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6d, $\mathcal{N} = (1, 0)$ Coulomb branch anomaly matching

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Abstract: 6d QFTs are constrained by the analog of 't Hooft anomaly matching: all anomalies for global symmetries and metric backgrounds are constants of RG flows, and for all vacua in moduli spaces. We discuss an anomaly matching mechanism for 6d $\mathcal{N} = (1, 0)$ theories on their Coulomb branch. It is a global symmetry analog of Green-Schwarz-West-Sagnotti anomaly cancellation, and requires the apparent anomaly mismatch to be a perfect square, $\Delta I_8 = \frac{1}{2}X_4^2$. Then ΔI_8 is cancelled by making X_4 an electric/magnetic source for the tensor multiplet, so background gauge field instantons yield charged strings. This requires the coefficients in X_4 to be integrally quantized. We illustrate this for $\mathcal{N} = (2, 0)$ theories. We also consider the $\mathcal{N} = (1, 0)$ SCFTs from N small E_8 instantons, verifying that the recent result for its anomaly polynomial fits with the anomaly matching mechanism.

Keywords: Supersymmetric gauge theory, Field Theories in Higher Dimensions, Anomalies in Field and String Theories

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Contents

1 Introduction

Brane constructions in a decoupling limit [1] led to the idea that there are local, interacting, 6d QFTs [2]. These theories cannot be formulated in any known, conventional lagrangian description, because they contain *interacting* two-form gauge fields, with self-dual field strength: the challenge is that the charged objects would be string-like, with self-dual electric-magnetic charges. Examples include the 6d $\mathcal{N} = (2, 0)$ theories, the $\mathcal{N} = (1, 0)$ include the theory of N small E_8 instantons[1] [1, 6, 7] and many others, obtained from decoupling limits of string, brane, M-theory, or F-theory constructions, see e.g. [8–15].

6d QFTs have chiral matter, so anomalies provide a useful handle. Gauge anomaly cancellation highly constrains the matter content [2, 9, 12, 16–19]. The analog of 't Hooft anomalies, for global symmetries, usefully constrains the low-energy theory: these anomalies must be constant along RG flows, and on the vacuum manifold, even if the symmetry is spontaneously broken. In the broken case, as in 4d [20], anomaly matching can require certain WZW-type low-energy interactions, to cancel apparent anomaly mismatches. This was discussed for 6d theories in [21], and applied to the case of $\mathcal{N} = (2, 0)$ theories on the Coulomb branch. We here apply analogous considerations to $\mathcal{N} = (1, 0)$ theories.

Consider a 6d, $\mathcal{N} = (1, 0)$ theory with a Coulomb branch moduli space of vacua, associated with $\langle\phi\rangle$ for the real scalar(s) of tensor multiplets. Let $\mathcal{S}^{\text{origin}}$ denote the low-energy theory at $\langle\phi\rangle = 0$. Moving to $\langle\phi\rangle \neq 0$, the theory reduces at low-energy as

$$\mathcal{S}^{\text{origin}} \qquad \to \qquad \mathcal{S}^{\text{away}} + \mathcal{S}[\mathrm{U}(1)] + \textit{anomaly matching terms.} \tag{1.1}$$

Here $\mathcal{S}[\mathrm{U}(1)]$ denotes a 6d $\mathcal{N} = (1, 0)$ tensor multiplet:[2] i.e. a real scalar, ϕ, a 2-form gauge field B with self-dual field strength H, and fermion superparters. The non-compact,

[1] Dimensionally reducing the small E_8 instanton theory to $d < 6$ gives theories that can be related to more conventional QFTs, e.g. [3–5].

[2] The notation is because it reduces, on an S^1, to a 5d $\mathcal{N} = 1$, U(1) vector multiplet.

real ϕ is the dilaton of spontaneously-broken conformal symmetry. The details of the $\to$ step in (1.1) involve integrating out poorly understood interactions, including effective strings coupling $\mathcal{S}^{away}$ to the B in $\mathcal{S}[\mathrm{U}(1)]$), with string tension $\sim \langle\phi\rangle \neq 0$. The *anomaly matching terms* in (1.1) are non-decoupling effects, regardless of how large ϕ is. Such anomaly-matching-derived interactions can provide useful clues about the dynamics.

