

Extremal hairy black holes

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ABSTRACT: We consider a gravitating system consisting of a scalar field minimally coupled to gravity with a self-interacting potential and an U(1) electromagnetic field. Solving the coupled Einstein-Maxwell-scalar system we find exact hairy charged black hole solutions with the scalar field regular everywhere. We go to the zero temperature limit and we study the effect of the scalar field on the near horizon geometry of an extremal black hole. We find that except a critical value of the charge of the black hole there is also a critical value of the charge of the scalar field beyond of which the extremal black hole is destabilized. We study the thermodynamics of these solutions and we find that if the space is flat then the Reissner-Nordström black hole is thermodynamically preferred, while if the space is AdS the hairy charged black hole is thermodynamically preferred at low temperature.

KEYWORDS: AdS-CFT Correspondence, Black Holes, Holography and condensed matter physics (AdS/CMT)

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1 Introduction

The gauge/gravity duality is a principle that it is well founded in string theory and connects a strongly coupled d -dimensional conformal field theory with a $(d+1)$ -dimensional gravity theory that is weakly coupled [1]. This principle has been applied to many field theories having gravity duals but its most noticeable application is in condensed matter physics. Recently there is a lot of activity in understating the properties of quantum liquids [2, 3]. Quantum liquids arise if we put a many-body system of a finite $U(1)$ charge density at zero temperature. Understanding the ground states of these finite density systems at strong coupling will give us information about the nature of these quantum liquids and will lead us to find applications in condensed matter systems.

According to gauge/gravity duality the gravity part of these systems is described by an extremal charged black hole in anti-de Sitter spacetime [4]. The metric of the extremal black hole has interesting properties. In the near horizon limit when the temperature goes to zero the horizon geometry is given by $AdS_2 \times R^2$ [5]. This happens because the charge of the black hole introduces another scale. The appearance of a new horizon geometry suggests that the boundary system could develop an enhanced symmetry. The other property is that the black hole has a finite horizon area at zero temperature. So we can assign a non-zero entropy at zero temperature.

The properties of the near horizon geometry were also used to explore the zero temperature limit of holographic fluids and superconductors [6]. It was shown in [7] that there is a critical temperature T_c where a charged scalar field condenses, and as $T_c \rightarrow 0$ there is a critical value q_c^2 of the charge of the scalar field attained in the zero temperature limit and this limit is determined solely by the AdS_2 geometry of the horizon. In connection to holographic superconductors the near horizon geometry of an extremal black hole in the presence of charged scalar fields was studied in [8]. Conditions for the existence of scalar

hair of neutral and charged scalar fields were derived. Also exact hairy black hole solutions were found without an electromagnetic field [9].

These developments put forward the necessity of a better understanding of the behaviour of matter fields near the horizon of a charged black hole at the zero temperature limit. This can be achieved if one has a fully back-reacted solution of the Einstein-Maxwell-scalar system. The main difficulty for such a construction is to evade the no-hair theorems and have a healthy behaviour of the scalar field: regular on the horizon and fall off sufficiently fast at large distances. The aim of this work is to study the properties and the behaviour of the fields in the near horizon geometry of a charged black hole as the temperature goes to zero. To achieve this we will use a profile for the scalar field and we will probe the near horizon geometry solving exactly the Einstein-Maxwell-scalar coupled differential equations. Exact solutions of this system without the electromagnetic field were found in [10].

Hairy black holes are interesting solutions of Einstein's Theory of Gravity and have been extensively studied over the years mainly in connection with the no-hair theorems. Then hairy black hole solutions were found in asymptotically flat spacetimes [11–13] but it was realized that these solutions were not physically acceptable as the scalar field was divergent on the horizon and stability analysis showed that they were unstable [14]. To remedy this a regularization procedure has to be used to make the scalar field finite on the horizon.

The easiest way to make the scalar field regular on the horizon is to introduce a scale in the gravity sector of the theory through a cosmological constant. Then various hairy black hole solutions were found [15]–[20]. A characteristic of these solutions is that the parameters connected with the scalar fields are connected in some way with the physical parameters of the hairy solution. This implies that it is not possible to continuously connect the hairy configuration with mass M and a configuration with the same mass and no scalar field.

Hairy solutions were also found of a scalar field coupled to a charged black holes. In [21] a topological black hole dressed with a conformally coupled scalar field and electric charge was studied. Phase transitions of hairy topological black holes were studied in [22, 23]. An electrically charged black hole solution with a scalar field minimally coupled to gravity and electromagnetism was presented in [24]. It was found that regardless the value of the electric charge, the black hole is massless and has a fixed temperature. The thermodynamics of the solution was also studied. Further hairy solutions were reported in [25]–[34] with various properties. More recently new hairy black hole solutions, boson stars and numerical rotating hairy black hole solutions were discussed [35–40]. Also the thermodynamics of hairy black holes was studied in [41].

In spite of this progress little are known on the behaviour of hairy black holes as the temperature goes to zero. To probe the near horizon limit of a charged black hole we introduce a profile of the scalar field that it falls off sufficiently fast outside the horizon. Then by solving the coupled Einstein-Maxwell-scalar system we find exact hairy charged black hole solutions with the scalar field regular everywhere. Then we go to the zero temperature limit and we study the effect of the scalar field on the near horizon geometry of an extremal black hole. We find that except a critical value of the charge of the black hole there is also a critical value of the charge of the scalar field away of which the extremal black

hole is destabilized. We also study the thermodynamics of these solutions and we find that if the space is flat then the Reissner-Nordström (RN) black hole is thermodynamically preferred, while if the space is AdS the hairy charged black hole is thermodynamically preferred at low temperature.

The work is organized as follows. In section 2 we present the general formalism and we derive the field equations. In section 3 we find exact hairy black hole solutions and we study their properties. In section 4 we study the effect of the scalar field on the near horizon geometry. In section 5 we study the thermodynamics of our solutions while in 6 are our conclusions.

2 General formalism

In this section we will review the general formalism discussed in [10] of a scalar field minimally coupled to curvature having a self-interacting potential $V(\phi)$, in the presence of an electromagnetic field. The Einstein-Hilbert action with a negative cosmological constant $\Lambda = -6l^{-2}/\kappa$, where l is the length of the AdS which has been incorporated in the potential as $\Lambda = V(0)$ ($V(0) < 0$) is

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2\kappa} R - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - V(\phi) \right), \quad (2.1)$$

where $\kappa = 8\pi G_N$, with G_N the Newton constant. The resulting Einstein equations from the above action are

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa (T_{\mu\nu}^{(\phi)} + T_{\mu\nu}^{(F)}), \quad (2.2)$$

the energy momentum tensors $T_{\mu\nu}^{(\phi)}$ and $T_{\mu\nu}^{(F)}$ for the scalar and electromagnetic fields are

$$\begin{aligned} T_{\mu\nu}^{(\phi)} &= \nabla_\mu \phi \nabla_\nu \phi - g_{\mu\nu} \left[\frac{1}{2} g^{\rho\sigma} \nabla_\rho \phi \nabla_\sigma \phi + V(\phi) \right], \\ T_{\mu\nu}^{(F)} &= F_\mu^\alpha F_{\nu\alpha} - \frac{1}{4} g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}. \end{aligned} \quad (2.3)$$

If we use eqs. (2.2) and (2.3) we obtain the equivalent equation

$$R_{\mu\nu} - \kappa (\partial_\mu \phi \partial_\nu \phi + g_{\mu\nu} V(\phi)) = \kappa \left(F_\mu^\alpha F_{\nu\alpha} - \frac{1}{4} g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \right). \quad (2.4)$$

