Revisiting $B \rightarrow \pi\pi\ell\nu$ at large dipion masses

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ABSTRACT: We revisit QCD factorization of $B \rightarrow \pi\pi$ form factors at large dipion masses, by deriving new constraints based on the analyticity properties of these objects. We then propose a parametrization of the form factors, inspired by the leading-twist QCD factorization formula, that incorporates all known analytic properties. This parameterization is used to interpolate between the QCDF results and the constraints from the B^* pole. Based on this interpolation, we predict the $B \rightarrow \pi\pi\ell\nu$ decay rate in a larger phase space region than previous studies could. We obtain a partially-integrated branching ratio up to $\mathcal{B} \simeq \mathcal{O}(10^{-6})$, which implies that a measurement of the non-resonant semileptonic decay is potentially within reach of the Belle II experiment.

KEYWORDS: QCD Phenomenology

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1 Introduction

Semileptonic $b \rightarrow u$ transitions are used to determine the Standard Model (SM) parameter $|V_{ub}|$. Contemporary determinations based on data provided by the B-factory experiments BaBar and Belle, as well as the LHCb experiment show tensions between determinations from exclusive and inclusive decays. Among the former, the decay $B \rightarrow \pi\ell\nu$ provides presently the most control over the hadronic matrix elements, whose knowledge is required for the $|V_{ub}|$ determinations. The matrix elements are known from lattice QCD [1–3] and Light-Cone Sum Rules (LCSRs) [4–6]. In addition, the decay $B \rightarrow \rho\ell\nu$ is widely discussed as a further promising channel. However, the ρ is not an asymptotic state, since it decays rapidly through the strong interaction. Alternative exclusive determinations of $|V_{ub}|$ or constraints on effects due to physics beyond the SM (BSM) can be obtained from e.g., the three-body decay $B_s \rightarrow K\ell\nu$ [7, 8]; the effective three-body decay $B_s \rightarrow K^*(\rightarrow K\pi)\ell\nu$ [9] (with a much narrower state K^* compared to the rather wide ρ); and the four-body decay $B \rightarrow \pi\pi\ell\nu$ [10–16]. In this article we will focus on a study of the hadronic matrix elements for the four-body decay, which are also an important input to studies of the fully-hadronic decay $B \rightarrow \pi\pi\pi$ [17, 18].

A QCD Factorization (QCDF) formula for $B \rightarrow \pi\pi$ form factors at large dipion masses was proposed in reference [13]. A major drawback to its phenomenological applications are the phase-space limits that are needed to ensure the factorization into the soft $B \rightarrow \pi$ form factor and pion and B -meson Light-Cone Distribution Amplitudes (LCDAs). Indeed, in reference [13] the partially-integrated branching ratio of $B \rightarrow \pi\pi\ell\nu$ decays was found to range from $4 \cdot 10^{-13}$ to $4 \cdot 10^{-10}$, depending on the applied phase-space cuts. Given the expected size of the Belle II data set, the smallness of the branching ratio draws into question the prospects for measurements in the QCDF-accessible phase space.

Due to the large number of independent kinematic variables in the four-body decay $B \rightarrow \pi\pi\ell\nu$, the analytic structure of the $B \rightarrow \pi\pi$ form factors is somewhat more complicated than for the $B \rightarrow \pi$ or $B \rightarrow \rho$ form factors. Following [10, 13] we assign the momenta as follows:

$$B^-(p) \rightarrow \pi^+(k_1)\pi^-(k_2)\bar{\nu}(q_1)\ell^-(q_2), \quad (1.1)$$

and define

$$\begin{aligned} q &= q_1 + q_2, & k &= k_1 + k_2, \\ \bar{q} &= q_1 - q_2, & \bar{k} &= k_1 - k_2. \end{aligned} \quad (1.2)$$

In the following we use k^2 , q^2 , and

$$\hat{q}^2 \equiv (p - k_2)^2 = (q + k_1)^2 \quad (1.3)$$

as the independent kinematic variables, with $q^2 \leq (M_B - \sqrt{k^2})^2$ and $\hat{q}^2 \leq M_B^2 - k^2$. The variable $\hat{q}^2$ can be exchanged for the dipion helicity angle θ_π [10, 13]

$$q \cdot \bar{k} = \frac{\sqrt{\lambda_B}}{2} \cos \theta_\pi \quad \text{for } k^2 \gg M_\pi^2, \quad (1.4)$$

with $\lambda_B = \lambda(q^2, k^2, M_B^2)$ the Källén function. We use both representations; particularly, we use the variable $\hat{q}^2$ to make the analytic structure of the form factors more readily understandable. Hadronic intermediate states, which may contribute to the $B \rightarrow \pi\pi\ell\nu$ decay in a dual way compared to the LO partonic picture, include:

$B \rightarrow R_n(\rightarrow \pi\pi)\ell\nu$. The $R_n = \rho, f_0, \rho', \dots$ are the light resonances arising from a branch cut in the variable k^2 . For a point-wise description of the k^2 spectrum detailed knowledge of the cut is required. However, in this work we focus on more inclusive observables far above the $\pi\pi$ threshold, which do not resolve the individual R_n resonances.

