty$ behaviour

$$
\begin{equation*}
R \approx \frac{4 D^{2}}{\epsilon(-x) e^{2 x}} . \tag{4.40}
\end{equation*}
$$

Hence we expect higher derivative corrections to our effective action (2.2) to be significant in the vicinity of the M2-particle. It would be interesting to investigate if those and the corresponding corrections to the probe brane action can be made more concrete using ideas of brane effective actions, along the lines of [23].

Next we would like to discuss the possible issue of closed timelike curves. Even though we know the solution only in a perturbation expansion, we will still be able to argue that it is actually free of closed timelike curves, making use of a result proven in [24]. It was shown there that, for solutions with two commuting isometries in a theory which obeys the null energy condition, as is the case for us, closed timelike curves must be absent if the component $g_{\psi \psi}$ is positive both in the vicinity of the symmetry axis and the boundary.

In our case, the symmetry axis is at the location of the M2-particle $x \rightarrow-\infty$, where we find, ${ }^{10}$ using (4.17) and the expansion (4.37),

$$
\begin{equation*}
g_{\psi \psi} \sim \frac{l^{2} \epsilon}{4 D^{2}}(-x) e^{2 x}+\mathcal{O}\left(e^{3 x}\right) \tag{4.41}
\end{equation*}
$$

which is indeed positive. Near the boundary $x \rightarrow 0_{-}$we have, using the expansion (4.28),

$$
\begin{equation*}
g_{\psi \psi} \sim \frac{\left(1-\frac{\epsilon}{2}\right)}{2(-x)}+\mathcal{O}(\ln (-x)) \tag{4.42}
\end{equation*}
$$

which is also positive since we argued that $\epsilon$ must be small in the regime of validity. We conclude that the requirements for the theorem of [24] are satisfied and that our solution is free of closed timelike curves.

### 4.5 Holographic interpretation: dual field theory

From our results on the near-boundary behaviour of $\Phi$ we can we can derive the asymptotic behaviour of the metric and other fields and interpret the solution holographically. It will sometimes be convenient to use instead of the dilaton $\tau_{2}$ the field

$$
\begin{equation*}
\Psi \equiv-\ln \tau_{2} \tag{4.43}
\end{equation*}
$$

which has a canonical kinetic term. Making the coordinate redefinitions

$$
\begin{align*}
x & =-\frac{1}{2}\left(1-\frac{\epsilon}{2}\right) y  \tag{4.44}\\
t & =\left(1-\frac{\epsilon}{2}\right)\left(x_{+}-x_{-}\right)  \tag{4.45}\\
\psi & =x_{-} \tag{4.46}
\end{align*}
$$

and using (4.16)-(4.18), (4.28), the fields in our solution have the following asymptotic expansions near the boundary $y \rightarrow 0$ :

$$
\begin{align*}
d s_{3}^{2} & =l^{2}\left[\frac{d y^{2}}{4 y^{2}}+\frac{g_{(0)}}{y}+g_{(2)++} d x_{+}^{2}+g_{(2)--} d x_{-}^{2}+\ln y \tilde{g}_{(2)--} d x_{-}^{2}+\mathcal{O}(y \ln y)\right]  \tag{4.47}\\
\Psi & =\Psi_{(0)}+y \Psi_{(2)}+\mathcal{O}\left(y^{2}\right)  \tag{4.48}\\
\tau_{1} & =\tau_{1(0)}  \tag{4.49}\\
\mathcal{A} & =\mathcal{A}_{(0)+} d x_{+}+\mathcal{A}_{(0)-} d x_{-} \tag{4.50}
\end{align*}
$$

[^6]These fit in the general Fefferman-Graham-type [26] expansions appropriate for asymptotically AdS solutions of the 3D theory (2.2) which we review in appendix B (see (B.6)-(B.9)), where we also perform in detail the holographic renormalization [27] of our 3D theory (2.2).

According to the standard AdS/CFT dictionary, the leading parts in these expansions are sources for various CFT operators: $g_{(0)}$ for the stress tensor $T, \Psi_{(0)}$ and $\tau_{1(0)}$ for two $(h, \bar{h})=(1,1)$ marginal operators $\mathcal{O}_{\Psi}$ and $\mathcal{O}_{\tau_{1}}$ respectively. Furthermore, as reviewed in appendix B , the component $\mathcal{A}_{-}$of the Chern-Simons gauge field plays the role of a source for one of the left-moving $\mathrm{SU}(2)$ R-symmetry currents, $J_{+}^{3}$, of the MSW theory. We find for our solution

$$
\begin{align*}
g_{(0)} & =d x^{+} d x^{-}  \tag{4.51}\\
\Psi_{(0)} & =-\ln V_{\infty}  \tag{4.52}\\
\tau_{1(0)} & =q_{\star} x_{-}  \tag{4.53}\\
\mathcal{A}_{(0)-} & =\frac{\epsilon}{2} \tag{4.54}
\end{align*}
$$

The first two expressions tell us that the CFT is defined on the flat cylinder with circumference ${ }^{11} 2 \pi$, while the second specifies the point in the CFT moduli space of marginal deformations by $\mathcal{O}_{\Psi}$. Both of these are unmodified by the addition of the M2-particle. More interesting are the last two relations (4.53), (4.54), which tell us that once the M2-charge $q_{\star}$ is nonzero, the dual CFT action is deformed by source terms for $\mathcal{O}_{\tau_{1}}$ and $J_{+}^{3}$ :

$$
\begin{equation*}
\delta S_{C F T}=-\int d x_{+} d x_{-}\left(\tau_{1(0)} \mathcal{O}_{\tau_{1}}+\mathcal{A}_{(0)-} J_{+}^{3}\right) \tag{4.55}
\end{equation*}
$$

The second term in (4.55) comes from the boundary term for the Chern-Simons field (B.24). It's somewhat suprising to find such a source for the R-current, since it was absent before adding the M2-particle and the Wilson line $\mathcal{A}$ is is not directly sourced by it. However, we see from (4.45) that adding the M2-particle induces a large coordinate transformation near the boundary which modifies the decomposition of $\mathcal{A}$ into left- and right moving pieces, which leads to the nonzero $\mathcal{A}_{(0)-}$ in (4.54).

A remark is in order regarding the special form of the source terms (4.52)-(4.54) in our solution. Generic non-constant sources in the dual field theory imply that translational invariance and therefore also conformal invariance is broken. This explicit symmetry breaking is encoded in a Ward identity for the divergence of the stress tensor [27], which we derived for our system from the bulk point of view in (B.34). Our solution on the other hand belongs to a subclass where the sources are of the form

$$
\begin{equation*}
\Psi_{(0)}=\text { constant }, \quad \partial_{+} \tau_{1(0)}=\partial_{+} \mathcal{A}_{(0)-}=0 \tag{4.56}
\end{equation*}
$$

Since the sources are purely rightmoving and the dual operators have $h=1$, we do expect to preserve the left-moving conformal symmetry. Deformations of this type are sometimes called null deformations and were studied in a holographic context in [28].

[^7]To be a bit more concrete, we substitute (4.56) in the expressions for the trace anomaly (B.26) and the Ward identity (B.34) for a flat boundary metric and obtain

$$
\begin{align*}
\left\langle T_{+-}\right\rangle & =0  \tag{4.57}\\
\partial_{-}\left\langle T_{++}\right\rangle & =0  \tag{4.58}\\
\partial_{+}\left\langle T_{--}\right\rangle & =-\frac{1}{2}\left\langle\mathcal{O}_{\tau_{1}}\right\rangle \tau_{1(0)}^{\prime} \tag{4.59}
\end{align*}
$$

The second line suggests that left-moving conformal invariance is preserved. In section 6 below we will find evidence that the deformation also preserves some supersymmetry, namely half of the left-moving $\mathrm{N}=4$ supersymmetry of the undeformed MSW theory. The last equation indicates that right-moving translation invariance (and hence conformal invariance) is broken. Nevertheless, $\partial_{+}\left\langle T_{--}\right\rangle$does vanish on states where $\left\langle\mathcal{O}_{\tau_{1}}\right\rangle=0$ which turns out to be the case for our solutions. This structure, where only one chiral sector of the CFT seems to be preserved/deformed is reminiscent of ideas of chiral [29] or warped [30] CFTs that appeared in other studies of extremal black holes. It would be interesting to investigate if such a connection can indeed be concretely realized.

