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A model for pseudo-Dirac neutrinos: leptogenesis and ultra-high energy neutrinos

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ABSTRACT: We propose a model where sterile neutrinos are introduced to make light neutrinos to be pseudo-Dirac particles. It is shown how tiny mass splitting necessary for realizing pseudo-Dirac neutrinos can be achieved. Within the model, we show how leptogenesis can be successfully generated. Motivated by the recent observation of very high energy neutrino events at IceCube, we study a possibility to observe the effects of the pseudo-Dirac property of neutrinos by performing astronomical-scale baseline experiments to uncover the oscillation effects of very tiny mass splitting. We also discuss future prospect to observe the effects of the pseudo-Dirac property of neutrinos at high energy neutrino experiments.

KEYWORDS: Beyond Standard Model, Neutrino Physics, Cosmology of Theories beyond the SM

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1 Introduction

Sterile neutrino not only can be a good candidate for dark matter [1–21] but also play an essential role in achieving smallness of neutrino masses [22–24] and baryogenesis via leptogenesis [25]. The sterile neutrino states can mix with the active neutrinos and such admixtures contribute to various processes which are forbidden in the Standard Model (SM), and affect the interpretations of cosmological and astrophysical observations. Thus, the masses of the sterile neutrinos and their mixing with the active neutrinos are subject to various experimental bounds as well as cosmological and astrophysical constraints [26–33].

While we do not have any clue to decide whether neutrinos are Dirac or Majorana particles, here we would like to investigate for neutrinos to be pseudo-Dirac particles [34, 35]. There have been several literatures to study neutrino as a pseudo-Dirac particle [36–47, 50]. Most of them have phenomenologically studied pseudo-Dirac neutrinos with very tiny mass splitting. In this work, we propose a model where sterile neutrinos are introduced to make light neutrinos to be pseudo-Dirac particles. We also show how tiny mass splitting necessary for realizing pseudo-Dirac neutrinos can be achieved. We also examine how leptogenesis can be successfully generated within the model.

From the phenomenological point of view, one of the most important questions must be how we can probe the pseudo-Dirac neutrinos. The magnitude of mass splittings for pseudo-Dirac neutrinos should be smaller than the solar, atmospheric and reactor neutrino mass scales, otherwise they should have affected neutrino oscillations for solar, atmospheric and terrestrial neutrinos. In order to investigate pseudo-Dirac neutrinos with very tiny mass splitting, we need to increase the propagation length of the neutrinos, and thus astrophysical/cosmic neutrinos detectable at neutrino telescope can provide us with the opportunity. In this work, we examine a possibility to observe the effects of the pseudo-Dirac property of neutrinos by performing astronomical-scale baseline experiments to uncover the oscillation effects of very tiny mass splitting. If the oscillation effects induced by pseudo-Dirac

neutrinos with very high energy and long trajectory are prominent, then they may affect the observables detected at neutrino telescope. The neutrino flavor composition detected from the ultra-high energy neutrino experiments can serve as the observable to probe the effects of the pseudo-Dirac neutrinos [51–67].

Recently, IceCube experiments announced the observation of very high energy neutrino events [68, 69]. Analyzing the high energy neutrino events observed at IceCube, the track-to-shower ratio of the subset with energy above 60 TeV has been studied in ref. [70]. They have shown that different production mechanisms for high energy neutrinos lead to different predictions of the ratio. Based on those results, we study how the oscillation effects induced by pseudo-Dirac neutrinos may affect the track-to-shower ratio. Given neutrino energy and mass splittings, the oscillation effects depend on neutrino trajectory in addition to neutrino mixing angles and CP phase. In our numerical analysis, we take the result of global fit to neutrino data for the input of neutrino mixing angles and CP phase. Thus, we examine how the oscillation peaks appear along with neutrino trajectory and discuss some implication on the numerical results.

This paper is organized as follows: in section 2, we describe a model which is an extension of the SM through the introduction of sterile neutrinos and show how pseudo-Dirac neutrinos can be realized. In section 3, we examine how leptogenesis can be successfully generated in this model. In section 4, we study how the pseudo-Dirac property of neutrinos can be probed through the results of high energy neutrino experiments. In section 5, we draw our conclusions.

2 A model for pseudo-Dirac neutrinos

In order to realize pseudo-Dirac neutrinos, let us consider the renormalizable Lagrangian given in the charged lepton basis as

$$-\mathcal{L} = \frac{1}{2} \overline{N_R^c} M_R N_R + \overline{L} \tilde{\Phi} Y_D N_R + \overline{L} \tilde{\Phi} Y_{DS} S + \overline{S^c} \Psi Y_S N_R + \frac{1}{2} \overline{S^c} \mu S + \text{h.c.}, \quad (2.1)$$

where L, N_R, S stand for $\text{SU}(2)_L$ left-handed lepton doublet, right-handed singlet and newly introduced singlet neutrinos, respectively, and $\tilde{\Phi} \equiv i\tau_2 \Phi^*$. M_R and μ are Majorana masses for the N_R and S fields, respectively. On top of the SM Higgs doublet $\Phi = (\phi^+, \phi^0)^T$, an $\text{SU}(2)_L$ singlet scalar field Ψ is introduced. Assigning quantum numbers $L : 1$, $N_R, S : 1$, $\Psi : -2$, and $\Phi : 0$ under the $\text{U}(1)_L$ (or $\text{U}(1)_{B-L}$) symmetry, the above Lagrangian is invariant under $\text{U}(1)_{B-L}$ when $\mu, M_R = 0$. So here the parameters μ, M_R reflect soft symmetry breaking of $\text{U}(1)_L$. When the scalar field Ψ attains a vacuum expectation value (VEV), it spontaneously breaks the $\text{U}(1)_L$ (or $\text{U}(1)_{B-L}$) symmetry, but does not break the electroweak gauge symmetry. Thus its VEV is not required to lie at the electroweak scale. Since the masses of Majorana neutrino N_R are much larger than those of Dirac and light Majorana ones, we can integrate out the heavy Majorana neutrinos in the Lagrangian eq. (2.1), resulting in the following effective Lagrangian for neutrino sectors,

$$\begin{aligned} -\mathcal{L}_{\text{eff}} = & \overline{\nu_L} \phi^0 Y_{DS} S - \frac{1}{2} \overline{\nu_L} \phi^0 Y_D M_R^{-1} Y_D^T \phi^0 \nu_L^c - \overline{\nu_L} \phi^0 Y_D M_R^{-1} Y_S^T \Psi S \\ & - \frac{1}{2} \overline{S^c} \Psi Y_S M_R^{-1} Y_S^T \Psi S + \frac{1}{2} \overline{S^c} \mu S + \text{h.c.}, \end{aligned} \quad (2.2)$$

where Y_D, Y_S, Y_{DS}, M_R and μ are all 3×3 matrices. When the scalar fields Φ and Ψ get VEVs, the mass matrix for light neutrino sector coming from the effective Lagrangian is given by