Let I_8^{origin} be the anomaly polynomial 8-form of $\mathcal{S}^{\text{origin}}$, and $I_8^{\text{away,naive}}$ that of $\mathcal{S}^{\text{away}} + \mathcal{S}[\mathrm{U}(1)]$. Any apparent mismatch, $\Delta I_8 \equiv I_8^{\text{origin}} - I_8^{\text{naive,away}}$ must be balanced by some remaining interactions in the low-energy theory. We here discuss an anomaly matching mechanism, which cancels ΔI_8 provided that it is a perfect square:

$$I_8^{\text{origin}} - I_8^{\text{away,naive}} \equiv \Delta I_8 = \frac{1}{2} X_4 \wedge X_4. \tag{1.2}$$

More generally, with multiple tensors, we need

$$\Delta I_8 = \frac{1}{2}\Omega_{IJ} X_4^I \wedge X_4^J \equiv \frac{1}{2}\vec{X} \wedge \cdot \vec{X}, \tag{1.3}$$

where the I index runs over the tensor multiplets, and Ω_{IJ} is a positive definite, symmetric metric on the space of tensor multiplets, which is implicit in the $\wedge\cdot$ product in (1.3).

The mechanism is analogous to that of [22, 23] for canceling anomalies of local symmetries. A reducible gauge anomaly I_8 can be cancelled via an additional tensor multiplet contribution ΔI_8 of the form[3] (1.3). This is achieved by making X_4^I into electric / magnetic sources for the tensor multiplet field strengths H^I. Our sign conventions[4] are such that Ω_{IJ} is positive definite. The full theory is then gauge anomaly free if $I_8 + \Delta I_8 = 0$.

We apply a similar mechanism to global symmetries; rather than canceling an unwanted I_8 of opposite sign, here the tensor multiplet's ΔI_8 provides the 't Hooft anomaly matching deficit. This is achieved by making $\vec{X}_4$ (the $\vec{\cdot}$ is shorthand for multiple tensors, i.e. the I index in (1.3)) act as electric / magnetic sources for the tensor multiplets, so

$$S_{\text{eff,low}} \supset -\int_{M_6} \vec{B}_2 \wedge \cdot \vec{X}_4, \tag{1.4}$$

and the magnetic dual effect (see section 2 for details)

$$d\vec{H} = \frac{1}{2} 2\pi \vec{X}_4, \qquad \text{so} \tag{1.5}$$

$$\vec{H}_3 = d\vec{B}_2 + \pi \vec{X}_3^{(0)}, \qquad \text{where} \qquad \vec{X}_4 = d\vec{X}_3^{(0)}. \tag{1.6}$$

Because $\vec{X}_3^{(0)}$ in (1.6) is not invariant under global symmetry background gauge transformations, $\vec{B}_2$ must also correspondingly transform, such that H is invariant, $\delta H = 0$:

$$\delta\vec{B}_2 = -\pi \vec{X}_2^{(1)}, \qquad \text{where} \qquad \delta\vec{X}_3^{(0)} \equiv d\vec{X}_2^{(1)}. \tag{1.7}$$

Then variation of (1.4) will compensate for the apparent discrepancy from (1.2).

[3] In [22, 23], the H^I also includes the tensor from the gravity multiplet, which has opposite chirality from those of the matter multiplets, and correspondingly enters into Ω_{IJ} with opposite signature [23]. Here we decouple gravity, so Ω_{IJ} has a definite signature. We take it to be positive.

[4] We take matter fermions to contribute positively to I_8, while gauginos contribute negatively. Then the positive ΔI_8 (1.3) from tensor multiplets can e.g. cancel a negative I_8 gauge anomaly.