### 4.6 Holographic one-point functions

Having determined some properties of the dual field theory in which our solution lives, we now turn to the determination of the holographic VEVs of various operators in the state encoded by our bulk solution. As derived in appendix B, these can be read off from the expansions (4.50) as follows:

$$
\begin{align*}
\left\langle T_{++}\right\rangle & =\frac{c}{12 \pi}\left(g_{(2)++}+\frac{1}{4} \mathcal{A}_{(0)+}^{2}\right), & & \left\langle T_{--}\right\rangle=\frac{c}{12 \pi}\left(g_{(2)--}+\tilde{g}_{(2)--}+\frac{1}{4} \mathcal{A}_{(0)-}^{2}\right)  \tag{4.60}\\
\left\langle T_{+-}\right\rangle & =0, & \left\langle J_{+}^{3}\right\rangle=\frac{c}{24 \pi} \mathcal{A}_{(0)+} & \left\langle\mathcal{O}_{\Psi}\right\rangle=-\frac{c}{12 \pi} \Psi_{(2)}, \tag{4.61}
\end{align*}\left\langle\mathcal{O}_{\tau_{1}}\right\rangle=0
$$

In particular we find, for our solution,

$$
\begin{align*}
g_{(2)++} & =-\frac{1}{4}\left(1-\frac{\epsilon}{2}\right)^{2}, \quad g_{(2)--}=-\frac{1}{4}\left(C+\frac{\epsilon^{2}}{12}+\epsilon^{2} \ln \left(\frac{1}{2}\left(1-\frac{\epsilon}{2}\right)\right)\right), \quad \tilde{g}_{(2)--}=-\frac{\epsilon^{2}}{4} \\
\Psi_{(2)} & =-\frac{\epsilon}{2}\left(1-\frac{\epsilon}{2}\right), \quad \mathcal{A}_{(0)+}=\left(1-\frac{\epsilon}{2}\right) \tag{4.62}
\end{align*}
$$

leading to the zero-mode VEVs

$$
\begin{align*}
h & =\left\langle L_{0}\right\rangle=0  \tag{4.63}\\
\bar{h} & =\left\langle\bar{L}_{0}\right\rangle=-\frac{c}{24}\left(C+\frac{5}{6} \epsilon^{2}+\epsilon^{2} \ln \left(\frac{1}{2}\left(1-\frac{\epsilon}{2}\right)\right)\right)  \tag{4.64}\\
j & =\left\langle\left(J^{3}\right)_{0}\right\rangle=\frac{c}{12}\left(1-\frac{\epsilon}{2}\right)  \tag{4.65}\\
\left\langle\left(\mathcal{O}_{\Psi}\right)_{0}\right\rangle & =\frac{c \epsilon}{12}\left(1-\frac{\epsilon}{2}\right)  \tag{4.66}\\
\left\langle\left(\mathcal{O}_{\tau_{1}}\right)_{0}\right\rangle & =0 \tag{4.67}
\end{align*}
$$

Some comments are in order. It is interesting that the left-moving weight $h$ vanishes exactly to to all orders $\epsilon$, indicating that our solution represents a Ramond sector ground state of the leftmoving superconformal algebra. We will find additional evidence for this interpretation from a Killing spinor analysis in section 6 . We further note that, due to the presence of the source term for the R -current in (4.55) the R -charge $j$ is smaller than it was in the background. As already anticipated in section 3, these charges differ from the naive probe computation already at first order in $q_{\star}$. This is related to spectral flow in the superconformal algebra: we could apply a spectral flow transformation in the bulk [20] to obtain a solution whose charges match (3.20), (3.22) to first order in $q_{\star}$.

To determine the right-moving weight $\bar{h}$, we substitute the value of the integration constant $C$ determined in (4.36). Somewhat surprisingly the order $\epsilon^{2}$ contributions to $\bar{h}$ cancel, while higher order corrections remain: ${ }^{12}$

$$
\begin{equation*}
\bar{h}=-\frac{c}{24}\left(1-\epsilon+\frac{\pi^{2}}{6} \epsilon^{3}-(0.819 \ldots) \epsilon^{4}+(0.621 \ldots) \epsilon^{5}\right)+\mathcal{O}\left(\epsilon^{6}\right) . \tag{4.68}
\end{equation*}
$$

To first order in $\epsilon$ this coincides with the probe approximation result (3.21) for an M2particle in the center of AdS.

Finally let us also discuss the 1 -parameter family of solutions, labeled by $\mu$ with $0<$ $\mu \leq 1$, obtained from the one above by shifting both $\chi$ and $\mathcal{A}$ by the same harmonic form $(\mu-1) d \psi$, leading to

$$
\begin{align*}
\tau & =q_{\star} \psi+i\left(V_{\infty}-q_{\star} x\right)  \tag{4.69}\\
d s_{3}^{2} & =\frac{l^{2}}{4}\left[-\left(d t+\left(\tilde{\Phi}^{\prime}+\mu\right) d \psi\right)^{2}+(1-\epsilon x) e^{-2 \tilde{\Phi}}\left(d x^{2}+d \psi^{2}\right)\right]  \tag{4.70}\\
\mathcal{A} & =d t+\mu d \psi \tag{4.71}
\end{align*}
$$

while the solution for $\tilde{\Phi}$ remains unmodified. Following our comments at the end of section 2, we propose that these solutions represent the backreaction of an M2-particle in the backgrounds which correspond to Ramond ground states with less than maximal R-charge. The transformation to Fefferman-Graham coordinates ( $y, x_{+}, x_{-}$) now reads

$$
\begin{align*}
x & =-\frac{1}{2}\left(\mu-\frac{\epsilon}{2}\right) y  \tag{4.72}\\
t & =\left(\mu-\frac{\epsilon}{2}\right)\left(x_{+}-x_{-}\right)  \tag{4.73}\\
\psi & =x_{-} . \tag{4.74}
\end{align*}
$$

A similar asymptotic analysis yields the result that the sources (4.51)-(4.54) are unmodified, hence the solutions for different values of $\mu$ represent states in the same boundary

[^8]theory. The operator VEVs (4.63)-(4.67) on the other hand change to
\[

$$
\begin{align*}
h & =0  \tag{4.75}\\
\bar{h} & =-\frac{c}{24}\left(C+\frac{5}{6} \epsilon^{2}+\epsilon^{2} \ln \left(\frac{1}{2}\left(\mu-\frac{\epsilon}{2}\right)\right)\right)  \tag{4.76}\\
j & =\frac{c}{12}\left(\mu-\frac{\epsilon}{2}\right)  \tag{4.77}\\
\left\langle\left(\mathcal{O}_{\Psi}\right)_{0}\right\rangle & =\frac{c \epsilon}{12}\left(\mu-\frac{\epsilon}{2}\right)  \tag{4.78}\\
\left\langle\left(\mathcal{O}_{\tau_{1}}\right)_{0}\right\rangle & =0 . \tag{4.79}
\end{align*}
$$
\]

Specifically, we find for the right-moving dimension

$$
\begin{equation*}
\bar{h}=\bar{h}_{\mu=1}-\frac{c \epsilon^{2}}{24} \ln \frac{\mu-\frac{\epsilon}{2}}{1-\frac{\epsilon}{2}} \tag{4.80}
\end{equation*}
$$

where $\bar{h}_{\mu=1}$ is the expression given in (4.68). In particular, we see that for $\mu \neq 1$, the order $\epsilon^{2}$ contribution to $\bar{h}$ no longer vanishes.

### 4.7 More general solutions

We end this section with some observations on the general solution to the equation (4.11), not necessarily obeying the conditions (4.12), (4.14) required to describe a backreacted M2particle. We will in particular comment on the interpretation of the solution describing the 3D Gödel universe which was was found in [6]. This subsection can easily be skipped by the reader mainly interested in the backreacted M2-particle solutions.

We begin by changing variables from $\tilde{\Phi}(x)$ to $X(s)$ defined by

$$
\begin{align*}
1-\epsilon x & =e^{-\epsilon s}  \tag{4.81}\\
X & =-\left(2 \tilde{\Phi}+3 \epsilon s+\ln \frac{3}{2}\right) \tag{4.82}
\end{align*}
$$

Note that this transformation has a well-defined $\epsilon \rightarrow 0$ limit. In terms of these variables (4.11) becomes an autonomous (i.e. with $s$-independent coefficients) equation for $X$ :

$$
\begin{equation*}
\ddot{X}+\epsilon \dot{X}+3\left(\epsilon^{2}-e^{X}\right)=0 \tag{4.83}
\end{equation*}
$$

where the dot means differentiation with respect to $s$. The translation symmetry in $s$ in this form corresponds to the symmetry (4.30) in the old variables.

Before continuing the analysis of (4.83) we note that we can also bring (4.83) to a (non-autonomous) first order form by taking the dependent variable to be $X$ and the independent variable to be $A(X)=\dot{X}$. The equation (4.83) then becomes

$$
\begin{equation*}
A \frac{d A}{d X}+\epsilon A=3\left(e^{X}-\epsilon^{2}\right) \tag{4.84}
\end{equation*}
$$

This is a particular case of Abel's equation of the second form. We have verified, using the techniques of [31], that it does not belong to a subclass which has been previously solved.

Returning to the autonomous form (4.83) of our equation, and defining $Y=\dot{X}$, the solutions to (4.83) can be pictured as flows in the $(X, Y)$ plane

$$
\begin{equation*}
(\dot{X}, \dot{Y})=\left(Y,-\epsilon Y+3\left(e^{X}-\epsilon^{2}\right)\right) \tag{4.85}
\end{equation*}
$$

The resulting flow diagram is shown in figure 3. All of these flows will turn out to correspond to supersymmetric solutions. The flow equation (4.85) has a fixed point at

$$
\begin{equation*}
X_{G}=\ln \epsilon^{2}, \quad Y_{G}=0 \tag{4.86}
\end{equation*}
$$

From looking at small fluctuations around the fixed point we see that there is one attractive and one repulsive direction. The tuned flows which begin or end precisely at the fixed point divide the ( $X, Y$ ) plane into 4 sectors, and the M2-particle solution of interest corresponds to a particular flow in sector I.

