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The 3.5 keV line from stringy axions

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ABSTRACT: An interesting result in particle astrophysics is the recent detection of an unexplained 3.5 keV line from galaxy clusters. A promising model, which can explain the morphology of the signal and its non-observation in dwarf spheroidal galaxies, involves a 7 keV dark matter particle decaying into a pair of ultra-light axions that convert into photons in the magnetic field of the clusters. Given that light axions emerge naturally in 4D string vacua, in this paper we present a microscopic realisation of this model within the framework of type IIB flux compactifications. Dark matter is a local closed string axion which develops a tiny mass due to subdominant poly-instanton corrections to the superpotential and couples via kinetic mixing to an almost massless open string axion living on a D3-brane at a singularity. The interaction of this ultra-light axion with photons is induced by U(1) kinetic mixing. After describing the Calabi-Yau geometry and the brane set-up, we discuss in depth moduli stabilisation, the resulting mass spectrum and the strength of all relevant couplings.

KEYWORDS: Compactification and String Models, Cosmology of Theories beyond the SM, Superstring Vacua

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1 Introduction

Recently several studies have shown the appearance of a photon line at $E \sim 3.5$ keV, based on stacked X-ray data from galaxy clusters and the Andromeda galaxy [1, 2]. The line has been detected in galaxy clusters by the X-ray observatories XMM-Newton, Chandra and Suzaku [3, 4] and in Andromeda with XMM-Newton [2]. The Hitomi satellite would have been able to study the 3.5 keV line with unprecedented energy resolution. However, unfortunately Hitomi was lost after only a few weeks in operation and the limited exposure time on the Perseus cluster only allows to put upper bounds on the 3.5 keV line which are consistent with the detection of the other satellites [5]. The findings of [1, 2] have inspired further searches in other astrophysical objects such as the galactic center [6–9], galaxies [10],

dwarfs [11–13] and other galaxy clusters [14, 15].¹ Currently, a compelling standard astrophysical explanation, e.g. in terms of atomic lines of the (cluster) gas is lacking.² This gives rise to the possibility that the 3.5 keV line is a signal related to dark matter (DM) physics.

A much explored model is that of dark matter decay, e.g. a sterile neutrino with mass $m_{\text{DM}} \sim 7 \text{ keV}$ decaying into an active neutrino and a photon [18, 19]. In this case, the photon flux from an astrophysical object is solely determined by the lifetime of the dark matter particle and the dark matter column density. The width of the line is due to Doppler broadening. There are several observational tensions, if one wants to explain the (non-)observation of the 3.5 keV line in currently analysed astrophysical objects. Most prominently, these are:

- Non-observation of the 3.5 keV line from dwarf spheroidal galaxies [11–13]. The dark matter density of these objects is rather well known and the X-ray background is low, making dwarf spheroidals a prime target for detecting decaying dark matter.
- Non-observation of the 3.5 keV line from spiral galaxies [10], where again the X-ray background is low. According to the dark matter estimates of [10], the non-observation of a 3.5 keV signal from spiral galaxies excludes a dark matter decay origin of the 3.5 keV line very strongly at 11σ .
- The radial profile of the 3.5 keV line in the Perseus cluster peaks on shorter scales than the dark matter profile, rather following the gas profile than the dark matter profile [1, 9]. However, the observed profile with Suzaku is only in mild tension with the dark matter profile [4].

These tensions, even though they could be potentially explained by uncertainties in the dark matter distributions in these objects [16], motivate different dark matter models than direct dark matter decay into a pair of 3.5 keV photons.

A dark matter model that is consistent with all the present (non-)observations was given in [20]. A dark matter particle with mass $m_{\text{DM}} \sim 7 \text{ keV}$ decays into an almost massless ($m_{\text{ALP}} \lesssim 10^{-12} \text{ eV}$) axion-like particle (ALP) with energy 3.5 keV which successively converts into 3.5 keV photons that are finally observed. Compared to direct dark matter decay into photons, the observed photon flux does not depend just on the dark matter column density, but also on the probability for ALPs to convert into photons. This is determined by the size and coherence scale of the magnetic field and the electron density in e.g. a galaxy cluster.

The 3.5 keV emission is stronger in astrophysical regions with relatively large and coherent magnetic field. This is verified by the experimental fact that cool core clusters like the Perseus cluster have stronger magnetic fields than non-cool core clusters and also a higher 3.5 keV flux is observed from such an object. Furthermore, the fact that central regions of a cool core cluster host particularly strong magnetic fields explains the radial morphology of the 3.5 keV flux from Perseus as the signal comes disproportionately from the

¹For a summary of observations and models on the 3.5 keV line see [16].

²See however [7, 17].

central region of the cluster. The model has made the prediction that galaxies can only generate a non-negligible 3.5 keV photon flux if they are spiral and edge-on as for instance the Andromeda galaxy [20]. In this case, the full length of the regions with regular magnetic field can be used efficiently for ALP to photon conversion. These predictions agree with the experimental results of non-observation of the 3.5 keV signal from generic (edge-on and face-on) spiral galaxies and dwarf galaxies [21].³

Given that the 4D low-energy limit of string compactifications generically leads to several light ALPs [23–27], it is natural to try to embed the model of [20] in string theory. This is the main goal of this paper where we focus in particular on type IIB flux compactifications where moduli stabilisation has already been studied in depth.

In 4D string models, ALPs can emerge either as closed string modes arising from the dimensional reduction of 10D anti-symmetric forms or as phases of open string modes charged under anomalous U(1) symmetries on stacks of D-branes [23–27]. Some of these modes can be removed from the low-energy spectrum by the orientifold projection which breaks $N = 2$ supersymmetry down to $N = 1$, others can be eaten up by anomalous U(1)’s via the Green-Schwarz mechanism for anomaly cancellation or can become as heavy as the gravitino if the corresponding saxions are stabilised by the same non-perturbative effects which give mass to the axions. However the axions enjoy a shift symmetry which is broken only at non-perturbative level. Therefore when the corresponding saxions are frozen by perturbative corrections to the effective action, the axions remain exactly massless at this level of approximation. They then develop a mass via non-perturbative effects which are however exponentially suppressed with respect to perturbative corrections. Hence whenever perturbative contributions to the effective scalar potential play a crucial rôle for moduli stabilisation, the axions are exponentially lighter than the associated saxions [28]. Notice that this case is rather generic in string compactifications for two main reasons: (i) if the background fluxes are not tuned, non-perturbative effects are naturally subleading with respect to perturbative ones; (ii) it is technically difficult to generate non-perturbative contributions to the superpotential which depend on all moduli (because of possible extra fermionic zero modes [29], chiral intersections with the visible sector [30] or non-vanishing gauge fluxes due to Freed-Witten anomaly cancellation [31]).

String compactifications where some moduli are fixed by perturbative effects are therefore perfect frameworks to derive models for the 3.5 keV line with light ALPs which can behave as either the 7 keV decaying DM particle or as the ultra-light ALP which converts into photons. The main moduli stabilisation mechanism which exploits perturbative corrections to the Kähler potential is the LARGE Volume Scenario (LVS) [32–34]. We shall therefore present an LVS model with the following main features (see figure 1 for a pictorial view of our microscopic setup):

- The underlying Calabi-Yau (CY) manifold is characterised by $h^{1,1} = 5$ Kähler moduli $T_i = \tau_i + ic_i$ where the c_i ’s are closed string axions while the τ_i ’s control the volume of 5 different divisors: a large four-cycle D_b , a rigid del Pezzo four-cycle D_s which

³Despite the successful interpretation of all these observations, this model would not be able to explain the dip around 3.5 keV in the Perseus AGN spectrum which might arise from Chandra data [22].

intersects with a ‘Wilson divisor’ D_p ($h^{0,1}(D_p) = 1$ and $h^{0,2}(D_p) = 0$) and two non-intersecting blow-up modes D_{q_1} and D_{q_2} .

- The two blow-up modes D_{q_1} and D_{q_2} shrink down to zero size due to D-term stabilisation and support D3-branes at the resulting singularities. These constructions are rather promising to build a semi-realistic visible sector with SM-like gauge group, chiral spectrum and Yukawa couplings [35, 36]. If D_{q_1} and D_{q_2} are exchanged by the orientifold involution, the visible sector features two anomalous U(1) symmetries (this is always the case for any del Pezzo singularity) [37–39], while if the two blow-up modes are separately invariant, one of them supports the visible sector and the other a hidden sector [40, 41]. Each of the two sectors is characterised by a single anomalous U(1) factor.
- A smooth combination of D_s and D_p is wrapped by a stack of D7-branes which give rise to string loop corrections to the Kähler potential K [42–44]. Moreover, non-vanishing world-volume fluxes generate moduli-dependent Fayet-Iliopoulos (FI) terms [45, 46]. An ED3-instanton wraps the rigid divisor D_s and generates standard T_s -dependent non-perturbative corrections to the superpotential W . A second ED3-instanton wraps the Wilson divisor D_p . Due to the presence of Wilson line modulini, this ED3-instanton contributes to the superpotential only via T_p -dependent poly-instanton effects [47, 48].
- At leading order in an inverse volume expansion, the moduli are fixed supersymmetrically by requiring vanishing D- and F-terms. These conditions fix the dilaton and the complex structure moduli in terms of three-form flux quanta together with the blow-up modes τ_{q_1} and τ_{q_2} in terms of charged open string fields, and hidden matter fields on the D7-stack in terms of τ_p .
- Quantum corrections beyond tree-level break supersymmetry and stabilise most of the remaining flat directions: α' corrections to K [49] and single non-perturbative corrections to W [50] fix τ_b , τ_s and c_s , while soft supersymmetry breaking mass terms and g_s loop corrections to K fix τ_p .
- Subdominant T_p -dependent poly-instanton corrections to W stabilise the local closed string axion c_p while a highly suppressed T_b -dependent non-perturbative superpotential fixes the bulk closed string axion c_b . Sequestered soft term contributions stabilise instead the radial component of U(1)-charged matter fields $C = |C| e^{i\theta}$ living on the D3-brane stacks.
- Both c_b and c_p are exponentially lighter than the gravitino, and so could play the rôle of the decaying DM particle with $m_{\text{DM}} \sim 7 \text{ keV}$. On the other hand the ultra-light ALP with $m_{\text{ALP}} \lesssim 10^{-12} \text{ eV}$ which converts into photons is given by the open string phase θ . Notice that if D_{q_1} and D_{q_2} are identified by the orientifold involution, there are two open string phases in the visible sector: one behaves as the standard QCD axion, which is however heavier than 10^{-12} eV , and the other is the ultra-light ALP

θ . If instead D_{q_1} and D_{q_2} are separately invariant under the involution, θ is an open string axion belonging to a hidden sector.

- The coupling of the closed string axions c_b and c_p to the open string ALP θ is induced by kinetic mixing due to non-perturbative corrections to the Kähler potential. However we shall show that the scale of the induced DM-ALP coupling can be compatible with observations only if the DM candidate is the local closed string axion c_p .
- If the ultra-light ALP θ belongs to the hidden sector, its coupling to ordinary photons can be induced by U(1) kinetic mixing which gets naturally generated by one-loop effects [51]. Interestingly, the strength of the resulting interaction can easily satisfy the observational constraints if the open string sector on the D3-brane stack is both unsequestered and fully sequestered from the sources of supersymmetry breaking in the bulk.
- The branching ratio for the direct axion DM decay into ordinary photons is negligible by construction since it is induced by kinetic mixing between Abelian gauge boson on the D7-stack and ordinary photons on the D3-stack which gives rise to an interaction controlled by a scale which is naturally trans-Planckian.

This paper is organised as follows. In section 2 we first discuss the phenomenology of the dark matter to ALP to photon model for the 3.5 keV line and its observational constraints, and then we describe how these phenomenological conditions turn into precise requirements on the Calabi-Yau geometry, the brane setup and gauge fluxes, the 4D fields which can successfully play the rôle of either the DM particle or the ultra-light ALP, the form of the various interactions and the resulting low-energy 4D supergravity. Section 3 provides a thorough discuss of moduli stabilisation showing how different sources of corrections to the effective action can fix all closed string moduli and the U(1)-charged open string modes. In section 4 we first derive the expressions for the canonically normalised fields and their masses and then we use these results to work out the strength of the DM-ALP coupling before presenting our conclusions in section 5. Several technical details are relegated to appendix A.

2 Phenomenology and microscopic realisation

In this section we first discuss the observational constraints of the model of [20] for the 3.5 keV line, and we outline the main phenomenological features of our embedding in LVS type IIB flux compactifications. We then provide the technical details of the microscopic realisation of the DM to ALP to photon model for the 3.5 keV line. We start by illustrating the geometry of the underlying Calabi-Yau compactification manifold. We then present the brane setup and gauge fluxes, and we finally describe the main features of the resulting low-energy 4D effective field theory.

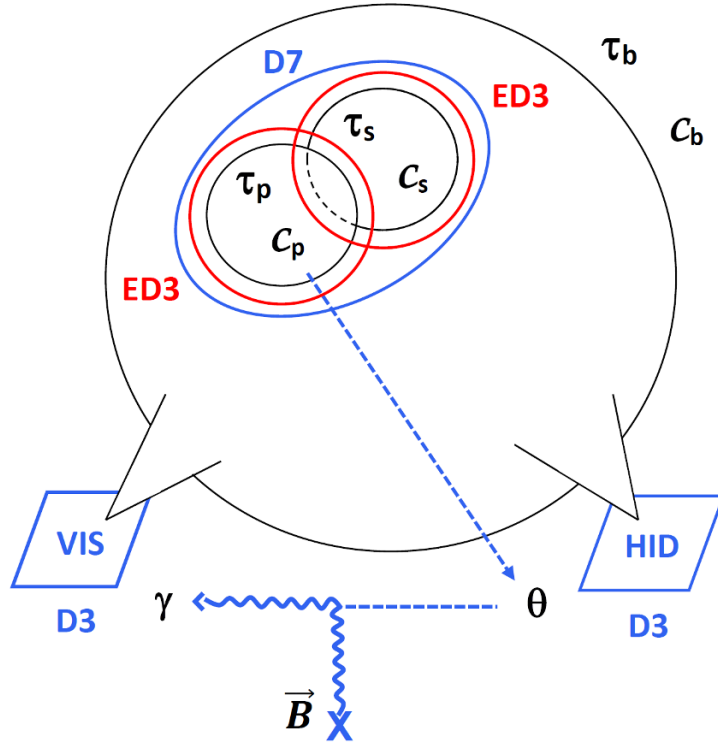


Figure 1. Pictorial view of our setup: a stack of D7-branes wraps the combination $\tau_s + \tau_p$, two ED3-instantons wrap respectively the rigid cycle τ_s and the Wilson divisor τ_p while two stacks of D3-branes at singularities support the visible and a hidden sector. The DM particle is the closed string axion c_p which acquires a 7 keV mass due to tiny poly-instanton effects and decays to the ultra-light open string ALP θ that gives the 3.5 keV line by converting into photons in the magnetic field of galaxy clusters.