We consider the following metric ansatz

$$ds^2 = -f(r) dt^2 + f^{-1}(r) dr^2 + a^2(r) d\sigma^2, \quad (2.5)$$

where $d\sigma^2$ is the metric of the spatial 2-section, which can have positive, negative or zero curvature, and $A_\mu = (A_t(r), 0, 0, 0)$ the scalar potential of the electromagnetic field. In the case of the metric of eq. (2.5), if we use eq. (2.4) we find the following three independent

differential equations

$$f''(r) + 2\frac{a'(r)}{a(r)}f'(r) + 2V(\phi) = A_t'(r)^2, \quad (2.6)$$

$$\frac{a'(r)}{a(r)}f'(r) + \left(\left(\frac{a'(r)}{a(r)} \right)^2 + \frac{a''(r)}{a(r)} \right) f(r) - \frac{k}{a(r)^2} + V(\phi) = -\frac{1}{2}A_t'(r)^2, \quad (2.7)$$

$$f''(r) + 2\frac{a'(r)}{a(r)}f'(r) + \left(4\frac{a''(r)}{a(r)} + 2(\phi'(r))^2 \right) f(r) + 2V(\phi) = A_t'(r)^2, \quad (2.8)$$

where k is the curvature of the spatial 2-section. All the quantities, in the above equations, have been rendered dimensionless via the redefinitions $\sqrt{\kappa}\phi \rightarrow \phi$ and $\kappa V \rightarrow V$. Now, if we eliminate the potential $V(\phi)$ from the above equations we obtain

$$a''(r) + \frac{1}{2}(\phi'(r))^2 a(r) = 0, \quad (2.9)$$

$$f''(r) - 2 \left(\left(\frac{a'(r)}{a(r)} \right)^2 + \frac{a''(r)}{a(r)} \right) f(r) + \frac{2k}{a(r)^2} = 2A_t'(r)^2, \quad (2.10)$$

where the potential can be determined from eq. (2.6) if the functions $a(r)$ and $f(r)$ are known. To find exact hairy black hole solutions the differential equations (2.6)–(2.8) have to be supplemented with the Klein-Gordon equation of the scalar field and the Maxwell equations which in general coordinates read

$$\begin{aligned} \square\phi &= \frac{dV}{d\phi}, \\ \nabla_\nu F^{\mu\nu} &= 0. \end{aligned} \quad (2.11)$$

3 A four-dimensional charged black holes with scalar hair

Following the general formalism developed in section 2 for a scalar field coupled minimally to gravity, we consider a particular profile of the scalar field. Consider the following ansatz for the scalar field

$$\phi(r) = \frac{1}{\sqrt{2}} \ln \left(1 + \frac{\nu}{r} \right), \quad (3.1)$$

where ν is a parameter controlling the behaviour of the scalar field and it has the dimension of length. Then from equation (2.9) and (2.11) we can determine the functions

$$\begin{aligned} a(r) &= \sqrt{r(r+\nu)}, \\ A_t(r) &= \frac{q}{\nu} \ln \left(\frac{r}{r+\nu} \right), \end{aligned} \quad (3.2)$$

analytically. We can also determine the metric function $f(r)$ analytically using equation (2.10). We find

$$\begin{aligned} f(r) &= -2\frac{q^2}{\nu^2} + C_1 r(r+\nu) - \frac{C_2(2r+\nu)}{\nu^2} + 2\frac{kr(2r+\nu)}{\nu^2} \\ &\quad - 2 \left(\frac{q^2(2r+\nu) + r(r+\nu)(C_2 + k\nu)}{\nu^3} + \frac{q^2 r(r+\nu) \ln \frac{r}{r+\nu}}{\nu^4} \right) \ln \frac{r}{r+\nu}, \end{aligned} \quad (3.3)$$

where $k = -1, 0, 1$ and C_1, C_2 are integration constants being proportional to the cosmological constant and to the mass respectively, and the potential is given by

$$\begin{aligned}
 V(\phi) = & \frac{1}{2\nu^4} e^{-2\sqrt{2}\phi} \left[\left(e^{\sqrt{2}\phi} - 1 \right)^2 \left(1 + 10e^{\sqrt{2}\phi} + e^{2\sqrt{2}\phi} \right) q^2 \right. \\
 & + e^{\sqrt{2}\phi} \nu \left(-6C_2 - 10k\nu - C_1\nu^3 - 4e^{\sqrt{2}\phi} \nu (4k + C_1\nu^2) + e^{2\sqrt{2}\phi} (6C_2 + 2k\nu - C_1\nu^3) \right) \\
 & + 2\sqrt{2}\phi \left(\left(1 + 4e^{\sqrt{2}\phi} + e^{2\sqrt{2}\phi} \right) q^2 \ln \frac{\nu}{e^{\sqrt{2}\phi} - 1} \right. \\
 & \left. \left. + 2e^{\sqrt{2}\phi} \left(\left(2 + \cosh\sqrt{2}\phi \right) \left(\nu C_2 + k\nu^2 - q^2 \ln \frac{e^{\sqrt{2}\phi} \nu}{e^{\sqrt{2}\phi} - 1} \right) + 6q^2 \sinh\sqrt{2}\phi \right) \right) \right], \quad (3.4)
 \end{aligned}$$

where $V(0) = \Lambda_{\text{eff}}$ as expected and also

$$C_1 + \frac{4k}{\nu^2} = -\frac{\Lambda_{\text{eff}}}{3} = \frac{1}{l^2}. \quad (3.5)$$

We see from the above relation that the parameter ν of the scalar field introduces a length scale connected with the presence of the scalar field in the theory. Besides, we know that

$$V''(\phi = 0) = m^2, \quad (3.6)$$

where m is the scalar field mass. Therefore, we obtain that the scalar field mass is given by

$$m^2 = \frac{2}{3} \Lambda_{\text{eff}} = -2l^{-2}, \quad (3.7)$$

which satisfies the Breitenlohner-Friedman bound that ensures the perturbative stability of the AdS spacetime [42].

One may wonder if in the limit of $\Lambda_{\text{eff}} \rightarrow 0$ and $\nu \rightarrow 0$ we recover the Reissner-Nordstrom (RN) black hole. Indeed from (3.3) if we fix the constant C_1 to $C_1 = -\frac{4k}{\nu^2}$ the function $f(r)$ can be written as

$$\begin{aligned}
 f(r) = & -\frac{2(q^2 + 5\nu + r(C_2 + k\nu))}{\nu^2} \\
 & - \frac{2\left(\nu(q^2(2r + \nu) + r(r + \nu)(C_2 + k\nu)) + q^2 r(r + \nu) \ln \frac{r}{r + \nu}\right) \ln \frac{r}{r + \nu}}{\nu^4}, \quad (3.8)
 \end{aligned}$$

and in the limit $\nu \rightarrow 0$ we recover the RN black hole

$$f(r) = k + \frac{q^2}{2r^2} - \frac{C_2}{3r}. \quad (3.9)$$

It is interesting to investigate the case with $\nu \neq 0$. To have a better understanding of the resulting geometry we make a change of coordinates $\rho = \sqrt{r(r + \nu)}$. Then the metric (3.3) can be written as

$$ds^2 = -\chi(\rho) dt^2 + \frac{4\rho^2/\nu^2}{4\rho^2/\nu^2 + 1} \frac{1}{\chi(\rho)} d\rho^2 + \rho^2 d\sigma^2, \quad (3.10)$$

where

$$\begin{aligned} \chi(\rho) = & k - \frac{2q^2}{\nu^2} + \rho^2 \left(C_1 + \frac{4k}{\nu^2} \right) - \left(k + \frac{C_2}{\nu} \right) \left(\sqrt{\frac{4\rho^2}{\nu^2} + 1} - 2\frac{\rho^2}{\nu^2} \ln \left(\frac{1 + \sqrt{\frac{4\rho^2}{\nu^2} + 1}}{-1 + \sqrt{\frac{4\rho^2}{\nu^2} + 1}} \right) \right) \\ & + \frac{2q^2}{\nu^2} \ln \left(\frac{1 + \sqrt{\frac{4\rho^2}{\nu^2} + 1}}{-1 + \sqrt{\frac{4\rho^2}{\nu^2} + 1}} \right) \left(\sqrt{\frac{4\rho^2}{\nu^2} + 1} - \frac{\rho^2}{\nu^2} \ln \left(\frac{1 + \sqrt{\frac{4\rho^2}{\nu^2} + 1}}{-1 + \sqrt{\frac{4\rho^2}{\nu^2} + 1}} \right) \right). \end{aligned} \quad (3.11)$$

The scalar field in the new coordinates reads

$$\phi(\rho) = \frac{1}{\sqrt{2}} \ln \left(\frac{1 + \sqrt{\frac{4\rho^2}{\nu^2} + 1}}{-1 + \sqrt{\frac{4\rho^2}{\nu^2} + 1}} \right). \quad (3.12)$$

Then it is clear that the cosmological constant is modified having contributions from the length scale introduced by the scalar field.