Let us comment on the solution corresponding to the fixed point (4.86). Translated back to the original variable $\tilde{\Phi}$ it corresponds to

$$
\begin{equation*}
\tilde{\Phi}_{G}=\frac{1}{2} \ln \frac{2(1-\epsilon x)^{3}}{3 \epsilon^{2}} . \tag{4.87}
\end{equation*}
$$

This solution, which is thus far the only known analytic solution to the equation (4.11), gives rise to the metric 3D Gödel universe and was studied in [6]. One of the key differences with the solutions studied in this paper is that it does not satisfy the condition (4.12) required to describe a single backreacted M2-particle. Nevertheless, it does allow for an interpretation in terms of (generalized) M2-particles in the following sense [14]. The 3D Gödel universe is highly symmetric, being a homogeneous space with isometry group $\mathrm{U}(1)_{L} \times \mathrm{SL}(2, \mathbb{R})_{R}$, while a single M2-particle only has the symmetry $\mathrm{U}(1)_{L} \times \mathrm{U}(1)_{R}$ as we argued in section 3. It was proposed in [14] that the Gödel solution instead comes from a smeared congruence of particles obtained by averaging over the action of $\operatorname{SL}(2, \mathbb{R})_{R}$. The resulting configuration preserves the same supersymmetry as a single M2-particle, while the bosonic symmetry is indeed enhanced to $\mathrm{U}(1)_{L} \times \mathrm{SL}(2, \mathbb{R})_{R}$. The stress tensor of the smeared congruence of particles is that of pressureless rotating dust which is well-known to give rise to the 3D Gödel universe.

It would be interesting to study the solutions corresponding to other sectors in the flow diagram of figure 3 , especially those in sector II since they also have an asymptotically $A d S_{3}$ region.

## 5 Backreacted M2-particle at finite radius

We now discuss our proposal for the backreacted solution corresponding to an M2-particle moving on a helical curve at radius $\rho_{0}$ in global $A d S_{3}$ as in figure 2(b). As argued in section 3 , in the probe approximation this solution can be obtained from the one at $\rho_{0}=0$ by applying the coordinate transformation (3.13). We also observed in (3.15) that this is a 'large' coordinate transformation which on the boundary reduces to a purely rightmoving $\mathrm{SL}(2, \mathbb{R})$ transformation.


Figure 3. Phase diagram for $\epsilon=0.5$. The blue flow line corresponds to the backreacted M2particle, while the fixed point indicated by the green dot corresponds to the 3D Gödel universe.

To obtain the corresponding backreacted solution, we propose to similarly perform a large coordinate transformation on the solution describing the M2-particle in the center of AdS. Concretely, we take our solution (4.16)-(4.18), expressed in the coordinates $\left(y, x_{+}, x_{-}\right)(4.74)$, and perform the transformation (3.13), where $\rho$ is related to $y$ as

$$
\begin{equation*}
y=4 e^{-2 \rho} \tag{5.1}
\end{equation*}
$$

Near the boundary this coordinate transformation acts as

$$
\begin{align*}
x_{-} & \rightarrow F\left(x_{-}\right)+\mathcal{O}\left(y^{2}\right)  \tag{5.2}\\
x_{+} & \rightarrow x_{+}-\frac{y}{2} \frac{F^{\prime \prime}\left(x_{-}\right)}{F^{\prime}\left(x_{-}\right)}+\mathcal{O}\left(y^{2}\right)  \tag{5.3}\\
y & \rightarrow F^{\prime}\left(x_{-}\right) y+\mathcal{O}\left(y^{2}\right) \tag{5.4}
\end{align*}
$$

where

$$
\begin{equation*}
F\left(x_{-}\right)=-i \ln \left(\frac{\cosh \rho_{0} e^{i x_{-}}+\sinh \rho_{0}}{\sinh \rho_{0} e^{i x_{-}}+\cosh \rho_{0}}\right) \tag{5.5}
\end{equation*}
$$

Applying this to the solution (4.16)-(4.18) for the M2-particle at $\rho_{0}=0$, with asymptotic behaviour (4.47)-(4.50), we easily read off the source terms and VEVs in the new solution. First of all, the solution represents a state in a different dual theory where the boundary sources $(4.53),(4.54)$ are changed to

$$
\begin{align*}
\tau_{1(0)} & =q_{\star} F\left(x_{-}\right)  \tag{5.6}\\
\mathcal{A}_{(0)-} & =\frac{\epsilon}{2} F^{\prime}\left(x_{-}\right) \tag{5.7}
\end{align*}
$$

Turning our attention to the operator VEVs in the new solution, we have already observed in section 4.6 that a coordinate transformation of the form (5.4) gives a new stress tensor VEV $\left\langle T_{--}\right\rangle$which is independent of $x_{+}$. Explicitly we find the transformation law

$$
\begin{equation*}
\left\langle T_{--}\right\rangle \rightarrow\left(F^{\prime}\right)^{2}\left\langle T_{--}\right\rangle-\frac{c}{24 \pi} S\left(F, x_{-}\right)+\frac{c}{12 \pi} \tilde{g}_{(2)--}\left(F^{\prime}\right)^{2} \ln F^{\prime} \tag{5.8}
\end{equation*}
$$

where $S\left(f, x_{-}\right)$is the Schwarzian derivative

$$
\begin{equation*}
S\left(F, x_{-}\right)=\frac{F^{\prime \prime \prime}}{F^{\prime}}-\frac{3}{2}\left(\frac{F^{\prime \prime}}{F^{\prime}}\right)^{2} \tag{5.9}
\end{equation*}
$$

The first two terms constitute the standard CFT stresstensor transformation law, while the anomalous last term is due to the fact that applying the conformal transformation gives a state in a different field theory. ${ }^{13}$ It would be interesting to derive the transformation (5.8) from the CFT side from the two-point function of $T_{--}$in the deformed theory (4.55). For the specific transformation (5.4) applied to our solution (4.47)-(4.50), (5.8) can be worked out a bit further to give

$$
\begin{equation*}
\left\langle T_{--}\right\rangle=\frac{1}{2 \pi}\left(\left(F^{\prime}\right)^{2}\left(\bar{h}_{\rho_{0}=0}+\frac{c}{24}\right)-\frac{c}{24}-\frac{c \epsilon^{2}}{24}\left(F^{\prime}\right)^{2} \ln F^{\prime}\right) \tag{5.10}
\end{equation*}
$$

where $\bar{h}_{\rho_{0}=0}$ is the rightmoving weight of the original solution (4.68) (or (4.80) for the more general solutions (4.16)-(4.18)). For the constant Fourier mode of this expression one finds
$\bar{h}=-\frac{c}{24}+\cosh 2 \rho_{0}\left(\bar{h}_{\rho_{0}=0}+\frac{c}{24}\right)+\frac{c \epsilon^{2}}{24}\left(2 \rho_{0} \cosh 2 \rho_{0}-e^{-4 \rho_{0}}{ }_{2} F_{1}^{(0,1,0,0)}\left(\frac{1}{2}, 2,1,1-e^{-4 \rho_{0}}\right)\right)$
Again, to linear order in $\epsilon$ this coincides with the probe result (3.21). The only other VEV which is modified compared to (4.75)-(4.79) is

$$
\begin{equation*}
\left\langle\mathcal{O}_{\Psi}\right\rangle=\frac{c}{24 \pi} \epsilon\left(1-\frac{\epsilon}{2}\right) F^{\prime} . \tag{5.12}
\end{equation*}
$$

## 6 Lift to 5 dimensions

We will now present the uplift of our 3D solutions to solutions of the 5 D supergravity theory which arises from dimensionally reducing 11D supergravity on a Calabi-Yau threefold. We will uncover some geometric structures present in our solutions and will show that these are precisely of the kind required for generic solutions with nontrivial hypermultiplets preserving at least one Killing spinor [32]. As promised, we will also explicitly construct the

[^9]full set of Killing spinors that preserve our solutions and discuss their properties. The current 5D setting would also be the natural starting point for constructing the corresponding asymptotically flat solutions, which we will not attempt in this work.

Dimensionally reducing 11-dimensional supergravity on a Calabi-Yau manifold $X$ gives ungauged 5 -dimensional $\mathrm{N}=1$ supergravity coupled to $h_{(1,1)}-1$ vector multiplets and $h_{(2,1)}+$ 1 hypermultiplets [33]. Of these hypermultiplets, one is the universal hypermultiplet whose couplings are independent of the topology of $X$. Our solutions fit within a consistent truncation where the vector multiplet scalars $Y^{I}$ are constant while, in the hypermultiplet sector, only one of the two complex scalars within the universal hypermultiplet is allowed to vary. This complex scalar is precisely our axion-dilaton field $\tau$. The action governing this truncation is given in (C.2) in appendix 6 , to which we also refer for more details on our conventions.