$B \rightarrow X_b(\rightarrow \ell\nu)\pi\pi$. Here, the $X_b = B^*, B_1, B$ is either the vector, axialvector or pseudoscalar resonance, respectively, that contributes to the q^2 spectrum of the individual form factors.

$B \rightarrow B^*(\rightarrow \pi_2\ell\nu)\pi_1$. The heavy vector meson B^* contributes here as a resonance in the $\hat{q}^2 \equiv (p - k_2)^2 = (q + k_1)^2$ spectrum. The aim of this article is to improve our description of the $B \rightarrow \pi\pi$ form factors by including *this* resonant contribution and to thereby extend the form factors' reach.

Our parametrization makes use of two conformal maps to accelerate convergence. The use of residues to improve our understanding of the form factors is an approach known from prior phenomenological studies, such as those deriving unitarity bounds for $b \rightarrow c$ form factors [19, 20] and $b \rightarrow u$ form factors [21].

In the phase space of interest — i.e. the phase space for the semileptonic B decay — the amplitudes are thus analytic functions of q^2 and $\hat{q}^2$, while they exhibit a branch cut

in k^2 that gives rise to resonant ρ , f_0 , and similar contributions. It then makes sense to perform the usual z -expansion, trading

$$q^2 \quad \text{for} \quad z \equiv z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}, \quad (1.5)$$

$$\hat{q}^2 \quad \text{for} \quad \hat{z} \equiv \hat{z}(\hat{q}^2) = \frac{\sqrt{\hat{t}_+ - \hat{q}^2} - \sqrt{\hat{t}_+ - \hat{t}_0}}{\sqrt{\hat{t}_+ - \hat{q}^2} + \sqrt{\hat{t}_+ - \hat{t}_0}}, \quad (1.6)$$

where

$$t_+ = (M_B + 2M_\pi)^2, \quad t_0 = 0, \quad (1.7)$$

$$\hat{t}_+ = (M_B + M_\pi)^2, \quad \hat{t}_0 = \hat{t}_+ - \sqrt{\hat{t}_+} \sqrt{\hat{t}_+ - M_{B^*}^2}. \quad (1.8)$$

Here t_+ and $\hat{t}_+$ denote the thresholds of the hadronic continuum in the respective channels and t_0 and $\hat{t}_0$ are the reference values that we will use for the *extrapolation* of the perturbative result to larger values of q^2 and $\hat{q}^2$. The expression for a generic dipion form-factor function will thus be given as

$$F(k^2, q^2, \hat{q}^2) \sim \sum_{i,j} z^i \hat{z}^j f_{ij}(k^2), \quad (1.9)$$

up to kinematic prefactors that arise from the definition of the form factors. The concrete parametrization that we propose, supplemented by appropriate “Blaschke factors” to account for subthreshold B -meson resonances, is discussed in section 2.1. We continue with our results for the B^* -pole residues in section 2.2. Our numerical results then follow in section 3, before we conclude in section 4.

2 Parametrization of the $B \rightarrow \pi\pi$ form factors and theoretical constraints

In the following we provide the rationale for our proposed parametrization, and provide constraints on its parameters based on two theoretical results. Common to the following discussions are the use of a basis of Dirac structures that define the $B \rightarrow \pi\pi$ form factors. This basis can be expressed in terms of orthogonal momenta, e.g. q , k_0 , $\bar{k}_\parallel$, and $\bar{q}_\perp$ [10]:

$$\begin{aligned} k_0^\mu &= k^\mu - \frac{k \cdot q}{q^2} q^\mu, \\ \bar{k}_\parallel^\mu &= \bar{k}^\mu - \frac{4(k \cdot q)(q \cdot \bar{k})}{\lambda_B} k^\mu + \frac{4k^2(q \cdot \bar{k})}{\lambda_B} q^\mu, \\ \bar{q}_\perp^\mu &= 2\epsilon^{\mu\alpha\beta\gamma} \frac{q_\alpha k_\beta \bar{k}_\gamma}{\sqrt{\lambda_B}}. \end{aligned} \quad (2.1)$$

The Dirac structures that project onto our choice of form factor basis then read:

$$\begin{aligned} \Gamma_t &= \frac{-\not{q}\gamma_5}{\sqrt{q^2}} & \Gamma_0 &= \frac{2\sqrt{q^2}\not{k}_0\gamma_5}{\sqrt{\lambda_B}} \\ \Gamma_\parallel &= \frac{\not{\bar{k}}_\parallel\gamma_5}{\sqrt{k^2}\sin^2\theta_\pi} & \Gamma_\perp &= \frac{i\not{\bar{q}}_\perp}{\sqrt{k^2}\sin^2\theta_\pi}. \end{aligned} \quad (2.2)$$