2.1 Observational constraints

The effective Lagrangian of the dark matter to ALP to photon model for the 3.5 keV line can be described as follows:

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{a_{\text{ALP}}}{4M}F^{\mu\nu}\tilde{F}_{\mu\nu} + \frac{1}{2}\partial_\mu a_{\text{ALP}}\partial^\mu a_{\text{ALP}} - \frac{1}{2}m_{\text{ALP}}^2 a_{\text{ALP}}^2 \\ & + \frac{a_{\text{DM}}}{\Lambda}\partial_\mu a_{\text{ALP}}\partial^\mu a_{\text{ALP}} + \frac{1}{2}\partial_\mu a_{\text{DM}}\partial^\mu a_{\text{DM}} - \frac{1}{2}m_{\text{DM}}^2 a_{\text{DM}}^2, \end{aligned} \quad (2.1)$$

where a_{ALP} is an ALP with mass m_{ALP} that converts into photons in astrophysical magnetic fields via the coupling suppressed by M . a_{DM} is a pseudoscalar which is the dark matter particle with mass $m_{\text{DM}} \sim 7$ keV. It decays via the kinetic mixing term in (2.1) with characteristic scale Λ . In order for ALP-photon conversion to be efficient in galaxy cluster magnetic field environments, we require $m_{\text{ALP}} \lesssim 10^{-12}$ eV which is the characteristic energy scale of the electron-photon plasma [20]. Otherwise, the ALP to photon conversion is suppressed by $\sim (10^{-12} \text{ eV}/m_{\text{ALP}})^4$. Therefore a_{ALP} is too light to be the standard QCD axion but it has instead to be a stringy axion-like particle.

The observed photon flux at an X-ray detector is given by:

$$F_{DM \rightarrow a_{\text{ALP}} \rightarrow \gamma} \propto \Gamma_{a_{\text{DM}} \rightarrow a_{\text{ALP}} a_{\text{ALP}}} P_{a_{\text{ALP}} \rightarrow \gamma} \rho_{\text{DM}}, \quad (2.2)$$

where ρ_{DM} is the dark matter column density and:

$$\Gamma_{a_{\text{DM}} \rightarrow a_{\text{ALP}} a_{\text{ALP}}} = \frac{1}{32\pi} \frac{m_{\text{DM}}^3}{\Lambda^2}, \quad (2.3)$$

is the dark matter decay rate and $P_{a_{\text{ALP}} \rightarrow \gamma}$ is the ALP to photon conversion probability. It is given as $P_{a_{\text{ALP}} \rightarrow \gamma} \propto M^{-2}$ and furthermore depends on the electron density in the plasma, the energy of the ALP/photon, the coherence length and the strength of the magnetic field. Hence, $F_{DM \rightarrow a_{\text{ALP}} \rightarrow \gamma} \propto \Lambda^{-2} M^{-2}$. For the ALP to photon conversion conditions in the Perseus cluster magnetic field, the observed 3.5 keV flux then implies [20]:

$$\Lambda \cdot M \sim 7 \cdot 10^{28} \text{ GeV}^2. \quad (2.4)$$

The scales M and Λ are subject to certain constraints. There is a lower bound $M \gtrsim 10^{11} \text{ GeV}$ from observations of SN1987A [52–54], the thermal spectrum of galaxy clusters [55] and active galactic nuclei [56–58]. This lower bound implies an upper bound on Λ via (2.4). To get sufficiently stable dark matter, we assume that the dark matter particle has a lifetime larger than the age of the universe, i.e. $\Lambda \gtrsim 5 \cdot 10^{12} \text{ GeV}$. This implies an upper bound on M via (2.4). To summarise, the parameters M and Λ have to satisfy (2.4) together with the following phenomenological constraints:

$$10^{11} \text{ GeV} \lesssim M \lesssim 10^{16} \text{ GeV}, \quad 5 \cdot 10^{12} \text{ GeV} \lesssim \Lambda \lesssim 7 \cdot 10^{17} \text{ GeV}. \quad (2.5)$$

Notice that ultra-light ALPs with intermediate scale couplings to photons will be within the detection reach of helioscope experiments like IAXO [59] and potentially light-shining-through-a-wall experiments like ALPS [60].

2.2 Phenomenological features

The phenomenological requirements for a viable explanation of the 3.5 keV line from dark matter decay to ALPs which then convert into photons, can be translated into precise conditions on the topology and the brane setup of the microscopic realisation. We shall focus on type IIB flux compactifications where moduli stabilisation has already been studied in depth. According to (2.4) and (2.5), we shall focus on the parameter space region where the DM to ALP coupling is around the GUT/Planck scale, $\Lambda \sim 10^{16}$ - 10^{18} GeV , whereas the ALP to photon coupling is intermediate: $M \sim 10^{11}$ - 10^{13} GeV . This region is particularly interesting since an ALP with this decay constant could also explain the diffuse soft X-ray excess from galaxy cluster via axion-photon conversion in the cluster magnetic field [61]. This phenomenological requirement, together with the observation that $m_{\text{DM}} \sim 10 \text{ keV}$ while $m_{\text{ALP}} \lesssim 10^{-12} \text{ eV}$, sets the following model building constraints:

- **ALP as an open string axion at a singularity:** from the microscopic point of view, a_{ALP} can be either a closed or an open string axion. In the case of closed string

axions, a_{ALP} could be given by the reduction of C_4 on orientifold-even four-cycles or by the reduction of C_2 on two-cycles duals to orientifold-odd four-cycles. As explained in [24, 27] and reviewed in appendix A.1, since axions are the imaginary parts of moduli, $T_i = \tau_i + i c_i$ (c_i is a canonically unnormalised axion), whose interaction with matter is gravitational, they tend to be coupled to photons with Planckian strength. However this is true only for bulk axions which have $M \simeq M_p$, while the coupling to photons of local axions, associated to blow-up modes of point-like singularities, is controlled by the string scale: $M \simeq M_s$. $M_s \sim M_p/\sqrt{\mathcal{V}}$ can be significantly lower than M_p if the volume of the extra dimensions in string units \mathcal{V} is very large, and so local closed string axions could realise $M \sim M_s \sim 10^{11}\text{--}10^{13}\text{ GeV}$.

A moduli stabilisation scheme which leads to an exponentially large \mathcal{V} is the LARGE Volume Scenario [32–34] whose simplest realisation requires a Calabi-Yau volume of the form:

$$\mathcal{V} = \tau_b^{3/2} - \tau_s^{3/2}. \quad (2.6)$$

The moduli are fixed by the interplay of the leading order α' correction to the Kähler potential and non-perturbative effects supported on the rigid cycle τ_s . The decay constant of the axionic partner of τ_s , which we denote as c_s , is set by the string scale, $M \sim M_s$, but this mode develops a mass of order the gravitino mass $m_{c_s} \sim m_{3/2} \sim M_p/\mathcal{V}$. The large divisor τ_b is lighter than the gravitino due to the underlying no-scale structure of the 4D effective field theory, $m_{\tau_b} \sim m_{3/2}/\sqrt{\mathcal{V}}$, but it has to be heavier than about 50 TeV in order to avoid any cosmological moduli problem. Hence the local axion c_s is much heavier than 10^{-12} eV , and so cannot play the rôle of a_{ALP} . Moreover, the bulk axion c_b cannot be the desired ALP as well since, even if it is almost massless, its coupling scale to photons would be too high: $M \sim M_p$.

We are therefore forced to consider an open string axion realisation for a_{ALP} . Anomalous U(1) factors appear ubiquitously in both D7-branes wrapped around four-cycles in the geometric regime and in D3-branes at singularities. In the process of anomaly cancellation, the U(1) gauge boson becomes massive by eating up an axion [62]. As explained in [28], the combination of axions which gets eaten up is mostly given by an open string axion for D7-branes and by a closed string axion for D3-branes. The resulting low-energy theory below the gauge boson mass, features a global U(1) which is an ideal candidate for a Peccei-Quinn like symmetry. In the case of D3-branes at singularities, the resulting D-term potential looks schematically like:

$$V_D = g^2 \left(q|\hat{C}|^2 - \xi \right)^2, \quad (2.7)$$

where we focused just on one canonically normalised charged matter field $\hat{C} = |\hat{C}| e^{i\theta}$ whose phase θ can play the rôle of an axion with decay constant set by the VEV of the radial part $|\hat{C}|$. The FI term $\xi \sim \tau_q/\mathcal{V}$ is controlled by the four-cycle τ_q which gets charged under the anomalous U(1) and whose volume resolves the singularity. A leading order supersymmetric solution fixes $|\hat{C}|^2 = \xi/q$, leaving a flat direction in the $(|\hat{C}|, \tau_q)$ -plane. This remaining flat direction is fixed by subdominant supersymmetry

breaking contributions from background fluxes which take the form [39]:

$$V_F(|\hat{C}|) = c_2 m_0^2 |\hat{C}|^2 + c_3 A |\hat{C}|^3 + O(|\hat{C}|^4), \quad (2.8)$$

where c_2 and c_3 are $\mathcal{O}(1)$ coefficients. If we parametrise the volume dependence of the soft scalar masses as $m_0 \sim M_p/\mathcal{V}^{\alpha_2}$ and the trilinear A-term as $A \sim M_p/\mathcal{V}^{\alpha_3}$, and we use the vanishing D-term condition to write τ_q in terms of $|\hat{C}|$ as $\tau_q \sim |\hat{C}|^2 \mathcal{V}$, the matter field VEV scales as:

$$\begin{aligned} (i) \text{ If } c_2 > 0 \quad & |\hat{C}| = 0 \quad \Leftrightarrow \quad \tau_q = 0, \\ (ii) \text{ If } c_2 < 0 \quad & |\hat{C}| \simeq \frac{M_p}{\mathcal{V}^{2\alpha_2 - \alpha_3}} \quad \Leftrightarrow \quad \tau_q \simeq \frac{1}{\mathcal{V}^{4\alpha_2 - 2\alpha_3 - 1}}. \end{aligned}$$

Only in case (ii) the matter field $|\hat{C}|$ becomes tachyonic and breaks the Peccei-Quinn symmetry, leading to a viable axion realisation. In the presence of flavour D7-branes intersecting the D3-brane stack at the singularity, the soft terms are unsequestered and $\alpha_2 = \alpha_3 = 1$ [63], giving $|\hat{C}| \sim M_p/\mathcal{V} \sim m_{3/2}$ and $\tau_q \simeq \mathcal{V}^{-1} \ll 1$ which ensures that τ_q is still in the singular regime. If the internal volume is of order $\mathcal{V} \sim 10^8$, the large modulus τ_b is heavy enough to avoid the cosmological moduli problem: $m_{\tau_b} \sim 100 \text{ TeV}$. In turn the gravitino mass, all soft terms and the axion decay constant $f_{a_{\text{ALP}}} = |\hat{C}|$ are around 10^9 GeV . Setting $\theta = a_{\text{ALP}}/f_{a_{\text{ALP}}}$, the axion to photon coupling then takes the form:

$$\frac{g^2}{32\pi^2} \frac{a_{\text{ALP}}}{f_{a_{\text{ALP}}}} F_{\mu\nu} \tilde{F}^{\mu\nu} \quad \Leftrightarrow \quad M = \frac{32\pi^2 f_{a_{\text{ALP}}}}{g^2} = \frac{32\pi^2 f_{a_{\text{ALP}}}}{g_s} \sim 10^{12} \text{ GeV}, \quad (2.9)$$

since for D3-branes the coupling $g^{-2} = \text{Re}(S) = g_s^{-1}$ is set by the dilaton S which controls also the size of the string coupling that we assume to be in the perturbative regime: $g_s \simeq 0.1$.

On the other hand, in the absence of flavour D7-branes the soft terms are sequestered with $\alpha_3 = 2$ and $\alpha_2 = 3/2$ or $\alpha_2 = 2$ depending on the form of the quantum corrections to the Kähler metric for matter fields and the effects responsible for achieving a dS vacuum [64, 65]. Notice that possible non-perturbative desequestering effects from couplings in the superpotential of the form $W_{\text{np}} \supset \mathcal{O}_{\text{matter}} e^{-a_s T_s}$ with $\mathcal{O}_{\text{matter}}$ a gauge-invariant operator composed of matter fields, cannot actually change the volume dependence of either the soft scalar masses or the A-terms [66]. Thus if $\alpha_2 = 3/2$ we have $f_{a_{\text{ALP}}} = |\hat{C}| \simeq M_p/\mathcal{V}$ and $\tau_q \sim \mathcal{V}^{-1} \ll 1$, while if $\alpha_2 = 2$ the open axion decay constant scales as $f_{a_{\text{ALP}}} = |\hat{C}| \simeq M_p/\mathcal{V}^2$ and $\tau_q \sim \mathcal{V}^{-3} \ll 1$. In both cases without flavour D7-branes the gaugino masses scale as $M_{1/2} \sim 0.1 M_p/\mathcal{V}^2$ and lie around the TeV scale for $\mathcal{V} \sim 10^7$. Considering this value of the volume, the axion-photon coupling therefore becomes:

$$(a) \text{ If } \alpha_2 = \frac{3}{2} \quad M = \frac{32\pi^2 f_{a_{\text{ALP}}}}{g_s} \sim 10^3 m_{3/2} \sim 10^{13} \text{ GeV}, \quad (2.10)$$

$$(b) \text{ If } \alpha_2 = 2 \quad M = \frac{32\pi^2 f_{a_{\text{ALP}}}}{g_s} \sim 10^3 \frac{m_{3/2}}{\mathcal{V}} \sim 10^6 \text{ GeV}. \quad (2.11)$$

- **ALP-photon coupling induced by U(1) kinetic mixing:** we have shown above that, if the matter field $|\hat{C}|$ charged under the anomalous U(1) develops a non-zero VEV due to a tachyonic soft scalar mass contribution, the open string axion θ can have an intermediate scale coupling to photons. However θ in general plays the rôle of the standard QCD axion which becomes much heavier than $m_{\text{ALP}} \lesssim 10^{-12}$ eV due to QCD instanton effects. Hence the simplest realisation of an ultralight ALP with the desired phenomenological features to reproduce the 3.5 keV line requires the existence of at least two open string axions. The Calabi-Yau volume (2.6) has then to be generalised to:

$$\mathcal{V} = \tau_b^{3/2} - \tau_s^{3/2} - \tau_{q_1}^{3/2} - \tau_{q_2}^{3/2}, \quad (2.12)$$

where τ_{q_1} and τ_{q_2} are both collapsed to a singularity via D-term fixing and support a stack of D3-branes. There are two possibilities to realise a viable a_{ALP} :