$$\mathcal{M}_\nu = \begin{pmatrix} M_{\nu\nu} & M_{\nu S} \\ M_{\nu S}^T & M_{SS} \end{pmatrix}, \quad (2.3)$$

in the $(\nu_L^c, S)^T$ basis, where $M_{\nu\nu}, M_{\nu S}$ and M_{SS} are 3×3 matrices and given respectively by

$$\begin{aligned} M_{\nu\nu} &= -m_D M_R^{-1} m_D^T, \\ M_{\nu S} &= m_{DS} - m_D M_R^{-1} m_S^T, \\ M_{SS} &= \mu - m_S M_R^{-1} m_S^T, \end{aligned} \quad (2.4)$$

where $m_D = Y_D \langle \phi^0 \rangle, m_S = Y_S \langle \Psi \rangle$ and $m_{DS} = Y_{DS} \langle \phi^0 \rangle$. Note that $M_{\nu\nu} = M_{\nu\nu}^T$ and $M_{SS} = M_{SS}^T$ are the symmetric 3×3 Majorana left- and right-handed neutrino mass matrices, respectively. Here we take $M_R \gg m_S \simeq m_D \gg \mu$, and neutrinos become pseudo-Dirac particles when $M_{\nu S}$ is dominant over $M_{\nu\nu}$ and M_{SS} in eq. (2.3), which reflects $m_{DS} \gg (m_D m_S)/M_R$. Then, the mass splitting of the light neutrinos depends on the diagonal elements $M_{\nu\nu}$ and M_{SS} in flavor space. And the mixing between active states and sterile ones given as $|\tan 2\theta| = |2M_{\nu S}/(M_{SS} - M_{\nu\nu})| \gg 1$ becomes almost maximal.

As shown by Lim and Kobayashi [71], the 6×6 matrix given in eq. (2.3) can be diagonalized by

$$W_\nu = X \cdot V, \quad \text{with} \quad X = \begin{pmatrix} U_L^* & 0 \\ 0 & U_R \end{pmatrix}, \quad V = \begin{pmatrix} V_1 & iV_1 \\ V_2 & -iV_2 \end{pmatrix}, \quad (2.5)$$

where the 3×3 matrix U_L corresponds to Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix, the 3×3 matrix U_R is an unknown unitary matrix and V_1 and V_2 are the diagonal matrices, $V_1 = \text{diag}(1, 1, 1)/\sqrt{2}, V_2 = \text{diag}(e^{-i\phi_1}, e^{-i\phi_2}, e^{-i\phi_3})/\sqrt{2}$ with ϕ_i being arbitrary phases. The dominant matrix $M_{\nu S}$ in eq. (2.3) can be real and positive diagonalized by biunitary transformation

$$U_L^\dagger M_{\nu S} U_R = \text{diag}(m_1, m_2, m_3) \equiv \hat{M}. \quad (2.6)$$

Keeping terms up to the first order in heavy Majorana mass, the Hermitian matrix $\mathcal{M}_\nu^\dagger \mathcal{M}_\nu$ can be real and positive diagonalized by a unitary transformation W_ν in eq. (2.5);

$$W_\nu^\dagger \mathcal{M}_\nu^\dagger \mathcal{M}_\nu W_\nu \equiv \hat{\mathcal{M}}_\nu^2 \simeq \begin{pmatrix} \hat{M}^2 + \hat{M}|\delta| & 0 \\ 0 & \hat{M}^2 - \hat{M}|\delta| \end{pmatrix}, \quad (2.7)$$

where $\delta \equiv \hat{M}_{\nu\nu}^* + \hat{M}_{SS}$, and $\hat{\mathcal{M}}_\nu \equiv W_\nu^T \mathcal{M}_\nu W_\nu = \text{diag}(m_{\nu 1}, m_{\nu 2}, m_{\nu 3}, m_{S1}, m_{S2}, m_{S3})$. Here $m_{\nu 2} = \sqrt{m_{\nu 1}^2 + \Delta m_{\text{Sol}}^2}$ and $m_{\nu 3} = \sqrt{m_{\nu 1}^2 + \Delta m_{\text{Atm}}^2}$, with Δm_{Sol}^2 and Δm_{Atm}^2 , respectively, being the solar and atmospheric mass-squared differences measured in neutrino oscillation experiments. As a result, the three active neutrino states emitted by weak interactions are described in terms of the mass eigenstates as

$$\nu_\ell = U_{\ell k} \frac{1}{\sqrt{2}} (\nu_k - iS_k^c), \quad (2.8)$$

where ℓ and k denote flavor and mass eigenstates, respectively, and $U \equiv U_L$ is the 3×3 leptonic PMNS mixing matrix. The diagonal matrix δ responsible for splitting the Dirac neutrino masses is given by

$$\delta = \hat{\mu} - \frac{(\hat{m}_D)^2}{\hat{M}_R} - \frac{(\hat{m}_S)^2}{\hat{M}_R}, \quad (2.9)$$

in which the hat stands for a diagonalized mass matrix: $\hat{m}_D = U_L^\dagger m_D U_R$, $\hat{m}_S = U_R^T m_S U_R$, $\hat{\mu} = U_R^T \mu U_R$, and $\hat{M}_R^{-1} = U_R^\dagger M_R^{-1} U_R^*$. It is easy to see that the size of δ is very small compared with the magnitude of $m_{D(S)}$ in the case that the lepton number violating parameter μ is small and $m_{D(S)} \ll M_R$. Then the mass squared difference between m_{ν_i} and m_{S_i} , $\Delta m_k^2 (= 2m_k|\delta_k|)$, can be small and thus the pairs of the active and sterile neutrinos can form pseudo-Dirac pairs. It is anticipated that $\Delta m_k^2 \ll \Delta m_{\text{Sol}}^2, |\Delta m_{\text{Atm}}^2|$, otherwise the effects of the pseudo-Dirac neutrinos should have been detected. But, in the limit that $\Delta m_k^2 = 0$, it is hard to discern the pseudo-Dirac nature of neutrinos. The largest m_k^2 value depends on the neutrino mass hierarchy: for normal neutrino mass hierarchy (NH), $m_3^2 \gtrsim \Delta m_{\text{Atm}}^2 \simeq 2.5 \times 10^{-3} \text{ eV}^2$ and $m_2^2 \gtrsim \Delta m_{\text{Sol}}^2 \simeq 7.5 \times 10^{-5} \text{ eV}^2$, while for inverted one (IH) $m_2^2 > m_1^2 \gtrsim 2.5 \times 10^{-3} \text{ eV}^2$. Thus, the upper bounds for the values of δ_k are given by

$$|\delta_1| \ll 3.8 \times 10^{-5} \text{ eV}^2/m_1, \quad |\delta_2| \ll 4.3 \times 10^{-3} \text{ eV}, \quad |\delta_3| \ll 7.5 \times 10^{-4} \text{ eV}, \quad (2.10)$$

for NH, and for IH

$$|\delta_{1,2}| \ll 7.5 \times 10^{-4} \text{ eV}, \quad |\delta_3| \ll 3.8 \times 10^{-5} \text{ eV}^2/m_3. \quad (2.11)$$