All the 3D solutions considered so far lift to 5D solutions of the following form: ${ }^{14}$

$$
\begin{array}{rlr}
d s_{5}^{2} & =\frac{l^{2}}{4}\left[-(d t+2 \Im m(\partial \Phi)+\Lambda)^{2}+\tau_{2} e^{-2 \Phi} d w d \bar{w}+d \theta^{2}+\sin ^{2} \theta(d \phi-\mathcal{A})^{2}\right] \\
\mathcal{A} & =d t+\Lambda, \quad d \Lambda=0, \quad \tau=\tau(w), \quad l=2\left(\frac{P^{3}}{6}\right)^{\frac{1}{3}} \\
F^{I} & =\frac{P^{I}}{2} \sin \theta d \theta \wedge(d \phi-\mathcal{A}), \quad Y^{I}=\frac{P^{I}}{l} \tag{6.2}
\end{array}
$$

where $\Phi$ is a solution of

$$
\begin{equation*}
4 \partial_{w} \partial_{\bar{w}} \Phi+\tau_{2} e^{-2 \Phi}=0 . \tag{6.3}
\end{equation*}
$$

The metric can be rewritten in the following way as a timelike fibration over a 4D base:

$$
\begin{equation*}
d s^{2}=-f^{2}(d t+\xi)^{2}+f^{-1} d s_{4}^{2} \tag{6.4}
\end{equation*}
$$

where

$$
\begin{align*}
d s_{4}^{2} & =-\frac{l^{2}}{8} \cos \theta\left(\tau_{2} e^{-2 \Phi} d w d \bar{w}+d \theta^{2}+\tan ^{2} \theta(d \phi+2 \Im m(\partial \Phi))^{2}\right)  \tag{6.5}\\
f & =-\frac{l}{2} \cos \theta  \tag{6.6}\\
\xi & =\tan ^{2} \theta d \phi+2 \sec ^{2} \theta \Im m(\partial \Phi)+\Lambda \tag{6.7}
\end{align*}
$$

Let us first discuss the geometry of the 4D-base space. We note that it is ambipolar, changing signature as $\theta$ varies between 0 and $\pi$, while nevertheless the full metric remains Lorentzian. When the axion-dilaton $\tau$ is constant, the base has a hyperkähler structure [34], which gets deformed in an interesting way for nonconstant holomorphic $\tau(w)$. We refer to refs. [14, 32] for more details on the general structure of such solutions. ${ }^{15}$ First of all, the

[^10]4D base is Kähler, with Kähler form

$$
\begin{equation*}
\Phi^{3}=-\frac{l^{2}}{8}\left[\cos \theta \tau_{2} e^{-2 \Phi} \frac{i}{2} d w \wedge d \bar{w}+\sin \theta d \theta \wedge(d \phi+2 \Im m(\partial \Phi))\right] \tag{6.8}
\end{equation*}
$$

It's straightforward to check that $\Phi^{3}$ is closed thanks to the equation (6.3). Adapted complex coordinates can be chosen to be $w$ and the combination

$$
\begin{equation*}
W=\ln \sin \theta-\Phi+i \phi . \tag{6.9}
\end{equation*}
$$

The Kähler potential is then given by

$$
\begin{equation*}
\mathcal{K}= \pm \frac{l^{2}}{8}\left[\sqrt{1-e^{2(\Re e(W)+\Phi)}}-\operatorname{arctanh} \sqrt{1-e^{2(\Re e(W)+\Phi)}}\right] \tag{6.10}
\end{equation*}
$$

where the upper (lower) sign holds in the patch where $\cos \theta>0(\cos \theta<0)$ respectively. Note that, for our rotationally invariant ansatz (4.16)-(4.18), where $\Phi$ depends only on $x=\Re e w$, the 4D base is actually a toric Kähler manifold, with $\partial / \partial_{\psi}$ and $\partial / \partial_{\phi}$ generating the torus action.

In addition to the Kähler form $\Phi^{3}$, there exist on the base two further selfdual twoforms $\Phi^{1}$ and $\Phi^{2}$ such that $\Phi^{i}, i=1,2,3$ satisfy the quaternionic algebra. The forms $\Phi^{1,2}$ are covariantly closed with respect to a $\mathrm{U}(1)$ connection built out of the axion-dilaton field. Defining $\Phi^{ \pm}=\Phi^{1} \pm i \Phi^{2}$, they satisfy

$$
\begin{equation*}
d \Phi^{ \pm} \mp i \frac{d \tau_{1}}{2 \tau_{2}} \wedge \Phi^{ \pm}=0 . \tag{6.11}
\end{equation*}
$$

Explicitly, the $\Phi^{ \pm}$are given by

$$
\begin{equation*}
\Phi^{+}=\frac{l^{2}}{32} \sqrt{\tau_{2}} e^{W} d w \wedge d W, \quad \Phi^{-}=\overline{\Phi^{+}} . \tag{6.12}
\end{equation*}
$$

We note that in these solutions the only Killing vector of the 4D metric which also leaves the axion-dilaton profile invariant is $\partial / \partial_{\phi}$. The form $\Phi^{+}\left(\Phi^{-}\right)$is not invariant under the corresponding isometry but carries charge 1 (resp. -1) and the Killing vector is therefore often called rotational. ${ }^{16}$

The function $f$ and 1-form $\xi$, which determine how the time coordinate is fibered, and the vector multiplet fields $Y^{I}, F^{I}$ obey a coupled set of BPS equations ${ }^{17}$ whose general solution was discussed in [35, 36].

Now let's turn to the full set of Killing spinors preserved by our solutions describing backreacted M2-particles. We derive these explicitly in appendix D to which refer for more details. It turns out that all our solutions preserve a set of Killing spinors which are constant on the 3D base and depend only on the two-sphere coordinates:

$$
\begin{equation*}
G_{0}^{\beta \gamma}=e^{-\frac{i}{2} \beta \phi \sigma_{3}} e^{\frac{i \theta}{2} \gamma^{\hat{\phi}}} g_{0}^{\beta \gamma} . \tag{6.13}
\end{equation*}
$$

[^11]Here, $\beta, \gamma= \pm 1$ and the $g_{0}^{\beta \gamma}$ are constant spinors defined in (D.12). These Killing spinors are periodic when going around on the boundary cylinder, $\psi \rightarrow \psi+2 \pi$, signalling that our solutions live in the Ramond sector of the dual theory, as does the D4-D0 black hole. When the M2-charge is turned off, $q_{\star}=0$, our solutions preserve all four $G_{0}^{\beta \gamma}$, which we interpret to correspond to the zero modes of the four leftmoving supercurrents of the $(4,0)$ theory in the Ramond sector. When the M2-charge is turned on, $q_{\star} \neq 0$, there is an extra projection condition

$$
\begin{equation*}
\left(1-2 \gamma_{\hat{w} \hat{w}} \sigma_{3}\right) G_{0}^{\beta \gamma}=0 \tag{6.14}
\end{equation*}
$$

which projects on the two Killing spinors (6.13) with $\beta=1$. Note that this is also the projection condition of $\kappa$-symmetry for a probe M2-particle placed in the $A d S_{3}$ background [6].

We propose that the holographic interpretation of the reduction in supersymmetries (6.14) when $q_{\star} \neq 0$ is that the deformation (4.55) of the boundary CFT breaks the number of preserved left-moving supersymmetries from four to two, and that our solutions represent left-moving Ramond states, preserving the two zero modes of the $\mathrm{N}=2$ supercurrents. We should also mention that, when the parameter $\mu$ in (4.16)-(4.18) is one, there are extra Killing spinors which do depend on $x_{+}$: four (resp. two) when $q_{\star}=0$ $\left(q_{\star} \neq 0\right)$. We interpret these as the extra mode number $\pm 1$ modes of the supercurrents preserved by the Ramond ground state with maximal R-charge, which can be obtained by spectral flow from the NS ground state also preserving eight (resp. four) supercharges.

## 7 Outlook

In this work we reported on progress towards constructing the fully backreacted microstate solutions arising in the black hole deconstruction proposal. We constructed in detail the M2-brane solution in the center of $\operatorname{AdS}$ and made a concrete proposal for the solutions describing M2-branes on helical curves. One of our main intentions was to show that these solutions are regular away from the M2-brane source, free of closed timelike curves and asymptotically AdS.

We also studied their holographic interpretation, identifying the dual field theory as a deformation of the MSW theory and computing the operator VEVs in the states dual to our solutions. These computations suggest that the solutions are to be interpreted as Ramond ground states in a dual field theory with a left-moving $\mathrm{N}=2$ superconformal symmetry. It would be interesting to get a more explicit picture of said deformations and states in the MSW sigma model [8].

To obtain the 4D ellipsoidal D2-brane solution depicted in figure 1(b) one would like to perform dimensional reduction along $\psi$ on our solutions. This would require some smearing of the M2 charge. One would expect that adding a further probe M2-brane to our backreacted solution at constant $w$ doesn't break any further supersymmetries and that it should be possible to smear our solutions on a helical curve along the $\psi$ direction to obtain a solution which is $\psi$-rotationally invariant. It would be interesting to work this out in detail.

As mentioned in the Introduction the configurations considered here do not include the backreaction effects of a fundamental string running between the D6 and anti-D6 centers,
which is required by tadpole cancellation. It would be of great interest to add this extra ingredient to our solutions.

Recently in the work [37, 38] it has been proposed that more generally configurations of M2-branes wrapping the nontrivial cycles in bubbling geometries [39] might possibly be the realization in the gravitational regime of the the pure Higgs states that can be identified in the associated quiver gauge theories [40, 41]. As our solutions describe the backreaction of a wrapped M2-brane in the simplest bubbling solution (in a decoupling limit), they can also be seen as a first sample calculation in this program.

