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$\alpha_s v^2$ corrections to η_c and χ_{cJ} production recoiled with a photon at e^+e^- colliders

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Abstract: We consider the production of η_c and χ_{cJ} states recoiled with a photon up-to $\mathcal{O}(\alpha_s v^2)$ at BESIII and B-factories within the frame of NRQCD factorization. With the corrections, we revisit the numerical calculations to the cross sections for $\eta_c(nS)$ and $\chi_{cJ}(mP)$ states. We argue that the search for XYZ states with even charge conjugation such as $X(3872)$, $X(3940)$, $X(4160)$, and $X(4350)$ recoiled with a photon at BESIII may help clarify the nature of these states. For completeness, the production of charmonium with even charge conjugation recoiled with a photon at B factories is also discussed.

Keywords: Phenomenological Models, NLO Computations

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Contents

1 Introduction

Non-relativistic quantum chromodynamics (NRQCD) is a rigorous and successful effect field theory that describes heavy quarkonium decay and production [1]. The color-octet mechanics (COM) is proposed in NRQCD. The infrared divergences in the decay widths of P-wave [2, 3] and D-wave [4–6] heavy quarkonium have been absorbed into the NRQCD matrix elements within NRQCD, and the infrared-safe decay rate can be obtained. But over the last decade, a comparison between leading order (LO) calculations and experimental measurements at e^+e^- colliders and at hadron colliders reveals large discrepancies.

In the e^+e^- annihilation experiment [7, 8], problems on NRQCD involving the inclusive and exclusive J/ψ production [9–14] had been solved by introducing higher-order corrections, including radiative corrections [15–26], relativistic corrections [27–35], and $\mathcal{O}(\alpha_s v^2)$ corrections [36, 37]. And the LO NRQCD calculations also encounter dilemmas in the heavy quarkonium production and polarization at hadron colliders especially in the large p_t region. The next-to-leading order (NLO) radiative corrections to the heavy quarkonium production [38–53] and polarization [54–59] at hadron colliders are significant. And the NLO relativistic corrections to J/ψ hadronic production are considered too [60–62]. $O(\alpha_s v^2)$ corrections to the decays of h_c, h_b and η_b are studied in refs. [63–65]. Actually, the corrections at higher-order (e.g., $\mathcal{O}(\alpha_s v^2)$, v^4), had been considered in many processes and contributed considerable effects. However, some drawbacks for fixed-order calculations involve the convergence for higher-order corrections and to which order should be considered within NRQCD. These problems can be understood by carrying out more higher-order calculations. More information about NRQCD can be found in ref. [66] and related papers.

The production of double charmonium at B factories aids in identifying charmonium or charmonium-like states with even charge conjugation; these particles are recoiled with J/ψ or $\psi(2S)$ [7, 8]. η_c, $\eta_c(2S)$, χ_{c0}, $X(3940)$ (decaying into $D\bar{D}^*$), and $X(4160)$ (decaying into $D^*\bar{D}^*$) have been observed, but χ_{c1} and χ_{c2} states are yet to be determined in double charmonium production at B factories. For the J^{PC} of photon is same with J/ψ, studies have focused on the production of quarkonium with even charge conjugation recoiled with a hard photon in the e^+e^- annihilation. The LO contribution for heavy quarkonium with even charge conjugation recoiled with a hard photon in the e^+e^- annihilation at the B factories and BESIII is a pure QED process [67, 68]. The NLO radiative corrections have been computed and analyzed [69–73]. And the NLO relativistic corrections have been computed too [70, 72]. Quarkonia with even charge conjugation are associated to the XYZ particles [74–76]. $X(3872)$, the well-known one of the XYZ particles [77], is supposed to χ'_{c1} state or the mixture of this state with other structure in some view [49, 78]. Recently, $X(3872)$ has been observed in photon-recoiled process with a statistical significance of 6.4σ at BESIII [79]. $X(3915)$ ($X(3945)$ or $Y(3940)$) and $Z(3930)$ are assigned as $\chi_{c0}(2P)$ and $\chi_{c2}(2P)$ states by the Particle Data Group (PDG) [80], however this identification may call into some questions [81]. The experimental results for states with even charge conjugation have elicited theoretical interest in the nature of charmonium-like states. The non-perturbative effects are strong because the energy region at BESIII approximates the threshold of charmonium states. Hence, the applicability of NRQCD is speculative within this region. However, some NRQCD-based calculations exhibit high compatibility with the data.

In this paper, the photon-recoiled η_c and χ_{cJ} production is studied based on our previous work [72]. We calculate the cross sections up-to the order of $\mathcal{O}(\alpha_s v^2)$ within the NRQCD. This study verifies the applicability of NRQCD at the threshold and determines the XYZ particles related to $\eta_c(nS)$ and $\chi_{cJ}(nP)$.

The paper is organized as follows. Section 2 introduces the framework of calculations, especially the method of the expansion up-to $(\alpha_s v^2)$ for the amplitudes. Section 3 presents the amplitude expansion and discussion on the cross sections for η_c and χ_{cJ} processes. Section 4 gives the numerical results up-to $\mathcal{O}(\alpha_s v^2)$. Finally, section 5 presents a summary.

2 The framework of the calculation

This section introduces the calculation method for the $\mathcal{O}(\alpha_s v^2)$ amplitude expansion to the process $e^+e^- \to \gamma^\star \to H(\eta_c, \chi_{cJ}) + \gamma$. The momenta of final states are stated as $H(p)$ and $\gamma(k)$. Cross section can be obtained applied with the amplitude expansion.

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2.1 Kinematics

In an arbitrary fame of the charmonium, the momenta of the charm and the anti-charm can be expressed by the meson momentum and their relative momentum,

$$\begin{aligned} p_c &= p/2 + q\,, \\ p_{\bar{c}} &= p/2 - q\,. \end{aligned} \tag{2.1}$$

The momenta p and q are orthogonal, i.e., $p{\cdot}q = 0$. In the meson rest frame, they can be written as, $p = (2E_q, \mathbf{0})$ and $q = (0, \mathbf{q})$. We calculated the amplitudes up-to the order $\mathcal{O}(\alpha_s v^2)$ using an orthodox method. In this method, the rest energy $E_q = \sqrt{m_c^2 + \mathbf{q}^2}$ of the charm/anti-charm should be expanded around the charm mass,

$$E_q = m_c + \frac{\mathbf{q}^2}{m_c^2}\frac{m_c}{2} + \mathcal{O}\left(\frac{\mathbf{q}^4}{m_c^4}\right). \tag{2.2}$$

The momenta of the final-state particles depend on E_q. For instance, the four-momenta of the particles in $\gamma^*(Q) \to H(p) + \gamma(k)$ in the center-of-mass system can be written as follows:

$$\begin{aligned} Q &= (\sqrt{s}, 0, 0, 0)\,, \\ p &= \left(\frac{s + 4E_q^2}{2\sqrt{s}}, 0, 0, \frac{s - 4E_q^2}{2\sqrt{s}}\right), \\ k &= \left(\frac{s - 4E_q^2}{2\sqrt{s}}, 0, 0, -\frac{s - 4E_q^2}{2\sqrt{s}}\right). \end{aligned} \tag{2.3}$$

Given the expression for E_q, the four-momenta can be expanded in terms of $\mathbf{q}^2/m_c^2$. For instance, the momenta of the final meson and the photon noted by p and k are expanded as the following expression,

$$\begin{aligned} p &= \left(\frac{s + 4m_c^2}{2\sqrt{s}}, 0, 0, \frac{s - 4m_c^2}{2\sqrt{s}}\right) + \frac{\mathbf{q}^2}{m_c^2}\frac{2m_c^2}{\sqrt{s}}(1, 0, 0, -1) + \mathcal{O}\left(\frac{\mathbf{q}^4}{m_c^4}\right) \\ &= p^{(0)} + \frac{\mathbf{q}^2}{m_c^2}p^{(2)} + \mathcal{O}\left(\frac{\mathbf{q}^4}{m_c^4}\right), \\ k &= \frac{s - 4m_c^2}{2\sqrt{s}}(1, 0, 0, -1) - \frac{\mathbf{q}^2}{m_c^2}\frac{2m_c^2}{\sqrt{s}}(1, 0, 0, -1) + \mathcal{O}\left(\frac{\mathbf{q}^4}{m_c^4}\right) \\ &= k^{(0)} + \frac{\mathbf{q}^2}{m_c^2}k^{(2)} + \mathcal{O}\left(\frac{\mathbf{q}^4}{m_c^4}\right). \end{aligned} \tag{2.4}$$

Therefore, the momenta with subscripts (0) or (2) are independent of $\mathbf{q}^2$. The scalar products of ($p^{(0)}$, $p^{(2)}$, k^0, and $k^{(2)}$) can be solved in a special frame. For instance, in the center-of-mass system, the relation $k^{(2)} = -p^{(2)}$ can be obtained to reduce the number of the independent momenta; all the three non-zero products are calculated as follows:

$$\begin{aligned} p^{(0)} \cdot p^{(0)} &= 4m_c^2\,, \\ p^{(0)} \cdot p^{(2)} &= 2m_c^2\,, \\ p^{(0)} \cdot k^{(0)} &= (s - 4m_c^2)/2\,. \end{aligned} \tag{2.5}$$

Studies on the $\mathcal{O}(\alpha_s v^2)$ corrections to the decay process of charmonium with massless final-states [63–65, 82] introduce a factor E_q/m_c to all external momenta. In our method, these momenta can be expanded as $p_i = p_i^{(0)} + \frac{\mathbf{q}^2}{m^2} p_i^{(2)} = p_i^{(0)}(1 + \frac{\mathbf{q}^2}{2m_c^2})$ with $p_i^{(2)} = p_i^{(0)}/2$. This equation indicates the compatibility of our method with that in the published works.