2.1 Inspiration by the QCD factorization formulas

We set out to produce a parametrization of the dipion form factor that is compatible with the QCDF formula at large dipion masses, but which can also be augmented with further constraints. To obtain better insight into the analytic dependence on the kinematic variables, we start from the QCDF expressions to leading order in α_s and to leading twist as given in [13]:

$$\langle \pi^+(k_1) \pi^-(k_2) | \bar{\psi}_u \Gamma \psi_b | B^-(p) \rangle = \frac{2\pi f_\pi}{k^2} \xi_\pi(\hat{q}^2) \int_0^1 du \phi_\pi(u) T_\Gamma^I(u, k^2, q^2, \hat{q}^2) + \dots \quad (2.3)$$

for any current Γ . The function T_Γ^I encodes the perturbative description of the dynamics related to the hadronic subprocesses $Y_b \rightarrow \pi^+ \ell^- \bar{\nu}$ in

$$B^-(p) \longrightarrow [Y_b(\hat{q}) \rightarrow \pi^+(k_1) \ell^-(q_1) \bar{\nu}(q_2)] \pi^-(k_2). \quad (2.4)$$

The simplest contributing hadronic states are given by

$$(i) : \quad Y_b = B^*, \quad (ii) : \quad Y_b = B^{(*)} \pi. \quad (2.5)$$

They imply a simple pole at $\hat{q}^2 = M_{B^*}^2$ and a cut for $\hat{q}^2 \geq (M_B^{(*)} + M_\pi)^2$. Notice that these should be taken care of by a realistic implementation of the $B^- \rightarrow \pi^-$ form factor as a function of $\hat{q}^2$, respectively by its $\hat{z}$ -expansion.

We investigate which terms are required in our parametrization by taking a closer look at the results for $\Gamma = \Gamma_\perp$ as an example:

$$\frac{1}{\sqrt{k^2}} T_\perp^I = \sqrt{\lambda_B} \frac{i\alpha_s C_F}{2N_C} \frac{f_2(u)}{M_B^2 - \hat{q}^2}, \quad (2.6)$$

with

$$f_2(u) = \frac{1}{\bar{u}} \frac{M_B^2 - \hat{q}^2}{M_B^2 - \bar{u} q^2 - u \hat{q}^2}. \quad (2.7)$$

Concerning the q^2 and $\hat{q}^2$ dependence in the subsequent decay, we may improve the description of the form factors by including appropriate Blaschke factors for the B^* resonance in both channels below the $B\pi\pi(B\pi)$ continuum thresholds,

$$P_{B^*} \equiv \frac{1 - z z_{B^*}}{z - z_{B^*}}, \quad z_{B^*} = z(M_{B^*}^2, t_0), \quad (2.8)$$

$$\hat{P}_{B^*} \equiv \frac{1 - \hat{z} \hat{z}_{B^*}}{\hat{z} - \hat{z}_{B^*}}, \quad \hat{z}_{B^*} = \hat{z}(M_{B^*}^2, \hat{t}_0), \quad (2.9)$$

respectively. These are implemented as

$$\frac{1}{\sqrt{k^2}} T_\perp^I = \sqrt{\lambda_B} \frac{i\alpha_s C_F}{2N_C} P_{B^*} \hat{P}_{B^*} \sum_{i,j} z^i \hat{z}^j f_{ij}^\perp(k^2). \quad (2.10)$$

Matching eq. (2.10) onto eq. (2.6), we find that the z -expansion makes the integration over the quark momentum fraction u in the convolution with the pion LCDA rather simple, because the k^2 -dependence decouples from the convolution integral.

As mentioned in the introduction, various $X_b = B^*, B_1, B$ states contribute as one-body hadronic intermediate states to the dispersive representation of the form factors as functions of q^2 . In principle, one would need to specify which exact resonant state contributes. Since we work at very small values of $q^2 \ll \{M_{B^*}^2, M_{B_1}^2, M_B^2\}$ we replace the individual poles with one effective pole at $q^2 = M_{B^*}^2$ by means of the Blaschke factor P_{B^*} . This choice corresponds to the resonant contribution in the $B \rightarrow \pi$ vector form factor f_+ , which we use later on for numerical predictions of the QCDF formulas and analyticity constraints.

The discussion above can be generalized to all four (axial)vector form factors. Our parametrizations then read:

$$F_t^{\text{param}} \equiv P_{B^*} \hat{P}_{B^*} \frac{M_B^3}{k^2 \sqrt{q^2}} \left[a_t + b_t \frac{M_B^2 - k^2}{M_B^2} + c_t \frac{(M_B^2 - k^2)^2}{M_B^4} \right], \quad (2.11)$$

$$F_0^{\text{param}} \equiv P_{B^*} \hat{P}_{B^*} \frac{M_B^5}{k^2 \sqrt{q^2} \lambda_B} \left[a_0 + b_0 \frac{M_B^2 - k^2}{M_B^2} + c_0 \frac{(M_B^2 - k^2)^2}{M_B^4} \right], \quad (2.12)$$

$$F_\perp^{\text{param}} \equiv P_{B^*} \hat{P}_{B^*} \frac{\sqrt{\lambda_B}}{M_B \sqrt{k^2}} \left[a_\perp + b_\perp \frac{M_B^2 - k^2}{M_B^2} + c_\perp \frac{(M_B^2 - k^2)^2}{M_B^4} \right], \quad (2.13)$$

$$F_\parallel^{\text{param}} \equiv P_{B^*} \hat{P}_{B^*} \frac{M_B}{\sqrt{k^2}} \left[a_\parallel + b_\parallel \frac{M_B^2 - k^2}{M_B^2} + c_\parallel \frac{(M_B^2 - k^2)^2}{M_B^4} \right]. \quad (2.14)$$

