A symmetry for ϵ_K

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ABSTRACT: We show a symmetry that, in the context of a composite Higgs with anarchic flavor, can suppress the dominant CP violating contributions to $K - \bar{K}$ mixing. Based on previous extensions of $SU(3)_c$, we consider the case in which the composite sector has a global $SU(6)$ symmetry, spontaneously broken to a subgroup containing $SU(3) \times SU(3)$. We show that the interactions with the Standard Model can spontaneously break the remaining symmetry to the diagonal subgroup, identified with the group of color interactions, and naturally suppress ϵ_K . We consider this scenario in the context of the Minimal Composite Higgs Model based on $SO(5)/SO(4)$ for the electroweak sector. By working in the framework of 2-site models, we compute the scalar potential, determine the conditions for a successful breaking of the symmetries and calculate the spectrum of lightest states. We find that ϵ_K can be suppressed and the top mass reproduced for a large region of the parameter space where the symmetries are dynamically broken. We also find a suppression of the Wilson coefficient of dipole operators, particularly interesting for the neutron dipole moments. Besides other new resonances, the model predicts the presence of a new singlet scalar state, generally lighter than the Higgs, that could have evaded detection at colliders.

KEYWORDS: Beyond Standard Model, Technicolor and Composite Models

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1 Introduction

The presence of a new strongly coupled field theory (SCFT) containing a Higgs-like resonance can provide a solution to the electroweak-Planck hierarchy problem as well as a rationale for the flavor hierarchies. One of the most attractive possibilities corresponds to the Higgs being a pseudo Nambu Goldstone Boson (pNGB) arising from the strong dynamics. Besides the Higgs, the SCFT provides new resonances at a scale of few TeV that cut off the momentum integrals in the Higgs potential. Generically, the presence of these states

induces corrections to the electroweak (EW) observables, that are in conflict with EW precision tests. Many of the constraints can be alleviated by the use of suitable symmetries [1, 2].

A key ingredient to obtain flavor hierarchies in composite Higgs models is partial compositeness [3, 4]. In partial compositeness hierarchies in the fermion masses can be easily obtained by assuming that the degree of compositeness of the fermions are hierarchical, without hierarchies or relations in the flavor structure of the SCFT. This assumption leads to non-universal couplings between the SM fermions and the resonances of the SCFT, such that virtual exchange of resonances induces flavor violating effects. Despite this violation, partial compositeness contains a built-in GIM-like mechanism suppressing flavor changing neutral currents (FCNC) [5, 6]. This setup is known as the anarchic approach to the flavor problem.

Although the GIM-like mechanism successfully suppresses most of the contributions to FCNC, the very stringent constraints from Kaon physics, in particular ϵ_K , push the scale of the composite resonances to $m_{cp} \sim 10 \div 30$ TeV, worsening the fine-tuning problem in the EW sector [7–9]. Some alternatives to avoid these constraints have been considered in the literature, such as the presence of flavor symmetries [7, 8] and hierarchies in the SCFT [10], as well as the presence of different dynamical scales [11]. These scenarios relax the constraints allowing m_{cp} to be lowered to a few TeV. In the context of anarchic flavor, the authors of ref. [12] have shown that enhancing the global symmetry of the SCFT from $SU(3)_c$ to $SU(3)_L \times SU(3)_R$, where the indices can be associated with the chiralities of the quarks, the scale can be lowered to $m_{cp} \sim 2$ TeV. In that scenario, masses for the SM quarks require an extension of the scalar sector of the theory, with new scalar fields charged under the extended symmetry. The masses of these states are expected to be of the same order as m_{cp} , introducing a sizable breaking of the enlarged symmetry and spoiling the mechanism protecting ϵ_K , unless a fine-tuning of several orders of magnitude is introduced [13].

In the present paper we will elaborate on the possibility of protecting ϵ_K from large corrections by considering an extension of $SU(3)_c$, within the context of an anarchic SCFT. We will work along the lines of ref. [12], but making the new scalar a pNGB, based on a global symmetry $SU(6)$ spontaneously broken by the strong dynamics to a subgroup containing $SU(3)_L \times SU(3)_R$. Besides this extension of the color symmetry group, we add the usual $SO(5) \times U(1)_X$ global symmetry that allows us to describe also the Higgs field as a pNGB. The interactions between the SCFT and the SM gauge and fermion fields generate a potential for both scalars at one-loop. The potential can trigger electroweak symmetry breaking (EWSB) and it can also spontaneously break $SU(3)_L \times SU(3)_R$ to the diagonal subgroup that is aligned with $SU(3)_c$. The size of the spontaneous breaking can be described in terms of the parameters ϵ_5 and ϵ_6 , that take values $0 \leq \epsilon_{5,6} \leq 1$, and parametrize the direction of the vacuum in the groups $SO(5)$ and $SU(6)$, respectively. As in the usual Minimal Composite Higgs Model (MCHM): $v_{SM} = \epsilon_5 f_5$, with f_5 the decay constant of the pNGB Higgs [14]. After spontaneous breaking of both symmetries fermion masses are successfully generated, leading to a realistic model. Quark masses, requiring breaking of both symmetries, are proportional to $\epsilon_5 \epsilon_6$. The dominant effect in ϵ_K is given by a contribution to the Wilson coefficient C_4 that, compared with the case without extended symmetry, is suppressed by $\epsilon_6^2(1 - \epsilon_6^2)$. In general, factors of $\epsilon_6 \sqrt{1 - \epsilon_6^2}$ are expected in operators with LR chiral structure, as for example magnetic dipole moments.

The values of ϵ_5 and ϵ_6 depend on the parameters of the theory, as well as on the representations of the fermions under $\text{SO}(5)$ and $\text{SU}(6)$. The representations of $\text{SU}(6)$ are fixed to cancel the dominant contributions to ϵ_K , according to [12]. We discuss the embedding into $\text{SO}(5)$ representations, and show that a realistic case is obtained for q_L and u_R embedded in the representations 14 and 1 of $\text{SO}(5)$ respectively. With this choice the potential can easily trigger spontaneous breaking of both symmetries, generating masses for the SM fields.

We use a very simple description of the above dynamics by considering a 2-site model; that is, a four dimensional theory containing the first level of resonances of the SCFT [15–18]. Within this framework we are able to calculate the one-loop Coleman-Weinberg potential that is dominated by the top and gauge contributions. We compute ϵ_5 and ϵ_6 , as well as the masses and mixing angle of the light scalars: the Higgs and the new neutral state. We find that, taking $f \simeq 1 \text{ TeV}$, the Higgs is usually somewhat heavier than 125 GeV, as expected in models with fermions in the 14. The other scalar is usually lighter, with a mass of order $50 \div 100 \text{ GeV}$ due to a suppressed quartic coupling, whose size is determined by the embedding of the quarks into the $\text{SU}(6)$ symmetry.

The paper is organized as follows: in section 2 we describe the symmetries of the theory assuming that the Higgs and the new scalar are pNGBs of a SCFT; we show the effective theory that is obtained after the heavy resonances of the SCFT are integrated out, codifying their effects in terms of a set of form factors. In section 3 we compute the one-loop potential by using the form factors of the effective theory, we analyse the conditions for spontaneous symmetry breaking and the spectrum of light scalar states; we also discuss the dependence of the potential with the $\text{SO}(5)$ embedding of the fermions. In section 4 we describe a 2-site model that provides a calculable framework for the above dynamics and we compute the contributions to some flavor observables. In section 5 we show numerical results for several interesting physical quantities, namely the spectrum of light states and the regions of the parameter space with spontaneous breaking of the symmetries. In section 6 we discuss some important issues, such as the neutron dipole moments, the degree of tuning of the model and the phenomenology of the new states at the LHC.

2 A symmetry for ϵ_K

We consider a new strongly interacting sector beyond the SM. It has a global symmetry $\text{SO}(5) \times \text{U}(1)_X \times \text{SU}(6)$.¹ The interactions produce bound states with masses of order TeV and spontaneously break the global symmetry to $\text{SO}(4) \times \text{U}(1)_X \times \text{SU}(3)_L \times \text{SU}(3)_R \times \text{U}(1)$. A set of NGBs Π_5 and Π_6 transforming non-linearly under the global $\text{SO}(5)$ and $\text{SU}(6)$, respectively, emerge from this breaking. The field Π_5 has the proper quantum numbers to be identified with the Higgs multiplet. The field Π_6 results in a new scalar field, with some components charged under the color interactions of the SM. The SM fermions and gauge bosons are elementary fields, and can be thought of as external sources probing the strong dynamics. Depending on the orientation of the vacuum compared to the SM gauge symmetry $\text{SU}(2)_L \times \text{U}(1)_Y \times \text{SU}(3)_c$, the EW and color symmetries can be broken or not.

¹As is well known, the extra $\text{U}(1)_X$ factor is required to obtain the proper normalization of hypercharge in the fermionic sector.

This direction is determined dynamically by the potential generated at loop level from the interactions between the SM fields and the strongly interacting sector that explicitly violate the global symmetry. The presence of this potential turns Π_5 and Π_6 into pNGBs.

The interactions between the SM gauge fields and the strongly interacting sector are introduced by the gauging of a subgroup of the global symmetry; in this way the elementary gauge fields are coupled to a subset of the conserved currents of the composite sector. The elementary fermions are coupled linearly with the strongly interacting sector, realizing partial compositeness.

To shorten the notation we define $G_6 \equiv \text{SU}(6)$ and $G_5 \equiv \text{SO}(5) \times \text{U}(1)_X$, and for the subgroups $H_6 \equiv \text{SU}(3)_L \times \text{SU}(3)_R \times \text{U}(1)$ and $H_5 \equiv \text{SO}(4) \times \text{U}(1)_X$. A possible orientation of the vacuum in the direction of G_6 can be described by the following matrix:

$$\Omega_0 = \begin{pmatrix} I_3 & 0 \\ 0 & -I_3 \end{pmatrix} \quad (2.1)$$

where I_3 is the identity matrix in three dimensions. Under G_6 this vacuum transforms as: $g_6 \Omega_0 g_6^\dagger$, with g_6 an element of G_6 in the fundamental representation. This vacuum preserves an H_6 subgroup. We identify $\text{SU}(3)_c$ with the diagonal subgroup $\text{SU}(3)_V$ contained in H_6 . Therefore, expanding around this vacuum one can classify the fields according to their representation properties under the SM group of strong interactions.

The NGBs Π_6 are given by a local element of G_6 of the form:

$$U_6 = e^{i\sqrt{2}\Pi_6/f_6}, \quad \Pi_6 = \Pi_6^{\hat{b}} T_6^{\hat{b}}, \quad (2.2)$$

with $T_6^{\hat{b}}$ the broken generators of G_6 . We will use the subindex 6 for elements related with this group, to distinguish them from the elements associated to G_5 . Only elements of the form U_6 transform the vacuum Ω_0 . U_6 transforms non-linearly under G_6 , as: $g_6 U_6 h_6^\dagger(g_6, \Pi_6)$, with $h_6 \in H_6$. Π_6 transforms linearly under H_6 , as a complex $(\bar{3}, 3)_{1/\sqrt{3}}$. We find it useful to define the matrix $\Phi_6 = \Pi_6|_{\text{fund}}$, where the subindex indicates that the generators $T_6^{\hat{b}}$ are in the fundamental representation. For the suitable basis defined in appendix A, Φ_6 can be written as:

$$\Phi_6 = \begin{pmatrix} 0 & \Phi_{(\bar{3},3)} \\ \Phi_{(\bar{3},3)}^\dagger & 0 \end{pmatrix} \quad (2.3)$$

with $\Phi_{(\bar{3},3)}$ a 3×3 complex matrix with eighteen real degrees of freedom, describing the NGB fields.

A representative orientation of the vacuum in the direction of G_5 can be parametrized by the vector [14]

$$\Sigma_0 = (0, 0, 0, 0, 1). \quad (2.4)$$

The vacuum Σ_0 transforms as: $g_5 \Sigma_0$, with g_5 an element of $\text{SO}(5)$ in the fundamental representation, Σ_0 is invariant under an H_5 subgroup. We identify the generators of the EW symmetry of the SM with a subset of the generators of H_5 . Expanding around this vacuum all the fields can be classified according to their representations under the EW gauge symmetry.

The NGB of G_5/H_5 are given by:

$$U_5 = e^{i\sqrt{2}\Pi_5/f_5}, \quad \Pi_5 = \Pi_5^{\hat{a}} T_5^{\hat{a}} \quad (2.5)$$

with $T_5^{\hat{a}}$ the broken generators of G_5 . In this case we use the subindex 5 for elements related with G_5 . Only elements of the form U_5 transform the vacuum. U_5 transforms non-linearly under G_5 , as: $g_5 U_5 h_5^\dagger(g_5, \Pi_5)$, with $h_5 \in H_5$, whereas Π_5 transforms with the fundamental representation $(2, 2)_0$ of H_5 . Similar to the case of Φ_6 , we define the matrix $\Phi_5 = \Pi_5|_{\text{fund}}$, with the generators $T_5^{\hat{a}}$ in the fundamental representation.

