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Neutrino μ - τ reflection symmetry and its breaking in the minimal seesaw

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ABSTRACT: In this paper, we attempt to implement the neutrino μ - τ reflection symmetry (which predicts $\theta_{23} = \pi/4$ and $\delta = \pm\pi/2$ as well as trivial Majorana phases) in the minimal seesaw (which enables us to fix the neutrino masses). For some direct (the preliminary experimental hints towards $\theta_{23} \neq \pi/4$ and $\delta \neq -\pi/2$) and indirect (inclusion of the renormalization group equation effect and implementation of the leptogenesis mechanism) reasons, we particularly study the breakings of this symmetry and their phenomenological consequences.

KEYWORDS: Discrete Symmetries, Neutrino Physics, CP violation

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1 Introduction

As is known, the discovery of neutrino oscillations indicates that neutrinos are massive and mixed [1]. On the one hand, the most popular way of generating neutrino masses is to invoke the seesaw mechanism which allows for a natural explanation of their smallness. In the type-I seesaw mechanism [2–6], one usually introduces three right-handed neutrino fields N_i (for $i = 1, 2, 3$). They not only take part in Yukawa interactions with the left-handed neutrino fields which lead to a Dirac mass matrix M_D , but also have a Majorana mass matrix M_N of themselves. Under the assumption of $M_N \gg M_D$, the effective mass matrix for light neutrinos is obtained as [2–6]

$$M_\nu = -M_D M_N^{-1} M_D^T. \quad (1.1)$$

The overall minus sign here is of no physical meaning and will be neglected in the following discussions. On the other hand, the neutrino mixing arises from a mismatch between their mass and flavor eigenstates and is described by a 3×3 unitary matrix $U = U_l^\dagger U_\nu$ [7, 8] with U_l and U_ν being respectively the unitary matrix for diagonalizing the charged-lepton mass matrix M_l and M_ν . In the standard parametrization, U reads [1]

$$U \equiv P_\phi V P_\nu = P_\phi \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} P_\nu. \quad (1.2)$$

Here $P_\phi = \text{Diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3})$ consists of three unphysical phases that can be removed via the rephasing of charged-lepton fields, while $P_\nu = \text{Diag}(e^{i\rho}, e^{i\sigma}, 1)$ contains two Majorana phases. Similar to the CKM matrix, V possesses a Dirac phase δ and three mixing angles θ_{ij} (for $ij = 12, 13, 23$). For convenience, the abbreviations $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$

have been used. Besides the mixing parameters, neutrino oscillations are also regulated by two neutrino mass-squared differences $\Delta m_{ij}^2 = m_i^2 - m_j^2$ (for $ij = 21, 31$) [9]

$$\Delta m_{21}^2 = (7.50 \pm 0.18) \times 10^{-5} \text{ eV}^2, \quad |\Delta m_{31}^2| = (2.524 \pm 0.039) \times 10^{-3} \text{ eV}^2. \quad (1.3)$$

In comparison, the absolute neutrino mass scale has to be inferred from non-oscillatory experiments,¹ and a definite result is still missing. Note that the sign of Δm_{31}^2 has not yet been determined, thereby allowing for two possible mass orderings $m_1 < m_2 < m_3$ (the normal hierarchy, NH for short) and $m_3 < m_1 < m_2$ (the inverted hierarchy, IH for short). It turns out that the fitted values of mixing parameters depend on the mass ordering in a certain way: θ_{13} , θ_{23} and δ take the values

$$\sin^2 \theta_{13} = 0.02166 \pm 0.00075, \quad \sin^2 \theta_{23} = 0.441 \pm 0.024, \quad \delta = 261^\circ \pm 55^\circ, \quad (1.4)$$

in the NH case, or

$$\sin^2 \theta_{13} = 0.02179 \pm 0.00076, \quad \sin^2 \theta_{23} = 0.587 \pm 0.022, \quad \delta = 277^\circ \pm 43^\circ, \quad (1.5)$$

in the IH case, whereas θ_{12} takes the value $\sin^2 \theta_{12} = 0.306 \pm 0.012$ in either case [9].

How to understand the observed neutrino mixing poses an interesting question. As one can see, the measured θ_{12} , θ_{23} and δ are close to some special values

$$\sin^2 \theta_{12} = \frac{1}{3}, \quad \sin^2 \theta_{23} = \frac{1}{2}, \quad \delta = -\frac{\pi}{2}. \quad (1.6)$$

These remarkable coincidences invite us to speculate that some flavor symmetry has played an important role in shaping the neutrino mixing.² In this connection, the μ - τ reflection symmetry [14] serves as a unique candidate: in the basis of M_l being diagonal, M_ν should stay invariant with respect to the transformations³

$$\nu_e \leftrightarrow \nu_e^c, \quad \nu_\mu \leftrightarrow \nu_\tau^c, \quad \nu_\tau \leftrightarrow \nu_\mu^c, \quad (1.7)$$

and have its elements $M_{\alpha\beta}$ (for $\alpha, \beta = e, \mu, \tau$) obeying the conditions

$$M_{e\mu} = M_{e\tau}^*, \quad M_{\mu\mu} = M_{\tau\tau}^*, \quad M_{ee} \text{ and } M_{\mu\tau} \text{ being real}. \quad (1.8)$$

Consequently, U_l is a diagonal phase matrix of no physical meaning $U_l = \text{Diag}(e^{i\varphi_1}, e^{i\varphi_2}, e^{i\varphi_3})$ where $\varphi_{1,2,3}$ can take arbitrary values. On the other hand, as a result of the six real conditions for M_ν given by eq. (1.8), there are six predictions [17]

$$\phi_1 = 0, \quad \phi_2 = -\phi_3, \quad \theta_{23} = \pi/4, \quad \delta = \pm\pi/2, \quad \rho, \sigma = 0 \text{ or } \pi/2, \quad (1.9)$$

for the mixing parameters of U_ν which itself takes a form as given by eq. (1.2). Furthermore, unlike the μ - τ interchange symmetry [18–21]⁴ which predicts $\theta_{13} = 0$, the μ - τ reflection

¹See e.g. refs. [10, 11].

²For a review with extensive references see refs. [12, 13].

³This operation is a combination of the μ - τ exchange and CP conjugate transformations — a specific example of the generalized CP transformations [15, 16].

⁴For a review with extensive references see ref. [22].

symmetry allows for an arbitrary θ_{13} . Thanks to these interesting consequences, this symmetry has been attracting a lot of attention recently.⁵

Nevertheless, the neutrino masses receive no clues from the μ - τ reflection symmetry. But they will be fixed if the smallest one (m_1 in the NH case, m_3 in the IH case) is to vanish, a possibility that is experimentally allowed and can be accommodated by the minimal seesaw⁶ where only two right-handed neutrino fields $N_{1,2}$ are involved. In this paper, we just attempt to combine the μ - τ reflection symmetry with the minimal seesaw so that both the neutrino mixing and mass spectrum can be settled. The implications of such a marriage are discussed in the next section. In consideration of the preliminary experimental results towards $\theta_{23} \neq \pi/4$ ⁷ and $\delta \neq -\pi/2$, we particularly explore the possible symmetry breakings and their impacts on the mixing parameters in section 3. Section 4 is devoted to the specific symmetry breaking triggered by the renormalization group equation (RGE) effect. In section 5, the operation of leptogenesis in this framework will be studied. Finally, our main results are summarized in section 6.