By using appropriate overall normalizations that reflect the dominant kinematic dependence of the QCDF results, we minimize the number of parameters needed later on in the fits. For all polarizations λ the coefficients $a_\lambda, b_\lambda, c_\lambda$ still require expansion in both z and $\hat{z}$. The precise type of expansion is not relevant at this point, and will be discussed in detail in section 3. Constraints on the B^* pole in the variable $\hat{z}$ can be readily included by replacing $\hat{P}_{B^*}$ with its residue:

$$\lim_{\hat{q}^2 \rightarrow M_{B^*}^2} (q^2 - M_{B^*}^2) \hat{P}_{B^*} = 4(M_{B^*}^2 - \hat{t}_+). \quad (2.15)$$

The QCDF expressions give good control over the behaviour in $\hat{z}$ for $0.25 \lesssim \hat{z} \lesssim 0.40$. However, our aim is to extrapolate from the region where QCDF is applicable to the larger $B \rightarrow \pi\pi$ phase space. This is achieved by imposing additional constraints at $\hat{z} < 0$, which are obtained from the B^* pole, as discussed in the next section. Our approach is best illustrated using $\cos \theta_\pi$ rather than $\hat{z}$. At small values of $k^2 \sim 7 \text{ GeV}^2$ the QCDF predictions are limited to the phase space $|\cos \theta_\pi| < 1/3$. The B^* -pole in the variable $\hat{q}^2$ then “lives” at unphysical values of $\cos \theta_\pi \sim 2$. These two constraints then anchor our parameterization on both sides of the QCDF-inaccessible phase space $1/3 < \cos \theta_\pi \leq 1$, therefore turning an extrapolation problem into an interpolation.

2.2 Analyticity constraints

The one-body contributions to the dispersion relation of the $B^- \rightarrow \pi^+ \pi^-$ form factors in the variable $\hat{q}^2$ yield:

$$\begin{aligned} F_\lambda(q^2, \hat{q}^2, k^2) &= \frac{1}{2\pi} \sum_{\lambda'} \int_{\hat{t}_+}^\infty d\hat{t} \frac{\langle \pi^+ | \bar{q} \Gamma_\lambda b | B^{*0} \rangle \langle B^{*0} \pi^- | B^- \rangle}{\hat{t} - \hat{q}^2} 2\pi \delta(\hat{t} - M_{B^*}^2) \\ &= \frac{-1}{2\pi} \int_{\hat{t}_+}^\infty d\hat{t} \frac{2\pi \delta(\hat{t} - M_{B^*}^2) \xi_{B^* \rightarrow \pi}(q^2) g_{B^* B \pi} M^\alpha(\lambda) p^\beta \left[-g_{\alpha\beta} + \frac{\hat{q}_\alpha \hat{q}_\beta}{\hat{q}^2} \right]}{\hat{t} - \hat{q}^2}. \end{aligned} \quad (2.16)$$

Here $\xi_{B^* \rightarrow \pi}$ refers to the soft form factor in $B^* \rightarrow \pi$ matrix elements, defined in complete analogy to the $B \rightarrow \pi$ form factors in the SCET limit [22]:

$$\begin{aligned} \langle \pi^+(k_1) | \bar{q} \Gamma_\lambda b | B^{*0}(\hat{q}, \lambda') \rangle &= \xi_{B^* \rightarrow \pi}(q^2 = (\hat{q} - k_1)^2) \text{tr} \left[\not{k}_1 \gamma_5 \Gamma_\lambda \frac{1 + \not{\epsilon}}{2} (-i \gamma^\alpha) \right] \epsilon_\alpha(\lambda') \\ &\equiv \xi_{B^* \rightarrow \pi}(q^2) M^\alpha(\lambda) \epsilon_\alpha(\lambda'), \end{aligned} \quad (2.17)$$

where λ' is the B^* polarization. In addition, we use the $B^* B \pi$ coupling [23]

$$\langle B^{*0}(\hat{q}, \lambda') \pi^-(k_2) | B^-(p) \rangle = -(p \cdot \epsilon^*(\hat{q}, \lambda')) g_{B^* B \pi}, \quad (2.18)$$

as well as the completeness relations

$$\sum_{\lambda'} \epsilon_\mu(\hat{q}; \lambda') \epsilon_\nu^*(\hat{q}; \lambda') = -g_{\mu\nu} + \frac{\hat{q}_\mu \hat{q}_\nu}{\hat{q}^2}. \quad (2.19)$$