JHEP10(2014)162

Because $\vec{B}_2$ has quantized charges, the coefficients in $\vec{X}_4$ must be correspondingly appropriately quantized. The general $\vec{X}_4$ can be expanded in characteristic classes

$$\vec{X}_4 = \vec{n}_{\text{grav}} \frac{p_1(T)}{4} + \vec{n}_{\text{SU}(2)_R} c_2(F_{\text{SU}(2)_R}) + \sum_i \vec{n}_i c_2(F_i), \tag{1.8}$$

$p_1(T)$ is the Pontryagin class for the rigid, background spacetime curvature, $p_1(T) \equiv \frac{1}{2}\,\text{tr}(R/2\pi)^2$, $c_2(R)$ and $c_2(F_i)$ are Chern classes of the $\text{SU}(2)_R$ and F_i flavor symmetry background field strengths. The Chern classes $c_2(R)$ and $c_2(F_i)$ will here always be normalized to integrate to one for the minimal associated instanton configuration in the background gauge fields; as we will discuss, the corresponding statement for $p_1(T)/4$ is less clear. Such background gauge field instanton configurations are codimension 4 strings,[5] with $\vec{H}$ charge given by $\vec{n}_{\text{SU}(2)_R}$ or $\vec{n}_i$ (the i index runs over all global symmetries). These charges must reside in an integral lattice, so there is a quantization condition

$$\vec{n}_{\text{SU}(2)_R} \in \vec{\mathbb{Z}}, \qquad \text{and} \qquad \vec{n}_i \in \vec{\mathbb{Z}}. \tag{1.9}$$

We expect that $\vec{n}_{\text{grav}}$ in (1.8) is also quantized, but are uncertain about the normalization.

Note also that the susy completion of (1.4) will give terms $\mathcal{L}_{\text{eff}} \sim -\phi F_{\mu\nu}F^{\mu\nu}$, as in [2], now coupling the real scalar ϕ of the tensor multiplets to the background field strengths.

The outline is as follows. In section 2, we elaborate on the above anomaly matching mechanism. In section 3, we discuss the $\mathcal{N} = (2,0)$ theories, from a $\mathcal{N} = (1,0)$ perspective. In section 4, we review the 6d $\mathcal{N} = (1,0)$ theories associated with small E_8 instantons, and their recently-obtained anomaly polynomial [27]. In section 5, apply the anomaly matching mechanism to the small E_8 instanton theory on its Coulomb branch.

Note added: just prior to posting this paper, the outstanding paper [28] appeared. It uses essentially the same kind of anomaly matching mechanism as discussed here, to derive new results for anomaly polynomials for many classes of $\mathcal{N} = (1,0)$ theories.

2 6d 't Hooft anomalies, and a new mechanism for their matching

By the descent procedure [29–32], the anomalous variation of the effective action of a 6d theory is given in terms of the anomaly polynomial[6] 8-form I_8:

$$\delta S_{\text{eff}} = 2\pi \int_{M_6} I_6^{(1)}, \qquad \text{where} \quad I_8 = dI_7^{(0)}, \qquad \text{and} \qquad \delta I_7^{(0)} = dI_6^{(1)}, \tag{2.1}$$

where δ denotes the variation, M_6 is 6d spacetime,[7] the subscript on $X_6^{(1)}$ is the form number, and the superscript the order in the gauge or global symmetry variation parameter.

[5] It would be interesting to consider the codimension 4 BPS soliton string configurations [24], and the analog of 't Hooft anomaly matching for the 2d string worldsheet [25, 26].

[6] The normalization of I_{d+2} is such that a Weyl fermion contributes $\widehat{A}(T)\,\text{tr}\, e^{iF/2\pi}|_{d+2}$.

[7] There would be a $(-1)^{d/2}$ factor in (2.1) in Minkowski M_d with mostly + signature [33]; we here use Euclidean signature to avoid writing the − sign.