At large distances the scalar field decouples and the metric goes to

$$\chi(\rho) = \left(C_1 + \frac{4k}{\nu^2} \right) \rho^2 + k - \frac{C_2 + k\nu}{3\rho} + \frac{q^2}{2\rho^2} + \mathcal{O}\left(\frac{1}{\rho^3}\right). \quad (3.13)$$

From the above relation we can see that the asymptotic behaviour can be RN anti-de Sitter, RN de Sitter or RN metric by depending of the sign of the term proportional to the effective cosmological constant $-\frac{\Lambda_{\text{eff}}}{3} = C_1 + \frac{4k}{\nu^2}$, as expected.

Then we can investigate if our system has a hairy charged black hole solution for $C_1 = -\frac{4k}{\nu^2}$, i.e. for $\Lambda_{\text{eff}} = 0$. In figure 1 we plot the behaviour of the metric function $f(r)$ of 3.3) for a choice of parameters $\nu = 3$, $C_2 = 1, 10$ and 100 , and $q = 0.1$ and $k = \pm 1, 0$. The metric function $f(r)$ changes sign for low values of r signalling the presence of an horizon for $k = 1$, while the potential asymptotically vanishes and the scalar field is regular everywhere outside the event horizon and null at large spatial distances as can be seen in figure 2. Also we check the behaviour of $f(r)$ for different values of q in figure 3. Additionally, we have checked the behaviour of the Kretschmann scalar $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}(r)$ outside the black hole horizon. As it is shown in figure 4 there is no curvature singularity outside the horizon for $k = 1$.

Therefore for $k = 1$, $\nu = 3$ and a small value q of the charge of the black hole we found a well behaved charged hairy black hole solution in asymptotically flat spacetime. This is an unexpected result. The presence of a cosmological constant in the gravity action introduces a scale which protects the scalar field from getting infinite on the horizon. All possible infinities are hidden behind the horizon. This is the case for most of the existing hairy black hole solutions [16, 18, 20, 21, 24, 25]. In our case the scalar field introduces a scale by itself which makes it regular on the horizon. Then depending on the appropriate choice of the parameters the scale of the cosmological constant is cancelled by the scale of the scalar field allowing in this way hairy black holes solutions in asymptotically flat spacetime. Note that this mechanism does not work if the charge is absent as in [10], because the possible examples of hairy black hole configurations violating the no-hair theorems and they were

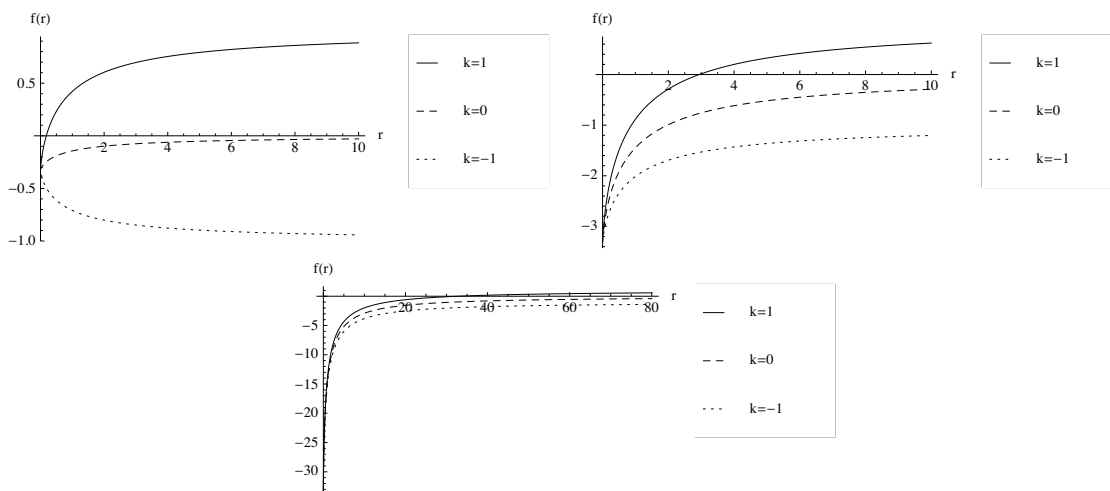


Figure 1. The behaviour of $f(r)$, for $\nu = 3$, $q = 0.1$, $C_2 = 1$ left figure, $C_2 = 10$ right figure, and $C_2 = 100$ bottom figure.

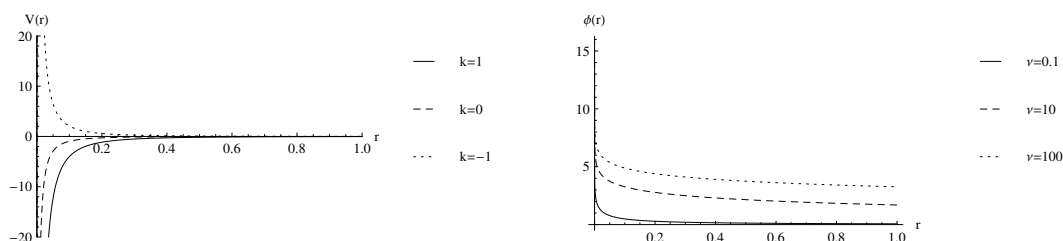


Figure 2. Left figure corresponds to the behaviour of $V(r)$, for $\nu = 3$, $C_2 = 1$, $q = 0.1$, and $k = 1, 0, -1$. Right figure corresponds to the behaviour of $\phi(r)$ for $C_2 = 1$, $q = 0.1$, and $\nu = 0.1, 3, 10$, and 100 .

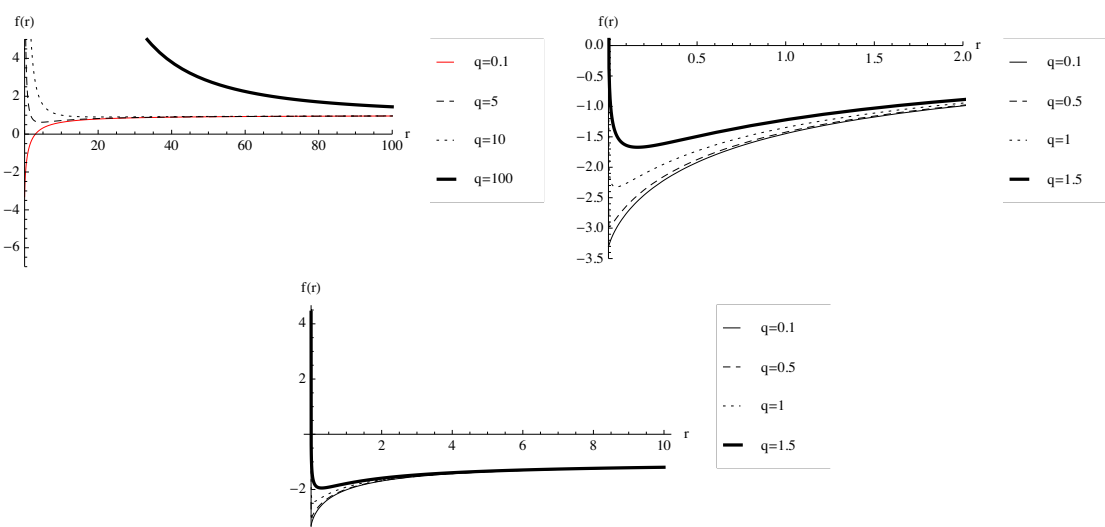


Figure 3. The behaviour of $f(r)$, for $\nu = 3$, $C_2 = 10$ and $k = 1$ left figure, $k = 0$ right figure, and $k = -1$ bottom figure for different values of q .