As we will show in section 3, virtual exchange of elementary fields can misalign the vacuum at loop level, breaking H_5 to $K_5 \equiv \text{SO}(3)$ and H_6 to $K_6 \equiv \text{SU}(3)_V$. Under K_5 the NGB Π_5 decomposes as: $(2, 2) \sim 1 \oplus 3$, whereas under K_6 the NGB Π_6 decomposes as: $(\bar{3}, 3) \sim 1 \oplus 8$, with the singlet and octet being complex fields. The singlets of Π_5 and Π_6 under K_5 and K_6 will be called h_5 and $h_6 e^{i\theta_6}$, respectively, and we define:

$$s_5 = \sin(h_5/f_5), \quad s_6 = \sin(h_6/\sqrt{6}f_6), \quad (2.6)$$

$$\epsilon_5 = \sin(v_5/f_5), \quad \epsilon_6 = \sin(v_6/\sqrt{6}f_6), \quad (2.7)$$

with $v_5 = \langle h_5 \rangle$ and $v_6 = \langle h_6 \rangle$ the vacuum expectation values of the singlets. By using these definitions the vacuum can be characterized by the following objects:

$$\Sigma_v = (0, 0, 0, \epsilon_5, \sqrt{1 - \epsilon_5^2}), \quad \Omega_v = \begin{pmatrix} (1 - 2\epsilon_6^2)I_3 & -i2\epsilon_6\sqrt{1 - \epsilon_6^2}I_3 \\ i2\epsilon_6\sqrt{1 - \epsilon_6^2}I_3 & -(1 - 2\epsilon_6^2)I_3 \end{pmatrix}. \quad (2.8)$$

As usual in models of $\text{SO}(5)/\text{SO}(4)$, ϵ_5 measures the misalignment of the vacuum, with no EWSB for $\epsilon_5 = 0$ and maximal breaking for $\epsilon_5 = 1$. ϵ_6 parametrizes the direction of H_6 inside G_6 . As we will discuss in the next section, the elementary fermions have well defined transformation properties under $\text{SU}(3)_L \times \text{SU}(3)_R \times \text{U}(1)$, thus the interactions between them and the SCFT select one H_6 inside G_6 , parametrized by $\epsilon_6 = 0$. For $\epsilon_6 > 0$ there is a misalignment of the H_6 preserved by the SCFT compared with the elementary one, whereas for $\epsilon_6 = 1$: $\text{SU}(3)_L \rightarrow \text{SU}(3)_R$ and $\text{SU}(3)_R \rightarrow \text{SU}(3)_L$. Notice that $\text{SU}(3)_c$ is always contained in H_6 , as the diagonal subgroup is invariant under the transformations that rotate Ω_0 to Ω_v .

2.1 Effective theory

Integrating out the heavy degrees of freedom of the strongly interacting sector one can obtain an effective theory for the elementary degrees of freedom and the NGBs. We find it useful to add spurious degrees of freedom to the elementary sector to extend its symmetry to $G_5 \times G_6$. Therefore one can write an effective Lagrangian that is formally invariant under this extended symmetry. In this procedure one has to choose the representations of the elementary fermions. We choose the fundamental representation of G_6 for the quarks, which under H_6 decomposes as: $6 \sim (3, 1)_{1/2\sqrt{3}} \oplus (1, 3)_{-1/2\sqrt{3}}$. We choose $(3, 1)$ for q_L and $(1, 3)$ for q_R to suppress the main contribution to ϵ_K [12]. The representations of G_5 are not fixed, the only constraint being that they have to contain the proper doublets and singlets of $\text{SU}(2)_L$ to match with the SM fields. As is well known, the choice of the representations

of G_5 has important consequences for the phenomenology; we will discuss their impact in the present model in section 3.

As usual, due to the non-linear transformation properties of the NGB, a $G_5 \times G_6$ invariant Lagrangian can be obtained by dressing the elementary fields with the NGB matrices U_5 and U_6 , and building with them a Lagrangian that superficially looks invariant under $H_5 \times H_6$.

Calling R a representation of G_5 , R decomposes under H_5 as $R \sim \oplus_j r_j$. Similarly, calling S a representation of G_6 , S decomposes under H_6 as $S \sim \oplus_k s_k$. Therefore an elementary fermion ψ in the representation R of G_5 and S of G_6 , decomposes under $H_5 \times H_6$ as: $\psi \sim \oplus_{r,s} \psi_{rs}$. Dressing ψ with the NGB matrices usually leads to fields that transform with reducible representations, therefore the building blocks for the effective theory are the projections of the dressed fields into the irreducible representations of the unbroken subgroup: $\tilde{\psi}_{rs} = (U_5^\dagger U_6^\dagger \psi)_{rs}$, where the generators contained in U_5 are in the representation R of G_5 and the generators contained in U_6 are in the fundamental representation of G_6 , as discussed previously. $G_5 \times G_6$ invariant operators can be obtained by considering products of factors $\tilde{\psi}_{rs}$, and selecting from these products the invariants of $H_5 \times H_6$. Using these objects, at the quadratic level in the elementary fields, the effective Lagrangian for the quarks is:

$$\begin{aligned} \mathcal{L}_{\text{eff}} \supset & \sum_{f=q,u,d} Z_f \bar{\psi}_f \not{p} \psi_f + \sum_{f=q,u,d} \sum_{r,s} \overline{(U_5^\dagger U_6^\dagger \psi_f)_{rs}} \not{p} \Pi_{rs}^f(p^2) (U_5^\dagger U_6^\dagger \psi_f)_{rs} \\ & + \sum_{f=u,d} \sum_{r,s} \overline{(U_5^\dagger U_6^\dagger \psi_q)_{rs}} M_{rs}^f(p^2) (U_5^\dagger U_6^\dagger \psi_f)_{rs} + \text{h.c.}, \end{aligned} \quad (2.9)$$

where Z_f are the factors of the elementary kinetic terms. $\Pi_{rs}^f(p^2)$ and $M_{rs}^f(p^2)$ are momentum dependent form factors that contain the information on the composite degrees of freedom; they are independent of the NGB fields. The components of the different factors of the dressed fermions $\tilde{\psi}_{rs}$ in eq. (2.9) are contracted in the usual way to obtain $H_5 \times H_6$ invariants. Generation indices are understood in eq. (2.9), and in equations below.

For the gauge sector, it is useful to notice that the adjoint representation of $SO(5)$ decomposes under $SO(4)$ as: $10 \sim \oplus_i t_i = (3, 1) \oplus (1, 3) \oplus (2, 2)$, whereas the adjoint of G_6 decomposes under H_6 as: $35 \sim \oplus_j u_j = (8, 1)_0 \oplus (1, 8)_0 \oplus (\bar{3}, 3)_{1/\sqrt{3}} \oplus (1, 1)_0$. Calling x_μ , a_μ and g_μ the fields of $U(1)_x$, $SO(5)$ and G_6 , respectively, the quadratic effective Lagrangian for the elementary gauge fields is:

$$\mathcal{L}_{\text{eff}} \supset \frac{1}{2} P^{\mu\nu} \left[\Pi^X(p^2) x_\mu x_\nu + \sum_t \Pi_t^A(p^2) (U_5^\dagger a_\mu)_t (U_5^\dagger a_\nu)_t + \sum_u \Pi_u^G(p^2) (U_6^\dagger g_\mu)_u (U_6^\dagger g_\nu)_u \right], \quad (2.10)$$

with $P_{\mu\nu} = \eta_{\mu\nu} - p_\mu p_\nu / p^2$, U_5 and U_6 in the adjoint representation, and t and u running over the decompositions of each adjoint under the subgroups as presented in the previous paragraph. There are also elementary kinetic terms for the gauge fields that were not written in eq. (2.10). The form factors $\Pi(p^2)$ encode the effects of the strongly coupled sector and are independent of the NGB fields.

There are also kinetic terms for the NGB fields that can be constructed, as usual, by making use of the Maurer-Cartan form d_μ defined from: $iU^\dagger D_\mu U = e_\mu^a T^a + d_{\mu\hat{a}} T^{\hat{a}} \equiv e_\mu + d_\mu$.

There is a d_μ^5 for the NGBs Π_5 and a d_μ^6 for the NGBs Π_6 . The NGBs kinetic terms are given by:

$$\mathcal{L}_{\text{eff}} \supset \frac{f_5^2}{4} d_{5,\mu}^{\hat{a}} d_5^{\hat{a},\mu} + \frac{f_6^2}{4} d_{6,\mu}^{\hat{b}} d_6^{\hat{b},\mu}. \quad (2.11)$$

Keeping only the dynamical fermions: $q_L = (u_L, d_L)$ contained in ψ_q , as well as u_R and d_R contained in ψ_u and ψ_d , eq. (2.9) leads to:

$$\begin{aligned} \mathcal{L}_{\text{eff}} \supset & \bar{q}_L \not{p} \Pi_q(\Pi_5, \Pi_6, p^2) q_L + \bar{u}_R \not{p} \Pi_u(\Pi_5, \Pi_6, p^2) u_R + \bar{d}_R \not{p} \Pi_d(\Pi_5, \Pi_6, p^2) d_R \\ & + \bar{q}_L M_u(\Pi_5, \Pi_6, p^2) u_R + \bar{q}_L M_d(\Pi_5, \Pi_6, p^2) d_R + \text{h.c.} \end{aligned} \quad (2.12)$$

Since $\Pi(\Pi_5, \Pi_6, p^2)$ and $M(\Pi_5, \Pi_6, p^2)$ depend on momentum and on the NGBs, they have non-trivial indices under $\text{SU}(3)_c \times \text{SU}(2)_L$. Using the form factors defined in eq. (2.9), they can be written as:

$$\Pi_f(\Pi_5, \Pi_6, p^2) = Z_f + \sum_{r,s} \Pi_{rs}^f(p^2) F_{rs}^f(\Pi_5, \Pi_6), \quad f = q, u, d, \quad (2.13)$$

$$M_f(\Pi_5, \Pi_6, p^2) = \sum_{r,s} M_{rs}^f(p^2) G_{rs}^f(\Pi_5, \Pi_6), \quad f = u, d, \quad (2.14)$$

where all the momentum dependence is codified in the form factors defined in eq. (2.9), and the NGB dependence is contained in the functions F_{rs}^f and G_{rs}^f . These functions can be obtained from the invariants built with the dressed quarks:

$$\begin{aligned} [F_{rs}^q(\Pi_5, \Pi_6)]_{\alpha\beta ij} &= \partial_{\bar{q}_L^{\alpha i}} \partial_{q_L^{\beta j}} \left[\overline{(U_5^\dagger U_6^\dagger \psi_q)_{rs}} (U_5^\dagger U_6^\dagger \psi_q)_{rs} \right], \\ [F_{rs}^f(\Pi_5, \Pi_6)]_{\alpha\beta} &= \partial_{\bar{f}_R^\alpha} \partial_{f_R^\beta} \left[\overline{(U_5^\dagger U_6^\dagger \psi_f)_{rs}} (U_5^\dagger U_6^\dagger \psi_f)_{rs} \right], \quad f = u, d, \\ [G_{rs}^f(\Pi_5, \Pi_6)]_{\alpha ij} &= \partial_{\bar{q}_L^{\alpha i}} \partial_{f_R^\beta} \left[\overline{(U_5^\dagger U_6^\dagger \psi_q)_{rs}} (U_5^\dagger U_6^\dagger \psi_f)_{rs} \right], \quad f = u, d, \end{aligned} \quad (2.15)$$

where $\alpha, \beta = 1, 2, 3$ are color indices, and $i, j = 1, 2$ are $\text{SU}(2)_L$ indices. It is straightforward to show that these functions factorize in one factor containing the Π_5 -dependence and another factor containing the Π_6 -dependence: $F_{rs}^f(\Pi_5, \Pi_6) = F_r^f(\Pi_5) F_s^f(\Pi_6)$ and similarly for G .

The dynamical gauge fields of QCD are associated to the generators of the diagonal subgroup $T_6^{V,a} \equiv T_6^{L,a} + T_6^{R,a}$, for $a = 1, \dots, 8$. In this case eq. (2.10) leads to:

$$\mathcal{L}_{\text{eff}} \supset \frac{1}{2} P^{\mu\nu} g_\mu^{V,a} \Pi_G^{ab}(\Pi_6, p^2) g_\nu^{V,b}, \quad (2.16)$$

where $\Pi_G(\Pi_6, p^2)$ is given by:

$$\Pi_G(\Pi_6, p^2) = Z_6 + \sum_s \Pi_s^G(p^2) F_s^G(\Pi_6), \quad (2.17)$$

with $Z_6 = g_{6,0}^{-2}$ being the coefficient of the elementary kinetic term, and

$$[F_s^G(\Pi_6)]_{ab} = \partial_{g^{V,a}} \partial_{g^{V,b}} \left[(U_6^\dagger g^V)_s (U_6^\dagger g^V)_s \right]. \quad (2.18)$$

A similar description can be done for the EW sector [14, 19].

In order to obtain the effective Lagrangian at quadratic level, it is useful to evaluate the scalar fields to their vev in eq. (2.12). In this case the functions F_{rs}^f of eq. (2.13) are proportional to the identity in color space,² thus they are color independent, as are the form factors. The functions G_{rs}^f of eq. (2.14) split into two 3×3 blocks, one proportional to the identity and the other vanishing. Moreover, the NGB matrices U_5 and U_6 can be resummed, leading to matrices that can be expressed as functions of ϵ_5 and ϵ_6 . Taking into account the representations of the quarks under G_6 and H_6 , and for generic representations of G_5 and H_5 , the form factors can be written as:

$$\begin{aligned}\Pi_{f_L}(p^2) &= Z_q + \sum_r \Pi_{r31}^q(p^2) F_r^{fL}(\epsilon_5) + \epsilon_6^2 \sum_r [\Pi_{r13}^q(p^2) - \Pi_{r31}^q(p^2)] F_r^{fL}(\epsilon_5), \\ \Pi_{f_R}(p^2) &= Z_f + \sum_r \Pi_{r13}^f(p^2) F_r^{fR}(\epsilon_5) + \epsilon_6^2 \sum_r [\Pi_{r31}^f(p^2) - \Pi_{r13}^f(p^2)] F_r^{fR}(\epsilon_5), \\ \Pi_{f_L f_R}(p^2) &= -i\epsilon_6 \sqrt{1 - \epsilon_6^2} \sum_r [M_{r31}^f(p^2) - M_{r13}^f(p^2)] G_r^{fL f_R}(\epsilon_5), \quad f = u, d, \quad (2.19)\end{aligned}$$

where 31 and 13 label the H_6 representations (3, 1) and (1, 3), respectively, omitting the parenthesis to simplify the notation. The functions $F_r(s_5)$ can be obtained from eqs. (2.13) and (2.14), and have been computed for several representations in the literature, see for example refs. [19–22] and appendix C.

As usual, the spectrum of fermions that mix with the SM ones corresponds to the zeroes of the following function:

$$\text{Zeroes}[\Pi_{f_L}(p^2)\Pi_{f_R}(p^2) - |\Pi_{f_L f_R}(p^2)|^2]. \quad (2.20)$$

The zero of each function that is closest to the origin gives the mass of the corresponding SM fermion.