JHEP10(2014)162

Now suppose that the theory has a moduli space of vacua, and the theory at the origin has anomaly polynomial $I_8^{\rm origin}$, while the theory away from the origin has a naively different anomaly polynomial $I_8^{\rm away,naive}$. The naive difference leads to an apparent mismatch

$$\Delta(\delta S_{\rm eff}) \equiv \delta S_{\rm eff}^{\rm origin} - \delta S_{\rm eff}^{\rm naive,away} = 2\pi \int_{M_6} \Delta I_6^{(1)}, \quad \text{with} \quad \Delta I_8 \equiv I_8^{\rm origin} - I_8^{\rm away,naive}. \tag{2.2}$$

The variation of the low-energy effective action must make up for this difference:

$$\delta S_{\rm eff,low} = 2\pi \int_{M_6} \Delta I_6^{(1)}. \tag{2.3}$$

As an example, consider $\mathcal{N} = (2,0)$ theories on their Coulomb branch:

$$\mathcal{T}[G] \qquad \to \qquad \mathcal{T}[H] \times \mathcal{T}[\mathrm{U}(1)] + \textit{anomaly matching interactions}. \tag{2.4}$$

Here $\mathcal{T}[G]$ denotes the $\mathcal{N} = (2,0)$ theory of ADE group type G, and $\mathcal{T}[\mathrm{U}(1)]$ denotes a free $\mathcal{N} = (2,0)$ tensor multiplet. The global $\mathrm{Sp}(2)_R \cong \mathrm{SO}(5)_R$ is broken in (2.4), as $\mathrm{SO}(5)_R \to \mathrm{SO}(4)_R$. The five real scalars $\phi^{A=1\ldots5}$ of $\mathcal{T}[\mathrm{U}(1)]$ can be regarded as a radial dilaton mode, for spontaneously broken conformal invariance, and Nambu-Goldstone boson modes $S^4 \cong \mathrm{SO}(5)_R/\mathrm{SO}(4)$. The $\mathrm{SO}(5)_R$ 't Hooft anomaly naively does not match, $\Delta I_8 = (c(G) - c(H)) p_2(F_{\mathrm{SO}(5)_R})/24$, where $p_2(F_{\mathrm{SO}(5)_R})$ is the 2nd Pontryagin class of the $\mathrm{SO}(5)_R$ background field strength, and the needed term (2.3) comes from [21]

$$S_{\rm eff,low} \supset 2\pi \frac{c(G) - c(H)}{6} \int_{M_7} \Omega_3(\phi, A) \wedge d\Omega_3(\phi, A), \tag{2.5}$$

with $d\Omega_3 = \phi^*(\omega_4)$ the volume form on the S^4 Nambu-Goldstone manifold, and $\partial M_7 = M_6$. It was conjectured in [21] that $c(G) = |G| h_G$, which fits with the $G = \mathrm{SU}(N)$ cases [34], and also $\mathrm{SO}(2N)$ [35], as derived via M- theory M5 branes and bulk anomaly inflow.

The interaction (2.5) remains even when the global symmetry background is turned off, $F_{\mathrm{Sp}(2)} \to 0$. This is related to the fact that the 't Hooft anomaly difference, $\Delta I_8 \propto p_2(F_{\mathrm{Sp}(2)})$, is irreducible (i.e. it includes $\mathrm{tr}\, F^4_{\mathrm{Sp}(2)}$, not just $(\mathrm{tr}\, F^2_{\mathrm{Sp}(2)})^2$). This is similar to the 4d Wess-Zumino-Witten interaction [20] for matching the irreducible 't Hooft anomaly differences of non-Abelian $\mathrm{SU}(N \geq 3)$ global symmetries. Reducible t Hooft anomaly differences, on the other hand, lead to WZW-type interactions that become trivial when the background symmetry gauge fields are set to zero. That will be the case for the reducible differences (1.2) to be discussed here.