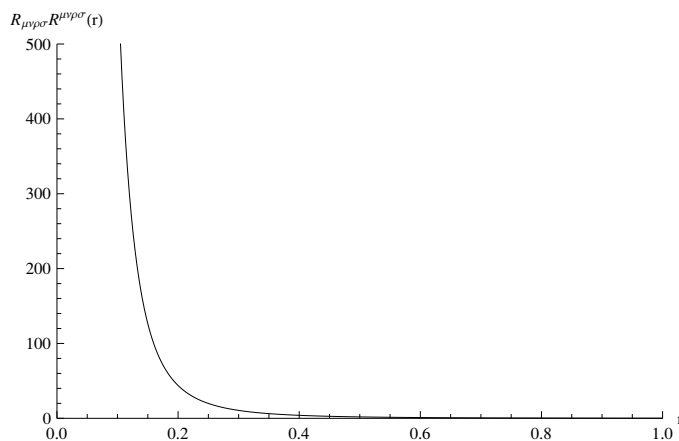


Figure 4. The behaviour of Kretschmann scalar $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}(r)$ as function of r for $\nu = 3$, $C_2 = 1$, $q = 0.1$ and $k = 1$.

not physically acceptable as the scalar field was divergent on the horizon and stability analysis showed that they were unstable [14].

For a massless scalar field minimally coupled to gravity it was shown in [43] that no hairy black hole solutions exist in asymptotically flat spacetimes. In our case the scalar field is coupled minimally to gravity but it has a non-trivial self-interaction potential. As can be seen in the second graph of figure 2 for small values of the parameter ν for which our system has hairy solutions, the scalar field goes to zero very fast outside the horizon of the black hole. Non-zero values of the scalar field are attended only for large values of the parameter ν .

A similar mechanism works in a class of Hordenski theories where a scale is introduced in the scalar sector and hairy black hole solutions can be found in asymptotically flat spacetime. In these theories there is a derivative coupling of a scalar field to Einstein tensor. The derivative coupling has the dimension of length square and it was shown that acts as an effective cosmological constant [44, 45]. Then in [30, 31] a gravitating system of vanishing cosmological constant consisting of an electromagnetic field and a scalar field coupled to the Einstein tensor was discussed. A RN black hole undergoes a second-order phase transition to a hairy black hole. The no-hair theorem is evaded due to the coupling between the scalar field and the Einstein tensor. Similar results were found in [46].

If we allow $\Lambda_{\text{eff}} \neq 0$ we find a general hairy charged black hole solution and if we take the limit $q \rightarrow 0$ we recover our previous solution found in [10]. Also, when $\nu \rightarrow 0$ we recover the RN-AdS black hole. Indeed from (3.3) if $C_1 = -\frac{4k}{\nu^2} - \frac{\Lambda_{\text{eff}}}{3}$ the function $f(r)$ can be written as

$$f(r) = -2\frac{q^2}{\nu^2} - \left(\frac{4k}{\nu^2} + \frac{\Lambda_{\text{eff}}}{3}\right)r(r+\nu) - \frac{C_2(2r+\nu)}{\nu^2} + 2\frac{kr(2r+\nu)}{\nu^2} - 2\left(\frac{q^2(2r+\nu) + r(r+\nu)(C_2 + k\nu)}{\nu^3} + \frac{q^2r(r+\nu)\ln\frac{r}{r+\nu}}{\nu^4}\right)\ln\frac{r}{r+\nu}, \quad (3.14)$$

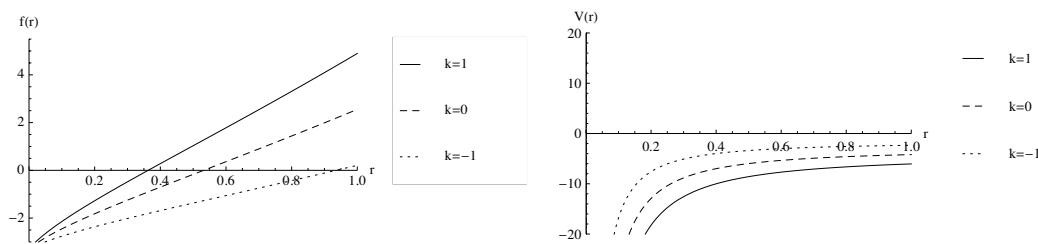


Figure 5. The behaviour of $f(r)$ and $V(\phi)$, for $\nu = 3$, $C_1 = 1$, $C_2 = 10$ and $q = 0.1$.

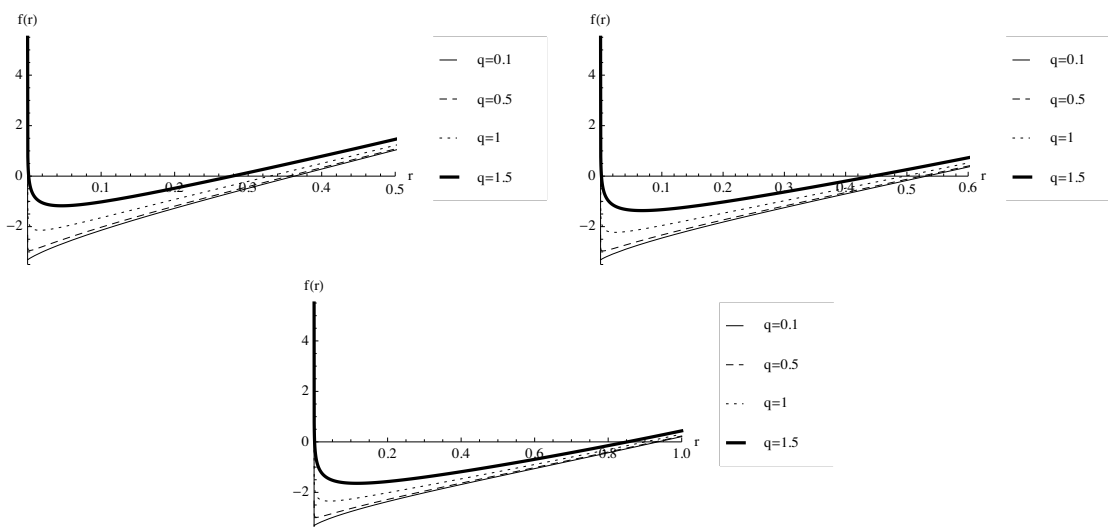


Figure 6. The behaviour of $f(r)$, for $\nu = 3$, $C_1 = C_2 = 1$, $k = 1$ left figure, $k = 0$ right figure, and $k = -1$ bottom figure for $q = 0.1, 0.5, 1, 1.5$.

and in the limit $\nu \rightarrow 0$ we recover the RN AdS black hole

$$f(r) = k - \frac{\Lambda_{\text{eff}}}{3} r^2 + \frac{q^2}{2r^2} - \frac{C_2}{3r} . \tag{3.15}$$

In figure 5 we plot the behaviour of the metric function $f(r)$ of (3.14) and the potential $V(r)$ for a choice of parameters $\nu = 3$, $C_1 = 1$, $C_2 = 10$ and $q = 0.1$ and $k = \pm 1, 0$. The metric function $f(r)$ changes sign for low values of r signalling the presence of an horizon, while the potential asymptotically tends to a negative constant (the effective cosmological constant), and the scalar field is regular everywhere outside the event horizon and null at large distances. Also we consider the metric function for different values of q in figure 6. We have also checked the behaviour of the Kretschmann scalar $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}(r)$ outside the black hole horizon. As it is shown in figure 7 there is no curvature singularity outside the horizon for $k = \pm 1, 0$.