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## A Near-brane expansion

In this appendix we will discuss how to set up a recursive expansion for the solutions to (4.11) near $x \rightarrow-\infty$ where the M2-particle is located. One finds that (4.11) is compatible with an $x \rightarrow-\infty$ expansion of the form

$$
\begin{equation*}
e^{\tilde{\Phi}}=e^{-m x} \sum_{n=0}^{\infty} Q_{n}\left(-x^{-1}\right) e^{m n x} \tag{A.1}
\end{equation*}
$$

where the positive number $m$ is a first integration constant. The $Q_{n}(v)$ turn out to be polynomials which can be recursively determined. Upon setting to zero an integration constant which can be absorbed in $m$, the equation for $Q_{0}$ is solved by

$$
\begin{equation*}
Q_{0}=D \tag{A.2}
\end{equation*}
$$

where $D$ is a second integration constant.
The remaining $Q_{n}(v)$ are determined recursively by

$$
\begin{align*}
& D(k-1) k m^{2} v Q_{k}-D v^{3}(-2 k m+m-2 v) \dot{Q}_{k}+D v^{5} \ddot{Q}_{k}  \tag{A.3}\\
& =-(v+\epsilon) \delta_{2, k}-\sum_{n=0}^{k-1}\left(m^{2}(n-1) v(2 n-k) Q_{n} Q_{k-n}-m(n-1) v^{3} Q_{n} \dot{Q}_{k-n}\right. \\
& \left.\quad-v^{3}(m(k-3 n+1)-2 v) \dot{Q}_{n} Q_{k-n}-v^{5} \dot{Q}_{n} \dot{Q}_{k-n}+v^{5} \ddot{Q}_{n} Q_{k-n}\right)
\end{align*}
$$

where the dot stands for differentiation with respect to $v$. By examining the solutions of the homogeneous equations one sees that their integration constants can be absorbed in
redefinitions of $m$ and $D$. One finds that $Q_{2 n+1}=0$ and the recursions (A.3) for the even $Q_{2 n}$ can be easily solved to high order. For example, the first terms are

$$
\begin{equation*}
e^{\tilde{\Phi}}=D e^{-m x}+\frac{x e^{m x}\left(m\left(\epsilon-\frac{1}{x}\right)-\frac{\epsilon}{x}\right)}{4 D m^{3}}+\mathcal{O}\left(e^{3 m x}\right) \tag{A.4}
\end{equation*}
$$

The asymptotic condition (4.12) determines the integration constant $m$ :

$$
\begin{equation*}
m=1 \tag{A.5}
\end{equation*}
$$

Comparing with the perturbative solution in $\epsilon$ gives the other integration constant $D$ to order $\epsilon^{2}$ as:

$$
\begin{equation*}
D=\frac{1}{2}+\frac{\epsilon}{4}+\left(\frac{1}{8}-\frac{\pi^{2}}{48}\right) \epsilon^{2}+\mathcal{O}\left(\epsilon^{3}\right) \tag{A.6}
\end{equation*}
$$

## B Holographic renormalization for 3D axion-dilaton gravity

Here we discuss the holographic renormalization for the 3D axion-dilaton theory defined by (2.2). The analysis is largely similar to the one for a massless scalar coupled to 3D gravity which was discussed in [27], while a discussion of holographic renormalization for higher-dimensional axion-dilaton theories appears in [42, 43]. Setting $\tau_{2}=e^{-\Psi}$, we start from the 3 D action:

$$
\begin{align*}
S=\frac{1}{16 \pi G_{3}} \int_{\mathcal{M}}[ & d^{3} x \sqrt{-G}\left(\mathcal{R}+\frac{2}{l^{2}}-\frac{1}{2} \partial_{\alpha} \Psi \partial^{\alpha} \Psi-\frac{e^{2 \Psi}}{2} \partial_{\alpha} \tau_{1} \partial^{\alpha} \tau_{1}+\frac{l}{2} \mathcal{A} \wedge d \mathcal{A}\right) \\
& \left.-2 \int_{\delta \mathcal{M}} \sqrt{-\gamma} K\right] \tag{B.1}
\end{align*}
$$

leading to the equations of motion

$$
\begin{align*}
\mathcal{E}_{\alpha \beta} & =\mathcal{R}_{\alpha \beta}+\frac{2}{l^{2}} G_{\alpha \beta}-\frac{1}{2} \partial_{\alpha} \Psi \partial_{\beta} \Psi-\frac{e^{2 \Psi}}{2} \partial_{\alpha} \tau_{1} \partial_{\beta} \tau_{1}=0  \tag{B.2}\\
\mathcal{E}_{\Psi} & =\square \Psi-e^{2 \Psi}\left(\partial \tau_{1}\right)^{2}=0  \tag{B.3}\\
\mathcal{E}_{\tau_{1}} & =\nabla^{\alpha}\left(e^{2 \Psi} \partial_{\alpha} \tau_{1}\right)=0 \tag{B.4}
\end{align*}
$$

We use a coordinate system in terms of which the metric looks like

$$
\begin{equation*}
d s_{3}^{2}=l^{2}\left(\frac{d y^{2}}{4 y^{2}}+\frac{1}{y} g_{i j}\left(x^{k}, y\right) d x^{i} d x^{j}\right) \tag{B.5}
\end{equation*}
$$

and assume the standard Fefferman-Graham expansion [26] for the fields near the boundary:

$$
\begin{align*}
g_{i j} & =g_{(0) i j}+y g_{(2) i j}+y \ln y \tilde{g}_{(2) i j}+\mathcal{O}\left(y^{2} \ln y\right)  \tag{B.6}\\
\Psi & =\Psi_{(0)}+y \Psi_{(2)}+y \ln y \tilde{\Psi}_{(2)}+\mathcal{O}\left(y^{2} \ln y\right)  \tag{B.7}\\
\tau_{1} & =\tau_{1(0)}+y \tau_{1(2)}+y \ln y \tilde{\tau}_{1(2)}+\mathcal{O}\left(y^{2} \ln y\right)  \tag{B.8}\\
\mathcal{A} & =\mathcal{A}_{(0)}+\mathcal{O}(y) \tag{B.9}
\end{align*}
$$

Substituting these in the equations of motion (B.4) and working out the leading terms one finds that the logarithmic coefficients $\tilde{g}_{(2)}, \tilde{\Psi}_{(2)}, \tilde{\tau}_{1(2)}$ are completely determined by the boundary values $g_{(0)}, \Psi_{(0)}, \tau_{1(0)}$ :

$$
\begin{align*}
\tilde{g}_{(2) i j} & =-\frac{1}{4}\left(\partial_{i} \Psi_{(0)} \partial_{j} \Psi_{(0)}+e^{2 \Psi_{(0)}} \partial_{i} \tau_{1(0)} \partial_{j} \tau_{1(0)}\right)+\frac{1}{8}\left(\left(\partial \Psi_{(0)}\right)^{2}+e^{2 \Psi_{(0)}}\left(\partial \tau_{1(0)}\right)^{2}\right) g_{(0) i j} \\
\tilde{\Psi}_{(2)} & =-\frac{1}{4} \square \Psi_{(0)}+\frac{e^{2 \Psi_{(0)}}}{4}\left(\partial \tau_{1(0)}\right)^{2}  \tag{B.10}\\
\tilde{\tau}_{1(2)} & =-\frac{1}{4} \square \tau_{1(0)}-\frac{1}{2} \partial_{i} \Psi_{(0)} \partial^{i} \tau_{1(0)} \tag{B.11}
\end{align*}
$$

where indices are raised and covariant derivatives taken with respect to the boundary metric $g_{(0)}$. We note that $\tilde{g}_{(2) i j}$ is traceless.

For the tensor $g_{(2) i j}$ on the other hand, only the trace and divergence are fixed by $g_{(0)}, \Psi_{(0)}, \tau_{1(0)}:$

$$
\begin{align*}
g_{(2)} & =-\frac{1}{2} R_{(0)}+\frac{1}{4}\left(\left(\partial \Psi_{(0)}\right)^{2}+e^{2 \Psi_{(0)}}\left(\partial \tau_{1(0)}\right)^{2}\right)  \tag{B.12}\\
\nabla^{j} g_{(2) i j} & =\partial_{i} g_{(2)}+\Psi_{(2)} \partial_{i} \Psi_{(0)}+\tau_{1(2)} \partial_{i} \tau_{1(0)} \tag{B.13}
\end{align*}
$$

As we shall see below, $g_{(2) i j}$ essentially encodes the expectation value of the CFT stress tensor. The functions $\Psi_{(2)}, \tau_{1(2)}$ are completely free and encode the expectation values of the operators dual to $\Psi, \tau_{1}$ respectively. As usual, these undetermined modes are fixed by physical requirements on the solution in the interior, such as regularity or, in our case, matching onto the proper source term.