Note that those values are very crucial for a successful low scale leptogenesis as will be discussed later. In case of $|\hat{\mu}| \ll |\hat{m}_D^2/\hat{M}_R|$, the very tiny mass splitting between active and sterile neutrinos arises from lepton number violating dimension-5 operators suppressed by a very high energy scale (e.g. the GUT scale or Planck scale): for example, assuming normal mass hierarchy and taking $M_R \sim \text{Planck mass} \sim 1.22 \times 10^{19} \text{ GeV}$, then the tiny mass splittings are $\Delta m_1^2 \ll 8.6 \times 10^{-8} y_1^2 \text{ eV}^2$, $\Delta m_2^2 \simeq 8.6 \times 10^{-8} y_2^2 \text{ eV}^2$, and $\Delta m_3^2 \simeq 5 \times 10^{-7} y_3^2 \text{ eV}^2$, where y_i are the diagonal entries of $\hat{Y}_D$ or $\hat{Y}_S$. On the other hand, for $|\hat{\mu}| \gg |\hat{m}_D^2/\hat{M}_R|$, the tiny mass splittings are governed by $\delta_k \simeq |\hat{\mu}_k|$. Interestingly enough, the upper bound for $|\delta_1|$ ($|\delta_3|$) could be large enough according to the lightest neutrino mass $m_{\nu_1} \approx m_1$ ($m_{\nu_3} \approx m_3$) for NH (IH). As will be seen in eq. (3.12), a successful TeV-scale leptogenesis could be viable even for a hierarchical heavy neutrino spectrum in such a way that, as the lightest neutrino mass gets lower, the corresponding scale of δ_1 or δ_3 increases.

3 Leptogenesis with pseudo-Dirac neutrinos

Now, let us consider how low scale leptogenesis¹ can be successfully generated in this scenario by decay of the lightest right-handed Majorana neutrino before the scalar fields get vacuum expectation values. In particular, there is a new contribution to the lepton asymmetry which is mediated by the extra singlet neutrinos.

¹See also leptogenesis in inverse seesaw neutrino models [72, 73].

Figure 1. Diagrams contributing to lepton asymmetry.

Without loss of generality, we can rotate and rephase the fields to make the mass matrices $M_{R_{ij}}$ and μ_{ij} real and diagonal. In this basis, the elements of Y_D and Y_S are in general complex. As shown in figure 1, the lepton number asymmetry from decay of the right-handed heavy neutrino into leptons and Higgs scalar required for baryogenesis is given by

$$\varepsilon_k = \sum_{\ell} \left[\frac{\Gamma(N_k \rightarrow L_{\ell} \Phi^*) - \Gamma(N_k \rightarrow \bar{L}_{\ell} \Phi)}{\Gamma_{\text{tot}}(N_k)} \right], \quad (3.1)$$

where N_k is the decaying right-handed neutrino and $\Gamma_{\text{tot}}(N_k)$ is the total decay rate. In addition to the diagrams of the standard leptogenesis scenario, there is a new contribution of the diagram which corresponds to the self energy correction of the vertex arisen due to the new Yukawa couplings with singlet neutrinos and Higgs sectors. Assuming that the masses of the Higgs sectors and extra singlet neutrinos are much smaller compared to that of the right-handed neutrino, to leading order, we have

$$\Gamma_{\text{tot}}(N_k) = \frac{(Y_D^{\dagger} Y_D + Y_S^{\dagger} Y_S)_{kk}}{4\pi} M_{R_k} \quad (3.2)$$

so that

$$\varepsilon_k = \frac{1}{8\pi} \sum_{j \neq k} ([g_V(x_j) + g_S(x_j)] \mathcal{T}_{kj} + g_S(x_j) \mathcal{S}_{kj}), \quad (3.3)$$

where $g_V(x_j) = \sqrt{x_j} \{1 - (1+x_j) \ln[(1+x_j)/x_j]\}$, $g_S(x_j) = \sqrt{x_j}/(1-x_j)$ with $x_j = M_{R_j}^2/M_{R_k}^2$ for $j \neq k$,

$$\mathcal{T}_{kj} = \frac{\text{Im}[(Y_D^{\dagger} Y_D)_{kj}^2]}{(Y_D^{\dagger} Y_D + Y_S^{\dagger} Y_S)_{kk}}, \quad \mathcal{S}_{kj} = \frac{\text{Im}[(Y_D^{\dagger} Y_D)_{kj} (Y_S^{\dagger} Y_S)_{kj}]}{(Y_D^{\dagger} Y_D + Y_S^{\dagger} Y_S)_{kk}}. \quad (3.4)$$

Notice that the term proportional to $\mathcal{S}_{kj}$ comes from the interference of the tree-level diagram with new contribution mediated by S .

The newly generated $B - L$ asymmetry is given as $Y_{B-L}^{\text{SM}} = -\eta \varepsilon_1 Y_{N_1}^{\text{eq}}$, where $Y_{N_1}^{\text{eq}}$ is the number density of the right-handed heavy neutrino at $T \gg M_{R_1}$ in thermal equilibrium given as $Y_{N_1}^{\text{eq}} \simeq \frac{45}{\pi^4} \frac{\zeta(3)}{g_* k_B} \frac{3}{4}$ with Boltzmann constant k_B and the effective number of degree of freedom g_* ($g_* = 217/2$ for the SM) [74]. The efficient factor η can be computed through a set of coupled Boltzmann equations which take into account processes that create or washout the asymmetry. For successful leptogenesis, the size of the denominator of ε_1 should be constrained by the out-of-equilibrium condition, $\Gamma_{N_1} < H|_{T=M_{R_1}}$, where Γ_{N_1} is the total decay width of N_1 and $H(T = M_{R_1}) = \sqrt{\frac{4\pi^3 g_*}{45}} \frac{M_{R_1}^2}{M_{\text{Pl}}}$ is the Hubble parameter at temperature $T = M_{R_1}$. The efficiency in generating the resultant baryon asymmetry is usually controlled by the parameter defined as

$$K \equiv \frac{\sum_{\ell} \Gamma(N_1 \rightarrow L_{\ell} \Phi^*)}{H(T = M_{R_1})} = \frac{\tilde{m}_1}{m_*}. \quad (3.5)$$