2 μ - τ reflection symmetry in the minimal seesaw

Conversely, one can reconstruct an M_ν of the μ - τ reflection symmetry in terms of the U_ν characterized by eq. (1.9) and neutrino masses by means of the relation $M_\nu = U_\nu \text{Diag}(m_1, m_2, m_3) U_\nu^T$. In the situation of one neutrino mass being vanishing, the reconstructed M_ν in such a way will take a form as

$$\begin{aligned} M_{ee} &= \bar{m}_2 s_{12}^2 c_{13}^2 - m_3 s_{13}^2, \\ M_{e\mu} &= \frac{1}{\sqrt{2}} [\bar{m}_2 s_{12} (c_{12} + i s_{12} \bar{s}_{13}) + i m_3 \bar{s}_{13}] c_{13} e^{i\phi}, \\ M_{\mu\mu} &= \frac{1}{2} [\bar{m}_2 (c_{12} + i s_{12} \bar{s}_{13})^2 + m_3 c_{13}^2] e^{2i\phi}, \\ M_{\mu\tau} &= \frac{1}{2} [\bar{m}_2 (c_{12}^2 + s_{12}^2 s_{13}^2) - m_3 c_{13}^2], \end{aligned} \quad (2.1)$$

in the NH case, or

$$\begin{aligned} M_{ee} &= [m_1 c_{12}^2 + \bar{m}_2 s_{12}^2] c_{13}^2, \\ M_{e\mu} &= \frac{1}{\sqrt{2}} [m_1 c_{12} (-s_{12} + i c_{12} \bar{s}_{13}) + \bar{m}_2 s_{12} (c_{12} + i s_{12} \bar{s}_{13})] c_{13} e^{i\phi}, \\ M_{\mu\mu} &= \frac{1}{2} [m_1 (s_{12} - i c_{12} \bar{s}_{13})^2 + \bar{m}_2 (c_{12} + i s_{12} \bar{s}_{13})^2] e^{2i\phi}, \\ M_{\mu\tau} &= \frac{1}{2} [m_1 (s_{12}^2 + c_{12}^2 s_{13}^2) + \bar{m}_2 (c_{12}^2 + s_{12}^2 s_{13}^2)], \end{aligned} \quad (2.2)$$

in the IH case, where $\bar{m}_2 \equiv m_2 e^{2i\sigma}$ and $\bar{s}_{13} \equiv -i s_{13} e^{i\delta}$ have been defined. Note that $M_{e\mu}$ and $M_{\mu\mu}$ depend on the unphysical phase $\phi \equiv \phi_2 = -\phi_3$ whose value can be chosen freely

⁵For an incomplete list see refs. [23–40].

⁶For an incomplete list see refs. [41–61].

⁷In particular, a recent result reported by the NOvA collaboration ($\theta_{23} = 39.5^\circ \pm 1.7^\circ$ or $52.1^\circ \pm 1.7^\circ$ in the NH case) disfavors the possibility of $\theta_{23} = 45^\circ$ at a 2.6σ level [62].

$M_{\alpha\beta}(\text{eV})$ [δ, σ]	$M_{ee}(\times 10^{-2})$	$M_{e\mu}(\times 10^{-2}e^{i\phi})$	$M_{\mu\mu}(\times 10^{-2}e^{2i\phi})$	$M_{\mu\tau}(\times 10^{-2})$
$[\pi/2, 0]$	0.15	$0.28 + 0.54i$	$2.76 + 0.06i$	-2.15
$[-\pi/2, 0]$	0.15	$0.28 - 0.54i$	$2.76 - 0.06i$	-2.15
$[\pi/2, \pi/2]$	-0.37	$-0.28 + 0.49i$	$2.16 - 0.06i$	-2.76
$[-\pi/2, \pi/2]$	-0.37	$-0.28 - 0.49i$	$2.16 + 0.06i$	-2.76

Table 1. The possible values of $M_{\alpha\beta}$ for various combinations of δ and σ in the NH case.

$M_{\alpha\beta}(\text{eV})$ [δ, σ]	$M_{ee}(\times 10^{-2})$	$M_{e\mu}(\times 10^{-2}e^{i\phi})$	$M_{\mu\mu}(\times 10^{-2}e^{2i\phi})$	$M_{\mu\tau}(\times 10^{-2})$
$[\pi/2, 0]$	4.86	$0.02 + 0.51i$	$2.45 + 0.01i$	2.56
$[-\pi/2, 0]$	4.86	$0.02 - 0.51i$	$2.45 - 0.01i$	2.56
$[\pi/2, \pi/2]$	1.86	$-3.21 + 0.20i$	$-1.01 - 0.68i$	-0.97
$[-\pi/2, \pi/2]$	1.86	$-3.21 - 0.20i$	$-1.01 + 0.68i$	-0.97

Table 2. The possible values of $M_{\alpha\beta}$ for various combinations of δ and σ in the IH case.

without affecting the physical results. And there is only one effective Majorana phase which we assign for m_2 (i.e., σ). Because of the condition $\text{Det}(M_\nu) = 0$,⁸ only five out of the six real components of these elements are independent. Taking the best-fit values for θ_{13} , θ_{12} , Δm_{21}^2 and $|\Delta m_{31}^2|$ as input, we give the values of $M_{\alpha\beta}$ for various combinations of δ and σ (i.e., $[\delta, \sigma] = [\pi/2, 0], [-\pi/2, 0], [\pi/2, \pi/2]$ and $[-\pi/2, \pi/2]$) in the NH (table 1) and IH (table 2) cases. As is well known, the size of $|M_{ee}|$ which governs the rate of neutrino-less double beta decays⁹ is much larger in the IH case than in the NH case. In the NH case, the elements exhibit a hierarchical structure as $|M_{\mu\mu}| \sim |M_{\mu\tau}| \gg |M_{ee}| \sim |M_{e\mu}|$, implying that they might have received contributions from different levels [67]. In the IH case, $|M_{ee}|$ (so does $|M_{e\mu}|$ for $\sigma = 0$) becomes comparable to $|M_{\mu\mu}|$ and $|M_{\mu\tau}|$. But for $\sigma = \pi/2$, $M_{e\mu}$ will have a magnitude much smaller than the other ones as a result of the heavy cancellation between its two components respectively associated with m_1 and $\bar{m}_2$. By choosing the value of ϕ in such a way that the phase of $M_{e\mu}$ or $M_{\mu\mu}$ cancels out, one may convert M_ν to a simpler form as given by tables 3–4.