1. The two blow-up modes τ_{q_1} and τ_{q_2} are exchanged by the orientifold involution [37–39]. The resulting quiver gauge theory on the visible sector stack of D3-branes generically features two anomalous U(1) symmetries. This is for example always the case for del Pezzo singularities. Hence the visible sector is characterised by the presence of two open string axions: one behaves as the QCD axion while the other can be an almost massless a_{ALP} with $M \sim 10^{11}$ – 10^{12} GeV as in (2.9) or (2.10). In this case the matter field $|\hat{C}|$ which develops a VEV of order the gravitino mass has to be a Standard Model gauge singlet in order not to break any visible sector gauge symmetry at a high scale.
2. The two blow-up divisors τ_{q_1} and τ_{q_2} are invariant under the orientifold involution [40, 41]. Therefore one D3-stack has to reproduce the visible sector while the other represents a hidden sector. Each of the two sectors features an anomalous U(1) which gives rise to an open string axion with a coupling to the respective photons controlled by the scale M . The visible sector axion plays the rôle of the QCD axion while the hidden sector open string axion can behave as a_{ALP} . Its coupling to ordinary photons can be induced by a U(1) kinetic mixing of the form [51, 67, 68]:

$$\mathcal{L} \supset -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \frac{\chi}{2}F_{\mu\nu}G^{\mu\nu} - \frac{a_{\text{QCD}}}{4M_{\text{vis}}}F_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{a_{\text{ALP}}}{4M_{\text{hid}}}G_{\mu\nu}\tilde{G}^{\mu\nu}, \quad (2.13)$$

where we denoted the QCD axion as a_{QCD} , the kinetic mixing parameter as χ and the visible sector Maxwell tensor as $F_{\mu\nu}$ while the hidden one as $G_{\mu\nu}$. The kinetic mixing parameter is induced at one-loop level and scales as:

$$\chi \sim \frac{g_{\text{vis}}g_{\text{hid}}}{16\pi^2} = \frac{g_s}{16\pi^2} \simeq 10^{-3}. \quad (2.14)$$

After diagonalising the gauge kinetic terms in (2.13) via $G_{\mu\nu} = G'_{\mu\nu} + \chi F_{\mu\nu}$, a_{ALP} acquires a coupling to ordinary photons of the form:

$$\mathcal{L} \supset -\frac{\chi^2 a_{\text{ALP}}}{4M_{\text{hid}}}F_{\mu\nu}\tilde{F}^{\mu\nu} \quad \Leftrightarrow \quad M \simeq \frac{M_{\text{hid}}}{\chi^2} \gg M_{\text{hid}}. \quad (2.15)$$

Given that $M \gg M_{\text{hid}}$, a_{ALP} can be a hidden sector open string axion only in case (2.11) where the scale of the coupling to hidden photons of order $M_{\text{hid}} \sim 10^6 \text{ GeV}$ is enhanced via $U(1)$ kinetic mixing to $M \sim 10^{12} \text{ GeV}$ for the coupling to ordinary photons.

- **DM as a local closed string axion fixed by poly-instanton effects:** in order to produce a monochromatic 3.5 keV line, the DM mass has to be around $m_{\text{DM}} \sim 7 \text{ keV}$. Such a light DM particle can be a sterile neutrino realised as an open string mode belonging to either the visible or the hidden sector. However we shall focus on a more model-independent realisation of the decaying DM particle as a closed string axion. A generic feature of any 4D string model where the moduli are stabilised by perturbative effects, is the presence of very light axions whose mass is exponentially suppressed with respect to the gravitino mass [28]. Thus closed string axions are perfect candidates for ultra-light DM particles. In LVS models, there are two kinds of axions which remain light:

1. Bulk closed string axion c_b since the corresponding supersymmetric partner τ_b is fixed by α' corrections to the Kähler potential K . This axionic mode develops a tiny mass only via T_b -dependent non-perturbative contributions to the superpotential W : $m_{c_b} \sim m_{\tau_b} e^{-\pi\tau_b} \ll m_{\tau_b} \sim m_{3/2}/\sqrt{V}$.
2. Local closed string axion c_p whose associated modulus τ_p is stabilised by g_s loop corrections to K . This can happen for so-called ‘Wilson divisors’ D_p which are rigid, i.e. $h^{2,0}(D_p) = 0$, with a Wilson line, i.e. $h^{1,0}(D_p) = 1$ [48]. Under these topological conditions, an ED3-instanton wrapping such a divisor does not lead to a standard non-perturbative contribution to W but it generates a non-perturbative correction to another ED3-instanton wrapping a different rigid divisor τ_s . This gives rise to poly-instanton corrections to W of the form [47]:

$$W_{\text{np}} = A_s e^{-2\pi(T_s + A_p e^{-2\pi T_p})} \simeq A_s e^{-2\pi T_s} - 2\pi A_s A_p e^{-2\pi T_s} e^{-2\pi T_p}. \quad (2.16)$$

In LVS models, the blow-up mode τ_s is fixed by the dominant non-perturbative correction in (2.16) since the leading loop contribution to the scalar potential is vanishing due to the ‘extended no-scale’ structure [44]. Thus the corresponding axion c_s becomes too heavy to play the rôle of a_{DM} since it acquires a mass of the same order of magnitude: $m_{\tau_s} \sim m_{c_s} \sim m_{3/2}$. On the other hand, the T_p -dependent non-perturbative correction in (2.16) has a double exponential suppression, and so τ_p gets frozen by perturbative g_s effects [42, 43]. Given that c_p enjoys a shift symmetry which is broken only at non-perturbative level, this axion receives a potential only due to tiny poly-instanton contributions to W which make it much lighter than τ_p . Hence c_p is a natural candidate for a_{DM} since $m_{c_p} \sim m_{\tau_p} e^{-\pi\tau_p/2} \ll m_{\tau_p} \sim m_{3/2}$. Notice that the presence of a ‘Wilson divisors’ τ_p would modify the volume form (2.12) to [48]:

$$\mathcal{V} = \tau_b^{3/2} - \tau_s^{3/2} - (\tau_s + \tau_p)^{3/2} - \tau_{q_1}^{3/2} - \tau_{q_2}^{3/2}. \quad (2.17)$$

- **DM to ALP decay induced by non-perturbative effects in K :** a DM to ALP coupling controlled by the scale Λ of the form shown in (2.1) can arise from the kinetic mixing between a closed string DM axion and an open string ALP. Given that the kinetic terms are determined by the Kähler potential, a kinetic mixing effect can be induced by non-perturbative corrections to the Kähler metric for matter fields which we assume to take the form:⁴

$$K_{\text{np}} \supset B_i e^{-b_i \tau_i} \cos(b_i c_i) C \bar{C}, \quad (2.18)$$

where $i = b$ if a_{DM} is a bulk closed string axion or $i = p$ if a_{DM} is a local closed string axion fixed by poly-instanton effects. As we shall show in section 4.3 after performing a proper canonical normalisation of both axion fields, the resulting scale which controls the DM-ALP coupling is given by:

$$\Lambda \sim \begin{cases} \frac{e^{b_b \tau_b}}{B_b \mathcal{V}^{4/3}} M_p \sim \frac{e^{b_b \mathcal{V}^{2/3}}}{B_b \mathcal{V}^{4/3}} M_p \gg M_p & \text{for } a_{\text{DM}} = c_b \\ \frac{e^{b_p \tau_p}}{B_p \mathcal{V}^{7/6}} M_p \sim \frac{M_p}{B_p \mathcal{V}^{7/6 - \kappa/N}} & \text{for } a_{\text{DM}} = c_p \end{cases} \quad (2.19)$$

where $b_p = 2\pi/N$, $\kappa = \tau_p/\tau_s$ and we have approximated $\mathcal{V} \sim \tau_b^{3/2} \sim e^{2\pi\tau_s}$. From (2.19) it is clear that Λ can be around the GUT/Planck scale only if the DM particle is a local closed string axion stabilised by tiny poly-instanton corrections to W which can give it a small mass of order $m_{\text{DM}} \sim 7 \text{ keV}$.

2.3 Calabi-Yau threefold

As explained in section 2.2, the minimal setup which can yield a viable microscopic realisation of the $a_{\text{DM}} \rightarrow a_{\text{ALP}} \rightarrow \gamma$ model for the 3.5 keV line of [20], is characterised by a Calabi-Yau with $h^{1,1} = 5$ Kähler moduli and a volume of the form (2.17). A concrete Calabi-Yau threefold built via toric geometry which reproduces the volume form (2.17) for $h^{1,1} = 4$ (setting either $\tau_{q_1} = 0$ or $\tau_{q_2} = 0$) is given by example C of [48]. We therefore assume the existence of a Calabi-Yau threefold X with one large divisor controlling the overall volume D_b , three del Pezzo surfaces, D_s , D_{q_1} and D_{q_2} and a ‘Wilson divisor’ D_p .

We expand the Kähler form J in a basis of Poincaré dual two-forms as $J = t_b \hat{D}_b - t_s \hat{D}_s - t_{q_1} \hat{D}_{q_1} - t_{q_2} \hat{D}_{q_2} - t_p \hat{D}_p$, where the t_i ’s are two-cycle volumes and we took a minus sign for the rigid divisors so that the corresponding t_i ’s are positive. The Calabi-Yau volume then looks like:

$$\mathcal{V} = \frac{1}{6} \int_X J \wedge J \wedge J = \frac{1}{6} \left[k_{bbb} t_b^3 - k_{sss} (t_s + \lambda t_p)^3 - \mu t_p^3 - k_{q_1 q_1 q_1} t_{q_1}^3 - k_{q_2 q_2 q_2} t_{q_2}^3 \right], \quad (2.20)$$

⁴Similar non-perturbative corrections to K induced by ED1-instantons wrapped around two-cycles have been computed for type I vacua in [69] and for type IIB vacua in [70], while similar non-perturbative effects in K from an ED3-instanton wrapped around the K3 divisor in type I’ string theory, i.e. type IIB compactified on $K3 \times T^2/\mathbb{Z}_2$, have been derived in [71].

where the coefficients λ and μ are determined by the triple intersection numbers $k_{ijk} = \int_X \hat{D}_i \wedge \hat{D}_j \wedge \hat{D}_k$ as:

$$\lambda = \frac{k_{ssp}}{k_{sss}} = \frac{k_{spp}}{k_{ssp}} \quad \text{and} \quad \mu = k_{ppp} - \frac{k_{ssp}^3}{k_{sss}^2}.$$

The volume of the curve resulting from the intersection of the del Pezzo divisor D_s with the Wilson surface D_p is given by:

$$\text{Vol}(D_s \cap D_p) = \int_X J \wedge \hat{D}_s \wedge \hat{D}_p = -(k_{ssp}t_s + k_{spp}t_p) = -k_{ssp}(t_s + \lambda t_p). \quad (2.21)$$

The volume of this curve is positive and the signature of the matrix $\frac{\partial^2 \mathcal{V}}{\partial t_i \partial t_j}$ is guaranteed to be $(1, h^{1,1} - 1)$ (so with 1 positive and 4 negative eigenvalues) [72] if $k_{ssp} < 0$ while all the other intersection numbers are positive and $t_s + \lambda t_p > 0$.⁵ The four-cycle moduli can be computed as:

$$\tau_i = \frac{1}{2} \int_X J \wedge J \wedge \hat{D}_i, \quad (2.22)$$

and so they become:

$$\begin{aligned} \tau_b &= \frac{1}{2} k_{bbb} t_b^2, & \tau_{q_1} &= \frac{1}{2} k_{q_1 q_1 q_1} t_{q_1}^2, & \tau_{q_2} &= \frac{1}{2} k_{q_2 q_2 q_2} t_{q_2}^2, \\ \tau_s &= \frac{1}{2} (k_{sss} t_s^2 + k_{spp} t_p^2 + 2k_{ssp} t_s t_p) = \frac{1}{2} k_{sss} (t_s + \lambda t_p)^2, \\ \tau_p &= \frac{1}{2} (k_{ppp} t_p^2 + k_{ssp} t_s^2 + 2k_{spp} t_s t_p) = \frac{1}{2} k_{ssp} (t_s + \lambda t_p)^2 + \frac{1}{2} \mu t_p^2. \end{aligned} \quad (2.23)$$

The overall volume (2.20) can therefore be rewritten in terms of the four-cycle moduli as:

$$\mathcal{V} = \lambda_b \tau_b^{3/2} - \lambda_s \tau_s^{3/2} - \lambda_p (\tau_p + x \tau_s)^{3/2} - \lambda_{q_1} \tau_{q_1}^{3/2} - \lambda_{q_2} \tau_{q_2}^{3/2}, \quad (2.24)$$

where:

$$\lambda_i = \frac{1}{3} \sqrt{\frac{2}{k_{iii}}}, \quad \forall i = b, s, q_1, q_2, \quad \lambda_p = \frac{1}{3} \sqrt{\frac{2}{\mu}} \quad \text{and} \quad x = -\frac{k_{ssp}}{k_{sss}} > 0.$$

Notice that (2.24) reproduces exactly the volume form (2.17).

2.4 Brane set-up and fluxes

As explained in section 2.2, a_{ALP} can be realised as an open string axion belonging either to the visible sector or to a hidden sector. In the first case the two rigid divisors D_{q_1} and D_{q_2} are exchanged by a proper orientifold involution whereas in the second case they are invariant. As we shall see more in detail in section 3.1, these two blow-up modes shrink down to zero size due to D-term stabilisation and support a stack of D3-branes at the resulting singularity.

Full moduli stabilisation requires the presence of non-perturbative corrections to the superpotential. We shall therefore consider an ED3-instanton wrapped around the ‘small’

⁵This analysis includes example C of [48] where $k_{sss} = k_{spp} = -k_{ssp} = 9$ and $k_{ppp} = 0$.

rigid divisor D_s which generates a standard non-perturbative contribution to W , together with another ED3-instanton wrapped around the Wilson surface D_p which gives rise to poly-instanton effects. In order to make τ_p heavier than the DM axion c_p , we need also to include a D7-stack that generates τ_p -dependent string loop corrections to the Kähler potential. This can be achieved if a stack of D7-branes wraps the divisor D_{D7} (which we assume to be smooth and connected) given by:

$$D_{D7} = m_s D_s + m_p D_p, \quad \text{with } m_s, m_p \in \mathbb{Z}. \quad (2.25)$$

In what follows we shall assume the existence of a suitable orientifold involution and O7-planes which allow the presence of such a D7-stack in a way compatible with D7-tadpole cancellation. The cancellation of Freed-Witten anomalies requires to turn on half-integer world-volume fluxes on the instantons and the D7-stack of the form [31]:

$$F_{D7} = f_s \hat{D}_s + f_p \hat{D}_p + \frac{1}{2} \hat{D}_{D7}, \quad F_s = \frac{1}{2} \hat{D}_s, \quad F_p = \frac{1}{2} \hat{D}_p, \quad (2.26)$$

with $f_s, f_p \in \mathbb{Z}$. In order to guarantee a non-vanishing contribution to W , the total flux $\mathcal{F}_j = F_j - \iota_j^* B$ (with $\iota_j^* B$ the pull-back of the NSNS B -field on D_j) on both instantons has to be zero: $\mathcal{F}_s = \mathcal{F}_p = 0$. This can be achieved if the B -field is chosen such that:

$$B = \frac{1}{2} \hat{D}_s + \frac{1}{2} \hat{D}_p, \quad (2.27)$$

and the pull-back of $\hat{D}_s/2$ on D_p and of $\hat{D}_p/2$ on D_s are both integer forms since in this case we can always turn on integer flux quanta to cancel their contribution to the total gauge flux. This is indeed the case if, for an arbitrary integer form $\omega = \omega_i \hat{D}_i \in H^2(\mathbb{Z}, X)$ with $\omega_i \in \mathbb{Z}$, we have that:

$$\frac{1}{2} \int_X \hat{D}_s \wedge \hat{D}_p \wedge \omega = \frac{1}{2} (k_{ssp} \omega_s + k_{spp} \omega_p) \in \mathbb{Z}. \quad (2.28)$$

This condition can be easily satisfied if both k_{ssp} and k_{spp} are even.