For the P-wave states, the spin and orbital vectors must also be expanded by

$$
\begin{aligned}
\epsilon_s &= \epsilon_s^{(0)} + \frac{\mathbf{q}^2}{m_c^2}\epsilon_s^{(2)} + \mathcal{O}\left(\frac{\mathbf{q}^4}{m_c^4}\right), \\
\epsilon_L &= \epsilon_L^{(0)} + \frac{\mathbf{q}^2}{m_c^2}\epsilon_L^{(2)} + \mathcal{O}\left(\frac{\mathbf{q}^4}{m_c^4}\right).
\end{aligned}
\tag{2.6}
$$

Furthermore, they couple onto the total angular momentum J states ($J = 0, 1, 2$) as follows:

$$
\begin{aligned}
\mathcal{P}_0^{\alpha\beta} &\equiv \sum_{s_z L_z} \epsilon_s^{*\alpha}\epsilon_L^{*\beta}\langle 1s_z; 1L_z|00\rangle = \frac{1}{\sqrt{D-1}}\Pi^{\alpha\beta}, \\
\mathcal{P}_1^{\alpha\beta} &\equiv \sum_{s_z L_z} \epsilon_s^{*\alpha}\epsilon_L^{*\beta}\langle 1s_z; 1L_z|1J_z\rangle = \frac{i}{\sqrt{2}M}\epsilon^{\alpha\beta\kappa\lambda}p_\kappa\epsilon_\lambda^*(J_z), \\
\mathcal{P}_2^{\alpha\beta} &\equiv \sum_{s_z L_z} \epsilon_s^{*\alpha}\epsilon_L^{*\beta}\langle 1s_z; 1L_z|2J_z\rangle = \epsilon^{*\alpha\beta}(J_z).
\end{aligned}
\tag{2.7}
$$

The polarization is summed over all directions of the vector for the total angular momentum:

$$
\begin{aligned}
&\sum_{J_z}\epsilon^\alpha(J_z)\epsilon^{*\beta}(J_z) = \Pi^{\alpha\beta}, \\
&\qquad\sum_{J_z}\epsilon^{\alpha\beta}\epsilon^{*\alpha'\beta'} = \frac{1}{2}(\Pi^{\alpha\alpha'}\Pi^{\beta\beta'} + \Pi^{\alpha\beta'}\Pi^{\alpha'\beta}) - \frac{1}{D-1}\Pi^{\alpha\beta}\Pi^{\alpha'\beta'},
\end{aligned}
\tag{2.8}
$$

where Π can be expanded in terms of $\mathbf{q}$:

$$
\begin{aligned}
\Pi_{\alpha\beta} &\equiv -g_{\alpha\beta} + \frac{p_\alpha p_\beta}{p^2}, \\
\Pi_{\alpha\beta} &= \Pi_{\alpha\beta}^{(0)} + \frac{\mathbf{q}^2}{4m_c^2}(p_\alpha^{(0)}p_\beta^{(2)} + p_\alpha^{(2)}p_\beta^{(0)} - p_\alpha^{(0)}p_\beta^{(0)}) + \mathcal{O}\left(\frac{\mathbf{q}^4}{m_c^4}\right).
\end{aligned}
\tag{2.9}
$$

The second term vanishes in the rest frame of the meson, which is consistent with the independence of the polarization vectors to $\mathbf{q}^2$ in this frame.

2.2 Amplitude expansion

The amplitude of $e^+e^- \to \gamma H(\eta_c, \chi_{cJ})$ can be written as [30]

$$
\mathcal{M}(e^+e^- \to \gamma H) = L_\alpha \mathcal{M}^\alpha(\gamma^* \to \gamma H), \tag{2.10}
$$

where the leptonic part L_α is independent of $\mathbf{q}$. We only consider the hadronic part element $\mathcal{M}^\alpha(\gamma^* \to \gamma H)$ in the NRQCD frame. The Feynman diagrams are shown in figure 1. The amplitude can be written as [30]:

$$
\mathcal{M}(\gamma^* \to \gamma H) = \sqrt{2M_H}\sum_n d_n\langle H|\mathcal{O}_n^H|0\rangle, \tag{2.11}
$$

where the factor $\sqrt{2M_H}$ originates from the relativistic normalization. d_n is the short-distance coefficient that can be obtained by matching with the full QCD calculations on the intermediate $c\bar{c}$ production. And the $\langle H|\mathcal{O}_n^H|0\rangle$ represents the NRQCD long-distance matrix elements that are extracted from the experimental data or determined by potential model or lattice calculations. The present study concentrates on the corrections up-to the order $\mathcal{O}(\alpha_s v^2)$ under the color-singlet frame. The expansion is given as follows:

$$
\begin{aligned}
&\mathcal{M}(\gamma^* \to \gamma H) \\
&\quad = \sqrt{2M_H}\left[(d^{(0)} + d^{(\alpha_s)})\langle H|\mathcal{O}^H|0\rangle + (d^{(v^2)} + d^{(\alpha_s v^2)})\langle H|\mathcal{P}^H|0\rangle\right] \\
&\quad \approx 2\sqrt{m_c}\left(1 + \frac{\mathbf{q}^2}{4m_c^2}\right)\left[(d^{(0)} + d^{(\alpha_s)})\langle H|\mathcal{O}^H|0\rangle + (d^{(v^2)} + d^{(\alpha_s v^2)})\langle H|\mathcal{P}^H|0\rangle\right]. \qquad (2.12)
\end{aligned}
$$

The short-distance coefficients are obtained from the matching between the pQCD and the NRQCD calculations on the $c\bar{c}$ production,

$$
\begin{aligned}
&\mathcal{M}_s[\gamma^* \to \gamma + c\bar{c}]|_{\rm pQCD} \\
&\quad = (d_s^{(0)} + d_s^{(\alpha_s)})\langle c\bar{c}|\mathcal{O}^{c\bar{c}}({}^1S_0^{[1]})|0\rangle + (d_s^{(v^2)} + d_s^{(\alpha_s v^2)})\langle c\bar{c}|\mathcal{P}^{c\bar{c}}({}^1S_0^{[1]})|0\rangle \\
&\quad = \sqrt{2N_c}2E_q\left[(d_s^{(0)} + d_s^{(\alpha_s)}) + \mathbf{q}^2(d_s^{(v^2)} + d_s^{(\alpha_s v^2)} + d_s^{(\text{self.})})\right]. \\
&\mathcal{M}_t[\gamma^* \to \gamma + c\bar{c}]|_{\rm pQCD} \\
&\quad = (d_t^{(0)} + d_t^{(\alpha_s)})\langle c\bar{c}|\mathcal{O}^{c\bar{c}}({}^3P_J^{[1]})|0\rangle + (d_t^{(v^2)} + d_t^{(\alpha_s v^2)})\langle c\bar{c}|\mathcal{P}^{c\bar{c}}({}^3P_J^{[1]})|0\rangle \\
&\quad = \sqrt{2N_c}2E_q\left[|\mathbf{q}|(d_t^{(0)} + d_t^{(\alpha_s)}) + \mathbf{q}^3(d_t^{(v^2)} + d_t^{(\alpha_s v^2)} + d_t^{(\text{self.})})\right], \qquad (2.13)
\end{aligned}
$$

where $\mathcal{M}_s$ and $\mathcal{M}_t$ represent the amplitudes with the $c\bar{c}$ pair coupling to spin-singlet and spin-triplet polarization, respectively. The above NRQCD operators $\mathcal{O}$ and $\mathcal{P}$ are respectively defined as follows:

$$
\begin{aligned}
\mathcal{O}^{c\bar{c}}({}^1S_0^{[1]}) &= \psi^\dagger\chi, \\
\mathcal{P}^{c\bar{c}}({}^1S_0^{[1]}) &= \psi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)^2\chi, \\
\mathcal{O}^{c\bar{c}}({}^3P_0^{[1]}) &= \frac{1}{3}\psi^\dagger\left(\frac{-i}{2}\overleftrightarrow{\mathbf{D}}\cdot\sigma\right)\chi, \\
\mathcal{P}^{c\bar{c}}({}^3P_0^{[1]}) &= \frac{1}{3}\psi^\dagger\left[\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)^2\left(\frac{-i}{2}\overleftrightarrow{\mathbf{D}}\cdot\sigma\right)\right]\chi, \\
\mathcal{O}^{c\bar{c}}({}^3P_1^{[1]}) &= \frac{1}{2}\psi^\dagger\left(\frac{-i}{2}\overleftrightarrow{\mathbf{D}}\times\sigma\right)\chi, \\
\mathcal{P}^{c\bar{c}}({}^3P_1^{[1]}) &= \frac{1}{2}\psi^\dagger\left[\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)^2\left(\frac{-i}{2}\overleftrightarrow{\mathbf{D}}\times\sigma\right)\right]\chi, \\
\mathcal{O}^{c\bar{c}}({}^3P_2^{[1]}) &= \psi^\dagger\left(\frac{-i}{2}\overleftrightarrow{D}^{(i}\sigma^{j)}\right)\chi, \\
\mathcal{P}^{c\bar{c}}({}^3P_2^{[1]}) &= \psi^\dagger\left[\left(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\right)^2\left(\frac{-i}{2}\overleftrightarrow{D}^{(i}\sigma^{j)}\right)\right]\chi, \qquad (2.14)
\end{aligned}
$$