For 't Hooft anomaly discrepancies of the form (1.2) on the Coulomb branch (1.1), the needed compensating variation (2.3) is

$$\delta S_{\rm eff,low} = 2\pi \int_{M_6} \left(\frac{1}{2} \vec{X}_4 \wedge \cdot \vec{X}_4 \right)^{(1)} = \pi \int_{M_6} \vec{X}_4 \wedge \cdot \vec{X}_2^{(1)}, \tag{2.6}$$

where we define $\vec{X}_3^{(0)}$ and $\vec{X}_2^{(1)}$ via the usual descent notation, as in (2.1):

$$\vec{X}_4 \equiv d\vec{X}_3^{(0)}, \qquad \delta \vec{X}_3^{(0)} \equiv d\vec{X}_2^{(1)}. \tag{2.7}$$

JHEP10(2014)162

The variation (2.6) arises from the term (1.4) in the low-energy effective action. Unlike (2.5), the interaction (1.4) does not require going to 7d, and it is only non-zero if the global symmetry and metric background fields are non-zero; again, this is because ΔI_8 here is reducible. Also, the compact global symmetries are unbroken, so there are no Nambu-Goldstone bosons (though ϕ is a dilaton).

Note that a self-dual string's charge $\vec{Q}$ is quantized as[8]

$$d\vec{H} = \frac{1}{2}\, 2\pi\, \vec{Q}\, \delta(\Sigma_2 \hookrightarrow M_6), \qquad \vec{Q} \in \vec{\mathbb{Z}}, \tag{2.8}$$

which expresses the compactness of the gauge invariance of B. More generally, the lattice of allowed dyonic string charges must be self-dual [37]. The general 4-form $\vec{X}_4$ in (1.2) can be expanded as in (1.8), in terms of properly normalized characteristic classes. So $\int_{\Sigma_4} c_2(F_G) = 1$ for the minimal $\mathrm{SU}(2) \subset G$ instanton[9], where Σ_4 are the 4 Euclidean directions of an instanton configuration, transverse to the Σ_2 of a string in 6d. So $c_2(F_{\mathrm{SU}(2)_R})$ and $c_2(F_i)$ are smoothed-out versions of the $\delta(\Sigma_2 \hookrightarrow M_6)$ in (2.8), and the $\vec{n}_{\mathrm{SU}(2)_R}$ or $\vec{n}_i$ in (1.8) give the $\vec{Q}$ charge, hence their quantization conditions in (1.9).

The quantization of $\vec{n}_{\mathrm{grav}}$ in (1.8) and (1.9) is less clear, as it depends on what are the allowed gravitational analog of instanton configurations. For compact Σ_4 without boundary, $\int_{\Sigma_4} p_1 \in 24\mathbb{Z}$ if Σ_4 is spin (this follows from the spin 1/2 index theorem, since $\widehat{A} = 1 + p_1/24 + \dots$); for compact Σ_4 that is not necessarily spin, $\int_{\Sigma_4} p_1 \in 3\mathbb{Z}$. But here we are interested non-compact Σ_4, or Σ_4 with boundary, where the index theorems include boundary contributions, η, and the quantization conditions are weaker, see e.g. [40]. The Q contribution from n_{grav} could likewise have boundary contributions. We will not consider the n_{grav} quantization issue further here. We will see that the E_8 instanton example gives $n_{\mathrm{grav}} = 1$ with the normalization in (1.8).

3 $\mathcal{N} = (2,0)$ theories, regarded as a special case of $\mathcal{N} = (1,0)$

A $\mathcal{N} = (2,0)$ theory can be regarded as a special case of a $\mathcal{N} = (1,0)$ theory, where the global $\mathrm{Sp}(1)_R$ enhances to $\mathrm{Sp}(2)_R$. As reviewed around (2.5), the full $\mathrm{Sp}(2)_R$ has an irreducible ΔI_8. But ΔI_8 becomes reducible from the $\mathcal{N} = (1,0)$ perspective, as we then only turn on background gauge fields in an $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \subset \mathrm{SO}(5)_R$, and then

$$\Delta I_8 = \frac{\Delta c}{24} p_2(F_{\mathrm{SO}(5)_R}) \to \frac{\Delta c}{24} \left(c_2(F_{\mathrm{SU}(2)_L}) - c_2(F_{\mathrm{SU}(2)_R})\right)^2, \tag{3.1}$$

where $\Delta c \equiv c(G) - c(H)$, and we take $c(G) \equiv h_G |G|$. The ΔI_8 in (3.1) can of course still be matched via (2.5), taking the gauge fields there only in $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$.