The M_ν given by eqs. (2.1)–(2.2) can be viewed as a result of the minimal seesaw: the Dirac mass matrix coupling $N_{1,2}$ with the left-handed neutrino fields is assumed to appear

⁸Since the determinant of an M_ν obeying the μ - τ reflection symmetry is always real, this condition only amounts to one constraint.

⁹For a review with extensive references see refs. [63–66].

$M_{\alpha\beta}(\text{eV})$ [δ, σ]	$M_{e\mu}(\times 10^{-2})$	$M_{\mu\mu}(\times 10^{-2})$	$M_{e\mu}(\times 10^{-2})$	$M_{\mu\mu}(\times 10^{-2})$
$[\pi/2, 0]$	0.61	$-1.56 - 2.27i$	$0.28 + 0.54i$	2.76
$[-\pi/2, 0]$	0.61	$-1.56 + 2.27i$	$0.28 - 0.54i$	2.76
$[\pi/2, \pi/2]$	0.56	$-1.05 + 1.89i$	$-0.29 + 0.49i$	2.16
$[-\pi/2, \pi/2]$	0.56	$-1.05 - 1.89i$	$-0.29 - 0.49i$	2.16

Table 3. The possible values of $M_{e\mu}$ and $M_{\mu\mu}$ for various combinations of δ and σ in the NH case, after one of them is made real by a particular value of ϕ .

$M_{\alpha\beta}(\text{eV})$ [δ, σ]	$M_{e\mu}(\times 10^{-2})$	$M_{\mu\mu}(\times 10^{-2})$	$M_{e\mu}(\times 10^{-2})$	$M_{\mu\mu}(\times 10^{-2})$
$[\pi/2, 0]$	0.51	$-2.43 - 0.24i$	$0.02 + 0.51i$	2.45
$[-\pi/2, 0]$	0.51	$-2.43 + 0.24i$	$0.02 - 0.51i$	2.45
$[\pi/2, \pi/2]$	3.22	$-0.92 - 0.80i$	$-1.13 - 3.02i$	1.21
$[-\pi/2, \pi/2]$	3.22	$-0.92 + 0.80i$	$-1.13 + 3.02i$	1.21

Table 4. The possible values of $M_{e\mu}$ and $M_{\mu\mu}$ for various combinations of δ and σ in the IH case, after one of them is made real by a particular value of ϕ .

as [68]

$$M_D = \begin{pmatrix} a_1 \sqrt{M_1} & \sqrt{M_2} b_1 \\ e^{i\phi_a} a_2 \sqrt{M_1} & \sqrt{M_2} b_2 e^{i\phi_b} \\ e^{-i\phi_a} a_2 \sqrt{M_1} & \sqrt{M_2} b_2 e^{-i\phi_b} \end{pmatrix}, \quad (2.3)$$

with $a_{1,2}$, $b_{1,2}$, $\phi_{a,b}$ and $M_{1,2}$ being real parameters. Apparently, its elements satisfy the conditions of $M_{\mu i} = M_{\tau i}^*$ and M_{ei} being real (for $i = 1, 2$). It is easy to see that these conditions still hold when the right-handed neutrino fields experience an orthogonal basis transformation. So, without loss of generality, we choose to work in the basis where the Majorana mass matrix for $N_{1,2}$ is diagonal $M_N = \text{Diag}(M_1, M_2)$. By virtue of the seesaw formula in eq. (1.1), we arrive at an effective neutrino mass matrix

$$M_\nu = \begin{pmatrix} a_1^2 + b_1^2 & a_1 a_2 e^{i\phi_a} + b_1 b_2 e^{i\phi_b} & a_1 a_2 e^{-i\phi_a} + b_1 b_2 e^{-i\phi_b} \\ a_1 a_2 e^{i\phi_a} + b_1 b_2 e^{i\phi_b} & a_2^2 e^{2i\phi_a} + b_2^2 e^{2i\phi_b} & a_2^2 + b_2^2 \\ a_1 a_2 e^{-i\phi_a} + b_1 b_2 e^{-i\phi_b} & a_2^2 + b_2^2 & a_2^2 e^{-2i\phi_a} + b_2^2 e^{-2i\phi_b} \end{pmatrix}. \quad (2.4)$$

Diagonalizing this M_ν with a U_ν characterized by eq. (1.9) yields the mixing parameters

$$\begin{aligned}\tan \theta_{13} &= \frac{a_2^2 \sin(2\phi - 2\phi_a) + b_2^2 \sin(2\phi - 2\phi_b)}{-\sqrt{2} \sin \delta [a_1 a_2 \cos(\phi - \phi_a) + b_1 b_2 \cos(\phi - \phi_b)]}, \\ \tan 2\theta_{13} &= \frac{-2\sqrt{2} [a_1 a_2 \sin(\phi - \phi_a) + b_1 b_2 \sin(\phi - \phi_b)]}{\sin \delta [a_1^2 + b_1^2 - P]}, \\ \tan 2\theta_{12} &= \frac{-2\sqrt{2} \cos 2\theta_{13} [a_1 a_2 \cos(\phi - \phi_a) + b_1 b_2 \cos(\phi - \phi_b)]}{c_{13} [(a_1^2 + b_1^2) c_{13}^2 - P s_{13}^2 - Q \cos 2\theta_{13}]},\end{aligned}\quad (2.5)$$

and neutrino masses

$$\begin{aligned}\bar{m}_1 &= Q - \frac{\sqrt{2}}{c_{13} t_{12}} [a_1 a_2 \cos(\phi - \phi_a) + b_1 b_2 \cos(\phi - \phi_b)], \\ \bar{m}_2 &= Q + \frac{\sqrt{2} t_{12}}{c_{13}} [a_1 a_2 \cos(\phi - \phi_a) + b_1 b_2 \cos(\phi - \phi_b)], \\ \bar{m}_3 &= \frac{(a_1^2 + b_1^2) s_{13}^2 - P c_{13}^2}{\cos 2\theta_{13}},\end{aligned}\quad (2.6)$$

where $\bar{m}_1 = m_1 e^{2i\rho}$, $\bar{m}_3 = m_3 e^{2i\gamma}$ (for $\rho, \gamma = 0$ or $\pi/2$) and

$$\begin{aligned}P &= 2a_2^2 \sin^2(\phi - \phi_a) + 2b_2^2 \sin^2(\phi - \phi_b), \\ Q &= 2a_2^2 \cos^2(\phi - \phi_a) + 2b_2^2 \cos^2(\phi - \phi_b).\end{aligned}\quad (2.7)$$

For any given values of $a_{1,2}$, $b_{1,2}$ and $\phi_{a,b}$, one mass will necessarily vanish as promised by the minimal seesaw.¹⁰ The resulting θ_{13} , ϕ , θ_{12} and two non-zero masses can be calculated with the help of the other five equations.