where Pauli spinors ψ and χ describe the quark annihilation and the anti-quark creation, respectively. The gauge-covariant derivative operator $\overleftrightarrow{\mathbf{D}} = \overrightarrow{\mathbf{D}} - \overleftarrow{\mathbf{D}}$. The term $d^{(\text{self.})}$ originates from the one-loop self-energy corrections to the NRQCD matrix elements [1, 36, 63, 64] and in the $\overline{\text{MS}}$ scheme

$$\langle c\bar{c}|\mathcal{O}^{c\bar{c}}|0\rangle_{\overline{\text{MS}}} = \left(\langle c\bar{c}|\mathcal{O}^{c\bar{c}}|0\rangle\right)^{(0)} + \frac{2\alpha_s}{3\pi m_Q^2} C_F \frac{N_\epsilon}{\epsilon_{\text{IR}}} \left(\langle c\bar{c}|\mathcal{P}^{c\bar{c}}|0\rangle\right)^{(0)}, \tag{2.15}$$

where $N_\epsilon(m_Q) \equiv \left(\frac{4\pi\mu_r^2}{m_Q^2}\right)^\epsilon \Gamma(1+\epsilon)$. μ_r is the renormalization scale. Therefore,

$$d^{(\text{self.})} = \frac{2\alpha_s}{3\pi m_Q^2} C_F \left[\frac{1}{\epsilon_{\text{IR}}} + \ln 4\pi - \gamma_E + \ln\left(\frac{\mu_r^2}{m_Q^2}\right)\right] d^{(0)}. \tag{2.16}$$

This expression is satisfied for all ${}^1S_0^{[1]}$ and ${}^3P_J^{[1]}$ states. Therefore, $d^{(\text{self.})}$ contributes to the amplitude expansion for $\mathcal{O}(\alpha_s v^2)$. The factor $\sqrt{2N_c}2E_q$ in eq. (2.13) originates from the perturbative calculations on the LO $Q\overline{Q}$ NRQCD matrix elements. The extra factor $|\mathbf{q}|$ arises from the derivative operator for the P-wave NRQCD operator $\mathcal{P}$.

The covariant projection method is adopted to calculate the full QCD amplitudes as,

$$\begin{aligned}
\mathcal{M}_s[\gamma^* \to \gamma + c\bar{c}] &= \text{Tr}\{\mathcal{M}[\gamma^* \to \gamma + c + \bar{c}) \otimes \mathcal{P}_{00} \otimes \pi_1]\}\,, \\
\mathcal{M}_t[\gamma^* \to \gamma + c\bar{c}] &= \text{Tr}\{\mathcal{M}[\gamma^* \to \gamma + c + \bar{c}) \otimes \mathcal{P}_{1s_z} \otimes \pi_1]\}\,.
\end{aligned} \tag{2.17}$$

The color-singlet projection operator is defined as $\pi_1 = \mathbf{1}/\sqrt{N_c}$. The spin-singlet and spin-triplet projection operators are given as,

$$\begin{aligned}
\mathcal{P}_{00} &= \frac{1}{2\sqrt{2}(E_q + m_c)} (\not{p}_{\bar{c}} - m_c) \frac{(-\not{p} + 2E_q)\gamma_5(\not{p} + 2E_q)}{8E_q^2} (\not{p}_c + m_c)\,, \\
\mathcal{P}_{1s_z}(\epsilon_s) &= \frac{1}{2\sqrt{2}(E_q + m_c)} (\not{p}_{\bar{c}} - m_c) \frac{(-\not{p} + 2E_q)\not{\epsilon}_s(\not{p} + 2E_q)}{8E_q^2} (\not{p}_c + m_c)\,,
\end{aligned} \tag{2.18}$$

where $\mathcal{P}_{00}$ and $\mathcal{P}_{1s_z}$ are for the spin-singlet and spin-triplet states, respectively. These operators can be expanded up-to the $\mathbf{q}^2/m_c^2$ order applied with eqs. (2.1), (2.2), (2.4).

According to the matching expression eq. (2.13), the short-distance coefficients are calculated by

$$\begin{aligned}
d_s^{(0)} &= \frac{\mathcal{M}_s^{(0)}}{\sqrt{2N_c}2m_c}\Big|_{q\to 0}\,, \\
d_s^{(\alpha_s)} &= \frac{\mathcal{M}_s^{(\alpha_s)}}{\sqrt{2N_c}2m_c}\Big|_{q\to 0}\,, \\
d_s^{(v^2)} &= \frac{1}{2!}\frac{\partial^2}{\partial \mathbf{q}^2}\frac{\mathcal{M}_s^{(0)}}{\sqrt{2N_c}2E_q}\Big|_{q\to 0}\,, \\
d_s^{(\alpha_s v^2)} &= \frac{1}{2!}\frac{\partial^2}{\partial \mathbf{q}^2}\frac{\mathcal{M}_s^{(\alpha_s)}}{\sqrt{2N_c}2E_q} - d_s^{\text{self.}}\Big|_{q\to 0}\,, \\
d_t^{(0)} &= \epsilon_L^{(0)}\frac{\partial}{\partial|\mathbf{q}|}\frac{\mathcal{M}_t^{(0)}}{\sqrt{2N_c}2E_q}\Big|_{q\to 0}\,,
\end{aligned}$$

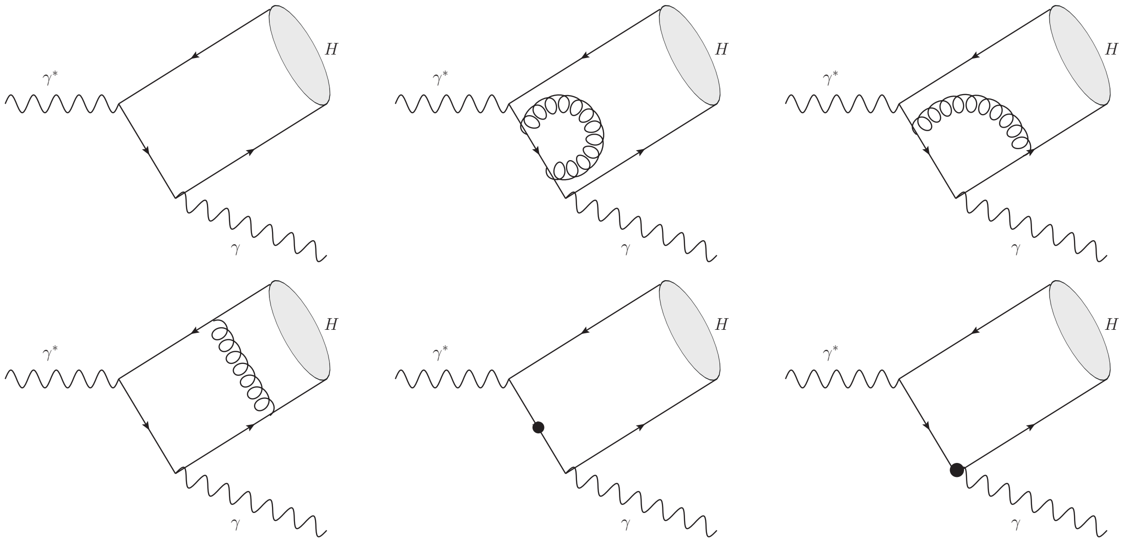

Figure 1. The typical born, loop, and counterterm Feynman diagrams. There are two diagrams for the born amplitude, six diagrams for the counterterm amplitude, and eight for the one-loop amplitude including two self-energy diagrams, four triangle diagrams, and two box diagrams.

$$
\begin{aligned}
d_t^{(\alpha_s)} &= \epsilon_L^{(0)} \frac{\partial}{\partial|\mathbf{q}|} \frac{\mathcal{M}_t^{(\alpha_s)}}{\sqrt{2N_c}2E_q}\Big|_{q\to 0}\,,\\
d_t^{(v^2)} &= \epsilon_L^{(0)} \frac{1}{3!}\frac{\partial^3}{\partial \mathbf{q}^3} \frac{\mathcal{M}_t^{(0)}}{\sqrt{2N_c}2E_q} + \epsilon_L^{(2)}\frac{d_t^{(0)}}{m_c^2}\Big|_{q\to 0}\,,\\
d_t^{(\alpha_s v^2)} &= \epsilon_L^{(0)} \frac{1}{3!}\frac{\partial^3}{\partial \mathbf{q}^3} \frac{\mathcal{M}_t^{(\alpha_s)}}{\sqrt{2N_c}2E_q} + \epsilon_L^{(2)}\frac{d_t^{(\alpha_s)}}{m_c^2} - d_t^{\text{self.}}\Big|_{q\to 0}\,,
\end{aligned}
\tag{2.19}
$$

where $\mathcal{M}^{(0)}$ and $\mathcal{M}^{(\alpha_s)}$ are defined by the born and one-loop amplitudes, respectively. The replacements are applied to resolve the amplitude expansion in term of the Lorentz vector q:

$$
q_\mu q_\nu \to \frac{\mathbf{q}^2}{D-1}\Pi_{\mu\nu}^{(0)}\,,
\tag{2.20}
$$

for S-wave states and

$$
q_\mu q_\nu q_\rho \to \frac{\mathbf{q}^3}{D+1}\left\{\Pi_{\mu\nu}^{(0)}[\epsilon_L^{(0)}]_\rho + \Pi_{\mu\rho}^{(0)}[\epsilon_L^{(0)}]_\mu + \Pi_{\nu\rho}^{(0)}[\epsilon_L^{(0)}]_\mu\right\}\,,
\tag{2.21}
$$

for P-wave states.