More directly, we can write (3.1) as $\Delta I_8 = \frac{1}{2} X_4^2$, and match it as in (1.4) and (1.5). Superficially, this does not fit with the quantization condition (1.9), since $\sqrt{\Delta c/12} \notin \mathbb{Z}$; e.g. for $\mathrm{SU}(N) \to \mathrm{SU}(N-1) \times \mathrm{U}(1)$, $\Delta c/3 = N(N-1)$, and for $E_8 \to E_7 \times \mathrm{U}(1)$, $\Delta c/6 = (29)^2$. A similar confusion appeared in [21] (with similar resolution as here), where

[8] The $\frac{1}{2}$ here is from the 6d string's Dirac quantization, $eg = \frac{1}{2}\, 2\pi\, n$, see e.g. [26, 33, 36].

[9] I. e. $c_2(F_G) = \lambda(G)^{-1}\, \frac{1}{2}\, \mathrm{tr}(F_G/2\pi)^2$, where $\lambda(G)$ can be computed as in e.g. [38, 39].

it was noted that (2.5) can be obtained by taking $d\Omega_3$ to source H_3 with coefficient α_m and $\star H_3$ with coefficient α_e, see also [41]. This seemed to require $\Delta c/12 = \alpha_e \alpha_m$, with $\alpha_e \neq \alpha_m$, apparently in conflict with self-duality of H_3, and unclear quantization of $\alpha_{e,m}$.

The point is simply that the metric Ω_{IJ}, implicit in (1.2) and (1.3), is not δ_{IJ}. Actually, $\Omega_{IJ} = C^{-1}_{IJ}$, the inverse Cartan matrix of the ADE group G (this is also seen in the related theories of five-branes at orbifold singularities, in [10]). E.g. for the $G = \mathrm{SU}(2)$ theory, $\Omega = \frac{1}{2}$, so (3.1) gives $X_4 = \sqrt{\Delta c/6}(c_2(F_{\mathrm{SU}(2)_L}) - c_2(F_{\mathrm{SU}(2)_R}))$, which satisfies (1.9) because here $\Delta c = 6$. More generally, as noted in [42] (or [43], for 2d Toda), the Freudenthal and de Vries strange formula implies that, for $G = A, D, E$, (where $|G| = r_G(h_G + 1)$)

$$\frac{c(G)}{12} \equiv \frac{h_G|G|}{12} = \frac{1}{12} f_{abc} f^{abc} = \tilde{\rho}\cdot\tilde{\rho}, \tag{3.2}$$

where f_{abc} are the group structure constants and $\tilde{\rho} = \frac{1}{2}\sum_{\alpha>0}\tilde{\alpha}$ is the Weyl vector. Then (3.1), with $\Omega_{IJ} = C^{-1}_{IJ}$ is indeed compatible with the quantization (1.9); it is just obscured a bit by focusing on partial breaking $G \to H \times \mathrm{U}(1)$.

4 Review: the small E_8 instanton theory, $\mathcal{E}_8[N]$, and its anomaly polynomial

We will illustrate the anomaly matching mechanism for the case $\mathcal{S}^{\text{origin}} = \mathcal{E}_8[N]$, i.e. the theory of N small E_8 instantons. Recall that the case of $N = 1$ small E_8 instanton has a Higgs branch $\mathcal{M}_{\text{Higgs}}$ that is the $29+1$ hypermultiplet-dimensional moduli space of an E_8 instanton. The $+1$ hypermultiplet here is the translational zero mode of the codimension 4 instanton. Likewise, for all $\mathcal{E}_8[N]$, it is convenient to add a free hypermultiplet, for the CM position of the N instantons. At the origin of the Higgs branch of $\mathcal{E}_8[N]$, there is an interacting SCFT, with an N real-dimensional, tensor-multiplet, Coulomb branch.