Our aim is now to relate the residue of the B^* pole to the residue of our parametrization, therefore anchoring it at large values of $\hat{q}^2 = M_{B^*}^2$. We determine the imaginary part of the residue on the B^* pole to be

$$\begin{aligned} \text{Im} \left[\text{Res}_{\hat{q}^2 \rightarrow M_{B^*}^2} F_\lambda(q^2, \hat{q}^2, k^2) \right] &= \text{Im} \left[\lim_{\hat{q}^2 \rightarrow M_{B^*}^2} (\hat{q}^2 - M_{B^*}^2) F_\lambda(q^2, \hat{q}^2, k^2) \right] \\ &= \xi_{B^* \rightarrow \pi}(q^2) g_{B^* B \pi} \text{Im} \left[S_\lambda(q^2, \hat{q}^2 = M_{B^*}^2, k^2) \right], \end{aligned} \quad (2.20)$$

where

$$S_\lambda \equiv M^\alpha(\lambda) p^\beta \left[-g_{\alpha\beta} + \frac{\hat{q}_\alpha \hat{q}_\beta}{M_{B^*}^2} \right]. \quad (2.21)$$

The soft form factor $\xi_{B^* \rightarrow \pi}$ is not well known. However, using heavy quark symmetry it can be related to the soft form factor $\xi_{B \rightarrow \pi}$, which can be identified with the $B \rightarrow \pi$ vector form factor f_+ [22]. Throughout we use the BCL parametrization for the form factor [24] with parameter values obtained from a LCSR study [5]. The relation between the soft form factors is subject to power corrections, which we estimate to be of the order of 30%.

For the different λ polarization, we obtain

$$S_t = -i \frac{M_B^2(M_B^2 - M_{B^*}^2)(M_{B^*}^2 - q^2) - k^2 M_{B^*}^2(M_B^2 + M_{B^*}^2)}{2M_B M_{B^*}^2 \sqrt{q^2}}, \quad (2.22)$$

$$S_0 = -i \frac{(k^2(M_B^2 + M_{B^*}^2) - (M_B^2 - q^2)(M_{B^*}^2 - M_B^2))(k^2 M_{B^*}^2 + M_B^2(q^2 - M_{B^*}^2))}{2M_B M_{B^*}^2 \sqrt{q^2} \sqrt{\lambda_B}}. \quad (2.23)$$

$$S_\perp = -i \frac{\sqrt{k^2} \sqrt{\lambda_B} (M_B^2 + M_{B^*}^2)}{4M_B M_{B^*}^2}, \quad (2.24)$$

$$S_\parallel = -i \frac{\sqrt{k^2} (M_B^4 + M_{B^*}^2(q^2 - k^2) + M_B^2(q^2 - 3M_{B^*}^2 - k^2))}{4M_B M_{B^*}^2}. \quad (2.25)$$

Finally, we equate the two different expressions for the residues through the statistical procedure outlined in section 3.

3 Phenomenological applications

We proceed in three steps. First, we produce a theoretical likelihood that incorporates information from the QCD factorization formulas as well as from the analyticity constraints. Second, we discuss the concrete parametrization and provide results for the parameters from a fit to the theoretical likelihood. Third, we produce numerical estimates of two integrated $B \rightarrow \pi\pi\ell\nu$ observables in various phase-space bins.

3.1 Theoretical likelihood

We use the QCDF expressions for the $B \rightarrow \pi\pi$ form factors to leading-order in α_s and to leading- and next-to-leading twist accuracy to produce synthetic data points. We generate these data points for the form factors at the following values of the kinematic variables k^2 , q^2 , and $\cos\theta_\pi$:

$$k^2 = 25 \text{ GeV}^2 \quad q^2 = 0.05 \text{ GeV}^2 \quad \cos\theta_\pi \in \{-1.0, -0.5, 0, +0.5, +1.0\} \quad (3.1)$$

$$k^2 = 19 \text{ GeV}^2 \quad q^2 \in \{0.05, 0.60\} \text{ GeV}^2 \quad \cos\theta_\pi \in \{-1.0, -0.5, 0, +0.5, +1.0\} \quad (3.2)$$

$$k^2 = 16 \text{ GeV}^2 \quad q^2 \in \{0.05, 1.50\} \text{ GeV}^2 \quad \cos\theta_\pi \in \{-1.0, -0.5, 0, +0.5, +1.0\} \quad (3.3)$$

$$k^2 = 7 \text{ GeV}^2 \quad q^2 \in \{0.05, 1.50, 6.00\} \text{ GeV}^2 \quad \cos\theta_\pi \in \{-0.33, 0, +0.33\}, \quad (3.4)$$

for a total of 34 QCDF data points per form factor. The smallest value of q^2 was chosen as roughly $\mathcal{O}(m_\mu^2)$, in order to regularize a divergence of kinematic origin in the form factors F_0 and F_t . Following [13] we do not use the QCDF factorisation results when the pion energies in the B-meson rest frame falls below a threshold of $\sim 1.2 \text{ GeV}$, which corresponds to a maximal value of $|\cos\theta_\pi| \simeq 0.33$ at $k^2 = 7 \text{ GeV}^2$.