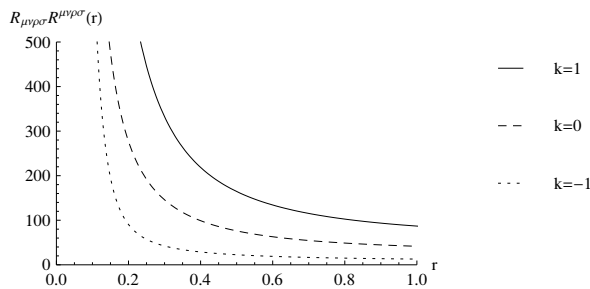


Figure 7. The behaviour of Kretschmann scalar $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}(r)$ as function of r for $\nu = 3$, $C_1 = C_2 = 1$ and $q = 0.1$.

4 Near-horizon geometry for extremal hairy black hole

In this section we will investigate what is the effect of the scalar field to the near horizon geometry of the hairy black hole solutions we discussed in the previous section as the temperature goes to zero. For the case of $\Lambda_{\text{eff}} \neq 0$ of an AdS hairy black hole solution the temperature is

$$T = \frac{f'(r_H)}{4\pi} \tag{4.1}$$

$$= \frac{1}{4\pi} \frac{-2q^2\nu^2 + C_1 r_H \nu^4 (r_H + \nu) - 2r_H (r_H + \nu) \ln\left(\frac{r_H}{r_H + \nu}\right) \left(k\nu(2r_H + \nu) - q^2 \ln\left(\frac{r_H}{r_H + \nu}\right)\right)}{r_H \nu (r_H + \nu) \left(\nu(2r_H + \nu) + 2r_H (r_H + \nu) \ln\left(\frac{r_H}{r_H + \nu}\right)\right)}.$$

Note that q has dimension of $[L]$ and it is convenient to parametrize it as

$$q = \sqrt{\frac{\nu r_* (r_* + \nu) (-C_1 \nu^3 + 2k(2r_* + \nu) \ln\left(\frac{r_*}{r_* + \nu}\right))}{2 \left(-\nu^2 + r_* (r_* + \nu) \left(\ln\left(\frac{r_*}{r_* + \nu}\right)\right)^2\right)}}, \tag{4.2}$$

where r_* is a length scale. So in the zero temperature limit $T = 0 \rightarrow r_H = r_*$ and the constant C_2 goes to

$$C_2 = \frac{-2k\nu^2 r_* + r_* (r_* + \nu) \ln\left(\frac{r_*}{r_* + \nu}\right) \left(C_1 \nu^3 - 2k(r_* + \nu) \ln\left(\frac{r_*}{r_* + \nu}\right)\right)}{-\nu^2 + r_* (r_* + \nu) \left(\ln\left(\frac{r_*}{r_* + \nu}\right)\right)^2}. \tag{4.3}$$

Also, in this limit the lapse function develops a double zero at the horizon

$$f(r) = \eta (r - r_*)^2 + \dots, \tag{4.4}$$

where η is given by

$$\eta = \frac{2k\nu^2 - C_1 \nu^4 - 2k \left(-\nu(2r_* + \nu) + r_* (r_* + \nu) \ln\left(\frac{r_*}{r_* + \nu}\right)\right) \ln\left(\frac{r_*}{r_* + \nu}\right)}{2r_* (r_* + \nu) \left(-\nu^2 + r_* (r_* + \nu) \left(\ln\left(\frac{r_*}{r_* + \nu}\right)\right)^2\right)}. \tag{4.5}$$

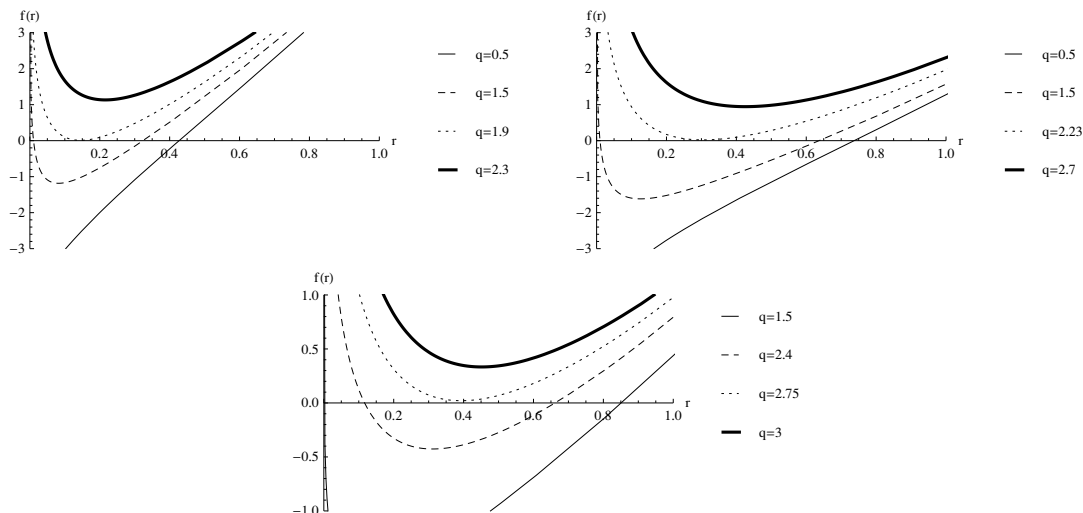


Figure 8. The behaviour of the lapse function $f(r)$ for different values of q , $C_1 = 1$, $C_2 = 10$. Left figure for $k = 1$ and $\nu = 2$. Right figure for $k = 0$ and $\nu = 2$, and bottom figure for $k = -1$ and $\nu = 3$.

Now, considering the scaling limit

$$r - r_* = \lambda \frac{R^2}{\chi}, \quad t = \lambda^{-1} \tau, \quad \lambda \rightarrow 0, \tag{4.6}$$

with χ and τ finite, where

$$R = \frac{1}{\sqrt{\eta}}, \tag{4.7}$$

we find that the metric (2.5) can be written as $AdS_2 \times \Omega_2$ [5], where Ω_2 is the base manifold

$$ds^2 = \frac{R^2}{\chi^2} (-d\tau^2 + d\chi^2) + r_*(r_* + \nu) d\Omega_2. \tag{4.8}$$

Note that the scale of the AdS_2 space R depends on the charge of the scalar field ν . In figure 8 we show the behaviour of the lapse function $f(r)$ for different values of q for fixed ν . We see that for each value of k there is critical value of q_c above which the extremal black hole is destabilized while below that value we depart from the extremal limit and we go to the charged hairy black hole solution. Similar behaviour we observe for the charge ν in figure 9, where we show the behaviour of the lapse function $f(r)$ for different values of ν for fixed q . Observe here that the behaviour of the lapse function $f(r)$ is more sensitive to k than in the q case. In summary, there is a pair of critical values of (q_c, ν_c) where the extremal black hole is formed and the near horizon geometry is given by $AdS_2 \times \Omega_2$. Then there is a range of values of (q, ν) which lead to AdS space or to no-extremal hairy solutions.

It is interesting to see how the temperature changes for various values of ν and for the fixed value of $q = q_c$ of the corresponding value of k . In figure 10 we see that there is a range of values of ν in which the temperature remains zero. Then above a critical value q_c

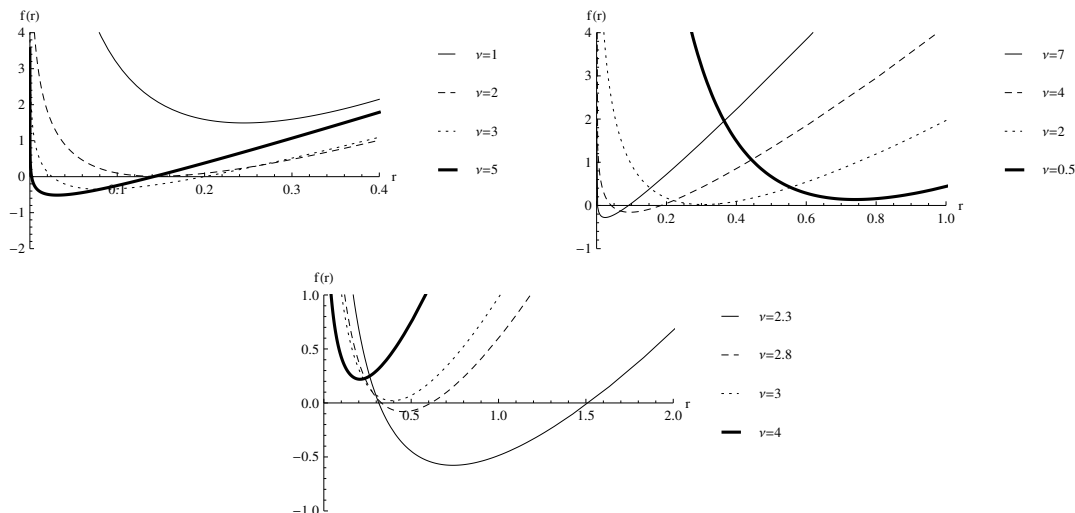


Figure 9. The behaviour of the lapse function $f(r)$ for different values of ν , $C_1 = 1$, $C_2 = 10$. Left figure for $k = 1$ and $q = 1.9$. Right figure for $k = 0$ and $q = 2.23$, and bottom figure for $k = -1$ and $q = 2.75$.