2.3 One-loop computation

The one-loop Feynman diagrams are shown in figure 1. The dimensional regularization scheme is selected here. The ultraviolet divergences in one-loop amplitude are canceled by the counterterms. The infrared divergences at the α_s order in one-loop amplitude are also canceled by the counterterm amplitude, and the additional infrared divergences at the order of $\alpha_s v^2$ are canceled by the one-loop self-energy corrections to the NRQCD matrix elements in eq. (2.15) and eq. (2.16). The real corrections need not to be included for the exclusive processes. We apply the method in ref. [83] to reduce the tensor integration. The relativistic expansion is done before dealing with the loop integrand. The on-mass-shell (OS) renormalization scheme is adopted and in this scheme the renormalization constants are chosen as

$$\begin{aligned}\delta Z_2^{\rm OS} &= -C_F\frac{\alpha_s}{4\pi}N_\epsilon\left(\frac{1}{\epsilon_{\rm UV}}+\frac{2}{\epsilon_{\rm IR}}+4\right),\\ \delta Z_{m_Q}^{\rm OS} &= -C_F\frac{\alpha_s}{4\pi}N_\epsilon\left(\frac{3}{\epsilon_{\rm UV}}+4\right),\end{aligned} \tag{2.22}$$

where $N_\epsilon(m_Q)$ has been previously defined and the renormalization scale μ_r is canceled by the loop and counterterm diagrams up-to the order of $\mathcal{O}(\alpha_s v^2)$. In the OS scheme, the diagrams for the external leg correction are not included. In our calculations, the 't Hooft-Veltman (HV) regularization scheme [94, 95] is adopted, in which γ^5 is defined as

$$\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = -\frac{i}{4!}\epsilon^{\mu\nu\rho\sigma}\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\,. \tag{2.23}$$

The traces involving more than four Dirac γ-matrices with a γ^5 are evaluated recursively by the West Mathematica programs [96]. Our strategy handing of γ^5 is coincident with ref. [63]. In the HV scheme, the Ward identities may be violated in the one-loop calculations, such as for the axial current known as the Adler-Bell-Jackiw anomalies, which arising from the symmetry breaking of γ_5 definitions in D-dimension as eq. (2.23). In our case, $\gamma* \to \gamma\eta_c$ process, for γ_5 appears outside of one-loop integrals, the amplitudes would satisfy the ward identities, that is seen as the short-distance results given in eq. (2.24) in the next section. More discussions of γ^5-scheme and the anomalous Ward identities could be refered to refs. [63, 91, 92, 94–99].

We use the `FeynArts` [84] package to generate Feynman diagrams and amplitudes, and the `FeynCalc` [85, 86] package and our self-written `Mathematica` package to handle the amplitudes and the phase space integrand.

2.4 Matching results for η_c

This subsection presents the matching results for the short-distance coefficients for η_c.

The final matching results of the coefficients are given in the appendix, where $r \equiv 4m_c^2/s$ and s is the squared beam energy. The coefficients are given as follows:[1]

$$\begin{aligned}d_s^{(0)} &= A^{(0)}\epsilon_1\,,\\ d_s^{(v^2)} &= A^{(v^2)}\epsilon_1 + A^{(0)}\epsilon_2/m_c^2\,,\\ d_s^{(\alpha_s)} &= A^{(\alpha_s)}\epsilon_1\,,\\ d_s^{(\alpha_s v^2)} &= A^{(\alpha_s v^2)}\epsilon_1 + A^{(\alpha_s)}\epsilon_2/m_c^2\,,\end{aligned} \tag{2.24}$$

[1]Here, we omit the third term of the coefficients in the orders of v^2 and $\alpha_s v^2$ as shown in the appendix to dilute the contribution of the relativistic renormalization in eq. (2.12) [30]. In other words, the below coefficients have included the contributions of the relativistic renormalization.

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where

$$\begin{aligned}\epsilon_1 &\equiv \epsilon^{\mu\nu\rho\tau}(\epsilon_Q^*)_\mu(\epsilon_k^*)_\nu k_\rho^{(0)} p_\tau^{(0)}\,,\\ \epsilon_2 &\equiv \epsilon_1^{(2)} = \epsilon^{\mu\nu\rho\tau}(\epsilon_Q^*)_\mu(\epsilon_k^*)_\nu (p^{(0)}+k^{(0)})_\rho p_\tau^{(2)}\,,\end{aligned}\tag{2.25}$$

where ϵ_Q and ϵ_k represent the polarization vector of the initial virtual photon and the final photon, respectively.

The coefficients in eq. (2.24) are provided in the high-energy region. In the limit $r \to 0$, the asymptotic behavior of these coefficients can be obtained. The lowest order of the coefficients is $\mathcal{O}(r)$; the higher-order contributions are omitted, and the reduced coefficients are given as follows:

$$\begin{aligned}d_s^{(0)} &= C\,r\epsilon_1 + \mathcal{O}(r^2)\,,\\ d_s^{(v^2)} &= -\frac{5}{12m_c^2}C\,r\epsilon_1 + \mathcal{O}(r^2)\,,\\ d_s^{(\alpha_s)} &= -\frac{C\,r\alpha_s\epsilon_1}{9\pi}[3(3-2\ln 2)\ln r + 9(\ln^2 2 - 3\ln 2 + 3) + \pi^2] + \mathcal{O}(r^2)\\ &\approx -\frac{C\,r\alpha_s\epsilon_1(-4.8\ln r - 22.5)}{9\pi} + \mathcal{O}(r^2)\,,\\ d_s^{(\alpha_s v^2)} &= \frac{C\,r\alpha_s\epsilon_1}{108m_c^2\pi}[3(27-10\ln 2)\ln r + (45\ln^2 2 - 75\ln 2 - 79) + 5\pi^2] + \mathcal{O}(r^2)\\ &\approx -\frac{C\,r\alpha_s\epsilon_1(5.0\ln r - 5.0)}{9\pi m_c^2} + \mathcal{O}(r^2)\,,\end{aligned}\tag{2.26}$$

where $C \equiv \frac{(4\pi\alpha)Q_c^2}{2m_c^3}$. The terms of ϵ_2 disappear in the expressions because ϵ_2 is suppressed by a factor of r than ϵ_1. The asymptotic behavior of $d^{(\alpha_s)}$ is consistent with that in ref. [70].

As mentioned in ref. [70], the asymptotic behavior of the coefficients for $r \to \infty$ corresponds to the process $\eta_c \to 2\gamma$. In this limit, we will get:[2]

$$\begin{aligned}\lim_{r\to\infty} A^{(0)} &= -C\,,\\ \lim_{r\to\infty} A^{(v^2)} &= \frac{17}{12m_c^2}C\,,\\ \lim_{r\to\infty} A^{(\alpha_s)} &= \frac{C\,\alpha_s(20-\pi^2)}{6\pi}\,,\\ \lim_{r\to\infty} A^{(\alpha_s v^2)} &= \frac{C\,\alpha_s}{216m_c^2\pi}(384\ln 2 - 844 + 63\pi^2)\,.\end{aligned}\tag{2.27}$$

Note that,

$$\lim_{r\to\infty}\epsilon_2 = \epsilon^{\mu\nu\rho\tau}(\epsilon_Q^*)_\mu(\epsilon_k^*)_\nu(k_\rho^{(2)}p_\tau^{(0)} + k_\rho^{(0)}p_\tau^{(2)}) = \epsilon_1\,.\tag{2.28}$$

[2] Compared with the previous results of the $\mathcal{O}(v^2)$ corrections with the di-photon decay process for η_c and χ_c [1, 82, 87], we find that the absolute values of our results differ by 1/4 which originates from the relativistic renormalization expansion (eq. (2.12)). Therefore the coefficients of the relativistic corrections for η_c, χ_c decay widths shown in table 3 differ by 1/2 from the previous works.

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Therefore, the NLO short-distance coefficients in v^2 are given by

$$\lim_{r\to\infty} d_s^{(v^2)} = \frac{5C}{12m_c^2}\epsilon_1 = -\frac{5}{12m_c^2}\lim_{r\to\infty} d^{(0)},$$
$$\lim_{r\to\infty} d_s^{(\alpha_s v^2)} = \frac{C\,\alpha_s}{m_c^2\pi}\left(\frac{16\ln 2}{9} - \frac{31}{54} + \frac{\pi^2}{8}\right). \tag{2.29}$$

The short-distance in $\mathcal{O}(\alpha_s v^2)$ is consistent with ref. [63], which contributes a factor of 1/4 if we disregard the contribution of the relativistic renormalization in eq. (2.12).