Proceeding as in [27], we regularize the action by cutting off the $y$ integral at $y=\delta \ll 1$. One finds for the regularized on-shell action

$$
\begin{equation*}
S_{\mathrm{reg}}=-\frac{l}{8 \pi G_{3}} \int d^{2} x\left[\int_{\delta} d y \frac{\sqrt{-g}}{y^{2}}+\left.2\left(\partial_{y} \sqrt{-g}-\frac{\sqrt{-g}}{y}\right)\right|_{y=\delta}\right] . \tag{B.14}
\end{equation*}
$$

Using (B.6) and (B.13) one finds that this contains the following divergent terms as $\delta \rightarrow 0$ :

$$
\begin{equation*}
S_{\text {div }}=\frac{l}{16 \pi G_{3}} \int_{\delta \mathcal{M}} d^{2} x \sqrt{-g_{(0)}}\left[\frac{2}{\delta}-\frac{1}{2}\left(R_{(0)}-\frac{1}{2}\left(\partial \Psi_{(0)}\right)^{2}-\frac{1}{2} e^{\left.\left.2 \Psi_{(0)}\left(\partial \tau_{1(0)}\right)^{2}\right) \ln \delta\right]}\right.\right. \tag{B.15}
\end{equation*}
$$

These divergences are cancelled if we add the boundary counterterm action

$$
\begin{equation*}
S_{\mathrm{ct}}=\frac{l}{16 \pi G_{3}} \int_{\delta \mathcal{M}} d^{2} x \sqrt{-g}\left[-\frac{2}{\delta}+\frac{1}{2}\left(R-\frac{1}{2}(\partial \Psi)^{2}-\frac{1}{2} e^{2 \Psi}\left(\partial \tau_{1}\right)^{2}\right) \ln \delta\right] \tag{B.16}
\end{equation*}
$$

We could of course have added further local finite terms as $\delta \rightarrow 0$, which in the dual field theory corresponds to using a different renormalization scheme. We then obtain the renormalized action

$$
\begin{equation*}
S_{\mathrm{ren}}=S_{\mathrm{reg}}+S_{\mathrm{ct}} . \tag{B.17}
\end{equation*}
$$

We can now determine the holographic one-point functions of various operators in our solution by varying the renormalized action with respect to the boundary sources. Here
we must be careful to vary the orginal action (B.1) and not the expression (B.14), which differs from it by terms proportional to the equations of motion whose variation is however not zero.

By varying with respect to the boundary metric we obtain the contribution to the expectation value of the stress tensor coming from bulk fields coupling to the metric (as we will see below, there is an extra contribution from the Chern-Simons gauge field):

$$
\begin{equation*}
\left\langle T_{i j}^{\mathrm{grav}}\right\rangle=-\lim _{\delta \rightarrow 0} \frac{2}{\sqrt{-g}} \frac{\delta S_{\mathrm{ren}}}{\delta g^{i j}}=\lim _{\delta \rightarrow 0}\left(T_{i j}^{\mathrm{reg}}+T_{i j}^{\mathrm{ct}}\right) \tag{B.18}
\end{equation*}
$$

We find

$$
\begin{align*}
T_{i j}^{\mathrm{reg}} & =\frac{1}{8 \pi G_{3}}\left(K_{i j}-K \gamma_{i j}\right)  \tag{B.19}\\
& =\frac{l}{8 \pi G_{3}}\left(g_{i j}^{\prime}+\frac{g_{i j}}{y}-g^{k l} g_{k l}^{\prime} g_{i j}\right)_{\mid y=\delta}  \tag{B.20}\\
& =\frac{l}{8 \pi G_{3}}\left(\frac{g_{(0) i j}}{\delta}+2 \tilde{g}_{(2) i j} \ln \delta+2 g_{(2) i j}+\tilde{g}_{(2) i j}-g_{(2)} g_{(0) i j}\right)  \tag{B.21}\\
T_{i j}^{\mathrm{ct}} & =\frac{l}{8 \pi G_{3}}\left(-\frac{g_{(0) i j}}{\delta}-2 \tilde{g}_{(2) i j} \ln \delta-g_{(2) i j}\right) \tag{B.22}
\end{align*}
$$

so that we obtain for the renormalized stresstensor

$$
\begin{equation*}
\left\langle T_{i j}^{\mathrm{grav}}\right\rangle=\frac{l}{8 \pi G_{3}}\left(g_{(2) i j}+\tilde{g}_{(2) i j}-g_{(2)} g_{(0) i j}\right) \tag{B.23}
\end{equation*}
$$

As reviewed in detail in [44], the inclusion of the Chern-Simons field $\mathcal{A}$ gives an extra contribution to the stress tensor. Since the two components of $\mathcal{A}$ are conjugate variables, we are to hold fixed only one of them, say $\mathcal{A}_{-}$, on the boundary. A correct variational principle for this boundary condition requires the addition of a metric-dependent boundary term

$$
\begin{equation*}
S_{\mathcal{A}}^{\mathrm{ct}}=-\frac{l}{64 \pi G_{3}} \int_{\delta \mathcal{M}} d^{2} x \sqrt{-g} g^{i j} \mathcal{A}_{i} \mathcal{A}_{j} \tag{B.24}
\end{equation*}
$$

which gives a contribution to the boundary stress tensor

$$
\begin{equation*}
\left\langle T_{i j}^{\mathcal{A}}\right\rangle=\frac{l}{32 \pi G_{3}}\left(\mathcal{A}_{(0) i} \mathcal{A}_{(0) j}-\frac{1}{2} \mathcal{A}_{(0)}^{k} \mathcal{A}_{(0) k} g_{(0) i j}\right) . \tag{B.25}
\end{equation*}
$$

The total stress tensor is then $\left\langle T_{i j}\right\rangle=\left\langle T_{i j}^{\text {grav }}\right\rangle+\left\langle T_{i j}^{\mathcal{A}}\right\rangle$. We note that the trace anomaly $\left\langle T_{i}^{i}\right\rangle$ is proportional to $g_{(2)}$ given in (B.26):

$$
\begin{equation*}
\left\langle T_{i}^{i}\right\rangle=-\frac{l}{32 \pi G_{3}}\left(\left(\partial \Psi_{(0)}\right)^{2}+e^{2 \Psi_{(0)}}\left(\partial \tau_{1(0)}\right)^{2}-2 R_{(0)}\right) \tag{B.26}
\end{equation*}
$$

The operator dual to $\mathcal{A}$ is an R-symmetry current $J^{3}$, and one finds for its holographic one point function

$$
\begin{equation*}
\left\langle J_{i}^{3}\right\rangle \equiv-\lim _{\delta \rightarrow 0} \frac{1}{\sqrt{-g}} \frac{\delta S_{\mathrm{ren}}}{\delta \mathcal{A}^{i}}=\frac{l}{16 \pi G_{3}} \mathcal{A}_{(0) i} . \tag{B.27}
\end{equation*}
$$

The variation of the renormalized action with respect to $\Psi$ and $\tau_{1}$ gives the renormalized one-point functions of the dual operators $\mathcal{O}_{\Psi}$ and $\mathcal{O}_{\tau_{1}}$ :

$$
\begin{align*}
& \left\langle\mathcal{O}_{\Psi}\right\rangle^{\mathrm{ren}}=-\lim _{\delta \rightarrow 0} \frac{1}{\sqrt{-g}} \frac{\delta S_{\mathrm{ren}}}{\delta \Psi}=\lim _{\delta \rightarrow 0}\left(\left\langle\mathcal{O}_{\Psi}\right\rangle^{\mathrm{reg}}+\left\langle\mathcal{O}_{\Psi}\right\rangle^{\mathrm{ct}}\right)  \tag{B.28}\\
& \left\langle\mathcal{O}_{\tau_{1}}\right\rangle^{\mathrm{ren}}=-\lim _{\delta \rightarrow 0} \frac{1}{\sqrt{-g}} \frac{\delta S_{\mathrm{ren}}}{\delta \tau_{1}}=\lim _{\delta \rightarrow 0}\left(\left\langle\mathcal{O}_{\tau_{1}}\right\rangle^{\mathrm{reg}}+\left\langle\mathcal{O}_{\tau_{1}}\right\rangle^{\mathrm{ct}}\right) \tag{B.29}
\end{align*}
$$

One finds

$$
\begin{array}{ll}
\left\langle\mathcal{O}_{\Psi}\right\rangle^{\mathrm{reg}}=-\frac{l}{8 \pi G_{3}}\left(\Psi_{(2)}+(1+\ln \delta) \tilde{\Psi}_{(2)}\right), & \left\langle\mathcal{O}_{\Psi}\right\rangle^{\mathrm{ct}}=\frac{l}{8 \pi G_{3}} \ln \delta \tilde{\Psi}_{(2)} \\
\left\langle\mathcal{O}_{\tau_{1}}\right\rangle^{\mathrm{reg}}=-\frac{l e^{2 \Psi_{(0)}}}{8 \pi G_{3}}\left(\tau_{1(2)}+(1+\ln \delta) \tilde{\tau}_{1(2)}\right), & \left\langle\mathcal{O}_{\tau_{1}}\right\rangle^{\mathrm{ct}}=\frac{l e^{2 \Psi_{(0)}}}{8 \pi G_{3}} \ln \delta \tilde{\tau}_{1(2)} \tag{B.31}
\end{array}
$$

with the upshot

$$
\begin{align*}
\left\langle\mathcal{O}_{\Psi}\right\rangle^{\mathrm{ren}} & =-\frac{l}{8 \pi G_{3}}\left(\Psi_{(2)}+\tilde{\Psi}_{(2)}\right)  \tag{B.32}\\
\left\langle\mathcal{O}_{\tau_{1}}\right\rangle^{\mathrm{ren}} & =-\frac{l e^{2 \Psi_{(0)}}}{8 \pi G_{3}}\left(\tau_{1(2)}+\tilde{\tau}_{1(2)}\right) \tag{B.33}
\end{align*}
$$

One further result we will need in the main text is the following. Taking the covariant derivative of the boundary stress tensor, we find the following relation

$$
\begin{equation*}
\nabla^{i}\left\langle T_{i j}\right\rangle=-\left\langle\mathcal{O}_{\Psi}\right\rangle \partial_{j} \Psi_{(0)}-\left\langle\mathcal{O}_{\tau_{1}}\right\rangle \partial_{j} \tau_{1(0)}+\nabla^{i}\left\langle T_{i j}^{\mathcal{A}}\right\rangle \tag{B.34}
\end{equation*}
$$

which reflects a Ward identity for the breaking of conformal invariance in the presence of sources in the dual theory [27].