In addition to the QCDF expressions, we also produce synthetic data points for the imaginary part of the residues of the form factors on the B^* pole. The theoretical expressions for the residue of the form factors are provided in section 2.2. Fixing $\hat{q}^2 = M_{B^*}^2$ still leaves two free kinematic variables. We choose the following values of k^2 and q^2 :

$$k^2 \in \{10, 13\} \text{ GeV}^2 \quad q^2 \in \{0.05, 1.50\} \text{ GeV}^2, \quad (3.5)$$

$$k^2 = 7 \text{ GeV}^2 \quad q^2 \in \{0.05, 1.50, 6.00\} \text{ GeV}^2. \quad (3.6)$$

By including the residues in the fit only for small values of k^2 we stabilize the fit and supplement information in the space region where the QCDF formulas lack predictive power.

For the production of all theory pseudo observables, both form factors and residues, we closely follow [13]. We use the publicly available EOS [27] software, which already features a numerical implementation of the QCDF expressions for the form factors. We extend EOS with an implementation of the B^* -pole residues. To produce the pseudo data points we follow a Bayesian approach. Our choice of the a-priori Probability Density Function (PDF) is summarized in table 1. We draw 10^6 samples from the prior PDF, which are then used to produce the same number of samples for each of the pseudo observables. The mean and parametric uncertainty are then estimated through the sample mean and sample covariance of the pseudo observables. Since both sets of predictions share a strong dependence on the value of the soft form factor ξ_π , we find that all pseudo observables are strongly correlated,

parameter	value/interval	unit	prior	source/comments
QCD input parameter				
$\alpha_s(m_Z)$	0.1184 ± 0.0007	—	gaussian @ 68%	[25]
μ	$M_B/2 \pm M_B/4$	GeV	gaussian [†] @ 68%	
$\overline{m}_{u+d}(2 \text{ GeV})$	7.8 ± 0.9	MeV	uniform @ 100%	see [5]
hadron masses				
M_B	5279.58	MeV	—	[25]
M_{B^*}	5324.65	MeV	—	[25]
M_π	139.57	MeV	—	[25]
parameters of the pion DAs				
f_π	130.4	MeV	—	[25]
$a_2^\pi(1 \text{ GeV})$	[0.09, 0.25]	—	uniform @ 100%	[26]
$a_4^\pi(1 \text{ GeV})$	[-0.04, 0.16]	—	uniform @ 100%	[26]
$\mu_\pi(2 \text{ GeV})$	2.5 ± 0.3	GeV	—	$M_\pi^2/(\overline{m}_{u+d})$
hadronic couplings				
$g_{B^*B\pi}$	30 ± 5	—	gaussian @ 68%	[5]

Table 1. The input parameters used in our numerical analysis. We express the prior distribution as a product of individual priors that are either uniform or gaussian. The uniform priors cover the stated intervals with 100% probability. The gaussian priors cover the stated intervals with 68% probability, and the central value corresponds to the mode of the prior. For practical purposes, random variates of the gaussian priors are only drawn from their respective 99% probability intervals. The prior for the parameters describing the $B \rightarrow \pi$ form factor f_+ are not listed here and taken from [5]. †: we restrict variates of the renormalization scale μ to the interval $[M_B/4, M_B]$ to avoid unphysical values.

with some correlation coefficients as large as 0.99. However, we find that the covariance matrix is still regular.

Both the QCDF expressions for the form factors from [13] as well as our results for the B^* -pole residues in section 2.2 only hold to leading power in an expansion in $1/m_b$ and $1/E_\pi$, the pion energy in the B -meson rest frame. In order to account for contributions beyond these leading-power expressions, we assign an ad-hoc systematic uncertainty of 30% of the central value. This systematic uncertainty is combined with the parametric uncertainty by adding their respective covariance matrices.

Due to the large dimensionality, we provide the combined multivariate Gaussian likelihood for all pseudo observables only in machine readable form, as part of the EOS software. The total number of observations in the likelihood is $N_{\text{obs}} = 4 \times 41 = 164$. The names of

the newly added EOS constraints with 30% added systematic uncertainty are

B->pipi::F_time[sys=0.30]@FvDV2018	for	F_t
B->pipi::F_long[sys=0.30]@FvDV2018	for	F_0
B->pipi::F_perp[sys=0.30]@FvDV2018	for	$F_\perp$
B->pipi::F_para[sys=0.30]@FvDV2018	for	$F_\parallel$

respectively.