2.2 Partial compositeness and flavor

Besides the group theoretical structure of the correlators, another useful piece of information arises from the hypothesis of partial compositeness [3, 4]. We define ϵ_f as the degree of compositeness of the fermions, with $\epsilon_f \ll 1$ for mostly elementary fermions and $\epsilon_f \sim 1$ for partially composite fermions [15]. Thus, to leading order in ϵ_f : $\Pi^f \propto \epsilon_f^2$ and $M^f \propto \epsilon_q \epsilon_f$.

There are two sources of flavor structure: the structure of the SCFT and the structure of partial compositeness contained in ϵ_f . Flavor anarchy corresponds to the assumption of anarchic flavor structure of the SCFT, meaning that there are no hierarchies in the couplings and masses of this sector. As is well known, in this case the hierarchies between the quark masses can be obtained by hierarchical degrees of compositeness ϵ_f . In addition, this mechanism can also lead to a realistic V_{CKM} . The hierarchy in ϵ_f can be naturally obtained in five dimensional models [4, 5, 23] and in cases where the SCFT has an approximate conformal symmetry at high energies [24, 25]. We will show more details in section 4.

² Π_q splits into four 3×3 blocks, with those in the diagonal proportional to the identity and the others vanishing.

A useful approximate expression for the masses of the SM fermions can be derived from the effective Lagrangian [14]. By canonically normalizing the kinetic term of the fermions and evaluating the scalars to their vev, we obtain:

$$m_f \simeq \frac{|\Pi_{f_L f_R}(0)|}{\left[\Pi_{f_L}(0) \Pi_{f_R}(0) - 2 |\Pi_{f_L f_R}(0)| \left. \frac{d|\Pi_{f_L f_R}(p^2)|}{dp^2} \right|_{p^2=0} \right]^{1/2}}, \quad (2.21)$$

where Π_{f_L} , Π_{f_R} and $\Pi_{f_L f_R}$ have been defined in eq. (2.19). Notice that, as expected, besides EWSB fermion masses require breaking of H_6 . Also, for $\epsilon_6 = 1$ the fermion masses vanish. Partial compositeness shows us that, to leading order in the mixing, the masses are of order

$$m_f \sim \epsilon_6 \sqrt{1 - \epsilon_6^2} G_r^{f_L f_R}(\epsilon_5) \epsilon_q m_r \epsilon_f, \quad (2.22)$$

with m_r the mass scale of the fermionic resonances of the SCFT. All the group structure is contained in the first three factors, whereas the flavor structure is contained in $\epsilon_q \epsilon_f m_r$. Comparing with the usual result in composite Higgs models, there is an extra factor $\epsilon_6 \sqrt{1 - \epsilon_6^2}$ that arises from the extended symmetry G_6 .

The Yukawa couplings with the scalars h_5 and h_6 can be computed by taking the derivative of m_f , as approximated in eq. (2.21), with respect to v_5 and v_6 , respectively:

$$y_{f,5} \simeq \frac{\partial m_f}{\partial v_5}, \quad y_{f,6} \simeq \frac{\partial m_f}{\partial v_6}. \quad (2.23)$$

The dependence of the Yukawa couplings on the parameters of the theory is encoded in the form factors, but the dependence on the vev's of the scalar fields is simple, it is encoded in ϵ_6 and in the functions $F_r(\epsilon_5)$ and $G_r(\epsilon_5)$ of eq. (2.19). These functions are straightforward to compute once the embedding of the fermions on G_5 is chosen, see appendix C. A good estimate of the size of the Yukawa couplings can be obtained by expanding eq. (2.23) to leading order in powers of $1/Z_f$, similar to an expansion in powers of the fermion mixing ϵ_f :

$$y_{f,5} \simeq m_f \frac{\sqrt{1 - \epsilon_5^2}}{f_5} \frac{d}{d\epsilon_5} \log \sum_r [M_{r31}^f(0) - M_{r13}^f(0)] G_r^{f_L f_R}(\epsilon_5), \quad (2.24)$$

$$y_{f,6} \simeq m_f \frac{1}{\sqrt{6} f_6} \frac{1 - 2\epsilon_6^2}{\epsilon_6 \sqrt{1 - \epsilon_6^2}}. \quad (2.25)$$

A non-vanishing $y_{f,5}$ requires $0 < \epsilon_6 < 1$. In the cases with just one non-trivial invariant function G_r , the dependence of the derivative on the form factors cancels out, and $y_{f,5}$ depends on the microscopic parameters of the theory only through ϵ_5 and the combination m_f/f_5 . The predictions for $y_{f,5}$ in the case of representations with custodial symmetry of $Z b_L \bar{b}_L$ have been reported in refs. [21, 22]. On the other hand, a non-vanishing $y_{f,6}$ requires $\epsilon_5 > 0$. Also, $y_{f,6}$ adopts a very simple form in this expansion, since all the dependence on the microscopic parameters of the theory has been absorbed in m_f/f_6 , and the dependence on ϵ_6 is shown explicitly. This dependence is determined by the embedding of the quarks in the extended symmetry G_6 .

We are also interested in contributions to the Wilson coefficients of four-fermion operators, in particular to the operator $\mathcal{O}_4 = (\bar{\psi}_q \psi_d)(\bar{\psi}_d \psi_q)$. However, instead of keeping track of the full NGB dependence, we are interested only in the coefficient evaluated in the vacuum of eq. (2.8), and keeping the dynamical fermions only. We obtain:

$$\mathcal{L}_{\text{eff}} \supset \epsilon_6^2 (1 - \epsilon_6^2) \Pi_{f_L f_R}^{(4)}(p^2) (\bar{f}_L f_R) (\bar{f}_R f_L), \quad (2.26)$$

where $\Pi_{f_L f_R}^{(4)}(p^2)$ is a form factor independent of ϵ_5 and ϵ_6 , and where flavor indices are understood. The Wilson coefficient C_4 is

$$C_4 = \epsilon_6^2 (1 - \epsilon_6^2) \Pi_{f_L f_R}^{(4)}(0). \quad (2.27)$$

Partial compositeness implies that $\Pi_{f_L f_R}^{(4)}$, and consequently also C_4 , is proportional to $(\epsilon_{f_L} \epsilon_{f_R})^2$. Eq. (2.27) must be compared with the usual result in composite Higgs models, without the extended symmetry G_6 , where the factor $\epsilon_6^2 (1 - \epsilon_6^2)$ is absent. In that case the bounds on the scale of the masses of the composite resonances arising from flavor physics are $m_{cp} \gtrsim 10 \div 30 \text{ TeV}$ [9]. In the present case, the extended symmetry of the SCFT gives an extra suppression factor to the contribution to C_4 , that can alleviate this bound. Up to corrections of numerical factors of $\mathcal{O}(1)$, that depend on the specific realization of the SCFT, for $\epsilon_6 \sqrt{1 - \epsilon_6^2} \simeq 0.1 \div 0.3$, the scale can be as low as $m_{cp} \sim 3 \text{ TeV}$. Moreover, as we will show in section 4, in 2-site models there is an extra factor 1/2 that relaxes $\epsilon_6 \sqrt{1 - \epsilon_6^2} \lesssim 0.2 \div 0.5$ for $m_{cp} \sim 3 \text{ TeV}$. Notice that not only a small ϵ_6 can suppress the contribution to C_4 , also for large ϵ_6 , near one, there can be a suppression.

The same factor involved in C_4 : $\epsilon_6 \sqrt{1 - \epsilon_6^2}$, suppresses the fermion masses, see eq. (2.22). Thus there is a tension between the large top mass and a small C_4 . We will elaborate more on this topic in section 4.

3 Radiative potential

The interactions between the fermions of the SM and the strongly interacting sector explicitly break $G_5 \times G_6$ to the subgroup $SU(2)_L \times U(1)_Y \times H_6$, generating a potential for the NGB fields at the one-loop level. This potential can misalign the vacuum and induce EW and H_6 spontaneous breaking. The interactions between the color gauge fields of the SM and the SCFT explicitly break G_6 to $SU(3)_V$, whereas the interactions with the EW gauge fields break G_5 to $SU(2)_L \times U(1)_Y$. In general the interactions with the gauge fields do not induce misalignment of the vacuum. Below we discuss the contributions of the fermions, as well as the contributions of the gauge sector with color. The contributions of the EW gauge fields have been discussed extensively; we refer the reader to the original articles on $SO(5)/SO(4)$ [14].

Let us start with the fermion contribution. We will consider only the effect of q_L and u_R of the third generation, since, having the largest mixing, they give the dominant contribution. We obtain:

$$V_f(\Pi_5, \Pi_6) = -2 \int \frac{d^4 p}{(2\pi)^4} \log \det \begin{pmatrix} \Pi_q & M_u \\ M_u^\dagger & \Pi_u \end{pmatrix}, \quad (3.1)$$

where, as shown in eqs. (2.13)–(2.15), the $SU(3)_c$ and $SU(2)_L$ indices of Π_q , Π_u and M_u , allow us to express them as 6×6 , 3×3 and 6×3 matrices, respectively.

Since the SM fermions are in full representations of $SU(3)_L \times SU(3)_R \times SU(2)_L \times U(1)_Y$, the one-loop potential can spontaneously break this symmetry. Expanding to fourth-order in powers of the NGB fields Π_5 and Π_6 , the most general potential is:

$$V_f(\Pi_5, \Pi_6) = \frac{1}{2}m_{f,5}^2\Pi_5^2 + \frac{1}{2}m_{f,6}^2\Pi_6^2 + \lambda_{f,5}\Pi_5^4 + \lambda_{f,6}(\Pi_6^4) + \lambda'_{f,6}(\Pi_6^4)' + \lambda_{f,56}\Pi_5^2\Pi_6^2 + \dots, \quad (3.2)$$

where the ellipsis stands for higher order terms. The indices of the NGB fields in the different products are contracted to form all the invariants allowed by the group structure, namely:

$$\Pi_5^2 = \text{tr}[\Phi_5^2] = \sum_a (\Pi_5^{\hat{a}})^2, \quad \Pi_5^4 = \text{tr}^2[\Phi_5^2] = \left[\sum_a (\Pi_5^{\hat{a}})^2 \right]^2, \quad (3.3)$$

$$\Pi_6^2 = \text{tr}[\Phi_6\Phi_6^\dagger] = 1/2 \sum_b (\Pi_6^{\hat{b}})^2, \quad (\Pi_6^4) = \text{tr}[\Phi_6\Phi_6^\dagger\Phi_6\Phi_6^\dagger], \quad (\Pi_6^4)' = \text{tr}^2[\Phi_6\Phi_6^\dagger]. \quad (3.4)$$

The quadratic and quartic coefficients can be expressed as integrals of the correlators of the effective theory. They depend on the representation chosen for the fermions.

For negative $m_{f,5}^2$ and $m_{f,6}^2$ and suitable quartic couplings, this potential can trigger spontaneous symmetry breaking of H_5 and H_6 .

Since the gauge interactions explicitly break G_6 to $SU(3)_V$, it is necessary to decompose the representation of Π_6 under this subgroup. Given that $(\bar{3}, 3) \sim 8 + 8 + 1 + 1$, we define: $\Phi_{(\bar{3},3)} \sim \phi_8 + I_3\phi_1$, where ϕ_8 and ϕ_1 are complex fields, and ϕ_1 can be identified with $h_6 e^{i\theta_6}$. In terms of these fields, the potential generated by virtual exchange of elementary gluons at one-loop is:

$$V_G(\Pi_6) = \int \frac{d^4p}{(2\pi)^4} \log \det \Pi_G, \quad (3.5)$$

where Π_G is an 8×8 matrix, as defined in eq. (2.17).

Expanding to fourth order in Π_6 :

$$\begin{aligned} V_G(\Pi_6) = & m_{g,1}^2\phi_1\phi_1^\dagger + m_{g,8}^2\text{tr}(\phi_8\phi_8^\dagger) + \lambda_{g,1}\text{tr}(\phi_8\phi_8^\dagger\phi_8\phi_8^\dagger) + \lambda_{g,2}\text{tr}^2(\phi_8\phi_8^\dagger) + \lambda_{g,3}\text{tr}(\phi_8^\dagger\phi_8^\dagger)\text{tr}(\phi_8\phi_8) \\ & + \lambda_{g,4}\text{tr}(\phi_8\phi_8^\dagger)\phi_1\phi_1^\dagger + \lambda_{g,5}\text{tr}(\phi_8^\dagger\phi_8^\dagger)(\phi_1)^2 + \lambda_{g,6}\text{tr}(\phi_8\phi_8)(\phi_1^\dagger)^2 + \lambda_{g,7}(\phi_1\phi_1^\dagger)^2 + \dots, \end{aligned} \quad (3.6)$$

where the ellipsis stands for higher order terms. Matching the quadratic and quartic coefficients with the expansion of eq. (3.5) we obtain:

$$\begin{aligned} m_{g,1}^2 &= 0, \quad \lambda_{g,7} = 0, \\ m_{g,8}^2 &= \frac{3}{f_6^2} \int_p \frac{2\Pi_{33}^G - \Pi_{81}^G - \Pi_{18}^G}{2Z_6 + \Pi_{81}^G + \Pi_{18}^G}, \\ \lambda_{g,1} &= -\frac{3}{2f_6^4} \int_p \frac{1}{(2Z_6 + \Pi_{81}^G + \Pi_{18}^G)^2} \left[(\Pi_{18}^G)^2 + 3(\Pi_{33}^G)^2 + (\Pi_{81}^G)^2 + (\Pi_{33}^G - 2\Pi_{11}^G)(\Pi_{18}^G + \Pi_{81}^G) \right. \\ &\quad \left. - \Pi_{18}^G\Pi_{81}^G + 2Z_6(\Pi_{81}^G + \Pi_{18}^G + 2\Pi_{11}^G - 4\Pi_{33}^G) \right], \end{aligned}$$

$$\begin{aligned}
 \lambda_{g,2} &= -\frac{1}{2f_6^4} \int_p \frac{1}{(2Z_6 + \Pi_{81}^G + \Pi_{18}^G)^2} \left[(21\Pi_{33}^G - 2\Pi_{11}^G)(\Pi_{81}^G + \Pi_{18}^G) - 33(\Pi_{33}^G)^2 - 5\Pi_{18}^G \Pi_{81}^G \right. \\
 &\quad \left. + 2Z_6(7\Pi_{81}^G + 7\Pi_{18}^G - 2\Pi_{11}^G - 12\Pi_{33}^G) \right], \\
 \lambda_{g,3} &= -\frac{1}{4f_6^4} \int_p \frac{1}{(2Z_6 + \Pi_{81}^G + \Pi_{18}^G)^2} \left[6(\Pi_{18}^G)^2 + 6(\Pi_{33}^G)^2 + 6(\Pi_{81}^G)^2 + (2\Pi_{11}^G - 33\Pi_{33}^G)(\Pi_{18}^G + \Pi_{81}^G) \right. \\
 &\quad \left. + 17\Pi_{18}^G \Pi_{81}^G + 2Z_6(\Pi_{81}^G + \Pi_{18}^G - 2\Pi_{11}^G) \right], \\
 \lambda_{g,4} &= -2\lambda_{g,5} = -2\lambda_{g,6} = \frac{6}{f_6^4} \int_p \frac{\Pi_{81}^G + \Pi_{18}^G - 2\Pi_{33}^G}{2Z_6 + \Pi_{81}^G + \Pi_{18}^G}. \tag{3.7}
 \end{aligned}$$

Similar to the case of the fermions, we have shortened the notation of the representations by omitting the parenthesis.