If we are to derive the allowed values of $a_{1,2}$, $b_{1,2}$ and $\phi_{a,b}$ from the measured θ_{13} , θ_{23} , Δm_{21}^2 and $|\Delta m_{31}^2|$, one just needs to confront the M_ν in eq. (2.4) with the results given by tables 1–2. Above all, it should be noted that both the ee and $\mu\tau$ elements of this M_ν are positive. In order for them to fit in with the corresponding results in tables 1–2, one must have $\sigma = \pi/2$ (or 0) and $M_\nu \rightarrow -M_\nu$ (or $+M_\nu$) in the NH (or IH) case. In light of the unphysical nature of ϕ , $\phi_{a,b} - \phi$ rather than $\phi_{a,b}$ will be treated as effective independent parameters. Recall that only five real components of the neutrino mass matrix elements are independent, so the free parameters are more than the constraint equations by one. For this reason, in figure 1 we choose to present the results for a_2 , $b_{1,2}$ and $\phi_{a,b} - \phi$ as functions of a_1 (which stands in an equivalent position as b_1). In the numerical calculations here and in the following, the best-fit values for θ_{13} , θ_{12} , Δm_{21}^2 and $|\Delta m_{31}^2|$ are input, whereas δ is specified as $-\pi/2$. We have only shown the results in the case of both a_1 and b_1 being positive. The results in the case of a_1 or (and) b_1 being negative can be obtained by simply making the replacement $a_1 \rightarrow -a_1$ combined with $(\phi_a - \phi) \rightarrow (\phi_a - \phi) + \pi$ or (and) $b_1 \rightarrow -b_1$ combined with $(\phi_b - \phi) \rightarrow (\phi_b - \phi) + \pi$. This is because the M_ν in eq. (2.4) keeps invariant under this kind of transformations. It is interesting to find that the possibility of

¹⁰We are left with the difference of the phases associated with two non-zero masses as the effective Majorana phase.

	b_1	$\phi_b - \phi$	a_2	b_2	$\phi_a - \phi$
NH	0.06	1.05	0.14	0.09	1.79, 4.93
IH	0.22	4.76	0.16	0.02	0, π

Table 5. The possible values of a_2 , $b_{1,2}$ and $\phi_{a,b} - \phi$ in the particular case of $a_1 = 0$.

$a_1 = 0$ is allowed. The possible values of a_2 , $b_{1,2}$ and $\phi_{a,b} - \phi$ in such a particular case are listed in table 5. If we further make one of $\phi_{a,b}$ vanish by giving ϕ an appropriate value, then we will reach the simplest M_D . In the NH case, for instance, a value of -1.79 or -4.93 for ϕ allows us to have $\phi_a = 0$. (In the meantime, ϕ_b is fixed to -0.74 or -3.88 .)

3 Breaking of the μ - τ reflection symmetry

In this section we study the possible breakings of μ - τ reflection symmetry and their impacts on the mixing parameters.¹¹ The most general perturbation to M_D

$$\delta M_D = \begin{pmatrix} \delta_{11}\sqrt{M_1} & \sqrt{M_2}\delta_{12} \\ \delta_{21}\sqrt{M_1} & \sqrt{M_2}\delta_{22} \\ \delta_{31}\sqrt{M_1} & \sqrt{M_2}\delta_{32} \end{pmatrix}, \quad (3.1)$$

can be decomposed into a symmetry-conserving part and a symmetry-violating part:

$$\delta M_D = \frac{1}{2} \begin{pmatrix} 2\text{Re}(\delta_{11})\sqrt{M_1} & 2\text{Re}(\delta_{12})\sqrt{M_2} \\ (\delta_{21} + \delta_{31}^*)\sqrt{M_1} & (\delta_{22} + \delta_{32}^*)\sqrt{M_2} \\ (\delta_{21}^* + \delta_{31})\sqrt{M_1} & (\delta_{22}^* + \delta_{32})\sqrt{M_2} \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 2i\text{Im}(\delta_{11})\sqrt{M_1} & 2i\text{Im}(\delta_{12})\sqrt{M_2} \\ (\delta_{21} - \delta_{31}^*)\sqrt{M_1} & (\delta_{22} - \delta_{32}^*)\sqrt{M_2} \\ (\delta_{31} - \delta_{21}^*)\sqrt{M_1} & (\delta_{32} - \delta_{22}^*)\sqrt{M_2} \end{pmatrix}. \quad (3.2)$$

And the full Dirac mass matrix can be reparameterized as

$$M'_D = M_D + \delta M_D = \begin{pmatrix} \hat{a}_1(1 + i\epsilon_1)\sqrt{M_1} & \hat{b}_1(1 + i\epsilon_2)\sqrt{M_2} \\ \hat{a}_2 e^{i\hat{\phi}_a}(1 + \epsilon_3)\sqrt{M_1} & \hat{b}_2 e^{i\hat{\phi}_b}(1 + \epsilon_4)\sqrt{M_2} \\ \hat{a}_2 e^{-i\hat{\phi}_a}(1 - \epsilon_3^*)\sqrt{M_1} & \hat{b}_2 e^{-i\hat{\phi}_b}(1 - \epsilon_4^*)\sqrt{M_2} \end{pmatrix}, \quad (3.3)$$

with

$$\begin{aligned} \hat{a}_1 &= a_1 + \text{Re}(\delta_{11}), & \hat{a}_2 e^{i\hat{\phi}_a} &= a_2 e^{i\phi_a} + \frac{\delta_{21} + \delta_{31}^*}{2}, \\ \hat{b}_1 &= b_1 + \text{Re}(\delta_{21}), & \hat{b}_2 e^{i\hat{\phi}_b} &= b_2 e^{i\phi_b} + \frac{\delta_{22} + \delta_{32}^*}{2}, \end{aligned} \quad (3.4)$$

and

$$\begin{aligned} \epsilon_1 &\equiv \frac{\text{Im}((M_D)_{e1})}{\text{Re}((M_D)_{e1})} = \frac{\text{Im}(\delta_{11})}{a_1 + \text{Re}(\delta_{11})}, & \epsilon_3 &\equiv \frac{(M_D)_{\mu 1} - (M_D)_{\tau 1}^*}{(M_D)_{\mu 1} + (M_D)_{\tau 1}^*} = \frac{\delta_{21} - \delta_{31}^*}{2a_2 e^{i\phi_a} + \delta_{21} + \delta_{31}^*}, \\ \epsilon_2 &\equiv \frac{\text{Im}((M_D)_{e2})}{\text{Re}((M_D)_{e2})} = \frac{\text{Im}(\delta_{21})}{a_1 + \text{Re}(\delta_{21})}, & \epsilon_4 &\equiv \frac{(M_D)_{\mu 2} - (M_D)_{\tau 2}^*}{(M_D)_{\mu 2} + (M_D)_{\tau 2}^*} = \frac{\delta_{22} - \delta_{32}^*}{2b_2 e^{i\phi_b} + \delta_{22} + \delta_{32}^*}. \end{aligned} \quad (3.5)$$

¹¹For some discussions about the breakings of μ - τ interchange symmetry see refs. [69, 70].

Figure 1. The allowed values of a_2 , $b_{1,2}$ and $\phi_{a,b} - \phi$ as functions of a_1 , with $\sigma = \pi/2$ (or 0) in the NH (or IH) case and $\delta = -\pi/2$.