We note that $K \ll 1$ corresponds to weak washout, whereas $K \gg 1$ to strong washout. To a good approximation the efficiency factor depends on the effective neutrino mass $\tilde{m}_1$ defined in the presence of the new Yukawa interactions with the coupling Y_S by

$$\tilde{m}_1 = \frac{(Y_D^{\dagger} Y_D + Y_S^{\dagger} Y_S)_{11}}{M_{R_1}} v^2 \sim 2 \delta_1, \quad (3.6)$$

which is a measure of the strength of the coupling of N_1 to the thermal bath. And the equilibrium neutrino mass is given by $m_* = \frac{16\pi^{5/2}}{3\sqrt{5}} \sqrt{g_*} \frac{v^2}{M_{\text{Pl}}} \simeq 1.08 \times 10^{-3} \text{ eV}$. Note here that the plausible range for $\tilde{m}_1$ is the one suggested by the range of the order of the δ_1 . In such a case, the decay rate is smaller than the expansion rate of the universe, and the particles come out of equilibrium and create a lepton asymmetry. So, the produced baryon asymmetry depends on the initial conditions in the weak washout regime. The efficiency factor for $0 < \eta < 1$ can be estimated by inserting this effective mass in the function [75]

$$\eta(x) \simeq \left(\frac{3.3 \times 10^{-3} \text{ eV}}{x} + \left(\frac{x}{5.5 \times 10^{-4} \text{ eV}} \right)^{1.16} \right)^{-1}, \quad (3.7)$$

valid for $M_{R_1} \ll 10^{14} \text{ GeV}$. Then, the baryon-to-photon ratio results in $\eta_B \simeq -0.97 \times 10^{-2} \times \eta(\tilde{m}_1) \times \varepsilon_1$. From the observed one in nine year WMAP data [76, 77] $\eta_B^{\text{WMAP}} = (6.19 \pm 0.14) \times 10^{-10}$, we can get the allowed range of the model parameter δ_i and some bounds on M_{R_1} , as will be shown later.

In a hierarchical pattern for right-handed neutrinos $M_{2,3} \gg M_1$, it is sufficient to consider the lepton asymmetry produced by the decay of the lightest right-handed neutrino N_{R_1} :

$$\varepsilon_1 \simeq \frac{3}{16\pi} \frac{M_{R_1}}{(m_D^{\dagger} m_D + m_S^{\dagger} m_S)_{11}} \text{Im}[(Y_D^{\dagger} M_{\nu\nu} Y_D^*)_{11}], \quad (3.8)$$

where $\langle \Psi \rangle \simeq \langle \phi^0 \rangle = v$ is used and the loop function g_V can be approximated as $g_V(x_j) = -\frac{3}{2\sqrt{x_j}}$ for $x_j \gg 1$. Using the relation below eq. (2.7), i.e. real and positive eigenvalues $\hat{M}_{\nu\nu} = U_L^{\dagger} M_{\nu\nu} U_L^*$, we have

$$\text{Im}[(\tilde{Y}_D^{\dagger} \hat{M}_{\nu\nu} \tilde{Y}_D^*)_{11}] = \sum_{j=1}^3 \text{Im}[(\tilde{Y}_D^{\dagger})_{1j}^2] (\hat{M}_{\nu\nu})_j \simeq \frac{1}{2} \sum_{j=1}^3 \text{Im}[(\tilde{Y}_D^{\dagger})_{1j}^2] |\delta_j|, \quad (3.9)$$

where $\tilde{Y}_D = U_L^\dagger Y_D$, and the third equality comes out from $|\hat{\mu}| \ll |\hat{m}_D^2/M_R|$ with $\hat{m}_D \sim \hat{m}_S$. By letting the three-vector $\hat{Y}_{1j}^\dagger = (\tilde{Y}_D^\dagger)_{1j} / \sqrt{(\tilde{Y}_D^\dagger \tilde{Y}_D)_{11}}$, and simply taking $(m_D^\dagger m_D)_{11} \simeq (m_S^\dagger m_S)_{11}$, the lepton asymmetry ε_1 in eq. (3.8) can be simplified as

$$\varepsilon_1 \simeq \frac{3M_{R_1}}{64v^2\pi} \sum_{j=1}^3 \text{Im}[(\hat{Y}_{1j}^\dagger)^2] |\delta_j| \leq \frac{3M_{R_1}}{64v^2\pi} \delta_{\max} = \varepsilon_1^{\max}, \quad (3.10)$$

where $\delta_{\max}$ is the heaviest $|\delta_j|$. Interestingly enough, we estimate what values of $\delta_{\max}$ can be obtained from the solar neutrino data and GRB neutrinos

$$\delta_{\max} = \begin{cases} \Delta m_1^2/2m_1 \approx 10^{-13} \text{ eV}^2/2m_1 \gtrsim 10^{-10} \text{ eV}, & \text{for NH} \\ \Delta m_3^2/2m_3 \approx 10^{-16} \text{ eV}^2/2m_3, & \text{for IH} \\ \Delta m_k^2/2m_k \approx 10^{-10} \text{ eV}, & \text{for QD} \end{cases} \quad (3.11)$$

in which $\Delta m_1^2 \sim 10^{-13} \text{ eV}^2$ and $\Delta m_3^2 \sim 10^{-16} \text{ eV}^2$ are taken, the QD stands for quasi-degenerate neutrino mass, and the lower bound for NH is achieved when both m_1 and m_2 are of the same order. The maximal CP asymmetry $\varepsilon_1^{\max}$ then yields the maximal baryon asymmetry $\eta_B^{\max}$ that can be produced in leptogenesis. The lower bound on $|\varepsilon_1|$ and the upper bound on $\delta_{\max}$ can be used to obtain a lower bound on M_{R_1}

$$M_{R_1} \gtrsim 1 \text{ TeV} \left(\frac{0.2 \text{ MeV}}{\delta_{\max}} \right) \left(\frac{|\varepsilon_1|}{10^{-7}} \right), \quad (3.12)$$

which means for $\delta_{\max} \sim 0.2 \text{ MeV}$, the lower bound on the scale of lightest heavy neutrino will be of the order of $\gtrsim 1 \text{ TeV}$ for a successful leptogenesis. Here the $\delta_{\max} \sim 0.2 \text{ MeV}$ in the normal neutrino mass hierarchy can be obtained for $\hat{m}_{D_1} \sim 0.3 \text{ GeV}$ and $|M_{R_1}| \sim 1 \text{ TeV}$ in the limit of $|\hat{\mu}_1| \ll |(\hat{m}_{D_1})^2/M_{R_1}|$. One can obtain the $\delta_{\max}$ for IH in similar way. On the other hand, in the case of QD, i.e. $m_{\nu_1} \approx m_{\nu_2} \approx m_{\nu_3}$, the lower bound on the scale of lightest heavy neutrino will be located around $M_R \sim \text{Planck scale}$ for a successful leptogenesis. In its simplest scenario, thermal leptogenesis, since the baryon asymmetry is produced during the radiation dominated era, the lower bound on $M_{R_1} \gtrsim 1 \text{ TeV}$ with $\delta_{\max} \sim 0.2 \text{ MeV}$ for a hierarchical neutrino mass translates into a lower bound on the reheating temperature after inflation.