2.5 Matching results for χ_{cJ}

This subsection presents the matching results for the short-distance coefficients for χ_{cJ}.

Similar to η_c case, the short-distance coefficients in the orders of v^2 and $\alpha_s v^2$ for χ_{cJ} are also written in two parts. All the coefficients are given as follows:

$$\begin{aligned} d_t^{(0)} &= B^{(0)}\epsilon_3\,, \\ d_t^{(v^2)} &= B^{(v^2)}\epsilon_3 + B^{(0)}\epsilon_4/m_c^2\,, \\ d_t^{(\alpha_s)} &= B^{(\alpha_s)}\epsilon_3\,, \\ d_t^{(v^2)} &= B^{(\alpha_s v^2)}\epsilon_3 + B^{(\alpha_s)}\epsilon_4/m_c^2\,, \end{aligned} \tag{2.30}$$

where

$$\begin{aligned} \epsilon_3 &\equiv (\epsilon_Q^*)_\mu(\epsilon_k^*)_\nu\mathcal{P}^{(0)}_{\alpha\beta}\,, \\ \epsilon_4 &\equiv (\epsilon_Q^*)_\mu(\epsilon_k^*)_\nu\mathcal{P}^{(2)}_{\alpha\beta}\,. \end{aligned} \tag{2.31}$$

The asymptotic behavior in the limit $r \to 0$ is also considered. For the ϵ_4 is higher order than ϵ_3 in r, then the coefficients are given as follows:

$$\lim_{r\to 0} d_t^{(0)} = F\epsilon_3(g^{\alpha\nu}g^{\beta\mu} - g^{\alpha\mu}g^{\beta\nu})\,, \tag{2.32}$$
$$\lim_{r\to 0} d_t^{(v^2)} = -\frac{F\epsilon_3}{20m_c^2}(11g^{\alpha\nu}g^{\beta\mu} - 11g^{\alpha\mu}g^{\beta\nu} + 2g^{\alpha\beta}g^{\mu\nu})\,,$$
$$\lim_{r\to 0} d_t^{(\alpha_s)} = \frac{F\alpha_s\epsilon_3}{9\pi}\{(g^{\alpha\nu}g^{\beta\mu} - g^{\alpha\mu}g^{\beta\nu})[3(3-2\ln 2)\ln r + 3(3\ln^2 2 - 5\ln 2 + 7)\pi^2]$$
$$+6g^{\alpha\beta}g^{\mu\nu}(1+2\ln 2)\}\,,$$
$$\lim_{r\to 0} d_t^{(\alpha_s v^2)} = \frac{F\alpha_s\epsilon_3}{540\pi m_c^2}\{6g^{\alpha\beta}g^{\mu\nu}[3(1-2\ln 2)\ln r + (9\ln^2 2 - 99\ln 2 - 93) + \pi^2]$$
$$+(g^{\alpha\nu}g^{\beta\mu} - g^{\alpha\mu}g^{\beta\nu})[9(45-22\ln 2)\ln r + (297\ln^2 2 - 75\ln 2 - 77 + 33\pi^2)]\}\,.$$

3 Cross section

The cross sections of the process $e^+e^- \to \gamma H$ are relative to the squared amplitudes of the process $\gamma^* \to \gamma H$,

$$\sigma[e^+e^- \to \gamma H] = \frac{1}{2s}\frac{2(D-2)(4\pi\alpha)}{(D-1)s}\int \Phi_2\overline{\sum}|\mathcal{M}(\gamma^* \to \gamma H)|^2. \tag{3.1}$$

$\overline{\sum}$ means summing NRQCD amplitudes $\mathcal{M}$ over the final-state color and polarization and averaging over the ones of the initial states. Where the differential two-body phase space in D dimensions can be solved:

$$\int \Phi_2 = \frac{1}{8\pi}\left(\frac{4\pi}{s}\right)^{\epsilon}\left(1-\frac{M_H^2}{s}\right)^{1-2\epsilon}\frac{\Gamma(1-\epsilon)}{\Gamma(2-2\epsilon)}$$
$$\approx \frac{1}{8\pi}\left(\frac{4\pi}{s}\right)^{\epsilon}(1-r)^{1-2\epsilon}\left[1-\frac{r(1-2\epsilon)}{1-r}\frac{\mathbf{q}^2}{m_c^2}\right]\frac{\Gamma(1-\epsilon)}{\Gamma(2-2\epsilon)}\,, \tag{3.2}$$

where $M_H \approx 2E_q$ has been chosen. This expression implies that the two-body phase space contributes another factor of $-r/(1-r)$ to the v^2 order cross section. This factor is linearly divergent near the low-energy threshold.

The results of short-distance amplitudes are obtained in the last section. Then the cross sections for η_c and χ_{cJ} states can be obtained as follows:

$$\sigma = \hat{\sigma}^{(0)}\left[1+\alpha_s c^{10} + (c^{02}+\alpha_s c^{12})\langle v^2\rangle\right]\langle 0|\mathcal{O}^H|0\rangle\,, \tag{3.3}$$

where $\hat{\sigma}^{(0)}$ is the LO short-distance cross section, and the matrix element $\langle v^2\rangle$ is defined as follows:

$$\langle v^2\rangle \equiv \frac{\langle 0|\mathcal{P}^H|0\rangle}{m_c^2\langle 0|\mathcal{O}^H|0\rangle}\,. \tag{3.4}$$

3.1 η_c

The LO short-distance cross section for η_c is given by:

$$\hat{\sigma}^{(0)}_{\eta_c} = \frac{(4\pi\alpha)^3 Q_c^4(1-r)}{6\pi m_c s^2}\,. \tag{3.5}$$

Figure 2 shows the coefficients c^{10}, c^{02}, and c^{11} for η_c production ranging r from 0 to 0.5, corresponding to the range from high-energy to low-energy. The $\mathcal{O}(\alpha_s v^2)$ correction is suppressed by $\alpha_s\langle v^2\rangle$ and negligibly contributes to the total cross section at $r=0.5$. Table 1 presents the asymptotic behaviors of the coefficients near the threshold. The coefficient c^{12} for the $\mathcal{O}(\alpha_s v^2)$ correction is about $4.8/\pi$ if the corrections from the phase space are not considered, the $\mathcal{O}(\alpha_s v^2)$ contribution without the phase space contributions is one fifth of the $\mathcal{O}(\alpha_s)$ contribution near the threshold if $\langle v^2\rangle = 0.2$. The phase space brings an additional linear singularity factor that markedly enhances the $\mathcal{O}(v^2)$ and $\mathcal{O}(\alpha_s v^2)$ corrections. However, the total coefficient of the singularity $1/(1-r)$ is $(4\alpha_s/\pi - 1)\langle v^2\rangle$ and there is a negative residue singularity. The $\mathcal{O}(\alpha_s v^2)$ correction becomes significant in the high-energy region and provides negative contribution under the same sign with the $\mathcal{O}(v^2)$ and $\mathcal{O}(\alpha_s)$ ones as table 2. For the B factories energy region ($r \approx 0.07$), the contributions of $\mathcal{O}(\alpha_s v^2)$ corrections are numerically suppressed by $\langle v^2\rangle$ than those from $\mathcal{O}(\alpha_s)$ corrections.

Table 3 lists the corresponding coefficients for the decay process $\eta_c \to \gamma\gamma$. The $\mathcal{O}(\alpha_s v^2)$ contribution slightly affects the decay rate, although our numerically calculated value is slightly larger than that from ref. [63]. However, the $\mathcal{O}(\alpha_s v^2)$ contribution can re-determine the elements $\langle v^2\rangle$ for the color-singlet S-wave states.