The total gauge flux on the D7-stack instead becomes:

$$\mathcal{F}_{D7} = f_s \hat{D}_s + f_p \hat{D}_p + \frac{1}{2} (m_s - 1) \hat{D}_s + \frac{1}{2} (m_p - 1) \hat{D}_p = f_s \hat{D}_s + f_p \hat{D}_p,$$

where without loss of generality, we have chosen $m_s = m_p = 1$ so that \mathcal{F}_{D7} is an integer flux. The presence of this flux has several implications:

- The blow-up moduli T_s and T_p get charged under the diagonal $U(1)$ of the D7-stack with charges:

$$q_i = \int_X \mathcal{F}_{D7} \wedge \hat{D}_{D7} \wedge \hat{D}_i = f_s (k_{ssi} + k_{spi}) + f_p (k_{spi} + k_{ppi}), \quad i = s, p, \quad (2.29)$$

which implies $q_p = \mu f_p - x q_s$.

- The coupling constant of the gauge theory living on D_{D7} acquires a flux-dependent shift of the form:

$$g_{D7}^{-2} = \tau_s + \tau_p - h(\mathcal{F}_{D7}) \operatorname{Re}(S), \quad (2.30)$$

where $\operatorname{Re}(S) = e^{-\varphi} = g_s^{-1}$ is the real part of the axio-dilaton while the flux-dependent shift reads:

$$h(\mathcal{F}_{D7}) = \frac{1}{2} \int_X \mathcal{F}_{D7} \wedge \mathcal{F}_{D7} \wedge \hat{D}_{D7} = \frac{f_s}{2} q_s + \frac{f_p}{2} q_p. \quad (2.31)$$

- \mathcal{F}_{D7} generates a moduli-dependent FI-term which looks like:

$$\xi_{D7} = \frac{1}{4\pi \mathcal{V}} \int_X J \wedge \mathcal{F}_{D7} \wedge \hat{D}_{D7} = \frac{1}{4\pi \mathcal{V}} (q_s t_s + q_p t_p). \quad (2.32)$$

- A non-vanishing gauge flux on D_{D7} might induce chiral intersections between the D7 stack and the instantons on D_s and D_p . Their net number is counted by the moduli U(1)-charges as:

$$I_{D7-E3} = \int_X \mathcal{F}_{D7} \wedge \hat{D}_{D7} \wedge \hat{D}_s = q_s, \quad I_{D7\text{-poly}} = \int_X \mathcal{F}_{D7} \wedge \hat{D}_{D7} \wedge \hat{D}_p = q_p. \quad (2.33)$$

The relations (2.33) imply that, whenever an instanton has a non-vanishing chiral intersection with a stack of D-branes, the four-cycle modulus T_{inst} wrapped by the instanton gets charged under the diagonal U(1) on the D-brane stack. Therefore a non-perturbative contribution to the superpotential of the form $W_{\text{np}} \supset e^{-T_{\text{inst}}}$ would not be gauge invariant. Thus a proper combination of U(1)-charged matter fields ϕ_i has to appear in the prefactor in order to make the whole contribution gauge invariant: $W_{\text{np}} \supset \prod_i \phi_i e^{-T_{\text{inst}}}$. If however the ϕ_i are visible sector matter fields, they have to develop a vanishing VEV in order not to break any Standard Model gauge group at high energies [30]. In our case the absence of chiral intersections between the instantons on D_s and D_p and the visible sector is guaranteed by the structure of the intersection numbers since $k_{sqij} = 0$ and $k_{pqij} = 0 \forall j$ for either $i = 1$ or $i = 2$.

On the other hand, as can be seen from (2.33), there are chiral intersections between the hidden D7-stack on D_{D7} and the two instantons on D_s and D_p . We could kill both of these intersections by setting $\mathcal{F}_{D7} = 0$. However this choice of the gauge flux on D_{D7} would also set to zero the FI-term in (2.32) which is instead crucial to make τ_p heavier than the DM axion c_p . We shall therefore perform a choice of the gauge flux \mathcal{F}_{D7} which sets $I_{D7-E3} = q_s = 0$ but leaves $I_{D7\text{-poly}} = q_p \neq 0$ so that ξ_{D7} can develop a non-trivial dependence on τ_p . This can take place if the flux quanta f_p and f_s are chosen such that:

$$f_p = -\frac{k_{sss} + k_{ssp}}{k_{ssp} + k_{spp}} f_s \quad \Leftrightarrow \quad q_s = 0 \quad \text{and} \quad q_p = \mu f_p. \quad (2.34)$$

The FI-term in (2.32) then becomes:

$$\xi_{D7} = \frac{q_p}{4\pi} \frac{t_p}{\mathcal{V}} = \frac{f_p \sqrt{2} \mu}{4\pi} \frac{\sqrt{\tau_p + x\tau_s}}{\mathcal{V}}, \quad (2.35)$$

while the shift of the gauge coupling in (2.31) simplifies to $h(\mathcal{F}_{D7}) = \frac{\mu}{2} f_p^2$. Due to non-zero chiral intersections between the D7-stack and the divisor D_p , the poly-instanton contribution to the superpotential comes with a prefactor that depends on a U(1)-charged matter

field ϕ . In section 3.1 we will show that the interplay between D-terms and string loop effects can fix ϕ at a non-zero VEV, so that the poly-instanton correction is non-vanishing. Notice that ϕ belongs to a hidden sector, and so it can safely develop a non-zero VEV at high energies without violating any phenomenological requirement.

2.5 Low-energy 4D theory

Type IIB string theory compactified on an orientifold of the Calabi-Yau threefold described in section 2.3 with the brane setup and gauge fluxes of section 2.4 gives rise to an $N = 1$ 4D supergravity effective field theory characterised by a Kähler potential K and a superpotential W of the form:

$$K = K_{\text{mod}} + K_{\text{matter}} \quad \text{and} \quad W = W_{\text{tree}} + W_{\text{np}}, \quad (2.36)$$

where:

- The moduli Kähler potential receives perturbative α' and g_s corrections beyond the tree-level approximation:

$$K_{\text{mod}} = K_{\text{tree}} + K_{\alpha'} + K_{g_s}, \quad (2.37)$$

with:

$$K_{\text{tree}} = -2 \ln \mathcal{V} + \frac{\tau_{q_1}^2}{\mathcal{V}} + \frac{\tau_{q_2}^2}{\mathcal{V}}, \quad K_{\alpha'} = -\frac{\zeta}{g_s^{3/2} \mathcal{V}}, \quad K_{g_s} = g_s \sum_i \frac{C_i^{\text{KK}} t_i^\perp}{\mathcal{V}}. \quad (2.38)$$

In (2.38) we neglected the tree-level Kähler potential for the dilaton $S = e^{-\varphi - i C_0}$ and the complex structure moduli U_{a-} , $a_- = 1, \dots, h_-^{1,2}$ and we expanded the effective theory around the singularities obtained by collapsing the two blow-up modes τ_{q_1} and τ_{q_2} (hence the volume \mathcal{V} in (2.38) should be thought of as (2.24) with $\tau_{q_1} = \tau_{q_2} = 0$). Moreover, we included only the leading order α' correction which depends on $\zeta = -\frac{\zeta(3)\chi(X)}{2(2\pi)^3}$ [49] since in the large volume limit higher derivative α' effects yield just subdominant contributions [73]. Finally in K_{g_s} we considered only string loop corrections arising from the exchange of Kaluza-Klein modes between non-intersecting stacks of D-branes and O-planes (C_i^{KK} are complex structure dependent coefficients and t_i^\perp is the two-cycle controlling the distance between two parallel stacks of D-branes/O-planes) while we did not introduce any g_s effects coming from the exchange of winding modes since these arise only in the presence of intersections between D-branes which are however absent in our setup [42–44].

- In the matter Kähler potential we focus just on the dependence on the matter fields which will develop a non-zero VEV. These are two U(1)-charged matter fields: $\phi = |\phi| e^{i\psi}$ which belongs to the hidden D7-stack on D_{D7} and $C = |C| e^{i\theta}$ which can be either a visible sector gauge singlet (if D_{q_1} and D_{q_2} are exchanged by the orientifold involution) or a hidden sector field (if both D_{q_1} and D_{q_2} are invariant under the orientifold involution) living on a D3-brane stack [74, 75]:

$$K_{\text{matter}} = \frac{\phi \bar{\phi}}{\text{Re}(S)} + \tilde{K}(T_i, \bar{T}_i) C \bar{C}. \quad (2.39)$$

In (2.39) we wrote down just the tree-level Kähler metric for ϕ while we shall consider both perturbative and non-perturbative corrections to the Kähler metric for C which we assume to take the form:

$$\tilde{K}(T_i, \bar{T}_i) = \frac{f(S, U)}{\mathcal{V}^{2/3}} + \tilde{K}_{\text{pert}} + B_i e^{-b_i \tau_i} \cos(b_i c_i) \quad \text{with } i = b, p, \quad (2.40)$$

where $f(S, U)$ is an undetermined function of the dilaton and complex structure moduli, \tilde{K}_{pert} represents perturbative corrections which do not depend on the axionic fields because of their shift symmetry and the last term is a non-perturbative correction which can in principle depend on either the large or the poly-instanton cycle. This term induces a kinetic mixing between the open string axion θ and either of the two ultra-light closed string axions c_b and c_p . As we shall see in section 3.1, the open string axion ψ gets eaten up by the anomalous U(1) on the D7-stack, and so light closed string axions cannot decay to this heavy mode. This is the reason why we did not include any non-perturbative effect in the Kähler metric for ϕ .

- The tree-level superpotential $W_{\text{tree}} = \int_X G_3 \wedge \Omega$, with Ω the Calabi-Yau (3, 0)-form, is generated by turning on background three-form fluxes $G_3 = F_3 - i S H_3$ and depends just on the dilaton and the U -moduli but not on the T -moduli [76].
- The non-perturbative superpotential receives a single contribution from the ED3-instanton wrapped around D_s together with poly-instanton effects from the ED3-instanton wrapped around the Wilson surface D_p and takes the same form as (2.16):

$$W_{\text{np}} = A_s e^{-2\pi T_s} - 2\pi A_s A_p e^{-2\pi T_s} e^{-2\pi T_p}. \quad (2.41)$$

The prefactors A_s and A_p depend on S and U -moduli. Given that T_p is charged under the anomalous diagonal U(1) on the D7-stack, A_p has to depend also on the charged matter field ϕ in order to make W_{np} gauge invariant. If we make the dependence of A_p on ϕ explicit by replacing $A_p \rightarrow A_p \phi^n$ with arbitrary n , and we use the fact that ϕ and T_p behave under a U(1) transformation as:

$$\delta\phi = i q_\phi \phi \quad \text{and} \quad \delta T_p = i \frac{q_p}{2\pi}, \quad (2.42)$$

the variation of W_{np} under a U(1) transformation becomes:

$$\delta W_{\text{np}} = W_{\text{np}} \left(n \frac{\delta\phi}{\phi} - 2\pi \delta T_p \right) = i W_{\text{np}} (n q_\phi - q_p). \quad (2.43)$$

Hence W is gauge invariant only if $n = q_p/q_\phi$. Notice that $n > 0$ since, as we shall see in section 3.1, a consistent D-term stabilisation can yield a non-zero VEV for ϕ only if q_ϕ and q_p have the same sign.

3 Moduli stabilisation

In this section we shall show how to stabilise all closed string moduli together with the two charged open string modes ϕ and C . The total $N = 1$ supergravity scalar potential

descending from the K and W described in section 2.5, includes both F- and D-term contributions of the form:

$$V = V_F + V_D = e^K \left(K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3|W|^2 \right) + \frac{g_{D7}^2}{2} D_{D7}^2 + \frac{g_{D3}^2}{2} D_{D3}^2, \quad (3.1)$$

where the Kähler covariant derivative is $D_I W = \partial_I W + W \partial_I K$, the gauge coupling of the field theory living on the D7-stack is given by (2.30) while $g_{D3}^2 = \text{Re}(S)$ for the quiver gauge theory on the D3-stack. The two D-term contributions look like:

$$D_{D7} = q_\phi \phi \frac{\partial K}{\partial \phi} - \xi_{D7}, \quad \text{and} \quad D_{D3} = q_C C \frac{\partial K}{\partial C} - \xi_{D3}, \quad (3.2)$$

where the FI-term for the D7-stack is given by (2.35) whereas the FI-term for the D3-brane stack is:

$$\xi_{D3} = q_i \frac{\partial K}{\partial T_{q_i}} = q_i \frac{\tau_{q_i}}{\mathcal{V}} \quad \text{for either } i = 1 \text{ or } i = 2. \quad (3.3)$$

In LVS models the Calabi-Yau volume is exponentially large in string units, and so $1/\mathcal{V} \ll 1$ is a small parameter which can be used to control the relative strength of different contributions to the total scalar potential (3.1). Let us analyse each of these contributions separately.