A few comments are in order. As expected, there is no contribution to the quadratic and quartic terms involving only the singlet. Besides, eqs. (3.2) and (3.7) are independent of θ_6 , which at this order remains as a NGB. In the specific realization of a two-site model (as well as simple extra dimensional models) $m_{g,8}^2 \geq 0$, and the gauge potential does not misalign the vacuum. For $m_{g,8}^2 > |m_{f,6}|^2$, only ϕ_1 has a negative mass term and can spontaneously break H_6 to $SU(3)_V$.

3.1 Symmetry breaking

We analyse in this section the potential for the singlets of $SU(2)_V$ and $SU(3)_V$: h_5 and h_6 . For small s_5 and s_6 the potential can be expanded in powers of these variables; to fourth order it results in:

$$V(s_5, s_6) \simeq \alpha_5 s_5^2 + \alpha_6 s_6^2 + \beta_5 s_5^4 + \beta_6 s_6^4 + \beta_{56} s_5^2 s_6^2, \tag{3.8}$$

where the coefficients α and β scale as f^4 , and are given by integrals of the correlators. As we will show in section 3.3, in general β_6 is suppressed compared with the other quartic couplings. This suppression will have deep implications for model building, as well as for the phenomenology of the scalars.

Minimizing for a non-trivial vacuum we obtain:

$$\epsilon_5 = \left(\frac{\alpha_6 \beta_{56} - 2\alpha_5 \beta_6}{4\beta_5 \beta_6 - \beta_{56}^2} \right)^{1/2}, \quad \epsilon_6 = \left(\frac{\alpha_5 \beta_{56} - 2\alpha_6 \beta_5}{4\beta_5 \beta_6 - \beta_{56}^2} \right)^{1/2}. \tag{3.9}$$

For $\beta_{56} \gg \beta_5, \beta_6$ the minimum is not stable and in general one of the symmetries is not spontaneously broken. As we discuss in the next subsection, this behavior has important consequences for model building.

3.2 Spectrum of neutral scalar states

From eqs. (3.8) and (3.9), trading α_5 and α_6 for ϵ_5 and ϵ_6 , one can obtain the mass matrix of the excitations of h_5 and h_6 . In the basis (h_5, h_6) the squared mass matrix is:

$$M^2 = \begin{pmatrix} 8\frac{\beta_5}{f_5^2}(1-\epsilon_5)^2\epsilon_5^2 & \frac{4}{\sqrt{6}}\frac{\beta_{56}}{f_5 f_6}\epsilon_5^2\sqrt{1-\epsilon_5^2}\epsilon_6^2\sqrt{1-\epsilon_6^2} \\ \frac{4}{\sqrt{6}}\frac{\beta_{56}}{f_5 f_6}\epsilon_5^2\sqrt{1-\epsilon_5^2}\epsilon_6^2\sqrt{1-\epsilon_6^2} & \frac{4}{3}\frac{\beta_6}{f_6^2}(1-\epsilon_6)^2\epsilon_6^2 \end{pmatrix}. \tag{3.10}$$

There are different interesting limits to study:

- $\epsilon_5^2 \ll 1$: the masses and mixing angles of the physical states are given by

$$m_1^2 \simeq 2 \frac{\epsilon_5^2}{f_5^2} \left(4\beta_5 - \frac{\beta_{56}^2}{\beta_6} \right), \quad m_2^2 \simeq \frac{4}{3} \frac{\epsilon_6^2(1 - \epsilon_6^2)}{f_6^2} \beta_6, \quad \theta \simeq \sqrt{\frac{3}{2(1 - \epsilon_6^2)}} \frac{\epsilon_5 f_6}{\epsilon_6 f_5} \frac{\beta_{56}}{\beta_6}. \quad (3.11)$$

In this case m_1 and the mixing are suppressed by powers of the small parameter ϵ_5 . The usual Higgs can be approximately identified with h_5 , and there is a new scalar state which is expected to be rather light also, since its mass is suppressed by $\epsilon_6^2(1 - \epsilon_6^2)$ and by the smaller quartic β_6 .

- $\beta_6 \ll \beta_5, \beta_{56}$: this case is particularly relevant because it is realized in some interesting models, according to the discussion of section 3.3. The masses and mixing angles of the physical states are given by

$$\begin{aligned} m_1^2 &\simeq 4\beta_5 \frac{\epsilon_5^2(1 - \epsilon_5^2)}{f_5^2} (1 + \sqrt{1 + c}), \\ m_2^2 &\simeq 4\beta_5 \frac{\epsilon_5^2(1 - \epsilon_5^2)}{f_5^2} (1 - \sqrt{1 + c}) + \frac{2}{3} \beta_6 \frac{\epsilon_6^2(1 - \epsilon_6^2)}{f_6^2} \frac{1 + \sqrt{1 + c}}{\sqrt{1 + c}}, \\ \theta &\simeq \frac{1}{\sqrt{2}} \frac{\sqrt{c}}{\sqrt{1 + c} - \sqrt{1 + c}}, \\ c &= \frac{1}{6} \frac{\epsilon_6^2(1 - \epsilon_6^2)\beta_{56}^2}{\epsilon_5^2(1 - \epsilon_5^2)\beta_5^2} \frac{f_5^2}{f_6^2}. \end{aligned} \quad (3.12)$$

It is worth analysing this result in some detail. Since the first term of m_2^2 is negative, the second term has to be greater than the first one to obtain a stable minimum. However the second term is subdominant compared to the first one, in the expansion of small β_6 . In this way one can see the tension introduced by the presence of a small β_6 and the requirement of spontaneous breaking of H_5 and H_6 by the potential. This scenario in general leads to m_2 smaller than m_1 , due to the suppression of β_6 compared with β_5 and due to the tuning required to obtain a suitable minimum.

- If in the previous case we consider $\beta_{56} \ll \beta_5$, we obtain

$$m_1^2 \simeq 8\beta_5 \frac{\epsilon_5^2(1 - \epsilon_5^2)}{f_5^2}, \quad m_2^2 \simeq \frac{4}{3} \beta_6 \frac{\epsilon_6^2(1 - \epsilon_6^2)}{f_6^2}, \quad \theta \simeq \frac{1}{2\sqrt{6}} \sqrt{\frac{1 - \epsilon_6^2}{1 - \epsilon_5^2}} \frac{\beta_{56}}{\beta_5} \frac{\epsilon_6 f_5}{\epsilon_5 f_6}. \quad (3.13)$$

This case is interesting because it is realized in several regions of the parameter space of the model studied in section 4. Since the mixing angle is small, the physical states can be approximated by h_5 and h_6 . The mass of h_5 is controlled by the quartic β_5 and suppressed by ϵ_5^2 , whereas the mass of h_6 is suppressed by β_6 , that was assumed to be smaller than the other quartic couplings, and by $\epsilon_6^2(1 - \epsilon_6^2)$.

The previous examples show that in general one expects the presence of two light scalar states. In the case of small mixing angle, there is a state with quantum numbers similar to the SM Higgs boson, whose mass is suppressed by ϵ_5^2 , similar to the MCHM. Compared with that scenario, there is a new state, whose mass is in general suppressed by the small quartic β_6 and by $\epsilon_6^2(1 - \epsilon_6^2)$.

3.3 Model building

In this subsection we discuss the impact of the fermion representations under G_5 on the potential $V(s_5, s_6)$. Using the form factors of eq. (2.19), and considering only the singlet scalar fields, the potential generated by exchange of virtual fermions at one-loop is:

$$V_f(s_5, s_6) = -2N_c \int_p [\log(p^2 \Pi_{u_L} \Pi_{u_R} - |\Pi_{u_L u_R}|^2) + \log p^2 \Pi_{d_L}]. \quad (3.14)$$

By expanding V_f of eq. (3.14) in powers of s_5 and s_6 to fourth order it is possible to express the quadratic and quartic coefficients of eq. (3.8) in terms of integrals of the form factors. A simplified description of V_f can be obtained by expanding also in powers of $1/Z_f$, or equivalently in powers of the fermion mixing ϵ_L and ϵ_R . As discussed in ref. [26], although for the third generation the mixing can be large, it can still provide a good estimate. Moreover, by performing numerical calculations to all orders in these parameters, we have checked that this expansion correctly describes the size of the coefficients.

Expanding in powers of s_5 , s_6 and $1/Z_f$, we have analysed models with fermions in representations 1, 4, 5, 10, 14 and 16 of G_5 . We find that: in all the cases α_5, α_6 and β_{56} are of $\mathcal{O}(Z_f^{-1})$, whereas β_6 is of $\mathcal{O}(Z_f^{-2})$. For models involving only representations 1 and 5 β_5 is of $\mathcal{O}(Z_f^{-2})$, for example $\psi_q \sim 5$ and $\psi_u \sim 5$ or 1. For models where at least one of the fermions, ψ_q or ψ_u , is in the representation 4, 10, 14 or 16, β_5 is of $\mathcal{O}(Z_f^{-1})$. If P_{LR} -symmetry is added to protect the $Z b_L \bar{b}_L$ coupling, representation 10 also gives β_5 of $\mathcal{O}(Z_f^{-2})$. As shown in section 3.1, for $\beta_{56} \gg \beta_5, \beta_6$ it is not possible to break both symmetries simultaneously, thus models involving only representations 1, 5 and 10 with P_{LR} -symmetry are not favored. By considering the 2-site models of section 4 we have computed the potential to all orders in s_5 , s_6 and Z_f , and we have checked that this is indeed the situation. Therefore, although the mixings of the quarks of the third generation are large, the simplified analysis allows for a correct selection of the best representations.

As is well known, the choice of the representation also affects flavor physics. In theories with a pNGB Higgs, fermion representations allowing more than one invariant Yukawa coupling induce Higgs mediated flavor violating processes that in general are incompatible with the current bounds from flavor physics for $f \sim \text{TeV}$ [27]. In order to proceed with our analysis, we will assume that, for a given type of fermion ψ_f , all the generations are in the same representation. Once the representations for ψ_q and ψ_u are chosen, it is straightforward to count the number of invariants. As done in eq. (2.9), a simple procedure is to decompose the representations of $G_5 \times G_6$ in irreducible representations of $H_5 \times H_6$: $\psi_f \sim \oplus_{rs} (\psi_f)_{rs}$, and count how many representations rs in the decomposition of ψ_q coincide with the ones in ψ_u . This number gives the number of independent invariants corresponding to Yukawa structures. There is one exception to this rule: if ψ_q and ψ_u are in the same representation of the group, there is one linear combination of the invariants that is independent of the pNGB, thus in this case one has to subtract one unit to the previous counting [28]. A similar situation holds for the down sector and leptons. Taking into account these results, as well as the results from the previous paragraph, an interesting set of representations is: $\psi_q \sim 14$, $\psi_u \sim 1$ and $\psi_d \sim 10$ of $SO(5)$. In this case there is just one non-trivial invariant for the Yukawa of the up-sector and one for the down-sector. Calling the invariants I_{rs} ,

where r labels the representation of $\text{SO}(4)$ and s the representation of $\text{SU}(3)_L \times \text{SU}(3)_R$, the invariant for the up-sector can be symbolically written as: $I_{(1,1),(3,1)} - I_{(1,1),(1,3)}$, and the one for the down-sector as: $I_{(2,2),(3,1)} - I_{(2,2),(1,3)}$. In this counting we have taken into account that, since q_L and q_R are in the same representation of G_6 , there is only one linear combination of invariants of H_6 , that we have chosen as: $I_{r(3,1)} - I_{r(1,3)}$.