for $k = 1, 0$ the hairy black hole is thermalized while for $k = -1$ as ν is lower below ν_c the hairy black hole researches its maximum temperature and then starts to cool down. This behaviour is interesting and it deserves to be studied further.

In the case of $\Lambda_{\text{eff}} = 0$ i.e. $C_1 = -\frac{4k}{\nu^2}$, we observe a similar behaviour. In this case a hairy black hole solution exist only for $k = 1$ (see figure 1). Then in figure 11 we can see that there is a critical value of q_c above which the extremal black hole is destabilized while below that value we depart from the extremal limit and we go to the charged hairy black hole solution. However in this case the extremal double horizon is larger than in the case of $\Lambda_{\text{eff}} \neq 0$. Also, the same behaviour is observed for ν so there is a pair of critical values of (q_c, ν_c) where the extremal black hole is formed and the near horizon geometry is given by $AdS_2 \times \Omega_2$. Then there is a range of values of (q, ν) which lead to flat space or to no-extremal hairy solutions. Also, we see that there is a range of values of ν in which the temperature remains zero, and a critical value ν_c above which the hairy black hole is thermalized. Observe that in this case the temperature remains constant above ν_c .

5 Thermodynamics

In this section we will study the thermodynamics of the found hairy black hole solutions. To apply the Euclidean formalism we will work in the $\rho = \sqrt{r(r + \nu)}$ coordinates in which the metric (3.10) can be written in the following form

$$ds^2 = N^2(\rho) g^2(\rho) d\tau^2 + g^{-2}(\rho) d\rho^2 + \rho^2 d\sigma^2, \tag{5.1}$$

where

$$N(\rho)^2 = \frac{\rho^2}{\left(\frac{\nu^2}{4} + \rho^2\right)}, \quad g^2(\rho) = \frac{\chi(\rho)}{\rho^2} \left(\frac{\nu^2}{4} + \rho^2\right). \tag{5.2}$$

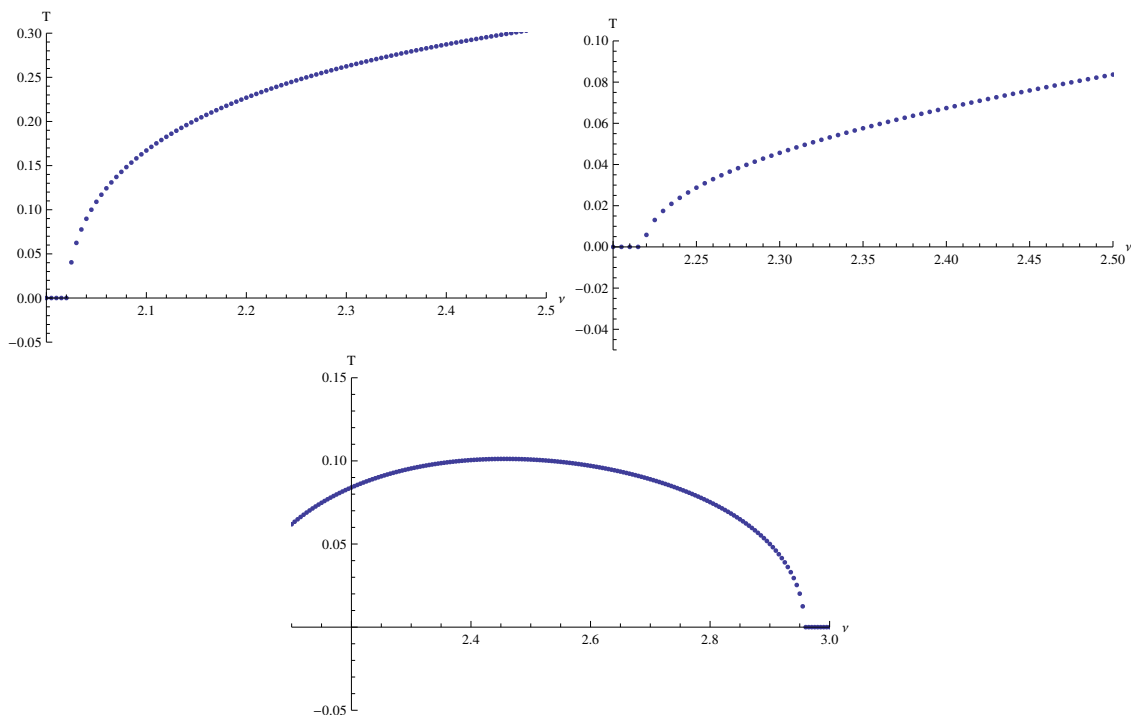


Figure 10. The behaviour of the temperature T as a function of ν , $C_1 = 1$, $C_2 = 10$. Left figure for $k = 1$ and $q = 1.9$. Right figure for $k = 0$ and $q = 2.23$, and bottom figure for $k = -1$ and $q = 2.75$.

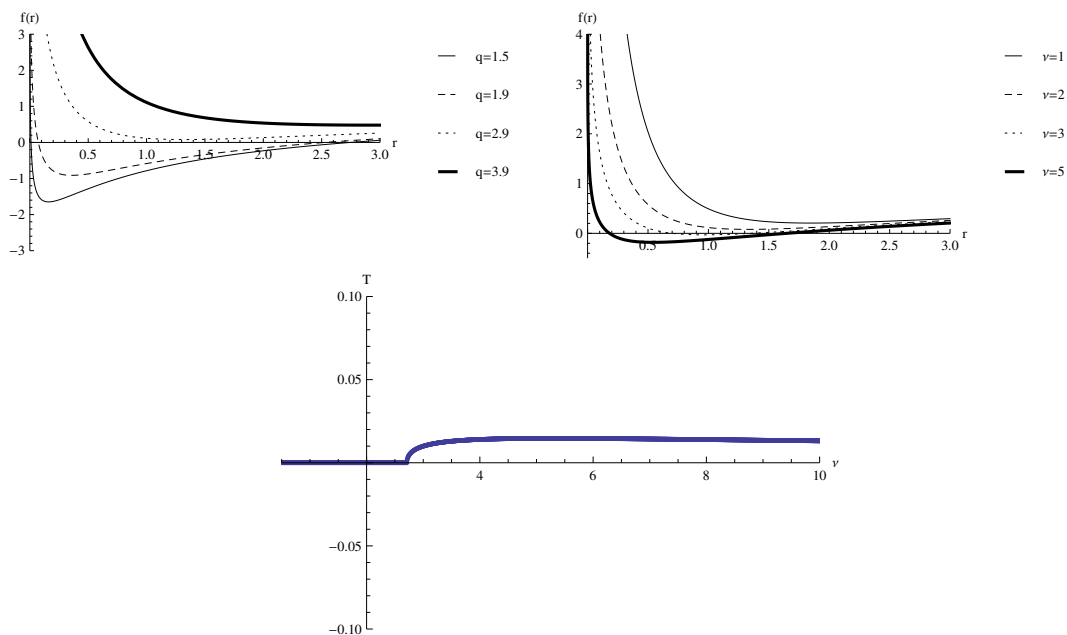


Figure 11. The behaviour of the lapse function $f(r)$ for different values of q , $k = 1$, $C_2 = 10$, and $\nu = 2$ (left figure). The behaviour of the lapse function $f(r)$ for different values of ν , $k = 1$, $C_2 = 10$, and $q = 2.9$ (right figure). The behaviour of the temperature T as a function of ν , $k = 1$, $C_2 = 10$, and $q = 2.9$ (bottom figure).