The dimensionless quantities $\epsilon_{1,2,3,4}$ measure the strength of symmetry breaking. They should be small (e.g., $|\epsilon_{1,2,3,4}| \leq 0.1$) in order for M'_D to assume an approximate μ - τ reflection symmetry.

Since ϵ_2 and ϵ_4 play equivalent roles as ϵ_1 and ϵ_3 , they will be assumed to vanish in the following discussions. For the sake of simplicity, the hat symbols on $\hat{a}_{1,2}$, $\hat{b}_{1,2}$ and $\hat{\phi}_{a,b}$ will also be neglected. Consequently, we are led to an effective neutrino mass matrix M'_ν of the form

$$\begin{aligned}
 M'_{ee} &= a_1^2 (1 + 2i\epsilon_1) + b_1^2, \\
 M'_{e\mu} &= a_1 a_2 e^{i\phi_a} (1 + i\epsilon_1 + \epsilon_3) + b_1 b_2 e^{i\phi_b}, \\
 M'_{e\tau} &= a_1 a_2 e^{-i\phi_a} (1 + i\epsilon_1 - \epsilon_3^*) + b_1 b_2 e^{-i\phi_b}, \\
 M'_{\mu\tau} &= a_2^2 [1 + 2i\text{Im}(\epsilon_3)] + b_2^2, \\
 M'_{\mu\mu} &= a_2^2 e^{2i\phi_a} (1 + 2\epsilon_3) + b_2^2 e^{2i\phi_b}, \\
 M'_{\tau\tau} &= a_2^2 e^{-2i\phi_a} (1 - 2\epsilon_3^*) + b_2^2 e^{-2i\phi_b},
 \end{aligned} \tag{3.6}$$

at the leading order. The unitary matrix U'_ν for diagonalizing M'_ν is expected to have some mixing parameters around the special values given by eq. (1.9), with the corresponding deviations

$$\begin{aligned}
 \Delta\phi_1 &= \phi'_1 - 0, & \Delta\phi_2 &= (\phi'_2 + \phi'_3)/2 - 0, & \Delta\theta_{23} &= \theta'_{23} - \pi/4, \\
 \Delta\delta &= \delta' - \delta, & \Delta\sigma &= \sigma' - \sigma,
 \end{aligned} \tag{3.7}$$

being some small quantities. By making series expansions for these mixing-parameter deviations in the diagonalization process, at the leading order we acquire the following relations connecting them with the symmetry-breaking parameters $\epsilon_{1,3}$

$$\begin{aligned}
 m_3 s_{13}^2 \Delta\delta + \bar{m}_2 s_{12}^2 \Delta\sigma &= \mp [a_1^2 \epsilon_1 - (a_1^2 + b_1^2) \Delta\phi_1], \\
 \sqrt{2} [(m_1 - \bar{m}_2) c_{12} s_{12} + i(m_1 c_{12}^2 + \bar{m}_2 s_{12}^2 + m_3) \bar{s}_{13}] \Delta\theta_{23} \\
 &\quad - \sqrt{2} (m_1 c_{12}^2 + \bar{m}_2 s_{12}^2 - m_3) \bar{s}_{13} \Delta\delta + 2\sqrt{2} \bar{m}_2 s_{12} (ic_{12} - s_{12} \bar{s}_{13}) \Delta\sigma, \\
 &= \mp 2 \left[a_1 a_2 e^{i(\phi_a - \phi)} (i\epsilon_1 + \epsilon_3 - i\Delta\phi_1 - i\Delta\phi_2) + b_1 b_2 e^{i(\phi_b - \phi)} (-i\Delta\phi_1 - i\Delta\phi_2) \right], \\
 &\quad - (m_1 s_{12}^2 + \bar{m}_2 c_{12}^2 - m_3) \Delta\theta_{23} + [(m_1 - \bar{m}_2) c_{12} s_{12} - i(m_1 c_{12}^2 + \bar{m}_2 s_{12}^2) \bar{s}_{13}] \bar{s}_{13} \Delta\delta \\
 &\quad + \bar{m}_2 c_{12} (ic_{12} - 2s_{12} \bar{s}_{13}) \Delta\sigma = \mp 2 \left[a_2^2 e^{2i(\phi_a - \phi)} (\epsilon_3 - i\Delta\phi_2) + b_2^2 e^{2i(\phi_b - \phi)} (-i\Delta\phi_2) \right]
 \end{aligned} \tag{3.8}$$

where $m_1 = 0$ (or $m_3 = 0$) in the NH (or IH) case and the values of $a_{1,2}$, $b_{1,2}$ and $\phi_{a,b} - \phi$ are the same as those presented in figure 1. The sign $\mp$ which takes $-$ (or $+$) in the NH (or IH) case arises from the aforementioned fact that M'_ν might need an overall sign change so as to fit in with the numerical results.

By solving these equations in a straightforward way, one will obtain the mixing-parameter deviations as some linear functions of ϵ_1 , $\text{Re}(\epsilon_3)$ and $\text{Im}(\epsilon_3)$. For illustration, in figure 2 we present the $\Delta\theta_{23}$, $\Delta\delta$ and $\Delta\sigma$ (as functions of a_1) arising from $\epsilon_1 = 0.1$, $\text{Re}(\epsilon_3) = 0.1$ and $\text{Im}(\epsilon_3) = 0.1$ in the NH and IH cases. Provided that the linear approximation holds to a good degree (i.e., the expected small quantities are really $\leq \mathcal{O}(0.1)$), the

$\Delta\theta_{23}$, $\Delta\delta$ and $\Delta\sigma$ generated by other values of ϵ_1 , $\text{Re}(\epsilon_3)$ and $\text{Im}(\epsilon_3)$ can be inferred by rescaling these results (according to the linear dependence of mixing-parameter deviations on symmetry-breaking parameters). The results in figure 2 tell us: (a) In the NH case, $\epsilon_1 = 0.1$ may give rise to a $|\Delta\delta|$ as large as 0.1. But the resulting $|\Delta\theta_{23}|$ and $|\Delta\sigma|$ are desparately small. (b) In the IH case, the $|\Delta\theta_{23}|$, $|\Delta\delta|$ and $|\Delta\sigma|$ from $\epsilon_1 = 0.1$ are $\simeq \mathcal{O}(0.01)$. (c) In the NH case, $\text{Re}(\epsilon_3) = 0.1$ likely leads to some considerable ($\simeq 0.1$ or so) $|\Delta\theta_{23}|$, $|\Delta\delta|$ and $|\Delta\sigma|$. (d) In the IH case, the $|\Delta\theta_{23}|$ and $|\Delta\delta|$ induced by $\text{Re}(\epsilon_3) = 0.1$ may reach 0.1 and 0.35 (but for distinct values of a_1), while $|\Delta\sigma|$ is rather small. (e)-(f) In both the NH and IH cases, $\text{Im}(\epsilon_3) = 0.1$ can result in considerable $|\Delta\delta|$ and $|\Delta\sigma|$ but relatively small $|\Delta\theta_{23}|$. In the particular case of $a_1 = 0$, $\text{Re}(\epsilon_3) = 0.1$ contributes $|\Delta\theta_{23}| \simeq 0.06$ (or 0.10), $|\Delta\delta| \simeq 0.21$ (or 0.01) and $|\Delta\sigma| \simeq 0.14$ (or 0.02) for NH (or IH), while $\text{Im}(\epsilon_3) = 0.1$ contributes $|\Delta\theta_{23}| \simeq 0.03$ (or 0.00), $|\Delta\delta| \simeq 0.12$ (or 0.10) and $|\Delta\sigma| \simeq 0.05$ (or 0.00). To summarize, ϵ_1 is unlikely to induce considerable mixing-parameter deviations, while $\text{Re}(\epsilon_3)$ is likely. Inversely, a considerable $\Delta\theta_{23}$ can be ascribed to $\text{Re}(\epsilon_3)$, while a considerable $\Delta\delta$ may result from any symmetry-breaking parameter.