Model 14-1. According to the discussion of the previous paragraph, we consider in detail the potential generated by fermions in the following representations of $\text{SO}(5)$: $\psi_q \sim 14$ and $\psi_u \sim 1$, we will refer to this case as 14-1. Under H_5 ψ_q decomposes as: $14 \sim (3,3) \oplus (2,2) \oplus (1,1)$, whereas ψ_u is a singlet $(1,1)$. These decompositions indicate the values that the subindex r can take in the expressions for the form factors defined in eqs. (2.13) and (2.14). For the representations of H_6 : $s = (3,1), (1,3)$. As done previously, to shorten the notation we will omit the parenthesis, thus we will write $\Pi_{ijk\ell}^f$, with i, j labeling the representation of H_5 , and k, ℓ that of H_6 , and similar for $M_{ijk\ell}^f$. We obtain:

$$\begin{aligned}
 m_{f,5}^2 &= -\frac{2}{f_5^2} \int_p \frac{3}{4} \frac{5\Pi_{1131}^q - 14\Pi_{2231}^q + 9\Pi_{3331}^q}{Z_q + \Pi_{2231}^q}, \\
 m_{f,6}^2 &= -\frac{2}{f_6^2} \int_p \left[\frac{\Pi_{2231}^q - \Pi_{2213}^q}{Z_q + \Pi_{2231}^q} + \frac{\Pi_{1131}^u - \Pi_{1113}^u}{Z_u + \Pi_{1113}^u} \right], \\
 \lambda_{f,5} &= -\frac{2}{f_5^4} \int_p \frac{-3}{16(Z_q + \Pi_{2231}^q)^2} [75(\Pi_{1131}^q)^2 - 140\Pi_{1131}^q \Pi_{2231}^q + 150\Pi_{1131}^q \Pi_{3331}^q + 44(\Pi_{2231}^q)^2 \\
 &\quad - 252\Pi_{2231}^q \Pi_{3331}^q + 123(\Pi_{3331}^q)^2 + 16Z_q(10\Pi_{1131}^q - 19\Pi_{2231}^q + 9\Pi_{3331}^q)], \\
 \lambda_{f,6} &= -\frac{2}{f_6^4} \int_p \left[\frac{(\Pi_{2231}^q - \Pi_{2213}^q)(2Z_q + 3\Pi_{2213}^q - \Pi_{2231}^q)}{12(Z_q + \Pi_{2231}^q)^2} + \frac{\Pi_{1113}^u - \Pi_{1131}^u}{12(Z_u + \Pi_{1113}^u)} - \frac{(\Pi_{1113}^u - \Pi_{1131}^u)^2}{8(Z_u + \Pi_{1113}^u)^2} \right], \\
 \lambda'_{f,6} &= 0, \\
 \lambda_{f,56} &= -\frac{2}{f_5^2 f_6^2} \int_p \frac{1}{32(Z_q + \Pi_{2231}^q)^2} [5\Pi_{1113}^q \Pi_{2231}^q - 5\Pi_{1131}^q \Pi_{2213}^q - 9\Pi_{2213}^q \Pi_{3331}^q + 9\Pi_{2231}^q \Pi_{3313}^q \\
 &\quad + Z_q(5\Pi_{1113}^q - 5\Pi_{1131}^q - 14\Pi_{2213}^q + 14\Pi_{2231}^q + 9\Pi_{3313}^q - 9\Pi_{3331}^q)].
 \end{aligned} \tag{3.15}$$

By keeping only the scalar fields h_5 and h_6 , and expanding in powers of s_5 and s_6 , one can also compute the potential $V(s_5, s_6)$ defined in eq. (3.8). For large Z_f , the leading non-trivial contribution to the coefficients of the potential are:

$$\begin{aligned}
 \alpha_5 &\simeq \frac{1}{2Z_q} \int_p (-9\Pi_{3331}^q + 14\Pi_{2231}^q - 5\Pi_{1131}^q), \\
 \alpha_6 &\simeq \int_p \left[\frac{4}{Z_q} (\Pi_{2231}^q - \Pi_{2213}^q) - \frac{2}{Z_u} (\Pi_{1131}^u - \Pi_{1113}^u) \right], \\
 \beta_5 &\simeq \frac{1}{2Z_q} \int_p (3\Pi_{3331}^q - 8\Pi_{2231}^q + 5\Pi_{1131}^q), \\
 \beta_6 &\simeq \int_p \left[-\frac{1}{Z_q^2} (\Pi_{2231}^q - \Pi_{2213}^q)^2 + \frac{1}{2Z_u^2} (\Pi_{1131}^u - \Pi_{1113}^u)^2 \right], \\
 \beta_{56} &\simeq \frac{1}{2Z_q} \int_p (9\Pi_{3331}^q - 9\Pi_{3313}^q - 14\Pi_{2231}^q + 14\Pi_{2213}^q + 5\Pi_{1131}^q - 5\Pi_{1113}^q).
 \end{aligned} \tag{3.16}$$

A few observations can be made: as already discussed, in this model all the coefficients are of $\mathcal{O}(Z_f^{-1})$, except for β_6 that is of $\mathcal{O}(Z_f^{-2})$; the sign of the quadratic coefficients is not fixed, thus for suitable regions of the parameter space it is possible to break both symmetries.

4 A 2-site model

Moose models can provide an effective description of a strongly coupled sector weakly coupled to external sources [29], they can also describe the lower level of resonances of extra-dimensional theories [15]. In this section we consider the simplest case given by 2-site models; by using this framework one can explicitly compute the potential as well as some relevant observables.

We follow the approach proposed in ref. [18] to describe NGB fields arising from the SCFT (see also [19]). The setup is given by two sites containing different sets of fields, and connected by a set of non-linear sigma model fields. The first site, also called site-0, contains the same field content and symmetries as the SM but without scalar fields. For convenience, as already described in section 2, we add spurious fields to extend the gauge symmetry of the SM to $G_{5,0} \times G_{6,0}$, with $G_5 = \text{SO}(5) \times \text{U}(1)_X$ and $G_6 = \text{SU}(6)$, the subindex labelling the site-0. The corresponding gauge couplings are: $g_{X,0}$, $g_{5,0}$ and $g_{6,0}$. Following the discussion of section 3.3, we put ψ_q , ψ_u and ψ_d in the representations $14_{2/3}$, $1_{2/3}$ and $10_{2/3}$ of G_5 , respectively. We take the fundamental representation of G_6 for the quarks, with q_L in $(3, 1)_{1/2\sqrt{3}}$ and q_R in $(1, 3)_{-1/2\sqrt{3}}$ of the subgroup H_6 , defined in the next paragraph.

In the second site, also called site-1, we put the fields that describe the first level of resonances of the SCFT. The spin one resonances can be described by the fields of the gauge symmetry $G_{5,1} \times G_{6,1}$, with gauge couplings $g_{X,1}$, $g_{5,1}$ and $g_{6,1}$. We add two sets of NGB scalar fields, Π_5^1 and Π_6^1 ,³ respectively describing the spontaneous breaking $G_{5,1}/H_{5,1}$ and $G_{6,1}/H_{6,1}$, by the strong dynamics, with $H_5 = \text{SO}(4) \times \text{U}(1)_X$ and $H_6 = \text{SU}(3)_L \times \text{SU}(3)_R \times \text{U}(1)$. Each spontaneous breaking is characterized by a scale $f_{5,1}$ and $f_{6,1}$; both scales are assumed to be of the same order. Thus we include in the Lagrangian of site-1 the kinetic terms of the gauge fields and the NGB scalars, that as usual can be expressed in terms of the Cartan-Maurer form.

In addition, there are some fermion multiplets: Ψ_q , Ψ_u and Ψ_d , that are in correspondence with the elementary ones, and are in the same representations of $G_5 \times G_6$ as the elementary fermions. The fermions of site-1 are vector-like, with masses arising from the strong dynamics. The non-linear transformation properties of the NGB fields allow us to write gauge invariant Yukawa interactions with the fermions on site-1, as shown below:

$$\mathcal{L}_1 \supset \sum_{f=q,u,d} \bar{\Psi}_f (i \not{D} - m_{f,1}) \Psi_f + f_1 \sum_{f=u,d} \sum_{rs} y_{rs}^f \overline{(U_5^\dagger U_6^\dagger \Psi_{qL})_{rs}} (U_5^\dagger U_6^\dagger \Psi_{fR})_{rs} + \text{h.c.}, \quad (4.1)$$

where y_{rs}^f are dimensionless Yukawa couplings, and f_1 is a dimensionful parameter of the same order as the NGB decay constants. The second term of eq. (4.1) generates Yukawa

³The index 1 indicates that these fields belong to site-1, and allows us to distinguish them from the non-linear sigma model fields connecting both sites that will be described below.

couplings for the elementary fermions after interactions between both sites. The representations chosen for the fermions allow for just one G_5 invariant for the Yukawa operator of the up sector, obtained by taking $r = (1, 1)$. Since a $10_{2/3}$ of G_5 decomposes under H_5 as: $10_{2/3} \sim (2, 2)_{2/3} \oplus (3, 1)_{2/3} \oplus (1, 3)_{2/3}$, there is also just one G_5 invariant for the down sector, obtained by taking $r = (2, 2)$. The index s takes the values $(3, 1)$ and $(1, 3)$. Since for $y_{r31}^f = y_{r13}^f$ the Yukawa is independent of Π_6 , the linear combination $y_{r31}^f - y_{r13}^f$ measures the Yukawa coupling involving Π_6 , whereas $y_{r31}^f + y_{r13}^f$ leads to a mixing between Ψ_q and Ψ_f . Thus in the present 2-site model the Yukawa couplings are proportional to $y_{r31}^f - y_{r13}^f$. These results are in agreement with the arguments of section 3.3 for the low energy effective theory.

In eq. (4.1) we have included only one chiral structure, avoiding terms with the opposite chiral structures: $(U^\dagger \Psi_{qR})_{rs} (U^\dagger \Psi_{fL})_{rs}$. As discussed in ref. [22], there is no fundamental reason for this, the motivation being that in this way one can obtain a finite one-loop potential.

All the couplings of site-1, generically called g_1 , are assumed to be larger than the SM ones but still perturbative: $g_{SM} \ll g_1 \ll 4\pi$. All the dimensionful parameters are assumed to be of order TeV, the masses of the fermions being $m_{f,1} \sim g_1 f_1$.

The two sites are connected by a set of non-linear sigma models: $\Pi_{X,0}$, $\Pi_{5,0}$ and $\Pi_{6,0}$, associated to the gauge symmetry groups. These fields parametrize the cosets $(G_{5,0} \times G_{5,1})/G_{5,0+1}$ and $(G_{6,0} \times G_{6,1})/G_{6,0+1}$, where $G_{5,0+1}$ and $G_{6,0+1}$ are the diagonal subgroups. The field $U_{5,0} = e^{i\sqrt{2}\Pi_{5,0}/f_{5,0}}$ transforms linearly under $G_{5,0} \times G_{5,1}$, as: $U_{5,0} \rightarrow \hat{g}_{5,0} U_{5,0} \hat{g}_{5,1}^\dagger$, with $\hat{g}_{5,0} \in G_{5,0}$ and $\hat{g}_{5,1} \in G_{5,1}$. Similar properties apply to $U_{6,0} = e^{i\sqrt{2}\Pi_{6,0}/f_{6,0}}$. Besides, these fields play an important role in the realization of partial compositeness, that is obtained by linear interactions between the fermions on site-0 and site-1, through the following terms:

$$\mathcal{L}_{\text{mix}} \supset f_0 \sum_{f=q,u,d} \lambda_f \bar{\psi}_f U_{X,0}^{2/3} U_{5,0} U_{6,0} \Psi_f + \text{h.c.} \quad (4.2)$$

with f_0 a dimensionful constant and λ_f a dimensionless coupling that controls the size of the mixing between the fermions. All the dimensionful parameters, namely: the decay constants and f_0 , are taken of order TeV, whereas the couplings λ_f can be hierarchical.

Let us briefly count the number of NGB fields. One can choose a gauge where the fields $\Pi_{X,0}$, $\Pi_{5,0}$ and $\Pi_{6,0}$ are removed. In this gauge it is straightforward to see that the gauge fields of the corresponding cosets become massive, with masses $m_1 \sim g_1 f_0 / \sqrt{2}$, whereas the gauge fields of the diagonal cosets remain massless. The NGBs $\Pi_{5,1}$ and $\Pi_{6,1}$ remain in the spectrum; $\Pi_{5,1}$ can be associated with the Higgs multiplet and $\Pi_{6,1}$ gives a new scalar multiplet. The previous gauge is not the unitary one, that can be obtained by demanding the absence of mixing terms between the gauge and the NGB fields, as described in refs. [9] and [30]. The decay constants of the physical NGB fields Π_5 and Π_6 are given by: $f_5^{-2} = f_{5,0}^{-2} + f_{5,1}^{-2}$ and $f_6^{-2} = f_{6,0}^{-2} + f_{6,1}^{-2}$, respectively.

The degree of compositeness of the light fermions is given by $\epsilon_f = \lambda_f / g_1$, where we have considered $f_0 \sim f_1$. The light fermions are mostly elementary if $\epsilon_f \ll 1$, leading to small mixing, whereas for $\epsilon_f \sim 1$ the mixing between the fermions on site-0 and site-1 is large, leading to partially composite states. We are interested in the anarchic scenario,

that in the 2-site model is obtained by taking structureless Yukawa couplings, with all the elements of the matrices being of the same order: $(y_{rs}^f)_{jk} \sim g_1$, where j and k are flavor indices. Keeping the NGB dependence, the mass matrices of the SM fermions are:

$$(m_f)_{jk} \sim n_f \epsilon_{qj} \epsilon_{fk} s_5 c_5 s_6 c_6 f_1 (y_{rs}^f)_{jk}, \quad n_u = \frac{\sqrt{5}}{2}, \quad n_d = \frac{1}{\sqrt{2}}, \quad (4.3)$$

thus a light fermion requires at least the mixing of one of the chiralities to be small, whereas the top requires both mixings of order one to be able to obtain a Yukawa coupling $y_t \sim 1$. Besides the masses, it is also necessary to reproduce the mixing angles of the CKM matrix. Taking hierarchical mixings ϵ_f leads to the hierarchies between the masses and CKM angles.⁴ Following refs. [7, 31] we take:

$$\frac{\epsilon_{q1}}{\epsilon_{q2}} \sim \lambda_C, \quad \frac{\epsilon_{q2}}{\epsilon_{q3}} \sim \lambda_C^2, \quad \frac{\epsilon_{q1}}{\epsilon_{q3}} \sim \lambda_C^3. \quad (4.4)$$

where λ_C is the Cabibbo parameter. By using $v_{\text{SM}} = \epsilon_5 f_5$, $m_t \sim v_{\text{SM}}/\sqrt{2}$ and $y_{rs}^u \sim g_1$, the following relations arise from anarchy:

$$\begin{aligned} \epsilon_6 \sqrt{1 - \epsilon_6^2} \epsilon_{q3} \epsilon_{u3} g_1 &\sim 1, & \epsilon_{d3} &\sim \epsilon_{u3} \frac{y^u}{y^d} \frac{m_b}{m_t}, \\ \epsilon_{u2} &\sim \frac{\epsilon_{u3}}{\lambda_C^2} \frac{m_c}{m_t}, & \epsilon_{d2} &\sim \frac{\epsilon_{u3}}{\lambda_C^2} \frac{y^u}{y^d} \frac{m_s}{m_t}, \\ \epsilon_{u1} &\sim \frac{\epsilon_{u3}}{\lambda_C^3} \frac{m_u}{m_t}, & \epsilon_{d1} &\sim \frac{\epsilon_{u3}}{\lambda_C^3} \frac{y^u}{y^d} \frac{m_d}{m_t}. \end{aligned} \quad (4.5)$$

The factor y^u/y^d in the second column is of $\mathcal{O}(1)$ if all couplings on site-1 are of the same order, but differs from this estimate if the Yukawa couplings of the down- and up-sectors on site-1 are taken different. The upper-left equation of (4.5) shows that $\epsilon_6 \sqrt{1 - \epsilon_6^2}$ cannot be too small if g_1 is required to be perturbative since in the most favorable case, for $\epsilon_{q3} \sim \epsilon_{u3} \sim 1$, it leads to: $g_1 \sim (\epsilon_6 \sqrt{1 - \epsilon_6^2})^{-1}$.