## C Review of 5D axion-dilaton solutions

In this appendix we review the 5D action and Killing spinor equations relevant for our solutions. We consider 11-dimensional supergravity compactified on a Calabi-Yau threefold with triple intersection form $D_{I J K}$. Dimensionally reducing to 5 D gives ungauged $\mathrm{N}=1$ supergravity coupled to $h_{(1,1)}-1$ vector multiplets and $h_{(2,1)}+1$ hypermultiplets, the action for which can be found in ${ }^{18}$ [32]. We make a consistent truncation of this theory where the vector multiplet scalars $Y^{I}$ (normalized such that $D_{I J K} Y^{I} Y^{J} Y^{K}=6$ ) are constant, and where all hypermultiplet scalars, apart from the axion-dilaton field $\tau$ in the universal hypermultiplet, are constant as well. The truncated theory has bosonic action

$$
\begin{align*}
S & =\int d^{5} x \sqrt{-g}\left[R-\frac{\partial_{\mu} \tau \partial^{\mu} \bar{\tau}}{2 \tau_{2}^{2}}-\frac{1}{4} a_{I J} F_{\mu \nu}^{I} F^{J \mu \nu}\right]+\frac{D I J K}{6} \int F^{I} \wedge F^{J} \wedge A^{K}  \tag{C.1}\\
a_{I J} & =-D_{I J K} Y^{K}+\frac{1}{4} Y_{I} Y_{J}, \quad Y_{I}=D_{I J K} Y^{J} Y^{K} \tag{C.2}
\end{align*}
$$

[^12]The resulting equations of motion are

$$
\begin{align*}
R_{\mu \nu}-\frac{\partial_{(\mu} \tau \partial_{\nu)} \bar{\tau}}{2 \tau_{2}^{2}}+\frac{1}{12} g_{\mu \nu} a_{I J} F_{\rho \sigma}^{I} F^{J \rho \sigma}-\frac{1}{2} a_{I J} F_{\mu \rho}^{I} F_{\nu}^{J \rho} & =0  \tag{C.3}\\
\partial_{\mu}\left(\sqrt{-g} g^{\mu \nu} \partial_{\nu} \tau\right)+i \sqrt{-g} g^{\mu \mu} \frac{\partial_{\mu} \tau \partial_{\mu} \tau}{\tau_{2}} & =0  \tag{C.4}\\
a_{I J} d\left(\star F^{J}\right)+\frac{D_{I J K}}{2} F^{J} \wedge F^{K} & =0 \tag{C.5}
\end{align*}
$$

Now let's discuss the supersymmetry variations of the fields. The supersymmetry parameter consists of two complex 4-component spinors

$$
\begin{equation*}
\epsilon=\binom{\epsilon^{1}}{\epsilon^{2}} \tag{C.6}
\end{equation*}
$$

which are related by a symplectic Majorana condition $\Gamma_{\mathcal{M}} \cdot \epsilon=\epsilon$, where $\Gamma_{\mathcal{M}}$ is an idempotent operator acting as

$$
\begin{equation*}
\Gamma_{\mathcal{M}} \cdot \epsilon=\gamma^{\hat{4}} \sigma_{2} \epsilon^{\star} \tag{C.7}
\end{equation*}
$$

and $\star$ denotes complex conjugation. After imposing the symplectic Majorana condition, $\epsilon$ contains a total of 8 independent real components.

The Killing spinor equations, i.e. the vanishing of the supersymmetry variations of the gravitino, gauginos and hyperinos, reduce to, respectively:

$$
\begin{align*}
{\left[\nabla_{\mu}+\frac{i Y_{I}}{48}\left(F^{I \nu \rho} \gamma_{\mu \nu \rho} F^{I}-4 F_{\mu}^{I}{ }^{\nu} \gamma_{\nu}\right)+i \frac{\partial_{\mu} \tau_{1}}{4 \tau_{2}} \sigma_{3}\right] \epsilon } & =0  \tag{C.8}\\
a_{I J} \not F^{I} \frac{\partial Y^{J}}{\partial \phi^{x}} \epsilon & =0  \tag{C.9}\\
\not \partial \bar{\tau} \epsilon^{1}=\not \partial \tau \epsilon^{2} & =0 \tag{C.10}
\end{align*}
$$

where $\phi^{x}$ are $n_{V}$ coordinates on the surface $D_{I J K} Y^{I} Y^{J} Y^{K}=6$.

## D Killing spinors

In this appendix we discuss how many supersymmetries are preserved by our solutions (4.16)-(4.18) for general values of the parameter $\mu$, and give explicit expressions for the Killing spinors. We look for supersymmetry parameters $G$ for which the gravitino, gaugino and hyperino variations (C.8)-(C.10) vanish. It's easy to see that the gaugino variation is automatically zero since the gauge fields are of the form $F^{I} \sim Y^{I} F$. Choosing the vielbein

$$
\begin{align*}
e^{\hat{t}} & =\frac{l}{2}\left(d(t+\mu \psi)+\Phi^{\prime} d \psi\right) & e^{\hat{\theta}}=\frac{l}{2} d \theta  \tag{D.1}\\
e^{\hat{x}} & =\frac{l}{2} \sqrt{\tau_{2}} e^{-\Phi} d x & e^{\hat{\phi}}=\frac{l}{2} d(\phi-t-\mu \psi)  \tag{D.2}\\
e^{\hat{\psi}} & =\frac{l}{2} \sqrt{\tau_{2}} e^{-\Phi} d \psi & \tag{D.3}
\end{align*}
$$

one finds, using (4.11), the following nonvanishing spin connection components:

$$
\begin{array}{rlrl}
\omega^{\hat{t} \hat{x}} & =-\frac{1}{2} \sqrt{\tau_{2}} e^{-\Phi} d \psi, & \omega^{\hat{t} \hat{\psi}} & =\frac{1}{2} \sqrt{\tau_{2}} e^{-\Phi} d x \\
\omega^{\hat{x} \hat{\psi}}=-\frac{1}{2} d(t+\mu \psi)+\frac{1}{2}\left(\Phi^{\prime}+\frac{q_{\star}}{\tau_{2}}\right) d \psi & \omega^{\hat{\theta} \hat{\phi}} & =-\cos \theta d(\phi-t-\mu \psi) \tag{D.5}
\end{array}
$$

After some algebra the gravitino and hyperino equations can then be rewritten as

$$
\begin{align*}
{\left[U \partial_{\mu} U^{-1}+\frac{q_{\star} \delta_{\mu}^{\psi} \gamma^{\hat{x} \hat{\psi}}}{4 \tau_{2}}\left(1-i \gamma^{\hat{x} \hat{\psi}} \sigma_{3}\right)\right] G } & =0  \tag{D.6}\\
q_{\star}\left(1-\gamma^{\hat{x} \hat{\psi}} \sigma_{3}\right) G & =0 \tag{D.7}
\end{align*}
$$

where

$$
\begin{equation*}
U=e^{\frac{i \theta}{2} \gamma^{\hat{\phi}}} e^{\frac{\phi-t-\mu \psi}{2}} \gamma^{\hat{\theta} \hat{\phi}} e^{\frac{t+\mu \psi}{2} \gamma^{\hat{\imath} \hat{\psi}}} \tag{D.8}
\end{equation*}
$$

From this we conclude that the solutions (4.16)-(4.18) preserve local Killing spinors of the form

$$
\begin{equation*}
G=U \cdot g \tag{D.9}
\end{equation*}
$$

where $g$ is a constant spinor satisfying the symplectic Majorana condition (C.7). When the M2-charge $q_{\star}$ is nonzero, $g$ should in addition satisfy the projection condition (D.7).