4 RGE triggered symmetry breaking

A flavor symmetry (FS) such as the μ - τ reflection symmetry under study is usually introduced at an energy scale Λ_{FS} much higher than the electroweak (EW) one $\Lambda_{\text{EW}} \sim 10^2 \text{ GeV}$ [12, 13]. Therefore, the RGE effects should be taken into account when the flavor-symmetry model is confronted with the low-energy experimental data [71]. During the evolution process, the significant difference between the Yukawa coupling of μ and that of τ may provide a unique source for symmetry breaking. This section is just devoted to such a specific breaking of the μ - τ reflection symmetry. At the one-loop level, the energy dependence of M_ν is described by [72–76]

$$16\pi^2 \frac{dM_\nu}{dt} = C \left(Y_l^\dagger Y_l \right)^T M_\nu + C M_\nu \left(Y_l^\dagger Y_l \right) + \alpha M_\nu, \quad (4.1)$$

where $t \equiv \ln(\mu/\mu_0)$ with μ denoting the renormalization scale, C and α read

$$\begin{aligned}
 C &= -\frac{3}{2}, & \alpha &\simeq -3g_2^2 + 6y_t^2 + \lambda, & \text{in the SM;} \\
 C &= 1, & \alpha &\simeq -\frac{6}{5}g_1^2 - 6g_2^2 + 6y_t^2, & \text{in the MSSM.}
 \end{aligned} \quad (4.2)$$

In the basis of M_l being diagonal, the Yukawa coupling matrix for charged leptons is given by $Y_l = \text{Diag}(y_e, y_\mu, y_\tau)$. Due to $y_e \ll y_\mu \ll y_\tau$, it is reasonable to neglect the contributions of y_e and y_μ . In eq. (4.1), the α -term is flavor universal and only contributes an overall rescaling factor I_α for the neutrino mass matrix, while the other two terms are able to modify its structure. Given an $M_\nu(\Lambda_{\text{FS}})$ of the form given by eq. (2.4) at Λ_{FS} , integration of eq. (4.1) enables us to derive the RGE-corrected neutrino mass matrix at Λ_{EW} [77, 78]

$$M_\nu(\Lambda_{\text{EW}}) = I_\alpha I_\tau^\dagger M_\nu(\Lambda_{\text{FS}}) I_\tau^* = I_\alpha \left[M_\nu(\Lambda_{\text{FS}}) - \Delta_\tau \begin{pmatrix} 0 & 0 & M_{e\tau} \\ 0 & 0 & M_{\mu\tau} \\ M_{e\tau} & M_{\mu\tau} & 2M_{\tau\tau} \end{pmatrix} \right], \quad (4.3)$$

Figure 2. The $\Delta\theta_{23}$, $\Delta\delta$ and $\Delta\sigma$ (as functions of a_1) induced by $\epsilon_1 = 0.1$, $\text{Re}(\epsilon_3) = 0.1$ and $\text{Im}(\epsilon_3) = 0.1$ in the NH and IH cases.

with $I_\tau \simeq \text{Diag}(1, 1, 1 - \Delta_\tau)$ and

$$I_\alpha = \exp \left(-\frac{1}{16\pi^2} \int_{\ln \Lambda_{\text{EW}}}^{\ln \Lambda_{\text{FS}}} \alpha \, dt \right), \quad \Delta_\tau = \frac{C}{16\pi^2} \int_{\ln \Lambda_{\text{EW}}}^{\ln \Lambda_{\text{FS}}} y_\tau^2 \, dt. \quad (4.4)$$

Obviously, Δ_τ measures the strength of symmetry breaking. Owing to the smallness of $y_\tau \simeq 0.01$ which gives $\Delta_\tau \simeq \mathcal{O}(10^{-5})$, the RGE effect is negligible in the SM. But in the MSSM, $y_\tau^2 = (1 + \tan^2 \beta) m_\tau^2 / v^2$ (with $v = 174 \text{ GeV}$ being the Higgs VEV) can be greatly enhanced by a large $\tan \beta$. To be explicit, the value of Δ_τ depends on $\tan \beta$ in a way as

$$\Delta_\tau \simeq 0.042 \left(\frac{\tan \beta}{50} \right)^2, \quad (4.5)$$

if we take $\Lambda_{\text{FS}} \simeq 10^{13} \text{ GeV}$ as an example.

Following the same approach as in the previous section, one can obtain the following relations connecting the mixing-parameter deviations with Δ_τ

$$\begin{aligned} m_3 s_{13}^2 \Delta\delta + \bar{m}_2 s_{12}^2 \Delta\sigma &= (m_1 c_{12}^2 + \bar{m}_2 s_{12}^2 - m_3 s_{13}^2) (-\Delta\phi_1), \\ 2 [(m_1 - \bar{m}_2) c_{12} s_{12} + i (m_1 c_{12}^2 + \bar{m}_2 s_{12}^2 + m_3) \bar{s}_{13}] \Delta\theta_{23} \\ &\quad - 2 (m_1 c_{12}^2 + \bar{m}_2 s_{12}^2 - m_3) \bar{s}_{13} \Delta\delta + 4 \bar{m}_2 s_{12} (i c_{12} - s_{12} \bar{s}_{13}) \Delta\sigma, \\ &= [i (m_{11} + m_3) \bar{s}_{13} - m_{12}] (\Delta_\tau - 2i \Delta\phi_1 - 2i \Delta\phi_2), \\ -2 (m_1 s_{12}^2 + \bar{m}_2 c_{12}^2 - m_3) \Delta\theta_{23} + 2 \bar{m}_2 c_{12} (i c_{12} - 2 s_{12} \bar{s}_{13}) \Delta\sigma \\ &\quad + 2 [(m_1 - \bar{m}_2) c_{12} s_{12} - i (m_1 c_{12}^2 + \bar{m}_2 s_{12}^2) \bar{s}_{13}] \bar{s}_{13} \Delta\delta \\ &= [m_1 s_{12}^2 + \bar{m}_2 c_{12}^2 + m_3 - 2i (m_1 - \bar{m}_2) c_{12} s_{12} \bar{s}_{13}] (\Delta_\tau - 2i \Delta\phi_2), \end{aligned} \quad (4.6)$$