Now, we go to Euclidean time $t \rightarrow it$ and we consider the action

$$I = -\frac{\beta\sigma}{4\pi} \int_{\rho_H}^{\infty} (N(\rho)\mathcal{H}(\rho) + A_t p') d\rho + B_{\text{surf}}, \quad (5.3)$$

where $\mathcal{H}(\rho)$ is the reduced Hamiltonian which satisfies the constraint $\mathcal{H}(\rho) = 0$ and $p(\rho) = \frac{\rho^2}{N(\rho)} A_t'$, $p'(\rho) = 0$. Also B_{surf} is a surface term, $\beta = 1/T$ is the period of Euclidean time and finally σ is the area of the spatial 2-section. We now compute the action when the field equations hold. The condition that the geometries which are permitted should not have conical singularities at the horizon imposes

$$T = \frac{N(\rho_H)g^2(\rho_H)'}{4\pi}. \quad (5.4)$$

So, by using the grand canonical ensemble we can fix the temperature and “voltage” $\psi = (A_t(\infty) - A_t(\rho_H))$. Then the variation of the surface term yields

$$\delta B_{\text{surf}} = \delta B_{\phi} + \delta B_G + \delta B_F, \quad (5.5)$$

where

$$\delta B_G = \beta\sigma [N\rho\delta g^2]_{\rho_H}^{\infty}, \quad (5.6)$$

$$\delta B_{\phi} = \beta\sigma [N\rho^2 g^2 \phi' \delta\phi]_{\rho_H}^{\infty}, \quad (5.7)$$

$$\delta B_F = \beta\sigma [A_t \delta p]_{\rho_H}^{\infty}. \quad (5.8)$$

Now, we will apply the above formalism to the cases $\Lambda_{\text{eff}} = 0$ and $\Lambda_{\text{eff}} \neq 0$. In both cases the variation of the fields at large distances yields

$$\delta B_{G\infty} = \beta\sigma \left[\left(-\frac{1}{3} + \mathcal{O}\left(\frac{1}{\rho^2}\right) \right) \delta C_2 + \left(-\frac{k}{3} + \mathcal{O}\left(\frac{1}{\rho}\right) \right) \delta\nu + \mathcal{O}\left(\frac{1}{\rho}\right) \delta q^2 \right], \quad (5.9)$$

$$\delta B_{\phi\infty} = \mathcal{O}\left(\frac{1}{\rho}\right) \delta\nu, \quad (5.10)$$

and

$$\delta B_{F\infty} = \mathcal{O}\left(\frac{1}{\rho^2}\right) \delta q^2 + \mathcal{O}\left(\frac{1}{\rho^4}\right) \delta\nu. \quad (5.11)$$

On the other hand,

$$\delta B_{\rho_H} = \sigma \left[-\frac{1}{2} N(\rho_H) \beta (g(\rho_H)^2)' \delta \rho_H^2 + \beta \psi \delta p \right]. \quad (5.12)$$

Thus, from the above expressions in both cases we deduce the surface terms at large distances

$$B_{\text{surf}\infty} = -\frac{\beta\sigma}{3} (C_2 + k\nu), \quad (5.13)$$

and at the horizon

$$B_{\text{surf}\rho_H} = -2\pi\sigma\rho_H^2 + \frac{\beta\psi\sigma q}{2}. \quad (5.14)$$

Therefore, the Euclidean action reads

$$I = -\frac{\beta\sigma}{3} (C_2 + k\nu) + 2\pi\sigma\rho_H^2 - \frac{\beta\psi\sigma q}{2}, \quad (5.15)$$

and as the Euclidean action is related to the free energy through $I = -\beta F$, we obtain

$$I = S - \beta\mathcal{M} + \beta\psi\mathcal{Q}, \quad (5.16)$$

where the mass \mathcal{M} is

$$\mathcal{M} = \frac{\sigma}{3} (C_2 + k\nu), \quad (5.17)$$

the entropy S is

$$S = 2\pi\sigma\rho_H^2, \quad (5.18)$$

and the electric charge \mathcal{Q} is

$$\mathcal{Q} = -\frac{\sigma q}{2}. \quad (5.19)$$

We will first discuss the case of $\Lambda_{\text{eff}} \neq 0$. The temperature is given by the relation (4.1), while the mass, entropy and electric charge can be written respectively as

$$\begin{aligned} \mathcal{M} = & \frac{\sigma}{3} \frac{\nu^2(-2q^2 + r_H(C_1\nu^2(r_H + \nu) + 2k(2r_H + \nu))) - 2\ln\left(\frac{r_H}{r_H + \nu}\right) \left(r_H(r_H + \nu)(k\nu^2 + q^2\ln\left(\frac{r_H}{r_H + \nu}\right)) + \nu q^2(2r_H + \nu)\right)}{\nu^2(2r_H + \nu) + 2\nu r_H(r_H + \nu)\ln\left(\frac{r_H}{r_H + \nu}\right)} \\ & + \frac{\sigma}{3} k\nu, \end{aligned} \quad (5.20)$$

$$S = 2\pi\sigma r_H(r_H + \nu), \quad (5.21)$$

$$\mathcal{Q} = \frac{\sigma}{2} \frac{\nu\psi}{\ln\left(\frac{r_H}{r_H + \nu}\right)}. \quad (5.22)$$

We will study possible phase transitions of the hairy black hole solutions to known black hole solutions without hair. For $\Lambda_{\text{eff}} \neq 0$ and in the absence of a scalar field the action (2.1) has as a solution the RN-AdS black hole with temperature, entropy, mass and electric charge given respectively by

$$\begin{aligned} T_{\text{RN}} &= \frac{1}{4\pi} \left(\frac{3\rho_+}{l^2} + \frac{k}{\rho_+} - \frac{\psi^2}{8\pi\rho_+} \right), & S_{\text{RN}} &= 2\pi\sigma\rho_+^2, \\ M_{\text{RN}} &= \sigma\rho_+ \left(k + \frac{\rho_+^2}{l^2} + \frac{\psi^2}{8\pi} \right), & Q_{\text{RN}} &= \frac{\sigma\rho_+\psi}{4\pi}. \end{aligned} \quad (5.23)$$

Then, the horizon radius $\rho_+ = \frac{2\pi l^2 T}{3} \left(1 + \sqrt{1 - \frac{3(k - \psi^2/(8\pi))}{4\pi^2 l^2 T^2}} \right)$ can be written as a function of the temperature and of the electric potential. Now in order to find phase transitions between the charged hairy and RN-AdS black hole, we must consider both black holes in a same grand canonical ensemble, i.e. at the same temperature T and electric potential ψ . Equating T and ψ for both black holes and by considering the free energy F

$$F = F(T, \phi) = M - TS - \psi\mathcal{Q}, \quad (5.24)$$

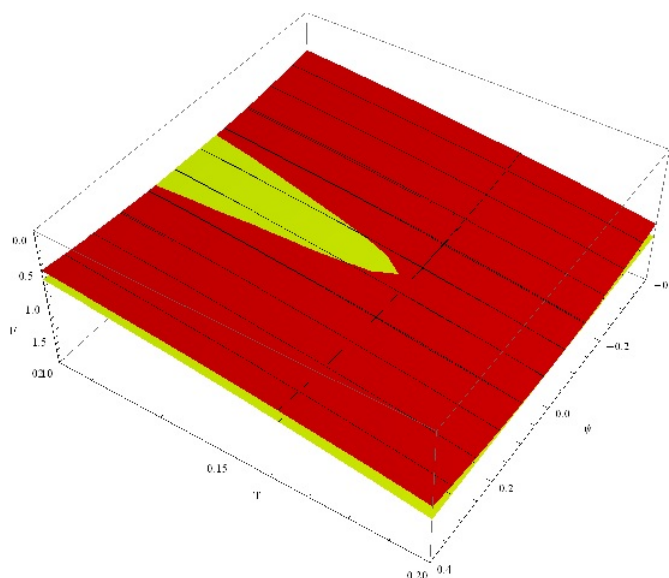


Figure 12. The behaviour of the free energy for the charged hairy black hole and the RN anti-de Sitter black hole as function of T and ψ with $k = -1$, $l = 1$, $\sigma = 1$, and $\nu = 0.5$. Red surface for charged hairy black hole and yellow surface for the RN anti-de Sitter black hole.

we plot the free energy F_0 for the charged black hole with scalar hair, and F_1 of the RN-AdS black hole in figure 12, in order to see the range of values of the electric potential ψ and of the temperature T of the black hole for which the phase transitions exist.