3.1 Stabilisation at $\mathcal{O}(1/\mathcal{V}^2)$

As can be seen from the volume scaling of the two FI-terms (2.35) and (3.3), the total D-term potential scales as $V_D \sim M_p^4/\mathcal{V}^2 \sim M_s^4$. Therefore its leading order contribution has to be vanishing since otherwise the effective field theory would not be under control since the scalar potential would be of order the string scale. As we shall see in more detail below, this leading order supersymmetric stabilisation fixes $|\phi|$ in terms of $\tilde{\tau}_p \equiv \tau_p + x\tau_s$ and τ_{q_i} in terms of $|C|$. The open string axion ψ and the closed string axion c_{q_i} are eaten up by the two anomalous U(1)'s living respectively on the D7 and D3-stack. Additional $\mathcal{O}(1/\mathcal{V}^2)$ tree-level contributions to the scalar potential arise from background fluxes which stabilise the dilaton and the complex structure moduli in a supersymmetric manner at $D_S W_{\text{tree}} = D_{U_a} W_{\text{tree}} = 0$ [76]. At this level of approximation the Kähler moduli are still flat due to the no-scale cancellation. They can be lifted by subdominant corrections to the effective action which can be studied by assuming a constant tree-level superpotential $W_0 = \langle W_{\text{tree}} \rangle$ that is naturally of $\mathcal{O}(1)$. Summarising the total $\mathcal{O}(1/\mathcal{V}^2)$ contribution to the scalar potential looks schematically like (we show the dependence just on the scalar fields which get frozen):

$$V_{\mathcal{O}(1/\mathcal{V}^2)} = V_D(|\phi|, \tau_{q_i}) + V_F^{\text{tree}}(S, U). \quad (3.4)$$

Let us focus in particular on the dynamics of the total D-term potential which from (2.35), (3.1) and (3.3) reads:

$$V_D = \frac{g_{D7}^2}{2} \left(q_\phi \frac{|\phi|^2}{\text{Re}(S)} - \frac{f_p \sqrt{2\mu}}{4\pi} \frac{\sqrt{\tilde{\tau}_p}}{\mathcal{V}} \right)^2 + \frac{g_{D3}^2}{2} \left(q_C \tilde{K}(T_i, \bar{T}_i) |C|^2 - q_i \frac{\tau_{q_i}}{\mathcal{V}} \right)^2. \quad (3.5)$$

Supersymmetry is preserved if:

$$q_\phi \frac{|\phi|^2}{\text{Re}(S)} = \frac{f_p \sqrt{2\mu}}{4\pi} \frac{\sqrt{\tilde{\tau}_p}}{\mathcal{V}} \quad \text{and} \quad q_C \tilde{K}(T_i, \bar{T}_i) |C|^2 = q_i \frac{\tau_{q_i}}{\mathcal{V}}. \quad (3.6)$$

These two relations fix one direction in the $(|\phi|, \tilde{\tau}_p)$ -plane and one direction in the $(|C|, \tau_{q_i})$ -plane. Each of these two directions corresponds to the supersymmetric partner of the axion which is eaten up by the relative anomalous U(1) gauge boson in the process of anomaly cancellation. The axions which become the longitudinal components of the massive gauge bosons are combinations of open string axions with decay constant f_{op} and closed string axions with decay constant f_{cl} . The Stückelberg mass of the anomalous U(1)'s scales as [80]:

$$M_{\text{U}(1)}^2 \simeq g^2 (f_{\text{op}}^2 + f_{\text{cl}}^2), \quad (3.7)$$

where:

$$\begin{aligned} \text{D7 case: } f_{\text{op}}^2 &= \frac{|\phi|^2}{\text{Re}(S)} = \frac{f_p \sqrt{2\mu}}{4\pi q_\phi} \frac{\sqrt{\tilde{\tau}_p}}{\mathcal{V}} \gg f_{\text{cl}}^2 = \frac{1}{4} \frac{\partial^2 K}{\partial \tau_p^2} = \frac{1}{4\sqrt{2\mu}} \frac{1}{\mathcal{V} \sqrt{\tilde{\tau}_p}}, \\ \text{D3 case: } f_{\text{op}}^2 &= \tilde{K}(T_i, \bar{T}_i) |C|^2 = \frac{q_i}{q_C} \frac{\tau_{q_i}}{\mathcal{V}} \ll f_{\text{cl}}^2 = \frac{1}{4} \frac{\partial^2 K}{\partial \tau_{q_i}^2} = \frac{1}{2\mathcal{V}}, \end{aligned} \quad (3.8)$$

for:

$$\tilde{\tau}_p \gg z_p \equiv \frac{\pi q_\phi}{2\mu f_p} \quad \text{and} \quad \tau_{q_i} \ll z_{q_i} \equiv \frac{q_C}{2q_i}. \quad (3.9)$$

In section 3.2 and 3.3 we shall explain how to fix the remaining flat directions, showing that the conditions in (3.9) can be satisfied dynamically. These conditions imply that for the D7 case the combination of axions eaten up is mostly given by the open string axion ψ , and so (3.6) should be read off as fixing $|\phi|$ in terms of $\tilde{\tau}_p$, while for the D3 brane case the combination of axions eaten up is mostly given by the closed string axion c_{q_i} which means that (3.6) fixes τ_{q_i} in terms of $|C|$. Notice that from (3.9) the U(1) gauge bosons acquire a mass of order the string scale: $M_{\text{U}(1)} \sim M_p/\sqrt{\mathcal{V}} \sim M_s$.

3.2 Stabilisation at $\mathcal{O}(1/\mathcal{V}^3)$

As we shall explain more in detail below, $\mathcal{O}(1/\mathcal{V}^3)$ effects arise from both the leading α' and $\tilde{\tau}_p$ -dependent g_s corrections to K in (2.38) together with the single instanton contribution in (2.41). They give rise to a scalar potential which depends on τ_s , c_s , τ_p and τ_b but not on the associated axions c_p and c_b since both T_p - and T_b -dependent non-perturbative corrections to W are much more suppressed due to the double exponential suppression of poly-instanton effects and the exponentially large value of $\tau_b \sim \mathcal{V}^{2/3}$. These $\mathcal{O}(1/\mathcal{V}^3)$ contributions alone would yield an AdS minimum which breaks supersymmetry spontaneously [32–34]. Additional contributions of the same order of magnitude can arise rather naturally from a hidden D7 T-brane stack [77] or from anti-D3 branes at the tip of a warped throat [50, 78, 79] and can be tuned to obtain a dS vacuum. The Kähler moduli develop non-zero F-terms and mediate supersymmetry breaking to each open string sector via gravitational interactions. Matter fields on the D7-stack are unsequestered, and so

acquire soft masses of order $m_{3/2}$. After using the vanishing D-term condition to write $|\phi|$ in terms of $\tilde{\tau}_p$, the resulting F-term potential for the matter fields also scales as $\mathcal{O}(1/\mathcal{V}^3)$. Thus the full $\mathcal{O}(1/\mathcal{V}^3)$ scalar potential behaves as:

$$V_{\mathcal{O}(1/\mathcal{V}^3)} = V_F^{\alpha'}(\mathcal{V}) + V_F^{g_s}(\mathcal{V}, \tilde{\tau}_p) + V_F^{E3}(\tau_s, c_s, \mathcal{V}) + V_F^{\text{matter}}(\mathcal{V}, \tilde{\tau}_p) + V_{\text{up}}(\mathcal{V}). \quad (3.10)$$

All these $\mathcal{O}(1/\mathcal{V}^3)$ contributions take the following precise form:

$$\begin{aligned} V_F^{\alpha'}(\mathcal{V}) &= \frac{3\zeta}{32\pi\sqrt{g_s}} \frac{W_0^2}{\mathcal{V}^3}, & V_F^{g_s}(\mathcal{V}, \tilde{\tau}_p) &= \frac{3g_s\lambda_p}{64\pi} (g_s C_p^{\text{KK}})^2 \frac{W_0^2}{\mathcal{V}^3 \sqrt{\tilde{\tau}_p}}, \\ V_F^{E3}(\tau_s, c_s, \mathcal{V}) &= \frac{4g_s\pi A_s^2}{3\lambda_s} \frac{\sqrt{\tau_s} e^{-4\pi\tau_s}}{\mathcal{V}} + g_s A_s \cos(2\pi c_s) \frac{W_0 \tau_s e^{-2\pi\tau_s}}{\mathcal{V}^2}, \\ V_F^{\text{matter}}(\mathcal{V}, \tilde{\tau}_p) &= m_{3/2}^2 \frac{|\phi|^2}{\text{Re}(S)} = \frac{3g_s\lambda_p}{64\pi z_p} \frac{W_0^2 \sqrt{\tilde{\tau}_p}}{\mathcal{V}^3}, \end{aligned} \quad (3.11)$$

where the string loop potential includes only the leading Kaluza-Klein contribution from K_{g_s} in (2.38) which is given by [44]:

$$V_F^{g_s}(\mathcal{V}, \tilde{\tau}_p) = \left(\frac{g_s}{8\pi}\right) (g_s C_p^{\text{KK}})^2 \frac{W_0^2}{\mathcal{V}^2} \frac{\partial^2 K}{\partial \tau_p^2},$$

and in V_F^{matter} we substituted the relation (3.6) which expresses $|\phi|$ in terms of $\tilde{\tau}_p$. Summing up the four contributions in (3.11), the total scalar potential at $\mathcal{O}(1/\mathcal{V}^3)$ has a minimum at (for $2\pi\tau_s \gg 1$):

$$\begin{aligned} c_s &= k + \frac{1}{2} \quad \text{with } k \in \mathbb{Z}, & \mathcal{V} &= \frac{3\lambda_s}{8\pi A_s} W_0 \sqrt{\tau_s} e^{2\pi\tau_s}, \\ \tau_s &= \left(\frac{\zeta}{2\lambda_s}\right)^{2/3} \frac{1}{g_s} (1 + \epsilon) \sim \frac{1}{g_s}, & \tilde{\tau}_p &= z_p (g_s C_p^{\text{KK}})^2 \sim \frac{1}{g_s}, \end{aligned} \quad (3.12)$$

for $C_p^{\text{KK}} \sim g_s^{-3/2} \gg 1$ and:

$$\epsilon = \left(\frac{2\lambda_p}{3\zeta z_p}\right) g_s^{3/2} \sqrt{\tilde{\tau}_p} \sim g_s^{5/2} C_p^{\text{KK}} \sim g_s \ll 1. \quad (3.13)$$

Notice that the condition $\tilde{\tau}_p \gg z_p$ in (3.9), which ensures that the closed string axion c_p is not eaten up by the anomalous $U(1)$ on the D7-stack and so can play the rôle of DM, can be easily satisfied if $C_p^{\text{KK}} \sim g_s^{-3/2} \gg 1$. We point out that the coefficients of the string loop corrections are complex structure moduli dependent, and so their values can be tuned by appropriate choices of background fluxes. Therefore for $z_p \sim \mathcal{O}(1)$, $\tilde{\tau}_p \sim \tau_s \sim \tau_p \sim g_s^{-1} \gg 1$. This behaviour justifies also the scaling of the small parameter ϵ in (3.13).

As stressed above, this minimum is AdS but can be uplifted to dS via several different positive definite contributions. Two examples which emerge rather naturally in type IIB flux compactifications are T-branes [77] or anti-D3 branes [50, 78, 79].

3.3 Stabilisation at $\mathcal{O}(1/\mathcal{V}^{3+p})$

Taking into account all contributions to the scalar potential up to $\mathcal{O}(1/\mathcal{V}^3)$, there are still four flat directions: the charged matter field $|C|$, the open string axion θ and the two closed string axions c_p and c_b . We shall now show how to stabilise the DM axion c_p and $|C|$ which sets the decay constant of the ALP θ and fixes τ_{q_i} from (3.6). The bulk closed string axion c_b receives scalar potential contributions only from T_b -dependent non-perturbative corrections, and so it is almost massless: $m_{c_b} \sim m_{\tau_b} e^{-\pi \mathcal{V}^{2/3}} \sim 0$.

The closed string axion c_p and the open string matter field $|C|$ receive a potential respectively via poly-instanton corrections to the effective action and soft supersymmetry breaking terms. As we shall see below, these terms scale as $\mathcal{O}(1/\mathcal{V}^{3+p})$ with $p > 0$. The only exception which leads to $p = 0$ is the case where flavour D7-branes desquester the open string sector on the D3-brane at a singularity. However, as shown in section 2.2, these effects would not modify the VEV of $|C|$ which sets the open string axion decay constant, and so, without loss of generality, we shall consider just the sequestered case. The resulting $\mathcal{O}(1/\mathcal{V}^{3+p})$ scalar potential looks schematically as (showing again just the dependence on the fields which get stabilised at this order in the inverse volume expansion of V):

$$V_{\mathcal{O}(1/\mathcal{V}^{3+p})} = V_F^{\text{poly}}(c_s) + V_F^{\text{matter}}(|C|). \quad (3.14)$$

The leading order expression of the C -dependent soft supersymmetry breaking terms is given by (2.8). A more complete expression in terms of the canonically normalised field $\hat{C} = |\hat{C}| e^{i\theta} = \sqrt{\tilde{K}} C$ (see section 4.1 for more details) is (the c_i 's are $\mathcal{O}(1)$ coefficients) [39]:

$$V_F(|\hat{C}|) = c_2 m_0^2 |\hat{C}|^2 + c_3 A |\hat{C}|^3 + c_4 \lambda |\hat{C}|^4 + \mathcal{O}(|\hat{C}|^5) + c_5 \frac{\tau_{q_i}^2}{\mathcal{V}^3} \left[1 + \mathcal{O}\left(\frac{1}{\mathcal{V}}\right) \right], \quad (3.15)$$

where the first three terms originate from expanding the F-term potential in powers of $|\hat{C}|$ up to fourth order, whereas the last term comes from the fact that the τ_{q_i} -dependent term in (2.38) breaks the no-scale structure. Using (3.6) we can rewrite the last term in (3.15) in terms of $|\hat{C}|$ and parameterising the soft terms in Planck units as $m_0 \sim \mathcal{V}^{-\alpha_2}$, $A \sim \mathcal{V}^{-\alpha_3}$ and $\lambda \sim \mathcal{V}^{-\alpha_4}$, we obtain (up to fourth order in $|\hat{C}|$):

$$V_F(|\hat{C}|) = \frac{c_2}{\mathcal{V}^{\alpha_2}} |\hat{C}|^2 + \frac{c_3}{\mathcal{V}^{\alpha_3}} |\hat{C}|^3 + \frac{k_4}{\mathcal{V}^{\alpha_4}} |\hat{C}|^4 \quad \text{with} \quad k_4 = c_4 \lambda + \frac{4c_5 z_{q_i}^2}{\mathcal{V}^{1-\alpha_4}}. \quad (3.16)$$

If the soft masses are non-tachyonic, the VEV of the matter field $|\hat{C}|$ is zero, and so the open string axion θ cannot play the rôle of the ALP a_{ALP} which gives the 3.5 keV line by converting into photons in astrophysical magnetic fields. On the other hand, as explained in section 2.2, if $c_2 < 0$ $|\hat{C}|$ can develop a non-vanishing VEV. Open string modes living on D3-branes localised at singularities are geometrically sequestered from the sources of supersymmetry breaking in the bulk, resulting in $\alpha_3 = 2$, $\alpha_4 = 1$ and $\alpha_2 = 3/2$ or $\alpha_2 = 2$ depending on the exact moduli dependence of \tilde{K}_{pert} in (2.40) and the details of the uplifting mechanism to a dS vacuum [64, 65]. The VEVs of $|\hat{C}|$ and τ_{q_i} from (3.6) are therefore:

$$\alpha_2 = \frac{3}{2} \text{ case:} \quad |\hat{C}| = f_{a_{\text{ALP}}} = \frac{M_p}{\mathcal{V}} \quad \Leftrightarrow \quad \tau_{q_i} = \frac{2 z_{q_i}}{\mathcal{V}} \ll z_{q_i}, \quad (3.17)$$

$$\alpha_2 = 2 \text{ case:} \quad |\hat{C}| = f_{a_{\text{ALP}}} = \frac{M_p}{\mathcal{V}^2} \quad \Leftrightarrow \quad \tau_{q_i} = \frac{2 z_{q_i}}{\mathcal{V}^3} \ll z_{q_i}, \quad (3.18)$$

where we have identified the open string axion θ with the ALP $a_{\text{ALP}} = f_{a_{\text{ALP}}} \theta$. Notice that the ALP decay constant in (3.17) reproduces exactly the ALP coupling to gauge bosons in (2.10) while the $f_{a_{\text{ALP}}}$ in (3.18) gives the coupling in (2.11). We stress that (3.17) and (3.18) show also how the condition $\tau_{q_i} \ll z_{q_i}$ in (3.9) is easily satisfied for $1/\mathcal{V} \ll 1$. This ensures that the blow-up mode τ_{q_i} is indeed collapsed to a singularity. Let us remind the reader that i can be either $i = 2$ or $i = 3$. When τ_{q_1} and τ_{q_2} are identified by the orientifold involution, an open string axion is the standard QCD axion a_{QCD} while the other is a_{ALP} with $|\hat{C}|$ a Standard Model gauge singlet with a large VEV. On the other hand, when the two blow-up modes τ_{q_1} and τ_{q_2} are separately invariant under the involution, \hat{C} belongs to a hidden sector and, as described in section 2.2, its axion θ has a coupling to ordinary photons of the form (2.15) which is induced by U(1) kinetic mixing.