This structure is evident in the M-theory realization, via N coincident M5 branes, which are also coincident with the end-of-the-interval [44] M9 brane. The E_8 gauge symmetry of the M9 brane becomes the global E_8 symmetry of the 6d SCFT in the decoupling limit. The 6d spacetime directions are $x^{0,1,2,3,4,5}$, and the M9 brane is at say $x^{11} = 0$. The Coulomb branch corresponds to moving the M5 branes to $\phi \sim x^{11} \neq 0$ (the Higgs branch corresponds to dissolving the M5s into E_8 instantons, necessarily at $x^{11} = 0$). The added free-hypermultiplet corresponds to the CM location of the M5 branes in the $x^{6,7,8,9}$ directions. By considering anomaly inflow, as in [34] but including the effect of the M9 brane, the anomaly polynomial of this theory was obtained in [27] to be

$$I_8[\mathcal{E}_8[N] + \text{f.h.}] = \frac{N^3}{6}\chi_4^2 + \frac{N^2}{2}\chi_4 I_4 + N(\frac{1}{2}I_4^2 - \frac{1}{48}\widehat{I}_8). \tag{4.1}$$

Here +f.h. denotes "free-hyper:" The notation in (4.1) is much as in [27]

$$\chi_4 \equiv c_2(F_{\mathrm{SU}(2)_L}) - c_2(F_{\mathrm{SU}(2)_R}), \tag{4.2}$$

$$I_4 \equiv -\frac{1}{2}c_2(F_{\mathrm{SU}(2)_R}) - \frac{1}{2}c_2(F_{\mathrm{SU}(2)_L}) + \frac{1}{4}p_1(T) + c_2(F_{E_8}), \tag{4.3}$$

$$\widehat{I}_8 \equiv \chi_4^2 + p_2(T) - \left(c_2(F_{\mathrm{SU}(2)_R}) + c_2(F_{\mathrm{SU}(2)_L}) - \frac{1}{2}p_1(T)\right)^2. \tag{4.4}$$

Our normalization is such that all $\int_{\Sigma_4} c_2(F) = 1$ for the minimal instanton configuration.

In this notation, the anomaly polynomial of the $\mathcal{N} = (2,0)$ theory of N M5 branes, keeping only $SO(4) \subset SO(5)_R$ background gauge fields, is [34]

$$I_8[\mathcal{T}[SU(N)]] + I_8[\mathcal{T}[U(1)]] = \frac{N^3}{24}\chi_4^2 - \frac{N}{48}\widehat{I}_8. \tag{4.5}$$

5 Anomaly matching for $\mathcal{E}_8[N]$ on its Coulomb branch

We consider the $\mathcal{E}_8[N]$ Coulomb branch associated with giving expectation value to just one of the N tensor multiplets. In the M5 realization, we move a single M5 to $x^{11} \neq 0$, leaving the other $N-1$ coincident with the M9 at $x^{11} = 0$. The breaking pattern is

$$\mathcal{E}_8[N] + \text{f.h.} \qquad \to \qquad \mathcal{E}_8[N-1] + 2(\text{f.h.}) + \mathcal{S}[U(1)] + \textit{anomaly matching terms}. \tag{5.1}$$