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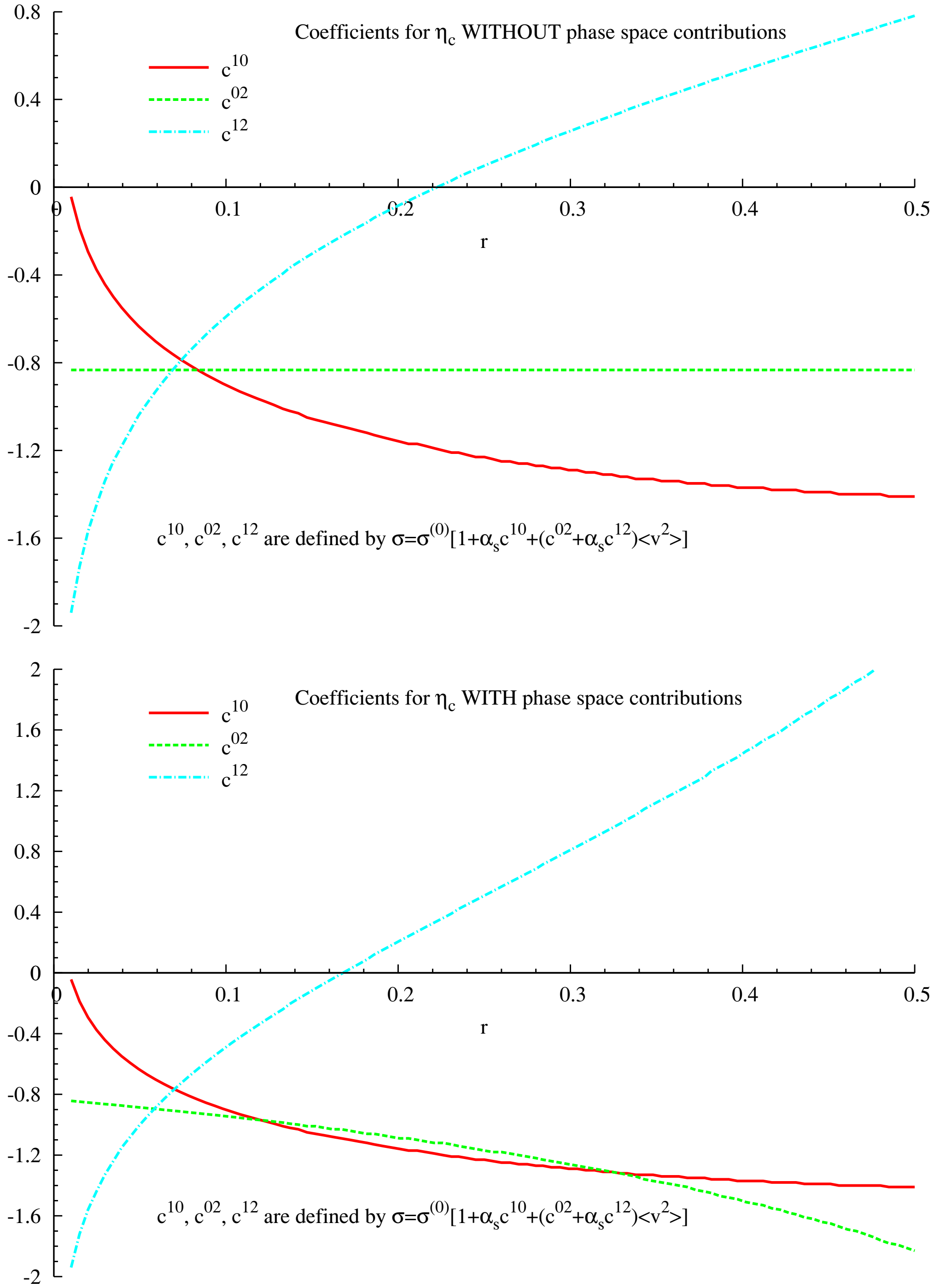

Figure 2. The higher order corrections for $e^+e^- \to \eta_c + \gamma$ as a function of r, where c^{10}, c^{02}, and c^{12} are defined by the expression $\sigma = \hat{\sigma}^{(0)}\left[1 + \alpha_s c^{10} + (c^{02} + \alpha_s c^{12})\langle v^2\rangle\right]\langle 0|\mathcal{O}^H|0\rangle$.

	$\lim_{r\to1} c^{02}$	$\lim_{r\to1} c^{10}$	$\lim_{r\to1} c^{12}$
η_c	$-\frac{5}{6}-\frac{1}{1-r}$	$-\frac{4}{\pi}$	$\frac{130}{27\pi}+\frac{4}{\pi(1-r)}$
χ_{c0}	$\frac{2}{1-r}-\frac{11}{10}-\frac{1}{1-r}$	$-\frac{16}{3\pi}$	$-\frac{32}{3\pi(1-r)}+\frac{160\ln[2(1-r)]-166}{45\pi}+\frac{16}{3\pi(1-r)}$
χ_{c1}	$\frac{2}{1-r}-\frac{13}{5}-\frac{1}{1-r}$	$-\frac{16}{3\pi}$	$-\frac{32}{3\pi(1-r)}+\frac{160\ln[2(1-r)]+389}{45\pi}+\frac{16}{3\pi(1-r)}$
χ_{c2}	$\frac{2}{1-r}-2-\frac{1}{1-r}$	$-\frac{16}{3\pi}$	$-\frac{32}{3\pi(1-r)}+\frac{160\ln[2(1-r)]+131}{45\pi}+\frac{16}{3\pi(1-r)}$

Table 1. The asymptotic behaviors of the coefficients near the threshold. The coefficients are defined in eq. (3.3). The last term in each cell of c^{02} and c^{12} originates from the phase space contributions.

η_c	$\lim_{r\to0} c^{02}$	$= -\frac{5}{6}$
	$\lim_{r\to0} c^{10}$	$= -\frac{2}{9\pi}[3(3-2\ln2)\ln r+9(\ln^2 2-3\ln2+3)+\pi^2]$ $\approx -0.34\ln r-1.59$
	$\lim_{r\to0} c^{12}$	$= \frac{1}{27\pi}[3(21-10\ln2)\ln r+15(3\ln^2 2-7\ln2)+28+5\pi^2]$ $\approx 0.50\ln r+0.31$
χ_{c0}	$\lim_{r\to0} c^{02}_{\chi_{c0}}$	$= -\frac{13}{10}$
	$\lim_{r\to0} c^{10}_{\chi_{c0}}$	$= -\frac{2}{9\pi}[3(1-2\ln2)\ln r+3\ln2(3\ln2-11)+\pi^2]$ $\approx 0.08\ln r+0.61$
	$\lim_{r\to0} c^{12}_{\chi_{c0}}$	$= \frac{1}{135\pi}[9(23-26\ln2)\ln r+3\ln2(117\ln2-443)-637+39\pi^2]$ $\approx 0.11\ln r-2.37$
χ_{c1}	$\lim_{r\to0} c^{02}_{\chi_{c1}}$	$= -\frac{11}{10}$
	$\lim_{r\to0} c^{10}_{\chi_{c1}}$	$= -\frac{2}{9\pi}[3(3-2\ln2)\ln r+3\ln2(3\ln2-5)+21+\pi^2]$ $\approx -0.34\ln r-1.75$
	$\lim_{r\to0} c^{12}_{\chi_{c1}}$	$= \frac{1}{135\pi}[9(39-22\ln2)\ln r+3\ln2(99\ln2-95)-637+33\pi^2]$ $\approx 0.50\ln r+1.36$
χ_{c2}	$\lim_{r\to0} c^{02}_{\chi_{c2}}$	$= -\frac{7}{10}$
	$\lim_{r\to0} c^{10}_{\chi_{c2}}$	$= -\frac{2}{9\pi}[3(1-2\ln2)\ln r+3\ln2(3\ln2+1)+18+\pi^2]$ $\approx 0.08\ln r-2.42$
	$\lim_{r\to0} c^{12}_{\chi_{c2}}$	$= \frac{1}{135\pi}[9(17-14\ln2)\ln r+3\ln2(63\ln2+79)+389+21\pi^2]$ $\approx 0.15\ln r+2.00$

Table 2. The asymptotic behaviors of the coefficients in the high-energy limit. The coefficients are defined in eq. (3.3). The asymptotic results for c^{10} are consistent with ref. [70] and for c^{02} are consistent with ref. [72].

	$\lim_{r\to\infty} c^{02}$	$\lim_{r\to\infty} c^{10}$	$\lim_{r\to\infty} c^{12}$
η_c	$-\frac{5}{6} \approx -0.83$	$\frac{1}{3\pi}[\pi^2 - 20] \approx -1.1$	$-\frac{1}{54\pi}[192\ln 2 + 21\pi^2 - 212] \approx -0.8$
χ_{c0}	$-\frac{11}{6} \approx -1.83$	$\frac{1}{9\pi}[3\pi^2 - 28] \approx -0.06$	$-\frac{1}{90\pi}[320\ln 2 + 65\pi^2 - 196] \approx -2.36$
χ_{c2}	$-\frac{3}{2} = -1.5$	$-\frac{16}{3\pi} \approx -1.7$	$-\frac{1}{135\pi}[48\ln 2 - 9\pi^2 - 1148] \approx 2.8$

Table 3. The asymptotic behaviors of the coefficients in the limt of $r \to \infty$. The coefficients are defined in eq. (3.3). These results are corresponding to the coefficients of the two-photon decay rates for η_c, χ_{c0} and χ_{c2}. $\chi_{c1} \to 2\gamma$ is forbade therefore the coefficients for it are not given.

3.2 χ_{cJ}

The LO short-distance cross section for χ_{cJ} is calculated as

$$\begin{aligned}
\hat{\sigma}^{(0)}_{\chi_{c0}} &= \frac{(4\pi\alpha)^3 Q_c^4 (1-3r)^2}{18\pi m_c^3 s^2 (1-r)}\,,\\
\hat{\sigma}^{(0)}_{\chi_{c1}} &= \frac{(4\pi\alpha)^3 Q_c^4 (1+r)}{3\pi m_c^3 s^2 (1-r)}\,,\\
\hat{\sigma}^{(0)}_{\chi_{c2}} &= \frac{(4\pi\alpha)^3 Q_c^4 (1+3r+6r^2)}{9\pi m_c^3 s^2 (1-r)}\,.
\end{aligned} \tag{3.6}$$

For χ_{c0}, the coefficients may be divergent at $r = 1/3$ for the LO short-distance coefficient reaches to zero at this point as eq. (3.6). Thus, we change eq. (3.3) into the following formula to define the coefficients:

$$\sigma_{\chi_{c0}} = \frac{(4\pi\alpha)^3 Q_c^4}{18\pi m_c^3 s^2}\left[c^{00} + \alpha_s c^{10} + (c^{02} + \alpha_s c^{12})\langle v^2\rangle\right]\langle 0|\mathcal{O}^H|0\rangle\,. \tag{3.7}$$

The redefined coefficients are shown as figure 3 and these coefficients are proportional to the corresponding short-distance cross sections. By a rough estimation, the LO cross sections are diluted by the sum of the $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(v^2)$ corrections as shown in figure. Furthermore, the $\mathcal{O}(\alpha_s v^2)$ contributes additional negative corrections. Thus, the total cross sections for χ_{c0} process may be small.