It will be useful to choose a specific basis of constant spinors by diagonalizing 3 commuting idempotent operators which also commute with the operator $\Gamma_{\mathcal{M}}$ appearing in the symplectic Majorana condition (C.7). We label these basis elements as $g_{\frac{1-\alpha}{2}}^{\beta \gamma}$, with $\alpha, \beta, \gamma= \pm 1$ and take them to satisfy

$$
\begin{align*}
i \gamma^{\hat{t}} g_{\frac{1-\alpha}{2}}^{\beta \gamma} & =\alpha g_{\frac{1-\alpha}{2}}^{\beta \gamma}  \tag{D.10}\\
i \gamma^{\hat{\theta} \hat{\phi}} \sigma_{3} g_{\frac{1-\alpha}{2}}^{\beta \gamma} & =\beta g_{\frac{1-\alpha}{2}}^{\beta \gamma}  \tag{D.11}\\
i \gamma^{\hat{\psi} \hat{\phi}} \sigma_{1} g_{\frac{1-\alpha}{2}}^{\beta \gamma} & =\gamma g_{\frac{1-\alpha}{2}}^{\beta \gamma} . \tag{D.12}
\end{align*}
$$

Spinors of the form (D.9) can then be rewritten as

$$
\begin{equation*}
G_{\frac{1-\alpha}{2}}^{\beta \gamma}=e^{\frac{i}{2}[\beta(1-\alpha)(t+\mu \psi)-\beta \phi] \sigma_{3}} e^{\frac{i \theta}{2} \gamma^{\phi}} g_{\frac{1-\alpha}{2}}^{\beta \gamma} . \tag{D.13}
\end{equation*}
$$

For $0<\mu<1$, only the local Killing spinors which obey $\alpha=1$ have a well-defined global periodicity under $\psi \rightarrow \psi+2 \pi$. In particular they are periodic on the boundary cylinder and should be interpreted as belonging to the Ramond sector of boundary theory. Explicitly they are given by

$$
\begin{equation*}
G_{0}^{\beta \gamma}=e^{-\frac{i}{2} \beta \phi \sigma_{3}} e^{\frac{i \theta}{2} \gamma^{\phi}} g_{0}^{\beta \gamma} . \tag{D.14}
\end{equation*}
$$

In the absence of an M2-particle, these are to be interpreted as the 4 zero modes of the supercurrents in the $(4,0)$ algebra preserved by the Ramond ground states. When the M2-charge $q_{\star}$ is nonzero, we have to impose the extra projection condition (6.14) which sets $\beta=1$ and the solution has only two Killing spinors $G_{0}^{+\gamma}$.

The case $\mu=1$ is special, as then even the spinors $G_{\frac{1-\alpha}{2}}^{\beta \gamma}$ with $\alpha=-1$ are periodic under $\psi \rightarrow \psi+2 \pi$. Without M2-charge, these four extra Killing spinors reflect the fact that the Ramond ground state with maximal R-charge preserves four nonzero modes of the supercurrents (as can be easily seen from its interpretation as the spectral flow of the NS ground state which is maximally supersymmetric). When $q_{\star} \neq 0$, the projection (6.14) condition imposes $\alpha=\beta$, so that we then preserve the four Killing spinors ${ }^{19} G_{0}^{+\gamma}, G_{1}^{-\gamma}$.

This analysis is now easily extended to the more general solutions describing an M2particle on a helical curve constructed in section 5 . Since these are obtained by performing a large coordinate transformation on the 3D coordinates ( $y, x_{+}, x_{-}$), these solutions preserve the same zero mode Killing spinors (D.14). Once again, for $\mu=1$ there are additional Killing spinors which do depend on the 3D coordinates.

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[^0]:    ${ }^{1}$ A first attempt in this direction appeared in [6], and we will comment in detail on the relation with present work below.
    ${ }^{2}$ The D2/M2 brane charges discussed here, denoted as $q_{A}$, should not be confused with the D2/M2 charge we introduce at a later stage, which we will denote by $q_{\star}$. The first type correspond to 2 -branes fully spatially wrapped inside the Calabi-Yau, while the second type corresponds to 2 -branes fully spatially extended in the $4 / 5$ external dimensions.
    ${ }^{3}$ For simplicity, we will not consider the effect of adding $D 2$ charge $q_{A}$.

[^1]:    ${ }^{4}$ A simple argument (for which we thank F. Denef) goes as follows: start from a D2 brane extended in the external 4D space, which does not envelop the D6 and anti-D6 centers. Such a brane wraps a contractible cycle and carries no net D2-charge. Expanding it so as to envelop the D6 and anti-D6 centers one obtains the D2-F1 configuration of interest whose net D2 charge must still vanish.

[^2]:    ${ }^{5}$ Invariance for the gauge field means that under an isometry it transforms by a corresponding gauge transformation that is well behaved on the boundary: $\mathcal{L}_{K} \mathcal{A}=d \lambda, \lim _{\rho \rightarrow \infty} \lambda \rightarrow$ cst.

[^3]:    ${ }^{6}$ The factor $2 \pi$ is introduced for convenience in order to reduce the number of $2 \pi$ factors in what follows. In the quantum theory, our $q_{\star}$ is quantized in units of $(2 \pi)^{-1}$.

[^4]:    ${ }^{7}$ We will find an interesting difference with the backreaction of codimension 2 axion-dilaton charged objects in asymptotically flat spacetimes which were constructed in [15]. In that situation, the additional holomorphic terms are required to obtain a solution with finite energy and can be seen as introducing further sources which break the rotational symmetry [21]. In our asymptotically AdS case, we will find that (4.2), without additional holomorphic terms, corresponds to an asymptotically AdS solution with finite energy. More precisely we will argue that it corresponds to a state in a dual field theory which is deformation of the MSW CFT.
    ${ }^{8}$ This statement is equivalent to the result that D7-brane sources do not produce conical singularities [22].

[^5]:    ${ }^{9}$ Note that the most general solution to (4.11) has two integration constants. As we demand the behaviour (4.15) for $x \rightarrow 0_{-}$, this fixes one of those two constants. This constant could easily be reinstated by making use of the following symmetry of (4.11):

    $$
    \begin{align*}
    x & \rightarrow\left(1-\epsilon c_{0}\right) x+c_{0}  \tag{4.29}\\
    \tilde{\Phi} & \rightarrow \tilde{\Phi}+\frac{3}{2} \ln \left(1-\epsilon c_{0}\right), \tag{4.30}
    \end{align*}
    $$

[^6]:    ${ }^{10}$ Note that near a regular timelike symmetry axis, closed timelike curves are guaranteed to be absent by the equivalence principle [25]; however since we have a curvature singularity on the axis we need to be more careful.

[^7]:    ${ }^{11}$ In fact, this was the reason for the making the rescaling (4.44).

[^8]:    ${ }^{12}$ We were able to evaluate the third order correction exactly, while numerical methods where used for the higher orders. Contrary to second order, a complete cancellation at third order does not arise. However, it seems suggestive that a partial cancellation still occurs, eliminating the rational term originating from the expansion of the logarithm in (4.64) and leaving a purely transcendental answer. It would be interesting to understand if the numerical results at higher order have a similar interpretation.

[^9]:    ${ }^{13}$ It's interesting to note that the anomalous term can be cancelled by accompanying (5.4) by a large gauge transformation of the form $\mathcal{A} \rightarrow \mathcal{A}+d\left(G\left(x_{-}\right)\right)$, where $G$ satisfies

    $$
    G^{\prime}=F^{\prime}\left(-\mathcal{A}_{(0)-}+\sqrt{\mathcal{A}_{(0)-}^{2}-4 \tilde{g}_{(2)--} \ln F^{\prime}}\right)
    $$

    This corresponds to changing the source (5.7) precisely such that $\left\langle T_{--}\right\rangle$has the desired transformation law. We will however refrain from doing this extra gauge transformation as it would obscure some nice properties of the Killing spinors to be discussed in section 6.

[^10]:    ${ }^{14}$ In this section, we adopt units in which the reduced 5D Planck length, $\tilde{l}_{5} \equiv \frac{l_{11}}{4 \pi V_{\infty}^{1 / 3}}$, is set to one.
    ${ }^{15}$ More specifically, our solutions are of the type of eqs. (3.54-3.58) in [14], with the parameters and coordinates appearing there related to the current ones as follows: $\kappa^{2}=-1, s_{2}=2, g\left(y_{2}\right)=\left(\frac{l^{2}}{8}\right)^{2}-y_{2}^{2} ; y_{2}=$ $\frac{l^{2}}{8} \cos \theta, \theta^{2}=\phi$.

[^11]:    ${ }^{16}$ The generalization of the analysis of [32] to include solutions with a rotational Killing vector was performed in [14].
    ${ }^{17}$ More precisely, eq. (2.8)-(2.11) in [14].

[^12]:    ${ }^{18}$ To obtain our conventions from those used in ref. [32], one should send $g_{\mu \nu} \rightarrow-g_{\mu \nu}, \gamma_{\mu} \rightarrow i \gamma_{\mu}, \gamma^{\mu} \rightarrow$ $-i \gamma^{\mu}$ to account for the fact that our metric signature is mostly plus, and replace the quantities $h^{I}$ and $C_{I J K}$ in [32] by $h^{I} \rightarrow Y^{I} / \sqrt{3}, C_{I J K} \rightarrow \sqrt{3} D_{I J K} / 2$. We denote tangent space indices with a hat, and use a representation where $\gamma^{\hat{0}, \hat{1}, \hat{2}, \hat{3}}$ are real and $\gamma^{\hat{4}}$ is imaginary, satisfying $\gamma^{\hat{0} \hat{1} \hat{2} \hat{4} \hat{4}}=i$.

[^13]:    ${ }^{19}$ In terms of the light-cone coordinates on the boundary, the Killing spinors $G_{1}^{-\gamma}$ are given by

    $$
    G_{1}^{-\gamma}=e^{-i \sigma_{2}\left[\left(1-\frac{\epsilon}{2}\right) x_{+}+\frac{\epsilon}{2} x_{-}+\frac{\beta \phi}{2}\right]} e^{\frac{i \theta}{2} \gamma^{\hat{\phi}}} g_{1}^{-\gamma} .
    $$