The f.h. on the l.h.s. of (5.1) is as in (4.1), and goes for the ride, and the other f.h. on the RSH arises in the low-energy theory. The anomaly polynomial I_8 of the l.h.s. of (5.1) is given in (4.1), and likewise for the $\mathcal{E}_8[N-1]+$f.h. on the r.h.s. , via $N \to N-1$, while that of f.h. $+\,\mathcal{S}[U(1)] = \mathcal{T}[U(1)]$ is given by setting $N = 1$ in (4.5). Thus the naive difference in anomalies between the l.h.s. and r.h.s. of (5.1) is

$$\begin{aligned}
\Delta I_8 &= \frac{1}{24}(4N^3 - 4(N-1)^3 - 1)\chi_4^2 + \frac{1}{2}(N^2 - (N-1)^2)\chi_4 I_4 + \frac{1}{2}I_4^2, \\
&= \frac{1}{8}(2N-1)^2\chi_4^2 + \frac{1}{2}(2N-1)\chi_4 I_4 + \frac{1}{2}I_4^2 \\
&= \frac{1}{2}\left(\left(N - \frac{1}{2}\right)\chi_4 + I_4\right)^2.
\end{aligned} \tag{5.2}$$

It's indeed a perfect square, as required. Moreover, writing this X_4 as in (1.2), the coefficients are indeed integrally quantized (the $\frac{1}{2}$'s in (5.2) all cancel or combine to 1)

$$X_4 = (N-1)c_2(F_{SU(2)_L}) - Nc_2(F_{SU(2)_R}) + \frac{1}{4}p_1(T) + c_2(F_{E_8}), \tag{5.3}$$

i.e. $n_{SU(2)_L} = N - 1$, $n_{SU(2)_R} = -N$, and $n_{E_8} = 1$: an $SU(2)_L$ instanton carries $N-1$ units of B-charge, an $SU(2)_R$ instanton has $-N$ units, and an E_8 instanton has 1 unit of B-charge. Also, $n_{\text{grav}} = 1$ here (recall the discussion at the end of sect. 2).

Consider e.g. the case of $N = 1$ small E_8 instanton where the theory on the r.h.s. of (5.1) is just the $\mathcal{N} = (1,0)$ tensor multiplet $\mathcal{S}[U(1)]$ and two free hypermultiplets. An $SU(2)_R$ instanton gives a string of B-charge -1, and an E_8 instanton gives one of B-charge $+1$. In the general N case, the $\mathcal{E}_8[N-1]$ theory at the origin evidently leads to an extra contribution to the B-charge of $\pm(N-1)$ for a $SU(2)_{L,R}$ instanton string.

Another breaking pattern is to give non-zero, coincident, expectation values to all N tensor multiplets of the $\mathcal{E}_8[N]$. In the M-theory realization, all N of the M5 branes are moved, together, away from the M9 brane. This gives the breaking pattern

$$\mathcal{E}_8[N] + \text{f.h.} \qquad \to \qquad \mathcal{T}[SU(N)] + \mathcal{T}[U(1)] + \textit{anomaly matching terms}, \tag{5.4}$$

where $\mathcal{T}$ denotes the $\mathcal{N} = (2,0)$ theories. The anomaly matching terms are a non-decoupling effect of the M9 brane. The rest of the low-energy theory on the r.h.s. of (5.4) has an approximate enhancement of $SO(4)_R \to S0(5)_R$, as part of the approximate, accidental enhancement of $\mathcal{N} = (1,0) \to \mathcal{N} = (2,0)$; the anomaly matching terms spoil this enhancement. The anomaly matching needed for (5.4), by (4.1) and (4.5), is

$$\Delta I_8 = \frac{N^3}{8}\chi_4^2 + \frac{N^2}{2}\chi_4 I_4 + \frac{N}{2} I_4 = \frac{N}{2}\left(\frac{N}{2}\chi_4 + I_4\right)^2 . \tag{5.5}$$

The $N = 1$ case of (5.4) and (5.5) coincides with the $N = 1$ case of (5.1) and (5.2). More generally, all N tensor multiplets on the r.h.s. of (5.4) participate in the anomaly matching mechanism, hence the overall N in (5.5), with an associated lattice of integral charges.

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JHEP10(2014)162