Integrating out the spin-one fields and fermions on site-1, one obtains the effective theory described in section 2. The correlators of eqs. (2.9) and (2.10) can be computed explicitly in terms of the parameters of the 2-site theory. We show them in appendix B.

4.1 Dominant contributions to ϵ_K

Integrating out the heavy resonances, new contributions to the Wilson coefficients C_i of dimension-6 operators contributing to $K - \bar{K}$ mixing are generated. The most dangerous contributions are those generated by exchange of massive gluons, that give a contribution to C_4 :

$$C_4 \sim \frac{2\epsilon_{qi}\epsilon_{qj}\epsilon_{di}\epsilon_{dj}}{f_{6,0}^2} \frac{f_{6,1}^2(\epsilon_6^2 - \epsilon_6^4)}{2(f_{6,0}^2 + f_{6,1}^2)}. \quad (4.6)$$

In the standard case with SU(3) global symmetry only, just the first factor is present.

⁴Although hierarchical λ_{fj} may look arbitrary, they appear naturally when considering the running of the couplings between the elementary fermions and the SCFT [4, 9]. A simple realization can be obtained in extra dimensional models [5, 23].

Eq. (4.6) is suppressed by a factor $(\epsilon_6^2 - \epsilon_6^4)$. However, trading ϵ_6 as well as the compositeness factors by the quark masses, and comparing with the standard case, only the last factor of eq. (4.6) remains

$$C_4 \sim C_4^{\text{SU}(3)} \frac{f_{6,1}^2}{2(f_{6,0}^2 + f_{6,1}^2)}. \quad (4.7)$$

Bounds on C_4^{sd} from ϵ_K usually require $m_{cp} \gtrsim 10 \div 30$ TeV. In the present case, taking $f_{6,0} \simeq f_{6,1}$, the scale of the masses of the composite gluons can be lowered to $\sim 5 \div 15$ TeV, whereas for $f_{6,0} \simeq 3f_{6,1}$ it can be lowered to $\sim 2.2 \div 6.7$ TeV.

As mentioned in section 2.1, the decomposition of the adjoint of G_6 under H_6 contains a $(1,1)_0$ - the generator of the $U(1)$ factor in H_6 . The presence of this state induces an extra contribution to C_5 , that is not suppressed by ϵ_6 . After trading the factors of mixing by the quark masses, a dependence on ϵ_6 is induced for C_5 . For $\epsilon_6 = 1/\sqrt{2}$ and $f_{6,0} \simeq f_{6,1}$:

$$C_5 = C_4^{\text{SU}(3)} \frac{1}{6}. \quad (4.8)$$

The contribution of C_5^{sd} to ϵ_K is suppressed by N_c , and in the present case by an extra factor $1/6$ compared with the usual case. Assuming that in the standard case $m_{cp} \gtrsim 10 \div 30$ TeV, the contributions of C_5^{sd} to ϵ_K require $m_{cp} \gtrsim 5 \div 9$ TeV. This bound can be as stringent as the bound from C_4^{sd} , and it can be even more stringent if ϵ_6 is near 0 or 1.

4.2 Estimation of the top mass

The mass of the top can be approximated by the coefficient $j = k = 3$ of eq. (4.3). Taking $\epsilon_{q3} \sim \epsilon_{u3} \sim 1$, $f_1 \simeq 1$ TeV, we obtain

$$y_{rs}^u \sim 2.4 \div 0.6, \quad (4.9)$$

where the largest value corresponds to $\epsilon_5 \sim 0.3$ and $\epsilon_6 \sim 0.2$ and the smallest one to $\epsilon_5 \sim 0.5$ and $\epsilon_6 \sqrt{1 - \epsilon_6^2} \sim 0.5$. Although there is an extra suppression from the factor $\epsilon_6 \sqrt{1 - \epsilon_6^2}$ compared with the usual case, $\mathcal{O}(1)$ Yukawa can lead to the top mass in the present model.

5 Numerical results

In this section we present some numerical results that arise from the 2-site model described in section 4. We focus first on the issue of spontaneous symmetry breaking of the extended symmetries: H_5 that contains the EW symmetry group, and H_6 that contains the color symmetry group. By performing a random scan over the parameters of the 2-site models, we have checked the properties of the potential discussed in section 3. In particular we have checked the predictions of section 3.3 for different combinations of the representations 1, 5, 10 and 14 of the fermions under $SO(5)$, and their impact on the possibility to obtain successful breaking of both symmetries. We have found that, in models with fermions

Figure 1. Contour plots for the values of ϵ_5 and ϵ_6 at the minimum of the potential (red dotted and blue solid lines respectively in plots labeled with an A) and for the masses of the Higgs, the new scalar h_6 and the top quark (red dotted, blue solid and black dashed lines respectively in plots labeled B) as functions of the mixings s_q and s_u for two different sets of parameters, labeled with 1 and 2, as described in the text.. In the A plots, the gray region corresponds to $\epsilon_6 = 1$, the light blue region to $\epsilon_6 = 0$ and the red region to $\epsilon_5 = 0$. All masses are expressed in GeV.

in combinations of the representations 1, 5, and 10, in general there is no spontaneous breaking at all, and in some cases only H_5 is broken.⁵ We have also checked that when the 14 is included there are suitable regions of the parameter space where the symmetry is spontaneously broken to $K_5 \times K_6$.

⁵We imposed Left-Right parity for the 10 of $SO(5)$.

In the rest of this section we will present the results for model 14-1 that, as discussed in section 3.3, is one of the most interesting models. We start by explaining our scan of the parameter space. It is useful to define mixing angles for the fermions by the following relation: $\tan \theta_f = \lambda_f f_1 / m_f$, with m_f the mass of the vector-like fermions on site-1. We will use s_f and c_f for $\sin \theta_f$ and $\cos \theta_f$ respectively; notice that with this parametrization the absence of mixing is given by $s_f = 0$, whereas large mixing corresponds to $s_f \simeq 1$. We have fixed the masses m_f and the Yukawa couplings y_{rs}^u , and we have varied s_q and s_u . For the bosonic sector we have fixed the ratio between the couplings g_0/g_1 and we have matched the coupling of the diagonal subgroups with the SM gauge couplings. We have also fixed the masses of the spin-one resonances, m_1 . In figure 1 we show the results obtained when scanning over s_q and s_u for a point with parameters $g_0/g_1 = 0.26$, $m_q = m_u = 2$ TeV, $m_1 = 2.5$ TeV, $y_{1131}^u = 0.4$, $y_{1113}^u = -0.82$ and $f_5 = f_6 = 1$ TeV (upper panels, labeled 1) and another with $g_0/g_1 = 0.25$, $m_q = 3$ TeV, $m_u = 1.4$ TeV, $m_1 = 3$ TeV, $y_{1131}^u = -0.7$, $y_{1113}^u = 0.5$ and $f_5 = f_6 = 1$ TeV (lower panels, labeled 2). The panels on the left (labeled A) show the expectation values obtained for s_5 (red dotted) and s_6 (blue solid), with the regions where $\epsilon_6 = 0$ colored in light blue, those where $\epsilon_5 = 0$ in red and the ones with $\epsilon_6 = 1$ in gray. The white, uncolored regions in each A panel are the regions where we found spontaneous symmetry breaking to $K_5 \times K_6$, and thus the most interesting phenomenologically. The panels labeled with a B show the masses of the Higgs boson (red dotted) and the h_6 scalar (blue solid). We have verified that the mixing angle between these scalars is very small (≤ 0.1 rad) and so their identification with the mass eigenstates is a reasonable approximation. It is worth noting that the mass of h_6 was always found to be smaller than that of the Higgs within the phenomenologically interesting region; this is a general feature of this model and a consequence of the discussion presented in section 3.2. The phenomenological implications of having this additional light scalar will be analysed in the following section. In the B panels we also show the mass of the SM top quark (black dashed), which is calculated as the mass of the lightest up-type quark. Even though the vev's and Higgs masses shown in these plots are not completely realistic, more realistic values could be found with a more extensive scan. All sharp angles and jagged lines in these plots are only a consequence of the limitations of the numerical calculations employed and do not carry any physical meaning.

6 Discussions

6.1 Neutron dipole moments

Besides the constraints from $K - \bar{K}$ mixing, the electromagnetic (EDM) and chromomagnetic dipole moments (CDM) of the neutron give the most important set of constraints in models with anarchic flavor [6, 8, 33, 34]. In theories where the SCFT has a global symmetry $SU(3)$, gauged by the color interactions, assuming that the dipole operators are induced at loop level the following bound is obtained: $f \gtrsim 4.5$ TeV [9]. As expected, in the presence of the larger $SU(6)$ symmetry, given the representations chosen for the fermions, the contribution to the Wilson coefficients of operators with LR chiral structures require insertions of the scalar Π_6 . The Wilson coefficient of the dipole operators $\bar{\psi}_f F_{\mu\nu} \sigma^{\mu\nu} \psi_f$ can

be estimated by considering a one-loop diagram with composite fermions and a Higgs in the loop [6, 31]. Each insertion of Yukawa coupling y_{rs}^f introduces a factor ϵ_6 , therefore the Wilson coefficient is of order $\sim g_{\text{SM}} \epsilon_q \epsilon_f \epsilon_5 \epsilon_6^3 (y_{rs}^f)^3 / (4\pi m_{cp})^2$. Trading the factors of compositeness by the masses, as in section 4.1, one obtains: $g_{\text{SM}} m_f \epsilon_6^2 / (4\pi f)^2$. Resumming insertions of ϵ_6 , the bound is relaxed as: $f / (\epsilon_6 \sqrt{1 - \epsilon_6^2}) \gtrsim 4.5 \text{ TeV}$. As an example, $f = 1 \text{ TeV}$ is compatible with the bounds for $\epsilon_6 \lesssim 0.23$ or $\epsilon_6 \gtrsim 0.97$.

6.2 Phenomenology of the scalar states

An interesting and distinctive phenomenology of the present model is given by the presence of the new scalar states. Let us briefly characterize this sector of the theory. As discussed in section 3.2, there are two light scalar states, corresponding to the excitations of the fields that acquire a vev at loop level, h_5 and h_6 . As mentioned in the previous section, we find that the mixing between these fields is small, and therefore, to leading order the mass eigenstates can be identified with h_5 and h_6 . Since h_5 arises from $\text{SO}(5)/\text{SO}(4)$, its interactions with the EW gauge bosons are similar to those of the Higgs in the MCHM. On the other hand, h_6 does not interact with the gauge bosons at tree level. The Yukawa couplings at low energies are shown in eqs. (2.24) and (2.25). In the full theory the scalars also interact with the resonances.

Let us characterize the creation and decay of these scalars at the LHC. As for the usual Higgs, the main creation channel of the scalars at the LHC is gluon fusion, with much smaller contributions from creation in association with $t\bar{t}$, as well as creation in association with EW gauge bosons for the case of h_5 . Gluon fusion is induced at loop level by virtual exchange of fermions $f^{(n)}$. In the limit of heavy fermions, $2m_f^{(n)} \gg m_j$, this coupling is proportional to $\sum_{n \geq 0} y_{f,j}^{(n)} / m_f^{(n)}$, where $j = 5, 6$ labels the scalars and quantities with $n = 0$ pertain to the would be zero mode corresponding to the SM fermion. By standard algebraic manipulations it is possible to rewrite this expression as $\text{tr}(Y_{f,j} \mathcal{M}_f^{-1})$, with $Y_{f,j} = \partial_{h_j} \mathcal{M}_f$ and $\mathcal{M}_f$ the fermion mass matrix of the full theory. As usual in models with NGB scalars, this trace leads to a compact expression:

$$\text{tr}(Y_{u,5} \mathcal{M}_u^{-1}) = \frac{1}{v_{\text{SM}}} \frac{1 - 2\epsilon_5^2}{\sqrt{1 - \epsilon_5^2}}, \quad (6.1)$$

$$\text{tr}(Y_{u,6} \mathcal{M}_u^{-1}) = \frac{1 - 2\epsilon_6^2}{\sqrt{6} f_6 \epsilon_6 \sqrt{1 - \epsilon_6^2}}. \quad (6.2)$$

The case of h_5 has been extensively studied, as well as its dependence on the fermion representations [21, 22, 32]. The coupling to h_6 has a similar behavior, but in terms of the parameters associated to G_6 . By comparing with eq. (2.25), one can see that the sum over states is saturated by the lightest fermion, in this case corresponding to the top.⁶ Given these results, creation of h_j by gluon fusion and $t\bar{t}h_j$ processes is similar to Higgs creation in the MCHM.

Since the couplings of h_5 are similar to the couplings of the Higgs in the MCHM, its decay width and branching ratios are similar to that case. On the other hand h_6 can decay

⁶The light quarks and the leptons do not contribute in the present setup, see ref. [22] for discussions.

at tree level to SM fermion-antifermion, with Yukawa coupling given by eq. (2.25). This Yukawa coupling can be written in terms of the usual SM Yukawa with the Higgs, up to a factor depending on ϵ_6 and f_6 :

$$y_{f,6} \simeq y_f^{\text{SM}} \delta, \quad \delta = \frac{v_{\text{SM}}}{f_6} \frac{1 - 2\epsilon_6^2}{\sqrt{6}\epsilon_6\sqrt{1 - \epsilon_6^2}}. \quad (6.3)$$

At loop level, for a light h_6 , the most important decay channels are pairs of gluons and photons. The amplitude of the former is modulated by eq. (6.2). The amplitude of the later is modulated by a similar factor, multiplied by the corresponding factors of electric charge. Compared with h_5 , in the one-loop level contribution to $h_6\gamma\gamma$ there is no contribution from virtual EW gauge bosons. For light h_6 we expect diphoton and $b\bar{b}$ final states to be the most relevant ones, whereas for a heavier scalar the final state $t\bar{t}$ will become important, and massive diboson states mediated by loops of fermions also will become available.