with $m_1 = 0$ (or $m_3 = 0$) in the NH (or IH) case. Solving these equations gives

$$\Delta\theta_{23} = +0.40 \Delta_\tau, \quad \Delta\delta = +0.66 \Delta_\tau, \quad \Delta\sigma = -0.02 \Delta_\tau, \quad (4.7)$$

in the NH case, or

$$\Delta\theta_{23} = -0.52 \Delta_\tau, \quad \Delta\delta = -0.06 \Delta_\tau, \quad \Delta\sigma = -0.16 \Delta_\tau, \quad (4.8)$$

in the IH case. We subsequently show the dependence of these mixing-parameter deviations on the value of $\tan \beta$ (which varies from 10 to 50) in figure 3. One can see that the mixing parameters are pretty stable against the RGE corrections. Even for $\tan \beta = 50$, one merely has $|\Delta\theta_{23}| \simeq 0.017$ (or 0.021), $|\Delta\delta| \simeq 0.028$ (or 0.003) and $|\Delta\sigma| \simeq 0.001$ (or 0.007) in the NH (or IH) case.

5 Leptogenesis and the symmetry breaking

Besides accommodating the smallness of neutrino masses, the seesaw mechanism can also explain the observed baryon-antibaryon asymmetry of the Universe via the leptogenesis mechanism [79]: the CP-violating, lepton-number-violating and out-of-equilibrium decays of N_i may generate a lepton-antilepton asymmetry which is eventually converted to the

Figure 3. The RGE-induced $\Delta\theta_{23}$, $\Delta\delta$ and $\Delta\sigma$ against $\tan\beta$ in the MSSM.

baryon-antibaryon asymmetry through the sphaleron process [80]. The essential CP violation is provided by the complex Yukawa couplings $Y_D = M_D/v$ which couple N_i with the left-handed neutrino fields, while the lepton-number violation originates from the Majorana mass terms of N_i . And the departure from thermal equilibrium can occur if N_i decay in a rate smaller than the expansion rate of the Universe when the temperature T drops to the mass scale of N_i . The produced amount of lepton-antilepton asymmetry is crucially dependent on the CP-violating asymmetries between the decays of N_i and their CP conjugate processes. In the minimal seesaw under study, we assume $N_{1,2}$ to have a strong mass hierarchy $M_1 \ll M_2$, in which case only the decay of N_1 is relevant for leptogenesis. The flavored CP-violating asymmetries ε_α are given by [81]

$$\varepsilon_\alpha \simeq -\frac{3}{16\pi(Y_D^\dagger Y_D)_{11}} \text{Im} \left[(Y_D^\dagger Y_D)_{12} (Y_D^\dagger)_{1\alpha} (Y_D)_{\alpha 2} \right] \frac{M_1}{M_2}, \quad (5.1)$$

for $\alpha = e, \mu, \tau$. A Y_D corresponding to the M_D in eq. (2.3) immediately yields [14]

$$\varepsilon_e = 0, \quad \varepsilon_\mu = -\varepsilon_\tau, \quad (5.2)$$

rendering the total CP-violating asymmetry $\varepsilon = \varepsilon_e + \varepsilon_\mu + \varepsilon_\tau$ vanishing. However, a successful leptogenesis is possible when the μ - τ reflection symmetry is broken [82] or (and) the flavor effects become relevant [83].

For the situation of $M_1 > 10^{12}$ GeV, the flavor effects are negligible. In order to achieve $\varepsilon \neq 0$, one has to break the μ - τ reflection symmetry. A Y'_D corresponding to the M'_D in eq. (3.3) (with $\epsilon_{2,4} = 0$ as assumed before) gives

$$\begin{aligned} \varepsilon \simeq & \frac{3M_1 [a_1 b_1 + 2a_2 b_2 \cos(\phi_a - \phi_b)]}{8v^2 (a_1^2 + 2a_2^2)} \\ & \times [a_1 b_1 \epsilon_1 + 2a_2 b_2 \sin(\phi_a - \phi_b) \text{Re}(\epsilon_3) + 2a_2 b_2 \cos(\phi_a - \phi_b) \text{Im}(\epsilon_3)], \end{aligned} \quad (5.3)$$

which means that ε is proportional to M_1 and a linear function of ϵ_1 , $\text{Re}(\epsilon_3)$ and $\text{Im}(\epsilon_3)$. The final baryon-to-entropy ratio can be written as [80]

$$Y_B \equiv \frac{n_B - n_{\bar{B}}}{s} \simeq -\frac{12}{37} \kappa \frac{\varepsilon}{g_*}. \quad (5.4)$$

Here $12/37$ is the efficiency factor of converting the lepton-antilepton asymmetry to the baryon-antibaryon asymmetry, whereas $g_* = 106.75$ is the effective number of relativistic degrees of freedom at $T = M_1$ in the SM. In particular, κ is the washout factor and can be parametrized as [80]

$$\kappa \simeq (2 \pm 1) \times 10^{-2} \times \left(\frac{0.01 \text{ eV}}{\tilde{m}_1} \right)^{1.1 \pm 0.1}, \quad (5.5)$$

with $\tilde{m}_1 = (Y_D^\dagger Y_D)_{11} v^2 / M_1$. In the present epoch (for $s = 7.04 n_\gamma$), the baryon-to-photon ratio is given by

$$\eta \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} \simeq 7.04 Y_B, \quad (5.6)$$

which has an observed value of $(6.08 \pm 0.09) \times 10^{-10}$ [84]. To figure out what kind of M_1 and symmetry-breaking parameters may give rise to the observed baryon-antibaryon asymmetry, in figure 4 we present the η (as functions of a_1) arising from some example values of them. The results show that a combination of $|M_1 \epsilon_1| \simeq \mathcal{O}(10^{11})$ (or $\mathcal{O}(10^{13})$) GeV, $|M_1 \text{Im}(\epsilon_3)| \simeq \mathcal{O}(10^{10})$ (or $\mathcal{O}(10^{13})$) GeV or $|M_1 \text{Re}(\epsilon_3)| \simeq \mathcal{O}(10^{10})$ (or $\mathcal{O}(10^{13})$) GeV can give successful leptogenesis in the NH (or IH) case. Clearly, it is much easier to gain the observed η in the NH case than in the IH case. In particular, a $|\text{Re}(\epsilon_3)|$ or $|\text{Im}(\epsilon_3)| \simeq \mathcal{O}(0.01)$ (or smaller if M_1 takes a value larger than 10^{12} GeV) is sufficient for generating the observed η in the NH case.