On the other hand, in a case $\delta_{\max} \sim 2 \times 10^{-4} \text{ eV}$, the lower bounds on the scale of lightest heavy neutrino will be of the order of $M_R \gtrsim 10^{12} \text{ GeV}$ for a successful leptogenesis, which translates into a lower bound on the reheating temperature after inflation.² In such a case, the lower bound on the reheating temperature can be relaxed by considering quasi-degenerate heavy Majorana neutrinos ($M_{R_1} \simeq M_{R_2}$) [78]. So in its form, thermal production of N_1 does not need too high reheating temperature and the Universe would not encounter the gravitino overproduction [79, 80]. In the following we will see this is the

²Such a large reheating temperature is potentially in conflict with bigbang nucleosynthesis (BBN) in supersymmetric models, where upper bound on the reheating temperature as low as 10^6 GeV unless $m_{3/2} \gg 1 \text{ TeV}$ have been obtained in supergravity (SUGRA) models [79].

case. As shown in [81–84], the new contributions to ε_1 could be important for the case of $M_{R_1} \simeq M_{R_2} < M_{R_3}$ for which the asymmetry is approximately given by

$$\varepsilon_1 \simeq \frac{M_{R_2}}{16\pi} \frac{\text{Im}[(Y_D^\dagger M_{\nu\nu} Y_D^*)_{11}] - \text{Im}[(Y_D^\dagger (m_D M_R^{-1} m_S^T) Y_S^*)_{11}]}{(m_D^\dagger m_D + m_S^\dagger m_S)_{11}} R, \quad (3.13)$$

where R is a resonance factor defined by $R \equiv |M_{R_1}|/(|M_{R_2}| - |M_{R_1}|)$. In the above equation the denominator can be expressed as

$$\begin{aligned} & \sum_j \left(\text{Im}[(\tilde{Y}_D^\dagger)^2]_{1j} (\hat{M}_{\nu\nu})_j + \text{Im}[(\tilde{Y}_D^\dagger)_{1j} (\tilde{Y}_S^\dagger)_{1j}] \{ \hat{M}_j - (\hat{m}_{DS})_j \} \right) \\ & \simeq \sum_j \left\{ \text{Im}[(\tilde{Y}_D^\dagger)^2]_{1j} + (\tilde{Y}_D^\dagger)_{1j} (\tilde{Y}_S^\dagger)_{1j} \right\} \frac{|\delta_j|}{2} \end{aligned} \quad (3.14)$$

where $\tilde{Y}_S = U_R^T Y_S$, and the second equality comes out from $|\hat{\mu}| \ll |\hat{m}_D^2/M_R|$ with $\hat{m}_D \sim \hat{m}_S$ leading to $\hat{M}_j - (\hat{m}_{DS})_j \simeq |\delta_j|/2$ as well as $(\hat{M}_{\nu\nu})_j \simeq |\delta_j|/2$. So, we obtain

$$\varepsilon_1 \simeq \frac{M_{R_2}}{32v^2\pi} \frac{\sum_j \left\{ \text{Im}[(\tilde{Y}_D^\dagger)^2]_{1j} + (\tilde{Y}_D^\dagger)_{1j} (\tilde{Y}_S^\dagger)_{1j} \right\} |\delta_j|}{(\tilde{Y}_D^\dagger \tilde{Y}_D + \tilde{Y}_S^\dagger \tilde{Y}_S)_{11}} R \leq \frac{M_{R_2}}{64v^2\pi} \delta_{\max} R. \quad (3.15)$$

Similar to the hierarchical case, the lower bound on M_{R_2} can be obtained by using both the upper bound on $|\varepsilon_1|$ and the lower bound on $\delta_{\max}$,

$$M_{R_2} \gtrsim 3 \times 10^{12-n} \text{ GeV} \left(\frac{2 \times 10^{-4} \text{ eV}}{\delta_{\max}} \right) \left(\frac{10^n}{R} \right) \left(\frac{|\varepsilon_1|}{10^{-7}} \right). \quad (3.16)$$

This lower bound on M_{R_2} further implies a lower bound of the reheating temperature after inflation, since the abundance of gravitinos is proportional to the reheating temperature. The degree of degeneracy between two heavy neutrinos $R = 10^{6-9}$ is required to achieve a successful leptogenesis, corresponding to the lower bound on $M_{R_2} \gtrsim 3 \times 10^{3-6} \text{ GeV}$ for $\delta_{\max} \sim 2 \times 10^{-4} \text{ eV}$.

4 Probing pseudo-Dirac neutrinos at astronomical-scale experiments

Now, let us consider how one can probe the effects of the pseudo-Dirac neutrinos. A possible way to probe the pseudo-Dirac neutrinos is to perform astronomical-scale baseline experiments to uncover the oscillation effects of very tiny mass splitting Δm_k^2 . With the help of the mixing matrix eq. (2.5), the flavor conversion probability between the active neutrinos follows from the time evolution of the state ν_k as,

$$P_{\ell\ell'} \equiv P_{\nu_\ell \rightarrow \nu_{\ell'}}(W_\nu, L, E) = \left| \left(W_\nu^* e^{-i \frac{\hat{\mathcal{M}}_k^2}{2E} L} W_\nu^T \right)_{\ell\ell'} \right|^2 = \frac{1}{4} \left| \sum_{k=1}^3 U_{\ell'k} \left\{ e^{i \frac{m_{\nu k}^2 L}{2E}} + e^{i \frac{m_{S_k}^2 L}{2E}} \right\} U_{\ell k}^* \right|^2, \quad (4.1)$$

where W_ν is the mixing matrix with which the weak gauge eigenstates, ν_ℓ , with flavor $\ell = e, \mu, \tau$ are composed of the mass eigenstates with definite masses, $n_k = (\nu_k \ S_k^c)^T$ ($k = 1, 2, 3$), given as $|\nu_\ell\rangle = \sum_k^{N_\nu=3} W_{\ell k}^* |n_k\rangle$.