The axionic partner c_p of the Kähler modulus τ_p which controls the volume of the Wilson divisor supporting poly-instanton effects, receives the following scalar potential contributions from the second term in (2.41) with $A_p \rightarrow A_p \phi^n$ and $n = q_p/q_\phi$:

$$V_F^{\text{poly}}(c_p) = -2g_s \pi A_s A_p \phi^n \left[\frac{8(1-x)\pi A_s}{3\lambda_s} \cos(2\pi c_p) \sqrt{\tau_s} e^{-2\pi\tau_s} + W_0 \cos[2\pi(c_s + c_p)] \frac{((1-x)\tau_s + \tilde{\tau}_p)}{\mathcal{V}} \right] \frac{e^{-2\pi\tau_s} e^{-2\pi\tau_p}}{\mathcal{V}},$$

which, after using the first D-term relation in (3.6) and substituting the VEVs in (3.12), reduces to (setting without loss of generality $\phi = |\phi|$ with $\psi = 0$):

$$V_F^{\text{poly}}(c_p) = \frac{A}{\mathcal{V}^{3+p}} \cos(2\pi c_p), \quad (3.19)$$

where:

$$A = \frac{3g_s \lambda_s A_p}{4} \left(\frac{3\lambda_p C_p^{\text{KK}}}{8\sqrt{z_p}} \right)^{n/2} \left(\frac{3\lambda_s \sqrt{\tau_s}}{8\pi A_s} \right)^\kappa \tilde{\tau}_p \sqrt{\tau_s} W_0^{2+\kappa},$$

with:

$$\kappa \equiv \frac{\tau_p}{\tau_s} > 0 \quad \text{and} \quad p = \frac{n}{2} + \kappa > 0. \quad (3.20)$$

Therefore the DM axion c_p is stabilised at $\mathcal{O}(1/\mathcal{V}^{3+p})$ at $c_p = 1/2 + k$ with $k \in \mathbb{Z}$ and $A > 0$.

4 Mass spectrum and couplings

In this section we shall first determine the expressions for all canonically normalised fields and their mass spectrum, and then we will compute the strength of the coupling of the light DM axion c_p to the open string ALP θ which is induced by non-perturbative corrections to the matter Kähler metric in (2.40).

4.1 Canonical normalisation

Similarly to the scalar potential, also the kinetic Lagrangian derived from the Kähler potential for the moduli given by the three terms in (2.38) and for the matter fields given

by (2.39), can be organised in an expansion in $1/\mathcal{V} \ll 1$. Hence the kinetic terms can be canonically normalised order by order in this inverse volume expansion. The detailed calculation is presented in appendix A.2 and here we just quote the main results which are useful to work out the strength of the DM-ALP coupling. The expressions for the canonically normalised fields at leading order look like (the moduli and the matter fields are dimensionless while canonically normalised scalar fields have standard mass dimensions):

$$\begin{aligned}
 \frac{|\hat{C}|}{M_p} &= \sqrt{2\tilde{K}}|C|, & a_{\text{ALP}} &= |\hat{C}|\theta = f_{a_{\text{ALP}}}\theta, & \frac{|\hat{\phi}|}{M_p} &= \sqrt{\frac{2}{\text{Re}(S)}}|\phi|, & \frac{\phi_{q_i}}{M_p} &= \frac{\tau_{q_i}}{\sqrt{\mathcal{V}}}, \\
 \frac{\phi_b}{M_p} &= \sqrt{\frac{3}{2}}\ln\tau_b, & \frac{a_b}{M_p} &= \sqrt{\frac{3}{2}}\frac{c_b}{\tau_b}, & \frac{\phi_s}{M_p} &= \sqrt{\frac{4\lambda_s}{3\mathcal{V}}}\tau_s^{3/4}, \\
 \frac{a_s}{M_p} &= \sqrt{\frac{3\lambda_s}{4\mathcal{V}\sqrt{\tau_s}}}c_s, & \frac{\tilde{\phi}_p}{M_p} &= \sqrt{\frac{4\lambda_p}{3\mathcal{V}}}\tilde{\tau}_p^{3/4}, & \frac{\tilde{a}_p}{M_p} &= \sqrt{\frac{3\lambda_p}{4\mathcal{V}\sqrt{\tilde{\tau}_p}}}\tilde{c}_p,
 \end{aligned} \tag{4.1}$$

where we did not include the axions ψ and c_{q_i} which are eaten up by two anomalous $U(1)$'s on the D7- and D3-brane stack respectively. Notice that the Kähler modulus $T_p = \tau_p + i c_p$ is given by the following combinations of the canonically normalised fields $\Phi_s = \phi_s + i a_s$ and $\tilde{\Phi}_p = \tilde{\phi}_p + i \tilde{a}_p$:

$$\tau_p = \tilde{\tau}_p - x\tau_s = \left(\frac{3\mathcal{V}}{4}\right)^{2/3} \left[\frac{1}{\lambda_p^{2/3}} \left(\frac{\tilde{\phi}_p}{M_p}\right)^{4/3} - \frac{x}{\lambda_s^{2/3}} \left(\frac{\phi_s}{M_p}\right)^{4/3} \right], \tag{4.2}$$

and:

$$c_p = \tilde{c}_p - xc_s = \sqrt{\frac{4\mathcal{V}}{3}} \left(\frac{\tilde{\tau}_p^{1/4}}{\sqrt{\lambda_p}} \frac{\tilde{a}_p}{M_p} - \frac{x\tau_s^{1/4}}{\sqrt{\lambda_s}} \frac{a_s}{M_p} \right). \tag{4.3}$$

4.2 Mass spectrum

The mass matrix around the global minimum and its eigenvalues are derived in detail in appendix A.3. Here we just show the leading order volume scaling of the mass of all moduli and charged matter fields for $g_s \sim 0.1$ (in order to trust our approach based on perturbation theory) and $\mathcal{V} \sim 10^7$. As explained in section 2.2, this choice of the internal volume leads naturally to TeV-scale soft terms for sequestered scenarios with D3-branes at singularities, while it guarantees the absence of any cosmological moduli problem for unsequestered cases with flavour D7-branes. The resulting mass spectrum looks like:

$$\begin{aligned}
 m_{c_{q_i}} &\sim m_{\tau_{q_i}} \sim m_\psi \sim m_{|\phi|} \sim M_s \sim g_s^{1/4} \sqrt{\pi} \frac{M_p}{\sqrt{\mathcal{V}}} \sim 10^{15} \text{ GeV}, \\
 m_{\tau_s} &\sim m_{c_s} \sim \sqrt{\frac{g_s}{8\pi}} \frac{M_p}{\mathcal{V}} \ln \mathcal{V} \sim 10^{11} \text{ GeV}, \\
 m_{3/2} &\sim \sqrt{\frac{g_s}{8\pi}} \frac{M_p}{\mathcal{V}} \sim 10^{10} \text{ GeV}, \\
 m_{\tilde{\tau}_p} &\sim \sqrt{\frac{g_s}{8\pi}} \frac{M_p}{\mathcal{V}\sqrt{\ln \mathcal{V}}} \sim 10^9 \text{ GeV},
 \end{aligned}$$

$$\begin{aligned}
 m_{\tau_b} &\sim \sqrt{\frac{g_s}{8\pi}} \frac{M_p}{\mathcal{V}^{3/2} \sqrt{\ln \mathcal{V}}} \sim 10^6 \text{ GeV}, \\
 m_{|C|} &\sim \sqrt{\frac{g_s}{8\pi}} \frac{M_p}{\mathcal{V}^2} \sim 1 \text{ TeV}, \\
 m_{c_p} &\sim \sqrt{\frac{g_s}{8\pi}} \frac{M_p}{\mathcal{V}^{1+p/2}} \sqrt{\ln \mathcal{V}} \sim 10 \text{ keV} \quad \text{for } p = \frac{9}{2}, \\
 m_\theta &\sim \frac{\Lambda_{\text{hid}}^2}{f_{a_{\text{ALP}}}} \lesssim 10^{-12} \text{ eV}, \\
 m_{c_b} &\sim \sqrt{\frac{g_s}{8\pi}} \frac{M_p}{\mathcal{V}^{2/3}} e^{-\pi \mathcal{V}^{2/3}} \sim 0,
 \end{aligned} \tag{4.4}$$

where we focused on the sequestered case with $\alpha_2 = 2$ illustrated in section 3.3 and Λ_{hid} represents the scale of strong dynamics in the hidden sector which gives mass to the open string axion $\theta = a_{\text{ALP}}/f_{a_{\text{ALP}}}$ whose decay constant is $f_{a_{\text{ALP}}} = |\hat{C}| \simeq M_p/\mathcal{V}^2$. As explained in section 2.2, this decay constant leads to a coupling to hidden photons controlled by the scale $M_{\text{hid}} \sim 10^6 \text{ GeV}$ that can yield a coupling to ordinary photons via U(1) kinetic mixing given by (2.15) which can be naturally suppressed by an effective scale of order $M \sim 10^{12} \text{ GeV}$. Notice that the DM axion c_p can acquire a mass from poly-instanton effects of order $m_{c_p} \sim 10 \text{ keV}$ if $p = \frac{n}{2} + \kappa = \frac{9}{2}$, which can be obtained for any $\mathcal{O}(1)$ value of n by appropriately choosing the flux dependent underlying parameters so that $\kappa \equiv \frac{\tau_p}{\tau_s} = \frac{1}{2}(9 - n)$.

4.3 DM-ALP coupling

As shown by the mass spectrum in (4.4) and by the coupling to ordinary photons in (2.15), the open string axion θ is a natural candidate for the ALP mode a_{ALP} which converts into photons in the magnetic field of galaxy clusters and generates the 3.5 keV line. However a monochromatic line requires the decay into a pair of ALP particles of a DM particle a_{DM} with mass $m_{\text{DM}} \sim 7 \text{ keV}$. According to the mass spectrum in (4.4) a_{DM} could be either the local closed string axion c_p or the bulk closed string mode c_b (if T_b -dependent non-perturbative effects do not suppress its mass too much). We shall now show that non-perturbative corrections to the matter Kähler metric in (2.39) can induce a coupling of the form $\frac{a_{\text{DM}}}{\Lambda} \partial_\mu a_{\text{ALP}} \partial^\mu a_{\text{ALP}}$ due to kinetic mixing between the closed string axion a_{DM} and the open string axion a_{ALP} . We shall also work out the value of the coupling Λ , finding that it can lie around the Planck/GUT scale only if the DM particle is the local axion c_p (c_b would give a trans-Planckian Λ). Finally we will explain how in our model a direct DM decay to photons induced by potential couplings of the form $\frac{a_{\text{DM}}}{4M_{\text{DM}}} F^{\mu\nu} \tilde{F}_{\mu\nu}$ is naturally suppressed by construction.

In order to compute the DM-ALP coupling, let us focus on contributions to the kinetic Lagrangian of the form:

$$\mathcal{L}_{\text{kin}} \supset \frac{\partial^2 K}{\partial C \partial \bar{C}} \partial_\mu C \partial^\mu \bar{C} = \tilde{K}(T_i, \bar{T}_i) (\partial_\mu |C| \partial^\mu |C| + |C|^2 \partial_\mu \theta \partial^\mu \theta). \tag{4.5}$$

If we now expand the closed string axions c_i and the charged open string mode $C = |C| e^{i\theta}$ around the minimum as:

$$c_i(x) \rightarrow \langle c_i \rangle + c_i(x), \quad |C|(x) \rightarrow \langle |C| \rangle + |C(x)|, \quad \theta(x) \rightarrow \langle \theta \rangle + \theta(x), \tag{4.6}$$

the kinetic terms (4.5) become:

$$\left[\langle \tilde{K} \rangle - B_i e^{-b_i \tau_i} \left(\cos(b_i \langle c_i \rangle) \frac{b_i}{2} \hat{c}_i^2 + \sin(b_i \langle c_i \rangle) b_i \hat{c}_i \right) \right] (\partial_\mu |C| \partial^\mu |C| + |C|^2 \partial_\mu \theta \partial^\mu \theta). \quad (4.7)$$

If we now express the open string mode C in terms of the canonically normalised fields \hat{C} and a_{ALP} using (4.1), (4.7) contains DM-ALP interaction terms of the form:

$$\frac{B_i}{2 \langle \tilde{K} \rangle} e^{-b_i \langle \tau_i \rangle} \left(\cos(b_i \langle c_i \rangle) \frac{b_i}{2} \hat{c}_i^2 + \sin(b_i \langle c_i \rangle) b_i \hat{c}_i \right) \partial_\mu a_{\text{ALP}} \partial^\mu a_{\text{ALP}}, \quad (4.8)$$

showing that, in order to obtain a three-leg vertex which can induce a two-body DM decay into a pair of ultra-light ALPs, the VEV of c_i has to be such that $b_i \langle c_i \rangle = (2k+1)\frac{\pi}{2}$ with $k \in \mathbb{Z}$. Let us therefore focus on this case and consider separately the two options with either $i = b$ or $i = p$:

- **$i = b$ case:** plugging in (4.8) the canonical normalisation for c_b from (4.1), we find a DM-ALP coupling of the form:

$$\frac{a_b}{\Lambda} \partial_\mu a_{\text{ALP}} \partial^\mu a_{\text{ALP}} \quad \text{with} \quad \Lambda = \frac{\sqrt{6} \langle \tilde{K} \rangle}{B_b b_b} \frac{e^{b_b \langle \tau_b \rangle}}{\langle \tau_b \rangle} M_p \sim \frac{e^{b_b \mathcal{V}^{2/3}}}{B_b \mathcal{V}^{4/3}} M_p \gg M_p, \quad (4.9)$$

which reproduces the value of Λ in (2.19) for $a_{\text{DM}} = c_b$. According to the phenomenological constraints discussed in section 2.1, c_b cannot play the rôle of the DM particle since the scale of its ALP coupling is trans-Planckian.