3.2 Fit of the $B \rightarrow \pi\pi$ form factor parameters

We now proceed to fit our expressions in eqs. (2.11)–(2.13) to the theory likelihood constructed in section 3.1. For this we need to define explicitly the expansion of the coefficient a_λ , b_λ and c_λ in z and $\hat{z}$. We use

$$x_\lambda = \sum_{i=0}^3 x_{0i}^\lambda \hat{z}^i + \sum_{i=0}^2 x_{1i}^\lambda z \hat{z}^i, \quad (3.7)$$

for all polarisations λ and all coefficients $x = a, b, c$. This amounts to 21 parameters per form factor, with θ representing the entire set of parameters. Our rationale for choosing these expansions is our power counting $\hat{z}^2 \simeq z$, and the fact that we can achieve a good fit with the smallest number of parameters per form factor, as outlined below.

We carry out a fit to all form factor parameters θ simultaneously, which amounts to a 84 dimensional fit. We use Minuit2 to find the best-fit point of the posterior PDF

$$P(\theta | \text{theory}) = \frac{P(\text{theory} | \theta) P_0(\theta)}{Z} \quad (3.8)$$

where $P_0(\theta)$ is the prior PDF, $P(\text{theory} | \theta)$ is the likelihood, and Z is the evidence. We find a total minimal $\chi^2 = 8.79$ for a total of $N_{\text{d.o.f.}} = N_{\text{obs}} - N_{\text{par.}} = 164 - 84 = 80$ degrees of freedom. The fit quality is therefore excellent, with a p value in excess of 99%.

We then use Minuit2's information on the parameters' uncertainties to inspire starting values for the prior intervals. Our final prior intervals are then chosen to include at least 99% of their respective one-dimensional marginalized posteriors. For latter purpose we produce 10^6 posterior samples. The sample mean and sample covariance of the posterior samples are provided as an EOS [27] constraint file in YAML format. The file is available as [supplementary material](#) attached to this article. We show plots of the form factors F_λ , normalized to the Blaschke factor $\hat{P}_{B^*}$, as functions of $\hat{z}$ and for fixed $q^2 = 1.5 \text{ GeV}^2$ and $k^2 = 7 \text{ GeV}^2$ in figure 1. The singularity due to the unphysical b -quark resonance in the QCDF results is clearly visible in the extrapolation of the QCDF formulas (dashed lines).

Figure 1. Plots of the $\hat{z}$ dependence of the form factors in the phase space point ($q^2 = 1.5 \text{ GeV}^2, k^2 = 7 \text{ GeV}^2$). Here $\hat{F}_\lambda \equiv F_\lambda / \hat{P}_{B^*}$ in order to be able to visualize the agreement between the parametrization and the residues on the B^* pole. The dashed lines show the QCDF predictions at LO in α_s , including both the twist-2 and twist-3 contributions. These predictions are valid in the region where $0.25 \leq \hat{z} \leq 0.33$. Beyond this region, QCDF breaks down and the curve shown should be understood as merely an extrapolation. Our fit result, based on the parametrization in eqs. (2.11)–(2.13), is shown as the black solid line. The residues following from section 2.2 are shown as red data points. The dotted vertical lines highlight $\hat{z}(q^2 = M_B^2)$, to show the unphysical pole emerging from the QCDF results at $q^2 \simeq M_B^2$. With these plots we deliberately show the phase-space points that correspond to the largest tensions ($< 2\sigma$) between the B^* residues and our fit.

3.3 Numerical results for $B \rightarrow \pi\pi\ell\nu$ observables

For the numerical illustration, we consider the three scenarios A, B and C discussed in [13]. These phase-space regions are defined as

$$\begin{aligned} 2M_B^2/3 \simeq 18.6 \text{ GeV}^2 \leq k^2 \leq M_B^2, \\ 0 \leq |\cos \theta_\pi| \leq 1, \end{aligned} \quad (\text{Region A})$$

$$\begin{aligned} M_B^2/2 \simeq 13.9 \text{ GeV}^2 \leq k^2 \leq 2M_B^2/3 \simeq 18.6 \text{ GeV}^2, \\ 0 \leq |\cos \theta_\pi| \leq 1, \end{aligned} \quad (\text{Region B})$$

$$\begin{aligned} M_B^2/4 \simeq 7 \text{ GeV}^2 \leq k^2 \leq M_B^2, \\ 0 \leq |\cos \theta_\pi| \leq 1/3. \end{aligned} \quad (\text{Region C})$$

In addition, we define three new scenarios which have an extended phase space compared to the previous ones.

$$\begin{aligned} M_B^2/4 \simeq 7 \text{ GeV}^2 \leq k^2 \leq M_B^2, \\ 0 \leq |\cos \theta_\pi| \leq 1, \end{aligned} \quad (\text{Region C'})$$

$$\begin{aligned} 4 \text{ GeV}^2 \leq k^2 \leq M_B^2, \\ 0 \leq |\cos \theta_\pi| \leq 1/3, \end{aligned} \quad (\text{Region D})$$

$$\begin{aligned} 4 \text{ GeV}^2 \leq k^2 \leq M_B^2, \\ 0 \leq |\cos \theta_\pi| \leq 1. \end{aligned} \quad (\text{Region D'})$$

For all regions $0.02 \text{ GeV}^2 \leq q^2 \leq (M_B - \sqrt{k^2})^2$ holds. Region C' corresponds to region C with an extended range for $|\cos \theta_\pi|$. Regions D and D' are entirely new, and they correspond to an extrapolation in k^2 with respect to regions C and C', respectively. In table 2, we present our results for two observables, the branching ratio $\mathcal{B}$ and the pionic forward-backward asymmetry A_{FB}^π as defined in [13], in each of the specified regions. For comparison, we also include the results obtained in [13] for the regions A, B, and C.