Neutrinos arriving at neutrino telescopes from astrophysical sources such as Gamma Ray Bursts (GRBs) [85, 86], active galactic nuclei [87–89], and type Ib/c supernova [90–93] travel large distances over ~ 100 Mpc. Neutrino telescope observes neutrinos from extragalactic sources located a few Gpc away from the earth and with neutrino energy $10^5 \text{ GeV} \lesssim E \lesssim 10^7 \text{ GeV}$. It has been shown [94] that inside the GRB sources $\int V_{C,N} dt \ll 1$ where the effective potentials due to the matter effects are $V_C = \sqrt{2} G_F n_e$ with n_e being the electron number density in matter and $V_N = -\sqrt{2} G_F n_n/2$ with n_n being the neutron number density in matter, so the matter effects inside the source are not relevant for neutrino oscillation, while inside the earth for $V_{C,N} \gg \Delta m_k^2/2E$ again the matter effect will not be significant because of the very tiny effective mixing angle. So, we only consider neutrino oscillation in vacuum for astrophysical neutrinos. Given neutrino trajectory L and energy E , the oscillation effects become prominent when $\Delta m_k^2 \sim E/4\pi L$, where $L \equiv L(z)$ is a distance-measure with redshift z given by [46, 47]

$$L(z) \equiv D_H \int_0^z \frac{dz'}{(1+z')^2 \sqrt{\Omega_m(1+z')^3 + \Omega_\Lambda}}, \quad (4.2)$$

where the Hubble length $D_H = c/H_0 \simeq 4.42 \text{ Gpc}$ with the present Hubble expansion rate $H_0 = 67.8 \pm 0.9 \text{ km s}^{-1} \text{ Mpc}^{-1}$ [48], the matter density of the Universe $\Omega_m = 0.306 \pm 0.007$, and the dark energy density of the Universe $\Omega_\Lambda = 0.694 \pm 0.007$ [49]. The asymptotic value of $L(z)$ is about 2.1 Gpc achieved by large value of z , which means that the smallest Δm_k^2 that can be probed with astrophysical neutrinos with E is 10^{-17} eV^2 (E/PeV) [46, 47]. Thus, astrophysical neutrinos with $L \simeq 1 \text{ Gpc}$ (the flight length) and energy $E \simeq 1 \text{ PeV}$ would be useful to probe the pseudo-Dirac property of neutrinos with very tiny mass splitting. In this case, to observe the oscillation effects, the oscillation lengths should not be much larger than the flight length before arriving at neutrino telescopes in earth, that is,

$$L_{\text{osc}}^k \simeq \left(\frac{0.8 \times 10^{-16} \text{ eV}^2}{\Delta m_k^2} \right) \left(\frac{E}{10^6 \text{ GeV}} \right) \text{ Gpc} \lesssim \text{Gpc} \quad (4.3)$$

which means that neutrino oscillations can be measurable only when $\delta_k \gtrsim 0.8 \times 10^{-15} \text{ eV}$. From eq. (4.3), we see that given the tiny mass splittings $\Delta m_k^2 = 10^{-16 \sim -17} \text{ eV}^2$ with the energies around TeV–PeV, a new oscillation curve at neutrino trajectory $\mathcal{O}(1) \text{ Gpc}$ is naively expected to occur. Since $E/\Delta m_k^2 \sim L(z) \gg E/\Delta m_{\text{Atm}}^2$, the probability of the oscillation $\nu_\mu \rightarrow \nu_\mu$ over the distance L is given approximately by

$$\begin{aligned} P(\nu_\mu \rightarrow \nu_\mu) \simeq & 1 - \frac{1}{2} c_{23}^4 \{ 4s_{23}^2 + \sin^2 2\theta_{12} c_{23}^2 + \sin 2\theta_{23} \sin 4\theta_{12} \cos \delta_{\text{CP}} s_{13} \} \\ & - s_{23}^4 \sin^2 \left(\frac{\Delta m_3^2 L}{4E} \right) - c_{23}^4 \left\{ s_{12}^4 \sin^2 \left(\frac{\Delta m_2^2 L}{4E} \right) + c_{12}^4 \sin^2 \left(\frac{\Delta m_1^2 L}{4E} \right) \right\} \\ & - \sin 2\theta_{23} \sin 2\theta_{12} \cos \delta_{\text{CP}} s_{13} c_{23}^2 \left\{ c_{12}^2 \sin^2 \left(\frac{\Delta m_2^2 L}{4E} \right) - s_{12}^2 \sin^2 \left(\frac{\Delta m_1^2 L}{4E} \right) \right\}, \end{aligned} \quad (4.4)$$

where we have ignored the terms proportional to $\sin^n \theta_{13}$ with $n \geq 2$. For the numerical analysis, we take the results from global fit of three-flavor oscillation parameters at 1 σ C.L. [95], which are given in table 1.

mass hierarchy	$\theta_{23}(^{\circ})$	$\theta_{12}(^{\circ})$	$\theta_{13}(^{\circ})$	$\delta_{\text{CP}}(^{\circ})$
normal	$42.3^{+3.0}_{-1.6}$	$33.48^{+0.78}_{-0.75}$	$8.50^{+0.20}_{-0.21}$	306^{+39}_{-70}
inverted	$49.5^{+1.5}_{-2.2}$	$33.48^{+0.78}_{-0.75}$	$8.51^{+0.20}_{-0.21}$	254^{+63}_{-62}

Table 1. Global fit of three-flavor neutrino oscillation parameters at 1σ .

Recently, authors in ref. [70] analyzed the high energy neutrino events observed by IceCube, aiming to probe the initial flavor of cosmic neutrinos. The expected number of events produced by an isotropic neutrino and antineutrino with flavor ℓ is given by

$$N = 4\pi T \int dE \Phi_{\ell}(E) A_{\ell}(E), \quad (4.5)$$

where T is the time of observation, $A_{\ell}(E)$ is the detector effective areas, and $\Phi_{\ell}(E)$ is the energy dependent isotropic flux of neutrinos and antineutrinos [70]. Then the track-to-shower ratio for the number of shower N_S and track events N_T in the IceCube detector³ can be expressed in terms of tiny mass splittings Δm_k^2 , flight length L , neutrino mixing angles and CP phase ($\theta_{12}, \theta_{23}, \theta_{13}, \delta_{\text{CP}}$), and initial flavor composition $\phi_{\ell'}^0$

$$\frac{N_T}{N_S} = \frac{a_{\mu} p_T \tilde{F}_{\mu}}{a_e \tilde{F}_e + a_{\mu} (1 - p_T) \tilde{F}_{\mu} + a_{\tau} \tilde{F}_{\tau}}, \quad (4.6)$$

where

$$\begin{aligned} \tilde{F}_{\ell} &= \sum_{\ell'k} |U_{\ell k}|^2 |U_{\ell' k}|^2 \phi_{\ell'}^0, \\ a_{\ell} &= 4\pi \int dE \cos^2 \left(\frac{\Delta m_k^2 L}{4E} \right) E^{-\alpha} A_{\ell}(E), \end{aligned} \quad (4.7)$$

with a spectral index α . Here p_T is the probability that an observed event produced by a muon neutrino is a track event, which is mildly dependent on energy and approximately equals to 0.8 [96]. Then above equation can be simplified to

$$\frac{N_T}{N_S} = \frac{\phi_{\mu}}{\frac{a_e}{a_{\mu} p_T} + \left(\frac{a_{\tau}}{a_{\mu} p_T} - \frac{a_e}{a_{\mu} p_T} \right) \phi_{\tau} + \left(\frac{1-p_T}{p_T} - \frac{a_e}{a_{\mu} p_T} \right) \phi_{\mu}}, \quad (4.8)$$

where $\phi_e = 1 - \phi_{\mu} - \phi_{\tau}$ with $\phi_{\ell} \equiv \tilde{F}_{\ell}/(\tilde{F}_e + \tilde{F}_{\mu} + \tilde{F}_{\tau})$ is assumed. By using the high energy neutrino events in the IceCube detector which lie in energies between 60 TeV and 3 PeV [68–70], eq. (4.8) shows directly that track-to-shower ratio N_T/N_S can give a new oscillation curve as a signal dependent on neutrino flight length if the neutrino mixing angles and CP phase, initial flavor composition, and tiny mass splittings are given as inputs.