- **$i = p$ case:** writing $b_p = \frac{2\pi}{N}$ and using the fact that the minimum for c_p lies at $\langle c_p \rangle = \frac{1}{2} + k_1$ with $k_1 \in \mathbb{Z}$, the condition $b_p \langle c_p \rangle = (2k_2+1)\frac{\pi}{2}$ with $k_2 \in \mathbb{Z}$ can be satisfied if $\frac{N}{2} = \frac{(2k_1+1)}{(2k_2+1)}$. Hence in the simplest case with $k_1 = k_2 = 0$ we just need $N = 2$. Plugging in (4.8) the canonical normalisation for c_p from (4.3), the DM-ALP coupling turns out to be:

$$\frac{\tilde{a}_p}{\Lambda} \partial_\mu a_{\text{ALP}} \partial^\mu a_{\text{ALP}} \quad \text{with} \quad \Lambda = \frac{\sqrt{3\lambda_p}}{\tilde{\tau}_p^{1/4}} \frac{\langle \tilde{K} \rangle}{B_p b_p} \frac{e^{b_p \langle \tau_p \rangle}}{\sqrt{\mathcal{V}}} M_p \sim \frac{M_p}{B_p \mathcal{V}^{7/6-\kappa/N}}, \quad (4.10)$$

which reproduces the value of Λ in (2.19) for $a_{\text{DM}} = c_p$. This scale of the DM-ALP coupling can easily be around the Planck/GUT scale. For example if $N = 2$ and the underlying parameters are chosen such that $\kappa \equiv \tau_p/\tau_s = 2$, $\Lambda \sim M_p/\mathcal{V}^{1/6} \sim 10^{17} \text{ GeV}$ for $\mathcal{V} \sim 10^7$ and $B_p \sim \mathcal{O}(1)$. Due to the poly-instanton nature of the non-perturbative effects supported by the Wilson divisor D_p , the prefactor B_p can however be exponentially small. Comparing T_p -dependent poly-instanton corrections to the superpotential in (2.41) with T_p -dependent non-perturbative corrections to the matter Kähler metric in (2.40), B_p at the minimum could scale as $B_p \sim \mathcal{O}(\mathcal{V}^{-1})$. In this case Λ can be below the Planck scale only if $\kappa \ll N$.

Let us conclude this section by showing that the branching ratio for direct DM decay into ordinary photons is negligible. Using the fact that the gauge kinetic function for the

D7-stack is given by $f_{D7} = T_s + T_p$ (we neglect the flux dependent shift) and the canonical normalisation (4.3), the closed string axion $c_p = \langle c_p \rangle + \hat{c}_p$ couples to Abelian gauge bosons living on the hidden D7-stack via an interaction term of the form:

$$\frac{\hat{c}_p}{4(\langle \tau_s \rangle + \langle \tau_p \rangle)} F_{\text{hid}}^{\mu\nu} \tilde{F}_{\mu\nu}^{\text{hid}} \sim \frac{\tilde{a}_p}{4M_s} F_{\text{hid}}^{\mu\nu} \tilde{F}_{\mu\nu}^{\text{hid}}. \quad (4.11)$$

One-loop effects generate a kinetic mixing between hidden photons on the D7-stack and ordinary photons on the D3-stack which is controlled by the mixing parameter $\chi \sim 10^{-3}$ given in (2.14). Thus the DM axion c_p develops an effective coupling to visible sector photons which from (2.15) looks like:

$$\frac{\tilde{a}_p}{4M_{\text{DM}}} F_{\mu\nu} \tilde{F}^{\mu\nu} \sim \frac{\chi^2 \tilde{a}_p}{4M_s} F_{\mu\nu} \tilde{F}^{\mu\nu} \quad \Leftrightarrow \quad M_{\text{DM}} \sim \frac{M_s}{\chi^2} \sim 10^5 \frac{M_p}{\sqrt{\mathcal{V}}} \sim 10^{20} \text{ GeV}, \quad (4.12)$$

which is naturally much larger than the scale Λ controlling the DM coupling to ALPs.

5 Conclusions

In this paper we described how to perform a successful global embedding in type IIB string compactifications of the model of [20] for the recently observed 3.5 keV line from galaxy clusters. The main feature of this model is the fact that the monochromatic 3.5 keV line is not generated by the direct decay of a 7 keV dark matter particle into a pair of photons but it originates from DM decay into ultra-light ALPs which subsequently convert into photons in the cluster magnetic field. Therefore the final signal strength does not depend just on the DM distribution but also on the magnitude of the astrophysical magnetic field and its coherence length which, together with the ALP to photon coupling, determine the probability for ALPs to convert into photons. These additional features make the model of [20] particularly interesting since it manages to explain not just the observation of a 3.5 keV line from galaxy clusters but also the morphology of the signal (e.g. the intensity of the line from Perseus seems to be picked at the centre where the magnetic field is in fact more intense) and its non-observation in dwarf spheroidal galaxies (due to the fact that their magnetic field is not very intense and has a relatively small spatial extension). These phenomenological features seem to make this model more promising than standard explanations where DM directly decays into a pair of photons.

Despite this observational success, the model of [20] for the 3.5 keV line did not have a concrete microscopic realisation. In this paper we filled this gap by describing how to construct an explicit type IIB Calabi-Yau compactification which can reproduce all the main phenomenological features of the DM to ALP to photon model. We focused in particular on LVS models since they generically lead to very light axions because some of the moduli are stabilised by perturbative corrections to the effective action. The DM particle is realised as a local closed string axion which develops a tiny mass due to poly-instanton corrections to the superpotential. By an appropriate choice of background and gauge fluxes, the DM mass can be around 7 keV. The ultra-light ALP is instead given by the phase of an open string mode living on D3-branes at singularities. The ALP decay constant is set by the radial part

of this open string mode which is charged under an anomalous $U(1)$. Thus the radial part gets fixed in terms of a moduli-dependent FI-term. In sequestered models with low-energy supersymmetry, the resulting decay constant is naturally in the right ballpark to reproduce a coupling to ordinary photons via $U(1)$ kinetic mixing which is around the intermediate scale, in full agreement with current observations. Notice that future helioscope experiments like IAXO might be able to detect ultra-light ALPs with intermediate scale couplings to photons [59]. Moreover the DM-ALP coupling is generated by kinetic mixing induced by non-perturbative corrections to the Kähler potential. For suitable choices of the underlying flux dependent parameters, the scale which controls the associated coupling can be around the GUT/Planck scale, again in good agreement with present observational constraints.

In this paper we discussed in full depth moduli stabilisation, the mass spectrum and the resulting strength of all relevant couplings but we just described the geometrical and topological conditions on the underlying Calabi-Yau manifold without presenting an explicit example built via toric geometry. This task is beyond the scope of our paper, and so we leave it for future work. Let us however stress that the construction of a concrete Calabi-Yau example with all the desired features for a successful microscopic realisation of our model for the 3.5 keV line is crucial to have a fully trustworthy scenario. Moreover it would be very interesting to have a more concrete computation of non-perturbative corrections to the 4D $N = 1$ Kähler potential.

Another aspect which would deserve further investigation is the cosmological history of our setup from inflation to the present epoch. Here we just point out that the rôle of the inflaton could be played by a small blow-up mode like τ_s [41, 81]. On the other hand, reheating might be due to the volume mode τ_b which gets displaced from its minimum during inflation [82] and later on decays giving rise to a reheating temperature of order $T_{\text{rh}} \sim 1 - 10 \text{ GeV}$ [83]. Such a low reheating temperature would dilute standard thermal WIMP dark matter and reproduce it non-thermally [83]. Given that in sequestered models with unified gaugino masses the WIMP is generically a Higgsino-like neutralino with an under-abundant non-thermal production in vast regions of the underlying parameter space [84, 85], an additional DM component in the form of a very light axion like c_p would be needed. Finally one should make sure that tight dark radiation bounds are satisfied since τ_b could decay both to a pair of ultra-light closed string axions c_b and to a pair of DM axions c_p which could behave as extra neutrino-like species [86, 87]. Notice however that the decay of τ_b to open string axions θ living on D3-branes at singularities is negligible since it is highly suppressed by sequestering effects [86]. The DM axions c_p are produced non-thermally at the QCD phase transition via the standard misalignment mechanism. Given that the decay constant of the local closed string axion c_p is of order the string scale which from (4.4) is rather high, i.e. $M_s \sim 10^{15} \text{ GeV}$, axion DM overproduction can be avoided only if the initial misalignment angle is very small. This might be due to a selection effect from the inflationary dynamics [88]. We finally stress that if inflation is driven by a blow-up mode like τ_s , the Hubble scale during inflation is rather low, $H \sim m_{\tau_b} \sim 10^6 \text{ GeV}$, and so axion isocurvature perturbations would not be in tension with CMB data [89].

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A Computational details

A.1 Closed string axion decay constants

In type IIB string compactifications on Calabi-Yau orientifolds axion-like particles emerge in the low-energy $N = 1$ effective field theory from the dimensional reduction of the Ramond-Ramond forms C_p with $p = 2, 4$. The Kaluza-Klein decomposition under the orientifold projection of these forms is given by [90]:

$$C_2 = c^{i-}(x) \hat{D}_{i-} \quad \text{and} \quad C_4 = c_{i+}(x) \tilde{D}^{i+} + Q_2^{i+}(x) \wedge \hat{D}_{i+} + V^{a+}(x) \wedge \alpha_{a+} - \tilde{V}_{a+}(x) \wedge \beta^{a+},$$

where $i_{\pm} = 1, \dots, h_{\pm}^{1,1}$, $a_+ = 1, \dots, h_+^{1,2}$, \tilde{D}^{i+} is a basis of $H_+^{2,2}$ dual to the $(1,1)$ -forms \hat{D}_{i+} and $(\alpha_{a+}, \beta^{a+})$ is a real, symplectic basis of $H_+^3 = H_+^{1,2} \oplus H_+^{2,1}$.

As explained in section 3.1, in our model the orientifold-odd axions c_{i-} , if present, are eaten up by anomalous $U(1)$'s in the process of anomaly cancellation. We shall therefore focus on the case with $h_-^{1,1} = 0$ where the Kähler moduli take the simple expression $T_i = \tau_i + i c_i$ with $i = 1, \dots, h_+^{1,1} = h^{1,1}$.

The coupling of orientifold-even closed string axions to $F \wedge F$ can be derived from the Kaluza-Klein reduction of the Chern-Simons term of the D-brane action. Moreover, the periods of the canonically unnormalised axions c_i are integer multiples of M_p and their kinetic terms read [27]:

$$\mathcal{L}_{\text{kin}} = K_{ij} \partial_\mu c_i \partial^\mu c_j = \frac{1}{8} \eta_i \partial_\mu c'_i \partial^\mu c'_i, \quad (\text{A.1})$$

where the c'_i 's are the axions which diagonalise the Kähler metric K_{ij} and η_i are its eigenvalues. A proper canonical normalisation of the kinetic terms can then be easily obtained by defining:

$$\frac{1}{8} \eta_i \partial_\mu c'_i \partial^\mu c'_i \equiv \frac{1}{2} \partial_\mu a_i \partial^\mu a_i \quad \text{with} \quad a_i = \frac{1}{2} \sqrt{\eta_i} c'_i, \quad (\text{A.2})$$

which shows that the canonically normalised axions a_i acquire periods of the form:

$$\frac{2}{\sqrt{\eta_i}} a_i = \frac{2}{\sqrt{\eta_i}} a_i + M_p \quad \Rightarrow \quad a_i = a_i + \frac{\sqrt{\eta_i}}{2} M_p. \quad (\text{A.3})$$

We can then set the conventional axionic period as:

$$a_i = a_i + 2\pi f_{a_i} \quad \text{with} \quad f_{a_i} = \frac{\sqrt{\eta_i} M_p}{4\pi}, \quad (\text{A.4})$$

where f_{a_i} is the standard axion decay constant. Closed string axions which propagate in the bulk of the extra dimensions have a decay constant of order the Kaluza-Klein scale

$M_{\text{KK}} \sim M_p/\mathcal{V}^{2/3}$, whereas the decay constant of closed string axions whose corresponding saxion parameterises the volume of localised blow-up modes is controlled by the string scale $M_s \sim M_p/\sqrt{\mathcal{V}}$:

$$f_{a_i} \simeq \begin{cases} M_p/\tau_i \sim M_{\text{KK}} & \text{bulk axion} \\ M_p/\sqrt{\mathcal{V}} \sim M_s & \text{local axion} \end{cases} \quad (\text{A.5})$$

Notice however that the axion coupling to the Abelian gauge bosons living on the D-brane wrapping the four-cycle whose volume is controlled by the associated saxion τ_i , is given by:

$$\frac{g_i^2}{32\pi^2} \frac{a_i}{f_{a_i}} F_{\mu\nu}^{(i)} \tilde{F}_{(i)}^{\mu\nu} = \frac{1}{32\pi^2} \frac{a_i}{\tau_i f_{a_i}} F_{\mu\nu}^{(i)} \tilde{F}_{(i)}^{\mu\nu}, \quad (\text{A.6})$$

since the gauge coupling is set by the saxion as $g_i^2 = \tau_i$. Hence combining (A.5) with (A.6) we realise that the coupling of bulk closed string axions to gauge bosons is controlled by $M \sim \tau_i f_{a_i} \sim M_p$, in agreement with the fact that moduli couple to ordinary matter with gravitational strength. On the other hand the coupling of local closed string axions to gauge bosons is set by the string scale M_s which in LVS models with exponentially large volume can be considerably smaller than the Planck scale.