According to the results of eqs. (6.2) and (6.3), the creation cross-section by gluon fusion and the partial widths of h_6 are similar to the ones of the SM Higgs, with a rescaling factor δ^2 , and changing the Higgs mass by the mass of h_6 . The only qualitative difference being that decays to pairs of massive gauge bosons are not present at tree level for h_6 .

In refs. [35] and [36], the ATLAS and CMS Collaborations have presented limits in the production cross-section of new scalar states decaying to photon pairs. In order to compare with those bounds, we consider a reference mass for h_6 of 80 GeV. In this case the production cross-section of h_6 is: $\sigma \simeq 45\text{pb} \times \delta^2$, and the BR to photon pair is of order 10^{-3} , leading to a bound $\delta \lesssim 1$. For $f_6 = 1\text{ TeV}$, this bound can be satisfied as long as $0.1 \lesssim \epsilon_6 \lesssim 0.99$. Therefore a new scalar state, somewhat lighter than the Higgs, could have evaded detection at the LHC.

Besides the neutral scalar fields already discussed, the present model contains an axion-like state, associated to the spontaneous breaking of the U(1) subgroup of H_6 . Indeed the vacuum expectation value v_6 spontaneously breaks this global U(1), and the scalar phase θ_6 becomes the associated NGB. In order to understand the phenomenology of this state, let us briefly discuss its interactions. Due to the embedding of the quarks into the fundamental representation of G_6 , the Left- and Right-handed quarks of the SM have opposite charges under U(1). The axion-like scalar has interactions with the fermions of the form: $\theta_6 h_5 \bar{q}_L \gamma^5 q_R$, that at one-loop level induce interactions $\theta_6 F_{\mu\nu} \tilde{F}^{\mu\nu}$ and $\theta_6 G_{\mu\nu} \tilde{G}^{\mu\nu}$ with photons and gluons. It is usually assumed that the axions acquire mass only through non-perturbative effects, however it has been shown recently that they can also obtain a mass at perturbative level [37, 38]. The simplest scenario is obtained by considering the presence of a 3-form field, that does not propagate since its equation of motion fixes its field strength to be a constant. It has been shown that the interactions of this form with the axion induce a potential, with a mass for the axion. If the dimensional coupling is of order TeV, the axion mass can be of order several hundred GeV to a few TeV. This scalar state could mix with the other scalars. It could also be produced by gluon and photon fusion, and decay to pairs of photons, gluons and fermion-antifermions. A precise description of its phenomenology requires assumptions on the specific realization, since there can be several 3-forms involved. We will not consider those details in the present work.

The new scalar sector also contains octets of $SU(3)_c$. The mass of these multiplets can be estimated from $m_{g,8}^2$ in eq. (3.7), leading to $m^2 \simeq \alpha_s/(4\pi)g_1^2 f_6^2 \sim \mathcal{O}(\text{TeV}^2)$. Being color octets, these states can be copiously produced by QCD interactions, with a large branching fraction to dijets. ATLAS and CMS have searched for narrow resonances in dijet final states [39–41], giving bounds on the mass of the scalar octets of order 3 TeV.

6.3 Tuning

The tuning can be roughly estimated as $(\epsilon_5 \epsilon_6)^{-2}$. The factor ϵ_5^{-2} is the usual tuning of the MCHM, that could be increased depending on the representations of the fermions [26]. EW precision tests usually require $\epsilon_5^2 \lesssim 0.1 \div 0.3$, leading to a tuning of order $4 \div 10$. Numerical calculations using different representations for the fermions show that the tuning varies between 5 and 10^3 in the MCHM [22]. The need in the present paper for the spontaneous breaking of another symmetry is expected to worsen the tuning, introducing an extra factor ϵ_6^{-2} . As discussed in section 4.1, by demanding the masses of the resonances to be of order 2.5 TeV, constraints from the Kaon system give the bounds: $\epsilon_6 \lesssim 0.2 \div 0.98$. In the most stringent case of $\epsilon_6 \lesssim 0.2$, we expect to increase the tuning by a factor of order 25, whereas in the most favorable case we expect no sizable rise of the tuning compared with the MCHM. Going beyond this general estimate requires considering the details of the model, in particular the embedding of the fermions, as well as numerical calculations.

Following the definition for the sensitivity parameter of refs. [26, 42, 43], we have computed the tuning in our model with the fermions q_L and u_R in the representations 14 and 1 of $SO(5)$. For the regions of the parameter space of figure 1 we find a tuning of order 50–300, with a few points where it rises to 400. One can compare this tuning with the results of ref. [22], that has reported a tuning of order $80 \div 300$ for the $MCHM_{14-1}$. However in that paper the Higgs, top and W masses were restricted to their physical values, selecting a region of the parameter space with non-natural cancellations in the scalar potential and thus increasing the tuning. In figure 1, the masses of the Higgs, top and EW gauge bosons are not fixed to their physical values. Constraining these masses requires non-trivial cancellations also in the model with the extended symmetry G_6 , and we expect a larger tuning in that case.

6.4 Other representations and symmetry groups

The need for a larger quartic coefficient β_6 suggests changing the representation of the quarks under G_6 . A larger β_6 could improve the tuning of the model by allowing a larger region in the parameter space with the right spontaneous breaking of symmetries. Cancellation of C_4 requires $q_L \sim (3, 1)$ and $q_R \sim (1, 3)$ of $SU(3)_L \times SU(3)_R$, thus the quark representations under $SU(6)$ must contain these representations of the subgroup $SU(3)_L \times SU(3)_R$. However we have not been able to find other small representations of $SU(6)$ that satisfy those conditions.

We have also explored the possibility of embedding the $SO(4)$ as well as the extended $SU(3)_L \times SU(3)_R$ symmetry groups into a single unified group. We have found several examples of groups that, after spontaneous breaking to $SO(4) \times SU(3)_L \times SU(3)_R$, deliver a NGB field transforming as a $(2, 2, 3, 3)$ of the unbroken subgroup, playing the role of the

Higgs field in the extended quark sector [13], the most interesting one being the exceptional group E_8 . However we have not been able to obtain suitable representations for the quarks. For example, for q_L , adding custodial symmetry to protect the $Zb_L\bar{b}_L$ coupling, one would require a representation containing a $(2,2,3,1)$ of the unbroken subgroup, whereas for u_R one would require $(1,1,1,3)$ or $(1,3,1,3)$. By exploring the lowest dimensional representations of the unified groups, we have not been able to find the proper representations for the quarks.

7 Conclusions

Flavor anarchic composite Higgs models with partial compositeness of the SM fermions offer a rationale to understand the hierarchies in the flavor of the quarks and leptons. The most stringent constraint from flavor physics, arising from ϵ_K in $K - \bar{K}$ system, pushes the scale of compositeness to $f \sim \mathcal{O}(10)\text{TeV}$, introducing a little hierarchy problem for a light Higgs. Ref. [12] showed that if the composite sector has an $SU(3)_L \times SU(3)_R$ global symmetry, with $SU(3)_c = SU(3)_{L+R}$, the main contribution to ϵ_K , given by the Wilson coefficient C_4^{sd} , can be suppressed. However quark masses require the presence of a new scalar field with a vacuum expectation value, whose presence can destabilize the cancellation protecting C_4 . We have embedded the global symmetry into a larger $SU(6)$ group, showing that a proper spontaneous breaking of the symmetries can occur dynamically. We have made an analysis of the potential, showing which are the representations of the fermions that can trigger this breaking. We have also shown that fermion masses can be reproduced, and the Wilson coefficient C_4 can be successfully suppressed, with a compositeness scale $f \sim \text{TeV}$. We have also briefly discussed the phenomenology at the LHC of the new light scalar states.

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A Fundamental representation of $SU(6)$

In this appendix we provide some basic ingredients of the group $SU(6)$. The generators of $SU(6)$ in the fundamental representation consist of thirty five 6×6 matrices, which are easily described as a combination of three by three blocks, i.e.:

$$T^i = \begin{bmatrix} A_{3 \times 3}^i & B_{3 \times 3}^i \\ C_{3 \times 3}^i & D_{3 \times 3}^i \end{bmatrix}, \quad (\text{A.1})$$

for the i th generator of $SU(6)$. After the dynamics of the SCFT break $SU(6)$ down, the generators can be organized into those of the preserved subgroup, $SU(3)_L \times SU(3)_R \times U(1)$ and those of the broken coset.

For the generators of $SU(3)_L \times SU(3)_R$, which we will consider to be the first 16 generators of the basis, $B^i = C^i = 0_{3 \times 3}$, $i = 1, \dots, 16$. Then, for those of $SU(3)_L$, $A^i = \frac{1}{2}\lambda_i$, with the λ matrices being the usual Gell-Mann matrices, and $D^i = 0_{3 \times 3}$, $i = 1, \dots, 8$. On the other hand, for $SU(3)_R$, $A^i = 0_{3 \times 3}$ and $D^i = \frac{1}{2}\lambda_{i-8}$, $i = 9, \dots, 16$. The generator of $U(1)$ is given by $B^{35} = C^{35} = 0_{3 \times 3}$, $A^{35} = -D^{35} = \frac{1}{2\sqrt{3}} I_{3 \times 3}$.

For the coset, the generators arrange into a complex $(3, \bar{3})$. In this case only blocks B and C will be populated by non-zero elements. Furthermore, the 18 generators of the complex $(3, \bar{3})$ can be organized into two octets and two singlets under the diagonal subgroup of $SU(3)_L \times SU(3)_R$, $SU(3)_V$. For the first octet one can choose $A^i = C^i = D^i = 0_{3 \times 3}$ and $B^i = K^{i-16}$, $i = 17, \dots, 24$; for the other $A^i = B^i = D^i = 0_{3 \times 3}$ and $C^i = K^{i-24}$, $i = 25, \dots, 32$. The matrix elements for the 8 three by three K^i blocks are as follows:

$$\begin{aligned} K_{(1,3)}^1 &= K_{(2,3)}^2 = K_{(3,1)}^3 = K_{(2,1)}^4 = -K_{(3,2)}^5 = -K_{(1,2)}^6 = \frac{i}{\sqrt{2}}, \\ K_{(1,1)}^7 &= K_{(3,3)}^7 = \frac{i}{2\sqrt{3}}, \quad K_{(2,2)}^7 = -\frac{i}{\sqrt{3}}, \quad K_{(1,1)}^8 = -K_{(3,3)}^8 = -\frac{i}{2}, \end{aligned}$$

with all other elements being zeroes. The two singlets can be described by $A^{33} = D^{33} = A^{34} = D^{34} = 0_{3 \times 3}$, $B^{33} = -C^{33} = iB^{34} = iC^{34} = -\frac{i}{2\sqrt{3}} I_{3 \times 3}$, with $I_{3 \times 3}$ the three by three identity matrix.

B Correlators in the symmetric vacuum for the 2-site model

Let's consider a 2-site model like the one used in this paper or the ones described in [18, 19, 22], where the composite sector has a global symmetry group G that is broken down spontaneously to an H subgroup and fermions transform under full irreducible representations (irreps) of G . If we focus solely on the top quark sector of the model, its fermion content consists of two chiral elementary multiplets, one containing the electroweak doublet, q_L , and the other containing the electroweak singlet, u_R , and two vector-like composite multiplets corresponding to the composite counterparts of the aforementioned elementary multiplets, whose chiral components we call $\Psi_{qL/R}$ and $\Psi_{uL/R}$ respectively. The composite multiplets Ψ_q and Ψ_u are in irreps of G , which we shall call α and β respectively. Assuming the partial compositeness scheme, we add spurious fields to the elementary multiplets q or u and embed them in the same irreducible representations of G as the composite ones; these elementary multiplets are called ψ_q and ψ_u . Let's define α_H as the set of irreps of H with a non-zero multiplicity in the decomposition of α , and similarly define β_H . For the model to contain Yukawa interactions, it is necessary that $\Gamma := \alpha_H \cap \beta_H \neq \emptyset$. Assuming that to be the case, the Lagrangian density for the fermionic sector of this model in the H -symmetric vacuum can be described as follows:

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{mass}} + \mathcal{L}_{\text{mix}} + \mathcal{L}_{\text{Yuk}} + \mathcal{L}_0 \tag{B.1} \\ \mathcal{L}_{\text{kin}} &= \bar{\psi}_{qL} \not{D} \psi_{qL} + \bar{\psi}_{uR} \not{D} \psi_{uR} + \bar{\Psi}_{qL} \not{D} \Psi_{qL} + \bar{\Psi}_{qR} \not{D} \Psi_{qR} + \bar{\Psi}_{uL} \not{D} \Psi_{uL} + \bar{\Psi}_{uR} \not{D} \Psi_{uR} \\ \mathcal{L}_{\text{mass}} &= -m_{q,1} \bar{\Psi}_{qL} \Psi_{qR} - m_{u,1} \bar{\Psi}_{uL} \Psi_{uR} + h.c. \end{aligned}$$

$$\mathcal{L}_{\text{mix}} = f_0 \lambda_q \bar{\psi}_{qL} \Psi_{qR} + f_0 \lambda_u \bar{\psi}_{uR} \Psi_{uL} + h.c.$$

$$\mathcal{L}_{\text{Yuk}} = f_1 \sum_{r \in \Gamma} y_r \overline{P_r(\Psi_{qL})} P_r(\Psi_{uR}) + h.c.$$

$$\mathcal{L}_0 = -\delta_{\alpha\beta} m_y \bar{\Psi}_{qR} \Psi_{uL} + h.c.$$

where P_r is the projector from the representation space of G to the subspace associated to the irrep r of H . Proceeding in a similar way it is straightforward to include the d_R sector, as well as the light generations of fermions.