Thus, from figure 12, we can see that there exists a phase transition, and the charged hairy black hole dominates for small temperatures, while for large temperatures the RN-AdS black hole would be preferred. Also, we can observe that the critical temperature at which this phase transition takes place depends on the ψ and as it can be seen in figure 13 it depends also on ν the charge of the scalar field at small temperatures. At zero temperature the RN-AdS is preferable which agrees with our previous results. This phase transition occurs for hyperbolic horizon $k = -1$, in agreement with the findings in [10, 23, 25] where only phase transitions of exact hairy black hole solutions to black hole solutions with hyperbolic horizons.

A similar analysis can be carried out for the case of $\Lambda_{\text{eff}} = 0$. In this case the temperature, mass, entropy and electric charge are given respectively by

$$T_{\text{RN}} = \frac{1}{4\pi\rho_+} \left(1 - \frac{\psi^2}{8\pi}\right), \quad S_{\text{RN}} = 2\pi\sigma\rho_+^2, \quad M_{\text{RN}} = \sigma\rho_+ \left(1 + \frac{\psi^2}{8\pi}\right), \quad Q_{\text{RN}} = \frac{\sigma\rho_+\psi}{4\pi}, \quad (5.25)$$

and the horizon radius is given by $\rho_+ = \frac{1}{4\pi T_{\text{RN}}} \left(1 - \frac{\psi^2}{8\pi}\right)$. In figure 14 we plot the free energy F_0 for the charged black hole with scalar hair and F_1 the free energy of the RN black hole. Then we can see that there not exists a phase transition, and the RN black hole dominates for all temperatures.

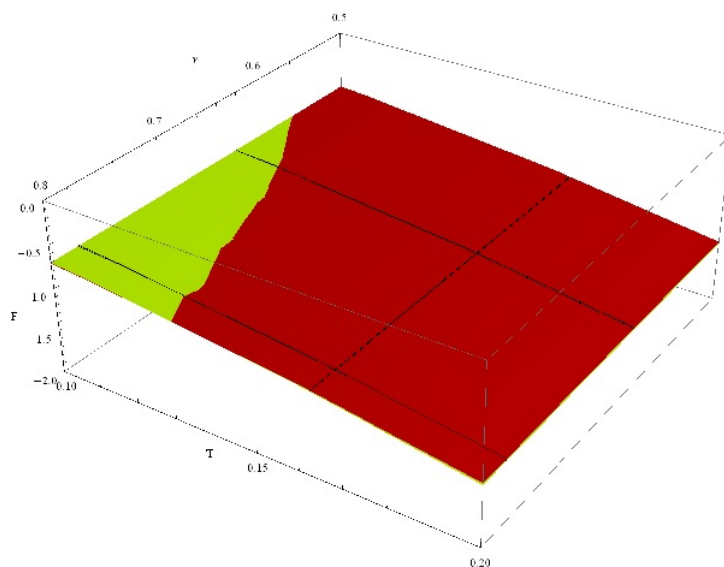


Figure 13. The behaviour of the free energy for the charged hairy black hole and the RN black hole as function of T and ν with $k = -1$, $\sigma = 1$, $l = 1$, and $\psi = 0.1$. Red surface for charged hairy black hole and yellow surface for the RN black hole.

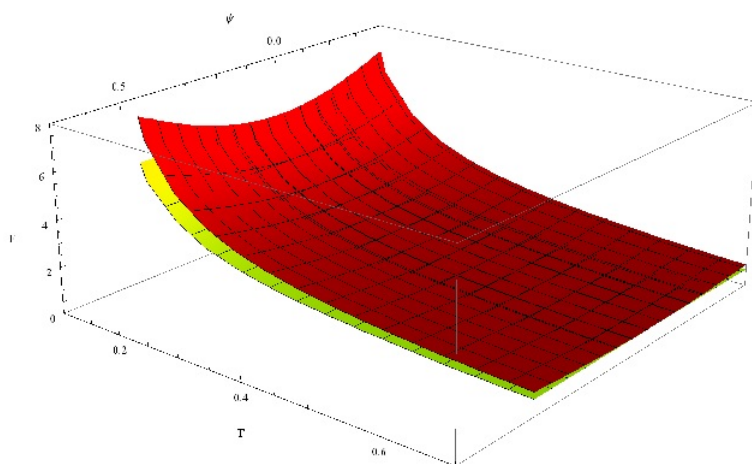


Figure 14. The behaviour of the free energy for the charged hairy black hole and the RN black hole as function of T and ν with $k = 1$, $\sigma = 4\pi$, and $\nu = 0.01$. Red surface for charged hairy black hole and yellow surface for the RN black hole, $C_1 = -\frac{4k}{\nu^2}$.

6 Conclusions

We have considered a gravitating system consisting of a scalar field minimally coupled to gravity with a self-interacting potential and an U(1) electromagnetic field. We solved exactly the coupled Einstein-Maxwell-scalar field equations with a profile of the scalar field which falls sufficient fast outside the black hole horizon. For a range of values of the scalar field parameter, which characterizes its behaviour, we found exact hairy charged black hole solutions with the scalar field regular everywhere.

The presence of the scalar field introduced a scale in the system, resulting in a redefinition of the cosmological constant to Λ_{eff} . If $\Lambda_{\text{eff}} \neq 0$ then hairy black hole solutions were found for $k = \pm 1, 0$ with the potential at large distances to tend to the effective cosmological constant, and the scalar field to be regular everywhere outside the event horizon and null at large distances. If $\Lambda_{\text{eff}} = 0$ then a hairy charged black hole solution was found for $k = 1$ in flat space. In both cases if the scalar field is decoupled then the RN black hole solution is recovered.

Because the scalar hair is non-zero only near the horizon of the black hole, we studied the effect of the scalar field on the near horizon limit of external black hole as the temperature goes to zero. We found that except a critical value of the charge of the black hole there exist also a critical value of scalar field beyond which the extremal black hole is destabilized. If $\Lambda_{\text{eff}} \neq 0$ it goes to an AdS space while if $\Lambda_{\text{eff}} = 0$ it goes to flat space.

Finally, we studied the thermodynamics of our hairy charge black hole solutions. In the case of $\Lambda_{\text{eff}} = 0$ we found that at all temperature the RN black hole solution is thermodynamically preferred over the hairy charge black hole solution. In the case of $\Lambda_{\text{eff}} \neq 0$ we found that the hairy charge black hole is thermodynamically preferred over the RN black hole at low temperature. This picture is in agreement with the findings of the application of the AdS/CFT correspondence to condensed matter systems. In these systems there is a critical temperature below which the system undergoes a phase transition to a hairy black hole configuration at low temperature. This corresponds in the boundary field theory to the formation of a condensation of the scalar field.

It would be interesting to study the stability of our solutions. This stability analysis will make our results robust specially in connection with the gauge/gravity applications to condensed matter systems. Also it would be interesting to extent this study to a gravitational system with a charged scalar field. In this case we expect to have a better stability behaviour of the system, because the electromagnetic force may balance the gravitational force. Work in this direction is in progress.

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