The coefficients for χ_{c1} and χ_{c2} processes are shown in figure 4 and figure 5, respectively. In the low-energy region ($0.3 < r < 0.5$), the $\mathcal{O}(\alpha_s v^2)$ corrections contribute the most. Meanwhile the $\mathcal{O}(v^2)$ and $\mathcal{O}(\alpha_s v^2)$ corrections increase faster with the addition of r, and they have different signs. As shown in table 1, the behaviors of the coefficients are similar for all the P-wave states. The coefficient c^{02} for the $\mathcal{O}(v^2)$ correction near the threshold has an additional linear singularity $1/(1-r)$ for the LO cross section. The coefficient c^{10} for the $\mathcal{O}(\alpha_s)$ corrections is a negative constant. In other words, the $\mathcal{O}(\alpha_s)$ contribution has the same rate to the corresponding LO cross section for different χ_{cJ} states. For the $\mathcal{O}(\alpha_s v^2)$ corrections, a logarithmic singularity term $\ln(1-r)$ apart from the linear singularity term also exists. We sum the linear singularity in the $\mathcal{O}(v^2)$ corrections and $\mathcal{O}(\alpha_s v^2)$ corrections and obtain the coefficient of the linear singularity as $(1 - 16\alpha_s/3/\pi)\langle v^2\rangle \approx -0.5\langle v^2\rangle$. The coefficient of the residual linear singularity is similar to that of η_c. The linear singularity half originates from the phase space. For the high-energy region, the $\mathcal{O}(\alpha_s v^2)$ corrections

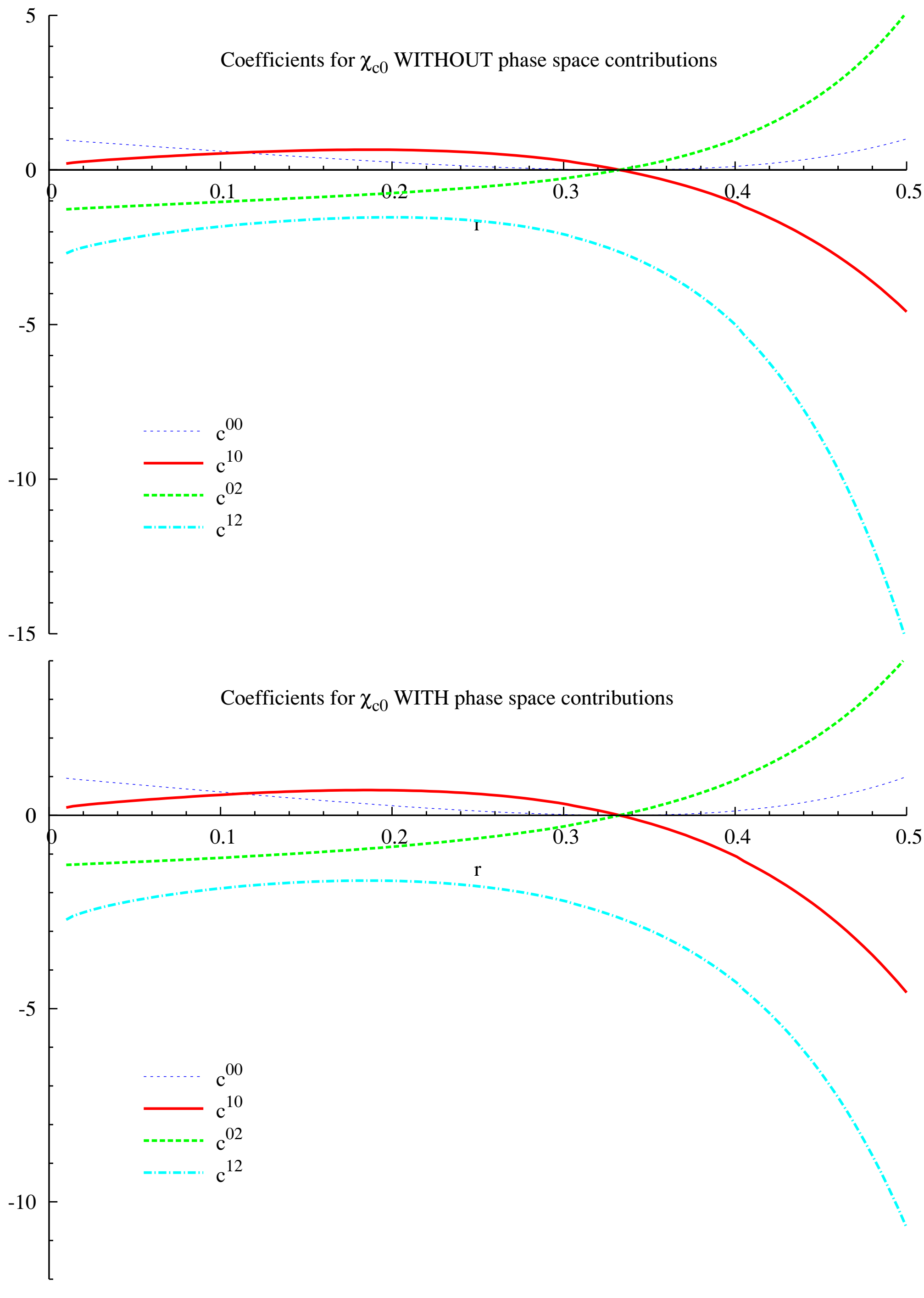

Figure 3. The higher order corrections for $e^+e^- \to \chi_{c0} + \gamma$ as a function of r, where c^{10}, c^{02}, and c^{12} are defined by the expression $\sigma = \frac{(4\pi\alpha)^3 Q_c^4}{18\pi m_c^3 s^2}\left[c^{00} + \alpha_s c^{10} + (c^{02} + \alpha_s c^{12})\langle v^2\rangle\right]\langle 0|\mathcal{O}^H|0\rangle$.

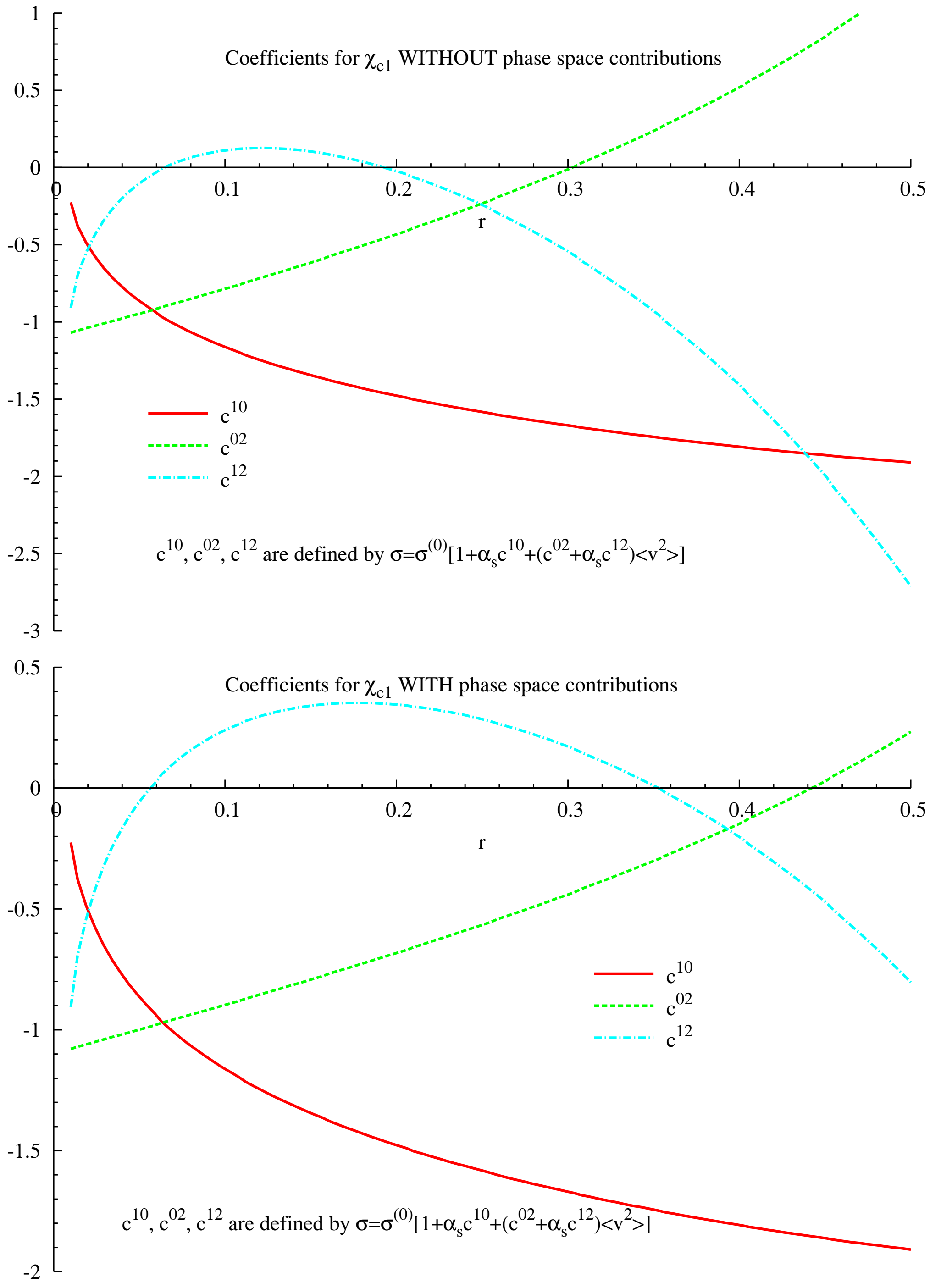

Figure 4. The higher order corrections for $e^+e^- \to \chi_{c1} + \gamma$ as a function of r, where c^{10}, c^{02}, and c^{12} are defined by the expression $\sigma = \hat{\sigma}^{(0)}\left[1 + \alpha_s c^{10} + (c^{02} + \alpha_s c^{12})\langle v^2\rangle\right]\langle 0|\mathcal{O}^H|0\rangle$.