In the limit of large or null mass splitting Δm_k^2 , there is no oscillation effects and thus the value of N_T/N_S becomes constant for a given data set of neutrino mixing angles

³We note that much larger detectors than the present IceCube would be required to get fully meaningful result for the test of our model in detail.

and CP phase. However, in the case that the oscillation effects are prominent, the value of N_T/N_S can be enhanced due to the new oscillatory term which depends on neutrino flight length, and small mass splittings. Thus, it is possible to probe the pseudo-Dirac property of neutrinos by measuring the deviation of N_T/N_S from the expectation without the oscillation arisen due to the tiny mass splitting. To see how large the value of N_T/N_S can be deviated by the oscillatory terms, we perform numerical analysis by taking the values of the neutrino mixing angles and CP phase from the global fit results at 1σ level [95] as shown in table 1. We expect that the different values of θ_{23} and δ_{CP} at 1σ level for normal and inverted mass orderings provide different predictions of the track-to-shower ratio, while normal and inverted mass orderings could not be distinguished with the data at 3σ level [95]. For the tiny mass splittings, we consider two cases: (i) all equivalent, $\Delta m_{1,2,3}^2 = \Delta m_k^2$, and (ii) hierarchical $\Delta m_i^2 \gg \Delta m_j^2$ for $(i > j)$.

Our numerical results depend on the initial flavor composition $\phi_e^0 : \phi_\mu^0 : \phi_\tau^0$ at the source which are relevant for the interpretation of observational data. We consider the well-known four production mechanisms for high energy neutrinos from which the flavor compositions are given as: (i) $(\frac{1}{3} : \frac{2}{3} : 0)$ for π decay, (ii) $(\frac{1}{2} : \frac{1}{2} : 0)$ for charmed mesons decay, (iii) $(1 : 0 : 0)$ for β decay of neutrons, and (iv) $(0 : 1 : 0)$ for π decay with damped muons. The tiny mass splittings Δm_k^2 can be searched for, looking at high energy cosmic neutrinos by measuring the track-to-shower ratio N_T/N_S as the function of L ($\log_{10}[\text{path length/Mpc}]$) in eq. (4.8). In the numerical analysis, we use the spectral index given by $\alpha = 2.2 \pm 0.4$ [96].

4.1 Results for the case of $\Delta m_{1,2,3}^2 = \Delta m_k^2$

As a benchmark point, we take the mass splittings $\Delta m_{1,2,3}^2$ to be 10^{-16} eV^2 . In figure 2, we present the track-to-shower ratio N_T/N_S for normal (inverted) mass ordering as a function of L ($\log_{10}[\text{path length/Mpc}]$). The red (dark black) and blue (light black) curves correspond to normal and inverted neutrino mass orderings, respectively, for $\alpha = 2.2$. The light red and light blue regions correspond to normal and inverted neutrino mass orderings, respectively, for $1.8 \lesssim \alpha \lesssim 2.6$. The gray shaded regions are forbidden by the measurement of $N_T/N_S = 0.18_{-0.05}^{+0.13}$ which is obtained in ref. [70]. In each panel, we present the initial flavor composition for neutrino flux. The width of each band in the panels represent the uncertainties in the measurements of neutrino mixing angles. From figure 2, we see that the bands for the cases with only one flavor in the initial flavor composition are wider than the others. This is because of the slightly hierarchical neutrino mixing angles and the initial flavor compositions with ϕ_ℓ in eq. (4.8): for example, the main reason in the different band widths of the left upper and lower panels in figure 2 is the initial flavor compositions with ϕ_ℓ , while the main reason in the different band widths of the left-lower and right-lower panels is the slightly hierarchical neutrino mixing angles. The results in the lower panels show that the predicted values of N_T/N_S for normal mass ordering are consistent with the boundaries of the allowed region of N_T/N_S . In the upper panels, we see that the oscillation peak occurs at the distance about 1.3 Gpc for both the tiny mass splittings $\Delta m_{1,2,3}^2 = 10^{-16} \text{ eV}^2$ and the deposited energies 60 TeV–3 PeV.

Figure 2. Plots of the track-to-shower ratio N_T/N_S for normal (inverted) mass ordering as a function of L ($\log_{10}[\text{path length/Mpc}]$) for $\Delta m_{1,2,3}^2 = 10^{-16} \text{ eV}^2$. Each panel corresponds to the specific initial flavor composition $(\phi_e^0 : \phi_\mu^0 : \phi_\tau^0)$ at the source. For three neutrino mixing angles and Dirac-type CP phase, we take the global fit results at 1σ [95]. Red and blue bands correspond to normal and inverted neutrino mass orderings, respectively, for $\alpha = 2.2$, whereas light red and light blue regions represent the corresponding results for $\alpha = 1.8\text{--}2.6$. Gray shaded regions represent the forbidden bound from $N_T/N_S = 0.18^{+0.13}_{-0.05}$ in ref. [70].

4.2 Results for the case of $\Delta m_i^2 \gg \Delta m_j^2$

In this case, we take $\Delta m_1^2 = 10^{-14} \text{ eV}^2$, $\Delta m_2^2 = 10^{-15} \text{ eV}^2$, and $\Delta m_3^2 = 10^{-16} \text{ eV}^2$ as a benchmark point. In the numerical analysis, we take the input values except for Δm_k^2 to be the same as in figure 2. In figures 3 and 4, we plot the track-to-shower ratio N_T/N_S as a function of L ($\log_{10}[\text{path length/Mpc}]$) for the inverted and normal neutrino mass orderings, respectively. Initial flavor compositions at the source are the same as in figure 2. Gray shaded regions represent the forbidden bound from $N_T/N_S = 0.18^{+0.13}_{-0.05}$ in ref. [70]. We can see from figure 3 that the results for the case with initial flavor composition $(\phi_e^0 : \phi_\mu^0 : \phi_\tau^0) = (0 : 1 : 0)$ at the source is not consistent with experimental results, whereas only the highest region for the case with $(\phi_e^0 : \phi_\mu^0 : \phi_\tau^0) = (1 : 0 : 0)$ is consistent with experimental results. As can be seen from figure 4, the predictions of N_T/N_S for the normal mass ordering are all consistent with experimental results. Different from the results of figure 2, the oscillation peaks in each panels in figures 3 and 4 are not so sharp. This means that the predictions of N_T/N_S in these cases are deviated from the case with no new oscillatory effect for rather wider regions of the parameter L .