For the new regions C', D, and D', we obtain results that are significantly larger than those in the QCDF regions A, B, and C. This is not surprising, since compared to region C, the new regions span a phase space which is larger by factors of 3, ~ 1.7 , and ~ 5 , respectively.

Compared to region C', the partially-integrated branching ratios in regions D and D' exhibit large uncertainties. This is caused by our extrapolation from data points at $k^2 \leq 7 \text{ GeV}^2$ down to 4 GeV^2 for which we cannot expect that our QCDF-inspired parametrization can still describe the form factors accurately. This can also be understood as a model error, since our parametrization does not (and presently cannot) account for the light and broad dipion resonances that contribute in the region $k^2 < 7 \text{ GeV}^2$. Including these resonances might further improve our description of the form factors, which requires extensive further studies. We leave these for future work.

phase space region	result of [13]		our result		unit
	central	unc.	central	unc.	
$\mathcal{B}(B^- \rightarrow \pi^+ \pi^- \mu^- \bar{\nu}_\mu) / V_{ub} ^2$					
(A)	2.93	$^{+1.00}_{-0.53}$	3.03	$^{+1.20}_{-0.96}$	10^{-8}
(B)	9.31	$^{+3.23}_{-1.47}$	12.30	$^{+3.90}_{-3.60}$	10^{-7}
(A+B)	9.60	$^{+3.38}_{-1.52}$	13.30	$^{+3.60}_{-3.90}$	10^{-7}
(C)	3.18	$^{+0.79}_{-0.71}$	4.88	$^{+3.90}_{-1.70}$	10^{-5}
(C')	—	—	1.61	$^{+1.80}_{-0.60}$	10^{-4}
(D)	—	—	0.74	$^{+3.40}_{-0.48}$	10^{-2}
(D')	—	—	2.86	$^{+16.00}_{-1.80}$	10^{-2}
$A_{\text{FB}}^\pi(B^- \rightarrow \pi^+ \pi^- \mu^- \bar{\nu}_\mu)$					
(A)	-1.96	$^{+0.16}_{-0.20}$	-1.72	$^{+1.70}_{-1.40}$	10^{-1}
(B)	-0.29	$^{+0.22}_{-0.22}$	+0.96	$^{+1.70}_{-1.30}$	10^{-1}
(A+B)	-0.32	$^{+0.20}_{-0.24}$	+1.07	$^{+1.80}_{-1.40}$	10^{-1}
(C)	+1.25	$^{+0.08}_{-0.11}$	+6.31	$^{+0.99}_{-2.40}$	10^{-1}
(C')	—	—	+6.31	$^{+0.99}_{-2.40}$	10^{-1}
(D)	—	—	+6.82	$^{+0.90}_{-0.56}$	10^{-1}
(D')	—	—	+6.82	$^{+0.90}_{-0.56}$	10^{-1}

Table 2. Results for the partially-integrated branching ratio $\mathcal{B}$ (in units of $|V_{ub}|^2$) and the pionic forward-backward asymmetry A_{FB}^π in different phase-space bins.

Exclusive determinations yield $|V_{ub}| \simeq 3.5 \cdot 10^{-3}$. Assuming this value, we find the 68% probability intervals $[4, 5.1] \cdot 10^{-7}$ and $[1.3, 23] \cdot 10^{-7}$ for the partially-integrated branching ratios in regions D and D', respectively. We emphasize that this prediction indicates that a measurement at the Belle II experiment is feasible. This shows that our strategy of including analyticity constraints is clearly beneficial, since it allows to consider a larger subset of the phase space.

4 Conclusion

We present an updated study of the form factors relevant for $B \rightarrow \pi\pi$ semileptonic decays, which were previously studied at large dipion masses in the framework of QCDF. These form factors feature interesting analytic properties. Combining the QCDF results with information on the B^* pole allows to interpolate the form factors in the kinematic variable $\hat{q}^2$. To this extent, we propose a parametrization that respects the dominant kinematic behavior of the QCDF formulas and has a pole at the B^* mass. Fitting this parametrization to all available predictions, we significantly extend the range of applicability of our theoretical framework. The relevance of this is illustrated by larger values of the partially-

integrated observables in the newly-defined phase space region. Our results indicate that, for moderately large dipion masses, a phenomenological study of the $B \rightarrow \pi\pi\ell\nu$ decay with the upcoming Belle II data set is feasible.

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