If M_1 turns out to be smaller than 10^{12} GeV, then the Yukawa interactions of charged leptons will enter in thermal equilibrium, making different lepton flavors distinguishable. In such a case, both the CP-violating asymmetries and washout factors associated with different flavors should be treated separately. Accordingly, the final baryon-to-entropy ratio can be rewritten as [85, 86]

$$Y_B \simeq -\frac{12}{37} \frac{K}{g_*}, \quad (5.7)$$

where the parameter K is defined as

$$K = (\epsilon_e + \epsilon_\mu) \kappa_f \left(\frac{417}{589} \tilde{m}_e + \frac{417}{589} \tilde{m}_\mu \right) + \epsilon_\tau \kappa_f \left(\frac{390}{589} \tilde{m}_\tau \right), \quad \text{if } 10^9 \text{ GeV} < M_1 < 10^{12} \text{ GeV};$$

$$K = \epsilon_e \kappa_f \left(\frac{151}{179} \tilde{m}_e \right) + \epsilon_\mu \kappa_f \left(\frac{344}{537} \tilde{m}_\mu \right) + \epsilon_\tau \kappa_f \left(\frac{344}{537} \tilde{m}_\tau \right), \quad \text{if } M_1 < 10^9 \text{ GeV}, \quad (5.8)$$

with

$$\kappa_f(\tilde{m}_\alpha) \simeq \left[\left(\frac{\tilde{m}_\alpha}{2.1 \times 10^{-3} \text{ eV}} \right)^{-1} + \left(\frac{5 \times 10^{-4} \text{ eV}}{\tilde{m}_\alpha} \right)^{-1.16} \right]^{-1}, \quad \text{for } \tilde{m}_\alpha \equiv \frac{|(Y_D)_{\alpha 1}|^2 v^2}{M_1}. \quad (5.9)$$

Figure 4. The η (as functions of a_1) arising from some example values of M_1 and symmetry-breaking parameters for the situation of $M_1 > 10^{12}$ GeV. The gray dashed line stands for the observed value of η .

For illustration, in figure 5 we show the η (as functions of a_1) arising from some example values of M_1 and symmetry-breaking parameters for the situation of $10^9 \text{ GeV} < M_1 < 10^{12} \text{ GeV}$. For comparison, the contributions from pure flavor effects (without symmetry-breaking effects) are also shown. It is easy to see that the flavor effects are more significant than the symmetry-breaking effects in the NH case, while the contrary is the case in the IH case. In the NH case the flavor effects themselves are competent for generating the observed η . But in the IH case the symmetry-breaking effects have to be invoked and the symmetry-breaking parameters should take some values at least $\mathcal{O}(0.1)$.

In order to see whether there exists a choice of basic parameters which can lead to sizable $\Delta\delta$, $\Delta\theta_{23}$ and successful leptogenesis, in figures 6–7 we present the resulting $\Delta\delta$ and $\Delta\theta_{23}$ from the parameter choices shown in figures 4–5 that can lead to successful leptogenesis. From figure 6 one finds that for the situation of $M_1 > 10^{12} \text{ GeV}$ a sizable $\Delta\delta$ can be generated in association with a realistic η from ϵ_1 in the NH case (see the sub-figure labelled as (a)), $\text{Re}(\epsilon_3)$ in the IH case (see the sub-figure labelled as (d)) or $\text{Im}(\epsilon_3)$ in the IH case (see the sub-figure labelled as (f)), but a sizable $\Delta\theta_{23}$ has no chance to arise along with a successful leptogenesis. The results in figure 7 tell us that for the situation of $10^9 \text{ GeV} < M_1 < 10^{12} \text{ GeV}$ all the parameter choices except for that shown in the sub-figure labelled as (b) may give rise to a sizable $\Delta\delta$, while only the parameter choice shown in the sub-figure labelled as (c) is capable of producing a sizable $\Delta\theta_{23}$.

6 Summary

The μ - τ reflection symmetry is powerful in shaping the neutrino mixing: it leads to the interesting results $\theta_{23} = \pi/4$ and $\delta = \pm\pi/2$ (which are close to the current experimental results) as well as trivial Majorana phases. On the other hand, the minimal seesaw has strong predictive power in settling the neutrino mass spectrum: it enforces the condition of $m_1 = 0$ or $m_3 = 0$. In this paper, an attempt of implementing the μ - τ reflection symmetry in the minimal seesaw has been made. Such a marriage results in $\sigma = \pi/2$ (or 0) and thus $|M_{ee}| \simeq 0.37$ (or 4.86) eV in the NH (or IH) case. Motivated by the preliminary experimental hints towards $\theta_{23} \neq \pi/4$ and $\delta \neq -\pi/2$, we particularly explore the possible symmetry breakings and their phenomenological consequences. Some parameters characterizing the breaking of μ - τ reflection symmetry are first defined and their implications for the mixing-parameter deviations then analyzed. It is found that ϵ_1 is difficult to produce considerable mixing-parameter deviations, while $\text{Re}(\epsilon_3)$ is relatively easy. Inversely, a considerable $\Delta\theta_{23}$ can be attributed to $\text{Re}(\epsilon_3)$, while a considerable $\Delta\delta$ may arise from any symmetry-breaking parameter.

As a unique example, the symmetry breaking triggered by the RGE effects is studied. It turns out that the mixing parameters are rather stable against the RGE corrections. Even for $\tan\beta \simeq 50$ in the MSSM, the RGE-induced mixing-parameter deviations are only of $\mathcal{O}(0.01)$. Finally, the operation of leptogenesis in the framework under study is discussed. For the situation of $M_1 > 10^{12} \text{ GeV}$ where one has a vanishing ε , the μ - τ reflection symmetry must be broken to make the leptogenesis mechanism work. For illustration, we give some example values of M_1 and symmetry-breaking parameters that

Figure 5. The η (as functions of a_1) arising from some example values of M_1 and symmetry-breaking parameters for the situation of $10^9 \text{ GeV} < M_1 < 10^{12} \text{ GeV}$. The gray dashed line stands for the observed value of η .

Figure 6. The resulting $\Delta\delta$ and $\Delta\theta_{23}$ (as functions of a_1) from the parameter choices shown in figure 4 that can lead to successful leptogenesis. The red line is used to denote the corresponding η .

Figure 7. The resulting $\Delta\delta$ and $\Delta\theta_{23}$ (as functions of a_1) from the parameter choices shown in figure 5 that can lead to successful leptogenesis. The red line is used to denote the corresponding η .

may give rise to the observed value of η . For the situation of $10^9 \text{ GeV} < M_1 < 10^{12} \text{ GeV}$, it is possible that the flavor effects themselves (without symmetry-breaking effects) are sufficient for producing the observed η in the NH case. But in the IH case, one has to turn to the symmetry-breaking effects for help. For both situations, it is easier to achieve a realistic value of η in the NH case than in the IH case. Last but not least, we point out that the mixing-parameter deviations can be connected to the implementation of leptogenesis, considering that they may originate from the same symmetry breaking. The results show that a sizable $\Delta\delta$ can be generated in association with the observed η in many cases, but a sizable $\Delta\theta_{23}$ can only arise along with a successful leptogenesis from $\text{Re}(\epsilon_3)$ in the NH case for the situation of $10^9 \text{ GeV} < M_1 < 10^{12} \text{ GeV}$.

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