A.2 Canonical normalisation

The kinetic terms for all Kähler moduli and the charged open string modes ϕ and C can be derived from the total Kähler potential $K = K_{\text{mod}} + K_{\text{matter}}$, where K_{mod} is given by the three contributions in (2.38) and K_{matter} is shown in (2.39) and (2.40), as follows:

$$\mathcal{L}_{\text{kin}} = \frac{\partial^2 K}{\partial \chi_i \partial \bar{\chi}_{\bar{j}}} \partial_\mu \chi_i \partial^\mu \bar{\chi}_{\bar{j}}, \quad (\text{A.7})$$

where χ_i denotes an arbitrary scalar field of our model. As can be seen from (2.39), the D7 open string mode ϕ mixes only with the dilaton S , and so can be easily written in terms of the corresponding canonically normalised field $\hat{\phi}$ as:

$$\frac{\hat{\phi}}{M_p} = \sqrt{\frac{2}{\text{Re}(S)}} \phi. \quad (\text{A.8})$$

From the first term in (2.38) we also realise that cross-terms between the blow-up mode τ_{q_i} and any of the other Kähler moduli are highly suppressed when evaluated at the minimum for $\tau_{q_i} \simeq 0$ (more precisely, as discussed in section 3.3, depending on the level of sequestering of soft masses, we can have either $\tau_{q_i} \sim \mathcal{V}^{-1} \ll 1$ or $\tau_{q_i} \sim \mathcal{V}^{-3} \ll 1$). Hence it is straightforward to write also τ_{q_i} in terms of the corresponding canonically normalised field ϕ_{q_i} as:

$$\frac{\phi_{q_i}}{M_p} = \frac{\tau_{q_i}}{\sqrt{\mathcal{V}}} \quad \text{for } i = 1, 2. \quad (\text{A.9})$$

The remaining fields T_b , T_s , T_p and C mix with each other, leading to a non-trivial Kähler metric whose components take the following leading order expressions for $\mathcal{V} \simeq \lambda_b \tau_b^{3/2} \gg 1$:

$$\begin{aligned}
 K_{T_i \bar{T}_j} &\simeq \frac{3}{8\mathcal{V}} \begin{pmatrix} \frac{2\lambda_b}{\sqrt{\tau_b}} & -\frac{3}{\tau_b} (\lambda_s \sqrt{\tau_s} + x \lambda_p \sqrt{\tilde{\tau}_p}) - \frac{3}{\tau_b} \lambda_p \sqrt{\tilde{\tau}_p} \\ -\frac{3}{\tau_b} (\lambda_s \sqrt{\tau_s} + x \lambda_p \sqrt{\tilde{\tau}_p}) & \frac{\lambda_s}{\sqrt{\tau_s}} + \frac{x^2 \lambda_p}{\sqrt{\tilde{\tau}_p}} & \frac{x \lambda_p}{\sqrt{\tilde{\tau}_p}} \\ -\frac{3}{\tau_b} \lambda_p \sqrt{\tilde{\tau}_p} & \frac{x \lambda_p}{\sqrt{\tilde{\tau}_p}} & \frac{\lambda_p}{\sqrt{\tilde{\tau}_p}} \end{pmatrix} \\
 K_{T_b \bar{C}} &\simeq -\frac{\tilde{K}}{2\tau_b} C, & K_{T_s \bar{C}} &\simeq \frac{\tilde{K}}{2\mathcal{V}} (\lambda_s \sqrt{\tau_s} + x \lambda_p \sqrt{\tilde{\tau}_p}) C, \\
 K_{T_p \bar{C}} &\simeq \frac{\tilde{K}}{2\mathcal{V}} \lambda_p \sqrt{\tilde{\tau}_p} C, & K_{C \bar{C}} &= \tilde{K}.
 \end{aligned}$$

In the large volume limit, different contributions to the kinetic Lagrangian can be organised in an expansion in $1/\mathcal{V} \ll 1$ as follows:

$$\mathcal{L}_{\text{kin}} = \mathcal{L}_{\text{kin}}^{\mathcal{O}(1)} + \mathcal{L}_{\text{kin}}^{\mathcal{O}(\mathcal{V}^{-1})} + \mathcal{L}_{\text{kin}}^{\mathcal{O}(\mathcal{V}^{-4/3})},$$

where, trading T_p for $\tilde{T}_p = T_p + x T_s$, we have:

$$\begin{aligned}
 \mathcal{L}_{\text{kin}}^{\mathcal{O}(1)} &= \frac{3}{4\tau_b^2} \partial_\mu \tau_b \partial^\mu \tau_b, \\
 \mathcal{L}_{\text{kin}}^{\mathcal{O}(\mathcal{V}^{-1})} &= \frac{3}{8\mathcal{V}} \left[\frac{\lambda_s}{\sqrt{\tau_s}} (\partial_\mu \tau_s \partial^\mu \tau_s + \partial_\mu c_s \partial^\mu c_s) + \frac{\lambda_p}{\sqrt{\tilde{\tau}_p}} (\partial_\mu \tilde{\tau}_p \partial^\mu \tilde{\tau}_p + \partial_\mu \tilde{c}_p \partial^\mu \tilde{c}_p) \right] \\
 &\quad - \frac{9}{4\mathcal{V}} \frac{\partial_\mu \tau_b}{\tau_b} \left(\lambda_s \sqrt{\tau_s} \partial^\mu \tau_s + \lambda_p \sqrt{\tilde{\tau}_p} \partial^\mu \tilde{\tau}_p \right), \\
 \mathcal{L}_{\text{kin}}^{\mathcal{O}(\mathcal{V}^{-4/3})} &= \frac{3}{4\tau_b^2} \partial_\mu c_b \partial^\mu c_b.
 \end{aligned}$$

At leading order the kinetic terms become canonical if τ_b is replaced by ϕ_b defined as:

$$\frac{\phi_b}{M_p} = \sqrt{\frac{3}{2}} \ln \tau_b, \quad (\text{A.10})$$

whereas $\mathcal{L}_{\text{kin}}^{\mathcal{O}(\mathcal{V}^{-1})}$ becomes diagonal if the small modulus T_s and the Wilson modulus \tilde{T}_p are substituted by:

$$\begin{aligned}
 \frac{\phi_s}{M_p} &= \sqrt{\frac{4\lambda_s}{3\mathcal{V}}} \tau_s^{3/4}, & \frac{a_s}{M_p} &= \sqrt{\frac{3\lambda_s}{4\mathcal{V}\sqrt{\tau_s}}} c_s, \\
 \frac{\tilde{\phi}_p}{M_p} &= \sqrt{\frac{4\lambda_p}{3\mathcal{V}}} \tilde{\tau}_p^{3/4}, & \frac{\tilde{a}_p}{M_p} &= \sqrt{\frac{3\lambda_p}{4\mathcal{V}\sqrt{\tilde{\tau}_p}}} \tilde{c}_p,
 \end{aligned} \quad (\text{A.11})$$

and the canonical normalisation (A.10) for τ_b gets modified by the inclusion of a subleading mixing with τ_s and $\tilde{\tau}_p$ of the form:

$$\frac{\phi_b}{M_p} = \sqrt{\frac{3}{2}} \ln \tau_b - \sqrt{\frac{2}{3}} \frac{1}{\mathcal{V}} \left(\lambda_s \tau_s^{3/2} + \lambda_p \tilde{\tau}_p^{3/2} \right). \quad (\text{A.12})$$

Finally the kinetic term in $\mathcal{L}_{\text{kin}}^{\mathcal{O}(\mathcal{V}^{-4/3})}$ are canonically normalised if the bulk axion c_b gets redefined as:

$$\frac{a_b}{M_p} = \sqrt{\frac{3}{2}} \frac{c_b}{\tau_b}. \quad (\text{A.13})$$

The U(1)-charged open string mode C appears in the kinetic Lagrangian only at $\mathcal{O}(|C|^2 \mathcal{V}^{-2/3})$ which according to (3.17) and (3.18) can scale as either $\mathcal{V}^{-8/3}$ or $\mathcal{V}^{-14/3}$. This part of the kinetic Lagrangian looks like:

$$\mathcal{L}_{\text{kin}} \supset \tilde{K} |C|^2 \left(\frac{\partial_\mu |C|}{|C|} \frac{\partial^\mu |C|}{|C|} + \partial_\mu \theta \partial^\mu \theta - \frac{\partial_\mu \tau_b}{\tau_b} \frac{\partial^\mu |C|}{|C|} \right), \quad (\text{A.14})$$

and becomes diagonal by redefining:

$$\frac{|\hat{C}|}{M_p} = \sqrt{2\tilde{K}} |C| \quad \text{and} \quad a_{\text{ALP}} = |\hat{C}| \theta = f_{a_{\text{ALP}}} \theta.$$

A.3 Mass matrix

As described in section 3.1, the moduli stabilised at tree-level are τ_{q_i} and $|\phi|$ while the corresponding axions are eaten up by two anomalous U(1)'s. Given that they fixed at $\mathcal{O}(1/\mathcal{V}^2)$, all these modes develop a mass of order the string scale:

$$m_{\tau_{q_i}} \sim m_{c_{q_i}} \sim m_{|\phi|} \sim m_\psi \sim M_s = g_s^{1/4} \sqrt{\pi} \frac{M_p}{\sqrt{\mathcal{V}}}. \quad (\text{A.15})$$

On the other hand, τ_b , τ_s , $\tilde{\tau}_p$ and the closed string axion c_s are stabilised at $\mathcal{O}(1/\mathcal{V}^3)$. The masses of the corresponding canonically normalised fields derived in appendix A.2 are given by the eigenvalues of the mass matrix evaluated at the minimum of the $\mathcal{O}(1/\mathcal{V}^3)$ scalar potential. The leading order contributions of all the elements of this 4×4 matrix read:

$$\begin{aligned} \frac{\partial^2 V}{\partial \phi_b \partial \phi_b} &= \left(\frac{g_s}{8\pi} \right) \frac{9 \lambda_s \tau_s^{3/2} W_0^2}{2 \mathcal{V}^3}, \\ \frac{\partial^2 V}{\partial \phi_b \partial \phi_s} &= \left(\frac{g_s}{8\pi} \right) \frac{3 \sqrt{2} \lambda_s \tau_s^{3/4}}{\sqrt{\mathcal{V}}} \left(\frac{W_0}{\mathcal{V}} \right)^2 (2\pi \tau_s), \\ \frac{\partial^2 V}{\partial \phi_s \partial \phi_s} &= \frac{\partial^2 V}{\partial a_s \partial a_s} = 4 \left(\frac{g_s}{8\pi} \right) \left(\frac{W_0}{\mathcal{V}} \right)^2 (2\pi \tau_s)^2, \\ \frac{\partial^2 V}{\partial \tilde{\phi}_p \partial \tilde{\phi}_p} &= \left(\frac{g_s}{8\pi} \right) \frac{1}{4 z_p \tilde{\tau}_p} \left(\frac{W_0}{\mathcal{V}} \right)^2, \\ \frac{\partial^2 V}{\partial \phi_b \partial \tilde{\phi}_p} &= \frac{\partial^2 V}{\partial \phi_b \partial a_s} = \frac{\partial^2 V}{\partial \phi_s \partial \tilde{\phi}_p} = \frac{\partial^2 V}{\partial \phi_s \partial a_s} = \frac{\partial^2 V}{\partial \tilde{\phi}_p \partial a_s} = 0, \end{aligned}$$

The eigenvalues of this mass matrix turn out to be:

$$\begin{aligned} m_{\phi_s}^2 &= m_{a_s}^2 = 4 \left(\frac{g_s}{8\pi} \right) \left(\frac{W_0}{\mathcal{V}} \right)^2 (2\pi \tau_s)^2 \simeq m_{3/2}^2 (\ln \mathcal{V})^2, \\ m_{\tilde{\phi}_p}^2 &= \left(\frac{g_s}{8\pi} \right) \frac{\pi}{2 z_p} \left(\frac{W_0}{\mathcal{V}} \right)^2 \frac{1}{2\pi \tilde{\tau}_p} \simeq \frac{m_{3/2}^2}{\ln \mathcal{V}} \quad \text{and} \quad m_{\phi_b}^2 = 0, \end{aligned} \quad (\text{A.16})$$

where the gravitino mass is given by:

$$m_{3/2}^2 = e^K |W|^2 \simeq \left(\frac{g_s}{8\pi}\right) \left(\frac{W_0}{\mathcal{V}}\right)^2. \quad (\text{A.17})$$

The mass of the canonically normalised large modulus ϕ_b becomes non-zero once we include subleading $1/(2\pi\tau_s) \sim 1/\ln \mathcal{V} \ll 1$ corrections to the elements of the mass matrix, and scales as (with c an $\mathcal{O}(1)$ numerical coefficient):

$$m_{\phi_b}^2 = c \lambda_s \tau_s^{3/2} \left(\frac{g_s}{8\pi}\right) \frac{W_0^2}{\mathcal{V}^3} \frac{1}{2\pi\tau_s} \simeq \frac{m_{3/2}^2}{\mathcal{V} \ln \mathcal{V}}. \quad (\text{A.18})$$

As explained in section 3.3, the charged matter field $|C|$ is fixed by soft supersymmetry breaking contributions to the scalar potential and can acquire a mass of order $m_{3/2}/\sqrt{\mathcal{V}}$ or $m_{3/2}/\mathcal{V}$ depending on the level of sequestering. The corresponding phase $\theta = a_{\text{ALP}}/f_{a_{\text{ALP}}}$ behaves as an open string ALP which develops a mass of order:

$$m_{a_{\text{ALP}}} \sim \frac{\Lambda_{\text{hid}}^2}{f_{a_{\text{ALP}}}} \sim \frac{\Lambda_{\text{hid}}^2}{|\hat{C}|}, \quad (\text{A.19})$$

where Λ_{hid} is the scale of strong dynamics effects in the hidden sector. In order to obtain a phenomenologically viable value $m_{a_{\text{ALP}}} \lesssim 10^{-12} \text{ eV}$, we need to have $\Lambda_{\text{hid}} \lesssim 10^4 \text{ eV}$ if $f_{a_{\text{ALP}}} \sim m_{3/2} \sim 10^{10} \text{ GeV}$ or $\Lambda_{\text{hid}} \lesssim 1 \text{ eV}$ if $f_{a_{\text{ALP}}} \sim m_{3/2}/\mathcal{V} \sim 1 \text{ TeV}$.

The DM axion c_p is stabilised by tiny poly-instanton corrections at $\mathcal{O}(1/\mathcal{V}^{3+p})$. Using the fact that $K_{T_p \bar{T}_p}^{-1} \sim \mathcal{V} \sqrt{\tilde{\tau}_p}$ and the expression (3.19) for the scalar potential for c_p , its mass can be easily estimated as:

$$m_{\tilde{a}_p}^2 \sim K_{T_p \bar{T}_p}^{-1} \frac{\partial^2 V_F^{\text{poly}}(c_p)}{\partial c_p^2} \sim \left(\frac{g_s}{8\pi}\right) \frac{W_0^2}{\mathcal{V}^{2+p}} 2\pi \tilde{\tau}_p \sim \frac{m_{3/2}^2}{\mathcal{V}^p} \ln \mathcal{V}. \quad (\text{A.20})$$

If the volume is of order $\mathcal{V} \sim 10^7$, this mass can be around 10 keV if $p = 9/2$. As explained in section 4.2 this value of p can be accommodated by an appropriate choice of underlying flux parameters. Finally the axion c_b of the large modulus $T_b = \tau_b + i c_b$ can receive a potential only from highly suppressed non-perturbative contributions to the superpotential of the form $W_{\text{np}} \supset A_b e^{-2\pi T_b}$ which can be shown to lead to a mass for the axion c_b that scales as:

$$m_{a_b}^2 \sim \left(\frac{g_s}{8\pi}\right) \frac{M_p^2}{\mathcal{V}^{4/3}} e^{-\frac{2\pi}{\lambda_b^{2/3}} \mathcal{V}^{2/3}} \sim 0. \quad (\text{A.21})$$

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