We now integrate the composite degrees of freedom using their tree-level equations of motion and calculate the correlators for the elementary fields that transform according to each of the irreps in α_H or β_H (both dynamical and spurious). After integration, the effective fermionic Lagrangian density for the elementary fields in this vacuum takes the form:

$$\mathcal{L}_{\text{eff}} = \sum_{r \in \alpha_H} \bar{\psi}_{qL}^r \not{p} (1 + \Pi_L^r) \psi_{qL}^r + \sum_{s \in \beta_H} \bar{\psi}_{uR}^s \not{p} (1 + \Pi_R^s) \psi_{uR}^s + \sum_{t \in \Gamma} (\bar{\psi}_{qL}^t \Pi_{LR}^t \psi_{uR}^t + \bar{\psi}_{uR}^t \Pi_{RL}^t \psi_{qL}^t) \quad (\text{B.2})$$

with the different correlators given by the following expressions:

$$\begin{aligned} \Pi_L^r &= -f_0^2 \lambda_q^2 \frac{p^2 - \mathbf{I}_\Gamma^r(m_{u,1}^2 + |f_1 y_r|^2)}{\delta_{\alpha\beta} [|m_y|^2 |f_1 y_r|^2 - p^2 |m_y|^2 + f_1 (y_r^* m_y + y_r m_y^*) m_{q,1} m_{u,1}] + (m_{q,1}^2 - p^2) (\mathbf{I}_\Gamma^r m_{u,1}^2 - p^2) - \mathbf{I}_\Gamma^r p^2 |f_1 y_r|^2} \\ \Pi_R^r &= -f_0^2 \lambda_u^2 \frac{p^2 - \mathbf{I}_\Gamma^r(m_{q,1}^2 + |f_1 y_r|^2)}{\delta_{\alpha\beta} [|m_y|^2 |f_1 y_r|^2 - p^2 |m_y|^2 + f_1 (y_r^* m_y + y_r m_y^*) m_{q,1} m_{u,1}] + (\mathbf{I}_\Gamma^r m_{q,1}^2 - p^2) (m_{u,1}^2 - p^2) - \mathbf{I}_\Gamma^r p^2 |f_1 y_r|^2} \\ \Pi_{LR}^r &= f_0^2 \lambda_q \lambda_u \frac{\delta_{\alpha\beta} m_y (|f_1 y_r|^2 - p^2) + \mathbf{I}_\Gamma^r m_{q,1} m_{u,1} f_1 y_r}{\delta_{\alpha\beta} [|m_y|^2 |f_1 y_r|^2 - p^2 |m_y|^2 + f_1 (y_r^* m_y + y_r m_y^*) m_{q,1} m_{u,1}] + (m_{q,1}^2 - p^2) (m_{u,1}^2 - p^2) - \mathbf{I}_\Gamma^r p^2 |f_1 y_r|^2} \\ \Pi_{RL}^r &= f_0^2 \lambda_q \lambda_u \frac{\delta_{\alpha\beta} m_y^* (|f_1 y_r|^2 - p^2) + \mathbf{I}_\Gamma^r m_{q,1} m_{u,1} f_1 y_r^*}{\delta_{\alpha\beta} [|m_y|^2 |f_1 y_r|^2 - p^2 |m_y|^2 + f_1 (y_r^* m_y + y_r m_y^*) m_{q,1} m_{u,1}] + (m_{q,1}^2 - p^2) (m_{u,1}^2 - p^2) - \mathbf{I}_\Gamma^r p^2 |f_1 y_r|^2} \end{aligned} \quad (\text{B.3})$$

where $\mathbf{I}_\Gamma^r$ is the characteristic or indicator function of the set Γ which takes the value 1 if $r \in \Gamma$ and is 0 otherwise.

Note that in the case $\alpha \neq \beta$ and $r \notin \Gamma$ the correlators reduce to:

$$\Pi_L^r = -\frac{f_0^2 \lambda_q^2}{(p^2 - m_{q,1}^2)}, \quad \Pi_R^r = -\frac{f_0^2 \lambda_u^2}{(p^2 - m_{u,1}^2)}, \quad \Pi_{LR}^r = \Pi_{RL}^r = 0. \quad (\text{B.4})$$

which are independent of r and thus the same for all irreps of H that don't belong to Γ .

Proceeding in a similar way for the gauge fields we obtain the following correlators in the $\text{SU}(6)$ -sector:

$$\Pi_{81}^G = \Pi_{18}^G = \Pi_{11}^G = \frac{p^2 f_{6,0}^2}{2p^2 - f_{6,0}^2 g_{6,1}^2}, \quad \Pi_{33}^G = \frac{f_{6,0}^2 (2p^2 - f_{6,1}^2 g_{6,1}^2)}{2(2p^2 - f_{6,0}^2 g_{6,1}^2 - f_{6,1}^2 g_{6,1}^2)}. \quad (\text{B.5})$$

The case of $\text{SO}(5) \times \text{U}(1)_X$ can be found, for example, in ref. [19].

C Invariants for the irreps of $\text{SO}(5)$

In this appendix, we present the explicit form of the $F_r^f(s_5)$ and $G_r^{ff'}(s_5)$ functions of eq. (2.19). Most of these had been calculated previously in the literature, see for example: [14, 19–22].

For the singlet of $\text{SO}(5)$ we have:

$1_{2/3}$ of $\text{SO}(5) \times \text{U}(1)_X$	u_L	d_L	u_R
$(1, 1)_{2/3}$ of $\text{SU}(2)_L \times \text{SU}(2)_R$	$F_{11}^{u_L}(s_5) = 0$	$F_{11}^{d_L}(s_5) = 0$	$F_{11}^{u_R}(s_5) = 1$

Now, keeping the same layout for the tables but simplifying the notation, the rest of the F_r^f functions are:

$4_{1/6}$	u_L	d_L	u_R	$5_{2/3}$	u_L	d_L	u_R	$10_{2/3}$	u_L	d_L	u_R
$(2, 1)_{1/6}$	$\frac{1+c_5}{2}$	$\frac{1+c_5}{2}$	$\frac{1-c_5}{2}$	$(2, 2)_{2/3}$	$\frac{1+c_5^2}{2}$	1	s_5^2	$(2, 2)_{2/3}$	$\frac{1+c_5^2}{2}$	c_5^2	$\frac{s_5^2}{2}$
$(1, 2)_{1/6}$	$\frac{1-c_5}{2}$	$\frac{1-c_5}{2}$	$\frac{1+c_5}{2}$	$(1, 1)_{2/3}$	$\frac{s_5^2}{2}$	0	c_5^2	$(3, 1)_{2/3}$	$\frac{s_5^2}{4}$	$\frac{s_5^2}{2}$	$\frac{(c_5^2-1)^2}{4}$
								$(1, 3)_{2/3}$	$\frac{s_5^2}{4}$	$\frac{s_5^2}{2}$	$\frac{(c_5^2+1)^2}{4}$

$14_{2/3}$	u_L	d_L	u_R
$(3, 3)_{2/3}$	$\frac{1}{4}(2 + 3c_5^2)s_5^2$	s_5^2	$\frac{15}{64}(1 - c_5^2 + s_5^2)^2$
$(2, 2)_{2/3}$	$\frac{1}{2}(c_5^4 + s_5^4 + c_5^2(1 - 2s_5^2))$	c_5^2	$\frac{5c_5^2s_5^2}{2}$
$(1, 1)_{2/3}$	$\frac{5c_5^2s_5^2}{4}$	0	$\frac{1}{64}(3 + 5c_5^2 - 5s_5^2)^2$

$16_{1/6}^{(q_L \in (2,1))}$	u_L	d_L	u_R
$(3, 2)_{1/6}$	$-\frac{15}{32}(c_5 - 1)(c_5 + 1)^2$	$-\frac{15}{32}(c_5 - 1)(c_5 + 1)^2$	$\frac{15}{32}(c_5 - 1)^2(c_5 + 1)$
$(2, 3)_{1/6}$	$\frac{15}{32}(c_5 - 1)^2(c_5 + 1)$	$\frac{15}{32}(c_5 - 1)^2(c_5 + 1)$	$-\frac{15}{32}(c_5 - 1)(c_5 + 1)^2$
$(2, 1)_{1/6}$	$\frac{1}{32}(5c_5 - 1)^2(c_5 + 1)$	$\frac{1}{32}(5c_5 - 1)^2(c_5 + 1)$	$-\frac{1}{32}(c_5 - 1)(5c_5 + 1)^2$
$(1, 2)_{1/6}$	$-\frac{1}{32}(c_5 - 1)(5c_5 + 1)^2$	$-\frac{1}{32}(c_5 - 1)(5c_5 + 1)^2$	$\frac{1}{32}(5c_5 - 1)^2(c_5 + 1)$

$16_{1/6}^{(q_L \in (2,3))}$	u_L	d_L	u_R
$(3,2)_{1/6}$	$\frac{1}{32}(11 - 13c_5 + 5c_5^2 - 3c_5^3)$	$\frac{1}{32}(11 - 13c_5 + 5c_5^2 - 3c_5^3)$	$\frac{15}{32}(c_5 - 1)^2(c_5 + 1)$
$(2,3)_{1/6}$	$\frac{1}{32}(11 + 13c_5 + 5c_5^2 + 3c_5^3)$	$\frac{1}{32}(11 + 13c_5 + 5c_5^2 + 3c_5^3)$	$-\frac{15}{32}(c_5 - 1)(c_5 + 1)^2$
$(2,1)_{1/6}$	$\frac{5}{32}(c_5 - 1)^2(c_5 + 1)$	$\frac{5}{32}(c_5 - 1)^2(c_5 + 1)$	$-\frac{1}{32}(c_5 - 1)(5c_5 + 1)^2$
$(1,2)_{1/6}$	$-\frac{5}{32}(c_5 - 1)(c_5 + 1)^2$	$-\frac{5}{32}(c_5 - 1)(c_5 + 1)^2$	$\frac{1}{32}(5c_5 - 1)^2(c_5 + 1)$

where we have used the shorthand notation $s_5 = \sin\left(\frac{h_5}{f_5}\right)$ and $c_5 = \cos\left(\frac{h_5}{f_5}\right)$

In a similar fashion, but with a different layout, the expressions for the $G_r^{u_L u_R}$ are shown in the following table:

		u_R in						
		$r \downarrow$	$1_{2/3}$	$4_{1/6}$	$5_{2/3}$	$10_{2/3}$	$14_{2/3}$	$16_{1/6}$
u_L in	$4_{1/6}$	$(2,1)_{1/6}$	-	$\frac{i}{2}s_5$	-	-	-	$-\frac{1}{8}(5c_5 + 1)s_5$
		$(1,2)_{1/6}$	-	$-\frac{i}{2}s_5$	-	-	-	$-\frac{i}{8}(5c_5 - 1)s_5$
	$5_{2/3}$	$(2,2)_{2/3}$	-	-	$\frac{s_5 c_5}{\sqrt{2}}$	$\frac{is_5}{2}$	$\frac{i\sqrt{5}}{2}c_5 s_5$	-
		$(1,1)_{2/3}$	$-\frac{s_5}{\sqrt{2}}$	-	$-\frac{s_5 c_5}{\sqrt{2}}$	-	$\frac{5s_5^3 - 4s_5}{4\sqrt{2}}$	-
	$10_{2/3}$	$(2,2)_{2/3}$	-	-	$\frac{s_5}{\sqrt{2}}$	$\frac{ic_5 s_5}{2}$	$\frac{i\sqrt{5}}{2}c_5^2 s_5$	-
		$(3,1)_{2/3}$	-	-	-	$-\frac{i}{4}s_5(c_5 - 1)$	-	-
		$(1,3)_{2/3}$	-	-	-	$-\frac{i}{4}s_5(c_5 + 1)$	-	-
	$14_{2/3}$	$(3,3)_{2/3}$	-	-	-	-	$\frac{i3\sqrt{5}}{8}c_5 s_5^3$	-
		$(2,2)_{2/3}$	-	-	$\frac{c_5 s_5}{\sqrt{2}}$	$\frac{i}{2}(s_5 - 2s_5^3)$	$\frac{i\sqrt{5}}{2}c_5 s_5(1 - 2s_5^2)$	-
		$(1,1)_{2/3}$	$-\frac{i\sqrt{5}}{2}c_5 s_5$	-	$-\frac{i\sqrt{5}}{2}c_5^2 s_5$	-	$-\frac{i\sqrt{5}}{8}c_5 s_5(4 - 5s_5^2)$	-
	$16_{1/6}^{(q_L \in (2,3))}$	$(3,2)_{1/6}$	-	-	-	-	-	$-\frac{\sqrt{5}}{32}(c_5 - 1)(3c_5 - 1)s_5$
		$(2,3)_{1/6}$	-	-	-	-	-	$\frac{\sqrt{5}}{32}(c_5 + 1)(3c_5 + 1)s_5$
		$(2,1)_{1/6}$	-	$-\frac{i\sqrt{5}}{8}(c_5 - 1)s_5$	-	-	-	$-\frac{\sqrt{5}}{32}(c_5 - 1)(5c_5 + 1)s_5$
		$(1,2)_{1/6}$	-	$-\frac{\sqrt{5}}{8}(c_5 + 1)s_5$	-	-	-	$-\frac{\sqrt{5}}{32}(c_5 + 1)(5c_5 - 1)s_5$
	$16_{1/6}^{(q_L \in (2,1))}$	$(3,2)_{1/6}$	-	-	-	-	-	$-\frac{15}{32}s_5^3$
		$(2,3)_{1/6}$	-	-	-	-	-	$\frac{15}{32}s_5^3$
		$(2,1)_{1/6}$	-	$\frac{i}{8}(5c_5 - 1)s_5$	-	-	-	$\frac{1}{32}(1 - 25c_5^2)s_5$
		$(1,2)_{1/6}$	-	$\frac{1}{8}(5c_5 + 1)s_5$	-	-	-	$-\frac{1}{32}(1 - 25c_5^2)s_5$

If a d_R quark was to be introduced in the model, the functions $F_r^{d_R}$ and $G_r^{d_L d_R}$ would also be needed. Their calculation is completely analogous to that of the functions presented here. For the specific case of the 14-1, if we are unwilling to mix the elementary q_L with a second composite multiplet in a different irrep of $SO(5) \times U(1)_X$ then the smallest irrep in which we can embed the d_R is the 10. For that minimal implementation, we obtain:

$$F_{22}^{d_R}(s_5) = \frac{s_5^2}{2}, \quad F_{31}^{d_R}(s_5) = \frac{(1 - c_5)^2}{4}, \quad F_{13}^{d_R}(s_5) = \frac{(1 + c_5)^2}{4},$$

$$G_{22}^{d_L d_R}(s_5) = \frac{s_5 c_5}{\sqrt{2}}.$$

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