Figure 3. Plots of N_T/N_S for inverted neutrino mass ordering as a function of $L(\log_{10}[\text{path length/Mpc}])$ for $\Delta m_1^2 = 10^{-14} \text{ eV}^2$, $\Delta m_2^2 = 10^{-15} \text{ eV}^2$, and $\Delta m_3^2 = 10^{-16} \text{ eV}^2$. The input values except for Δm_k^2 are taken to be the same as in figure 2. Gray shaded regions represent the forbidden bound from $N_T/N_S = 0.18^{+0.13}_{-0.05}$ in ref. [70]. Each panel corresponds to the specific initial flavor composition at the source, as in figure 2.

E (PeV)	(0.523,0.627)	(0.627,0.754)	(0.754,0.905)	(0.905,1.312)	(1.312,1.576)	(1.576,2.286)	(2.286,3)
Dev(%)	15.7	15.9	16.0	16.1	16.2	16.3	16.4

Table 2. Deviation of the prediction for N_T/N_S from the case without new oscillatory effect for $\Delta m_k^2 = 10^{-16} \text{ eV}^2$.

Up to this point, we have presented the numerical results for N_T/N_S as a function of L for a given Δm_k^2 and $60 \text{ TeV} \lesssim E \lesssim 3 \text{ PeV}$. However, the experimental results on high energy neutrinos released from IceCube are shown for narrow interval of energy from 60 TeV to 3 PeV. From our numerical results, we see that the amount to which the prediction for N_T/N_S is deviated from the one without new oscillatory effect depends on the energy scale. In table 2, we present how the prediction for N_T/N_S is deviated from the case without new oscillatory effect for $\Delta m_k^2 = 10^{-16} \text{ eV}^2$ and $L(z) = 1.1 \text{ Gpc}$. According to table 2, we see that the largest deviation occurs for $2.286 \lesssim E \lesssim 3 \text{ PeV}$. The deviation becomes smaller as E goes lower. Similar result is obtained for the hierarchical case. Thus, more PeV scale data would be desirable to test our model.

In order to probe the presence of pseudo-Dirac neutrino, observation of new oscillatory effects in N_T/N_S is essential. To do this, future experiments should precisely measure the

Figure 4. Plots of N_T/N_S for normal neutrino mass ordering as a function of $L(\log_{10}[\text{path length/Mpc}])$ for $\Delta m_1^2 = 10^{-14} \text{ eV}^2$, $\Delta m_2^2 = 10^{-15} \text{ eV}^2$, and $\Delta m_3^2 = 10^{-16} \text{ eV}^2$. The input values except for Δm_k^2 are taken to be the same as in 2. Gray shaded regions represent the forbidden bound from $N_T/N_S = 0.18^{+0.13}_{-0.05}$ in ref. [70]. Each panel corresponds to the specific initial flavor composition at the source, as in figure 2.

value of N_T/N_S . If the uncertainty in future measurements could be reduced by 50–60% from the current one without changing the central value, there would be a high chance to observe the new oscillatory effects via the oscillation peak for the case of degenerate Δm_k^2 , and we would be able to test the pseudo-Dirac property of neutrinos, particularly for the cases with two flavors in the initial flavor composition of neutrino flux. For the case of hierarchical Δm_k^2 , to test the model for pseudo-Dirac neutrino, we need to reduce the uncertainty by 40–70% depending on the initial flavor compositions.

As expected, for 3σ data of three neutrino oscillations one could not distinguish normal and inverted orderings for the track-to-shower ratio N_T/N_S , while the band width can be enlarged.

We can constrain mass squared differences Δm_1^2 and Δm_2^2 from the fact that the UHE neutrinos with energy 10^9 GeV are expected from the Greisen-Zatsepin-Kuzmin limit cosmic rays originated at distances of 100 Mpc [97]. To observe such UHE neutrinos through neutrino oscillation whose length is of order 100 Mpc, the required magnitudes of δ_1 and δ_2 can be estimated from

$$L_{\text{osc}}^{1,2} \simeq \left(\frac{0.8 \times 10^{-12} \text{ eV}^2}{\Delta m_{1,2}^2} \right) \left(\frac{E}{10^9 \text{ GeV}} \right) 100 \text{ Mpc} \lesssim 100 \text{ Mpc}, \quad (4.9)$$

which means that for such neutrinos with $E \sim 10^9$ GeV, oscillation length will be order of 100 Mpc for $\Delta m_{1,2}^2 \simeq 10^{-12} \text{ eV}^2$. In other words, $\delta_{1,2} \geq 4.6 \times 10^{-11} \text{ eV}$ (for normal mass hierarchy) and $\delta_{1,2} \geq 0.8 \times 10^{-11} \text{ eV}$ (for inverted mass hierarchy) are required for significant conversion of these neutrinos. Taking into account oscillation length of order the earth-sun distance 1 A.U. for neutrino energy 10^9 GeV, we estimate the mass splittings are so large $\Delta m_{1,2}^2 \simeq 16.6 \text{ eV}^2$ which contradicts with $\Delta m_k^2 \ll \Delta m_{\text{sol}}^2$. Thereby, electron neutrinos from the nearby sources and high energy can remain undepleted, but ones from extragalactic sources get depleted.

5 Conclusion

In this work, we have proposed a model where sterile neutrinos are introduced to make light neutrinos to be pseudo-Dirac particles. It has been shown how tiny mass splitting necessary for realizing pseudo-Dirac neutrinos can be achieved. Within the model, we have examined how leptogenesis can be successfully generated. Motivated by the recent observation of very high energy neutrino events at IceCube and the results for the track-to-shower ratio, N_T/N_S , of the subset with energy above 60 TeV studied in ref. [70], we have examined a possibility to observe the effects of the pseudo-Dirac property of neutrinos by performing astronomical-scale baseline experiments to uncover the oscillation effects of very tiny mass splitting. Using the result of global fit to neutrino data for the input of neutrino mixing angles and CP phase at 1σ C.L. and fixing neutrino energy and mass splittings, we have studied how the oscillation effects induced by pseudo-Dirac neutrinos may affect the track-to-shower ratio, and found that the oscillation peaks occur at the distance about 1.3 Gpc for $\Delta m_{1,2,3}^2 = 10^{-16} \text{ eV}^2$ and $60 \text{ TeV} \lesssim E_\nu \lesssim 3 \text{ PeV}$. If future experiments can precisely measure the value of N_T/N_S , whose uncertainty becomes reduced to about 40–70% depending on the initial flavor compositions at the source, we could test the pseudo-Dirac property of neutrinos particularly for the cases with two flavors in the initial flavor composition of neutrino flux. In fact, in order to obtain fully meaningful results for testing our model in detail, much larger detectors than the present IceCube would be required [98].

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