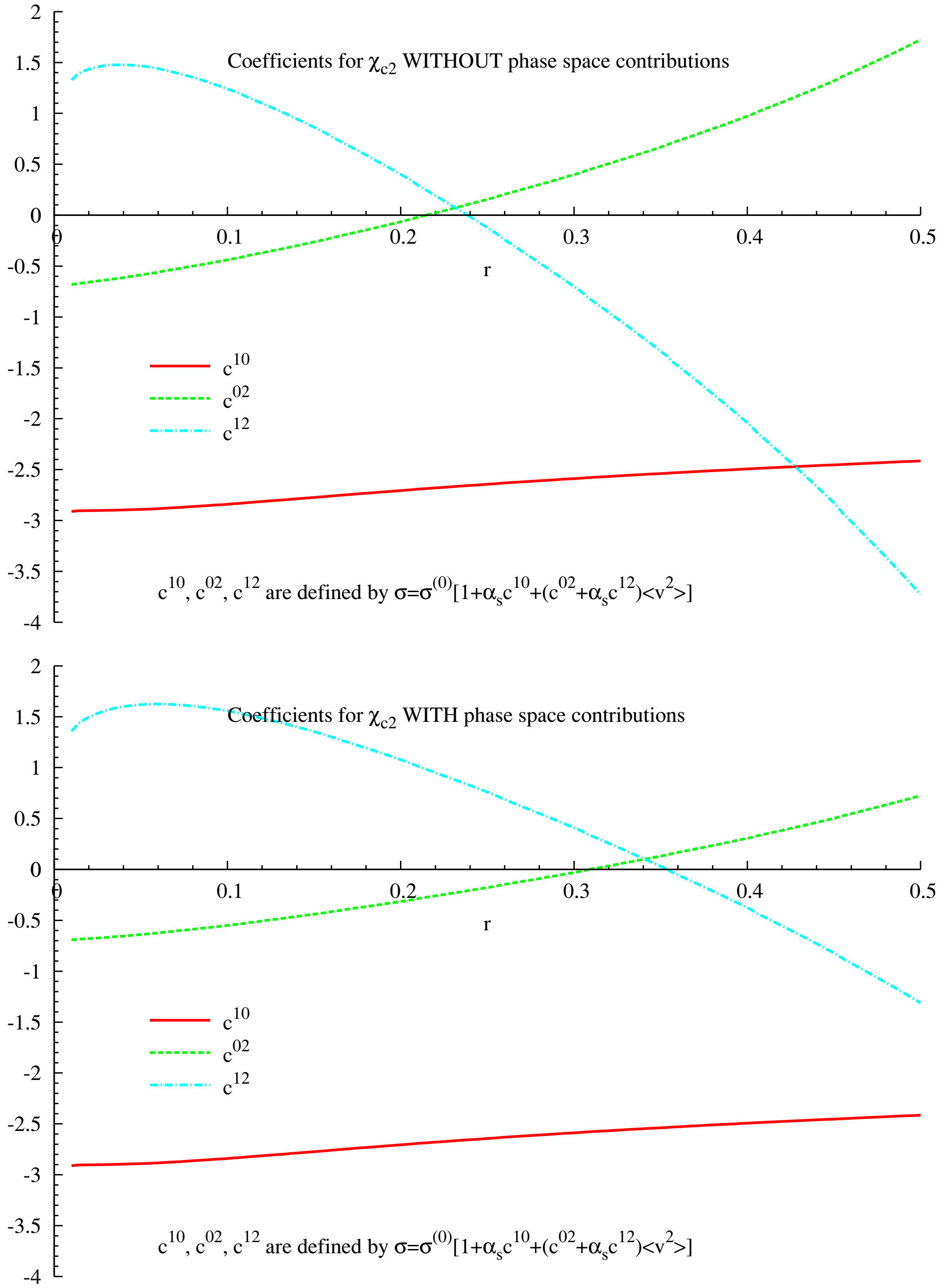

Figure 5. The higher order corrections for $e^+e^- \to \chi_{c2} + \gamma$ as a function of r, where c^{10}, c^{02}, and c^{11} are defined by the expression $\sigma = \hat{\sigma}^{(0)}\left[1 + \alpha_s c^{10} + (c^{02} + \alpha_s c^{12})\langle v^2 \rangle\right]\langle 0|\mathcal{O}^H|0\rangle$.

suppressed by the factor of $\alpha_s\langle v^2\rangle$. The numerical results in the high-energy approximation in table 2 also show that the $\alpha_s v^2$ corrections are much smaller than the α_s and v^2 corrections.

The coefficients corresponding to the two-photon decay for χ_{c0} and χ_{c2} are also given in table 3. By the rough estimation, we select α_s and $\langle v^2\rangle$ as a range of $0.2 \sim 0.3$. Therefore the $\mathcal{O}(\alpha_s v^2)$ corrections contribute $10\% \sim 20\%$ to the LO decay rate for $\chi_{c0} \to 2\gamma$ or $\chi_{c2} \to 2\gamma$. The $\mathcal{O}(\alpha_s v^2)$ corrections may also significantly affect the fit to the element $\langle v^2\rangle$ for χ_{cJ}.

4 Numerical results and discussion

In this section, we revisit the numerical calculations to the cross sections. In our numerical calculation, the total cross sections strongly depend on the input parameters (e.g., mass of the charm quark, long distance matrix elements, and the strong-coupling constant). The relativistic matrix elements can hardly be determined. In the consequent calculations for $\eta_c(1S)$, $\chi_{cJ}(1P)$ process, we select the fine structure constant $\alpha = 1/137$ and the charm quark mass as

$$m_c = 1.5 \pm 0.1\,\mathrm{GeV}, \tag{4.1}$$

for both $\eta_c(1S)$ and $\chi_{cJ}(1P)$ process. The strong-coupling constant is chosen as

$$\alpha_s = 0.23 \pm 0.03\,. \tag{4.2}$$

The matrix elements $\langle v^2\rangle$ are chosen as[3]

$$\begin{aligned}\langle v^2\rangle^{\eta_c} &= 0.15 \pm 0.1\,,\\ \langle v^2\rangle^{\chi_{cJ}} &= 0.20 \pm 0.1\,.\end{aligned} \tag{4.3}$$

The LO long-distance matrix elements are obtained from the radial wave functions at the origin in the potential model calculations [88] with the replacements

$$\begin{aligned}\langle 0|\mathcal{O}^{\eta_c(nS)}|0\rangle &= \frac{2N_c|R_{nS}(0)|^2}{4\pi}\,,\\ \langle 0|\mathcal{O}^{\chi_{c0}(mP)}|0\rangle &= \frac{6N_c|R'_{mP}(0)|^2}{4\pi}\,,\end{aligned} \tag{4.4}$$

[3]For the P-wave states, we can use our new up-to $\mathcal{O}(\alpha_s v^2)$ results for $\chi_{c0} \to \gamma\gamma$ and $\chi_{c2} \to \gamma\gamma$ to fit $\langle v^2\rangle$:

$$\begin{aligned}\Gamma[\chi_{c0} \to \gamma\gamma] &= \frac{6\pi Q_c^4\alpha^2}{m_c^4}\langle\mathcal{O}^{\chi_{c0}}\rangle[1 - 0.06\alpha_s - (1.83 + 2.36\alpha_s)\langle v^2\rangle^{\chi_{c0}} - 3a_8]\,,\\ \Gamma[\chi_{c2} \to \gamma\gamma] &= \frac{8\pi Q_c^4\alpha^2}{5m_c^4}\langle\mathcal{O}^{\chi_{c2}}\rangle[1 - 1.7\alpha_s - (1.5 - 2.8\alpha_s)\langle v^2\rangle^{\chi_{c2}} - 2.3a_8 - 1.7a_F]\,.\end{aligned}$$

The color-octet contributions in the above formula originate from ref. [87]. In the estimation, we ignore the v^2 corrections to the elements and assume $\langle\mathcal{O}^{\chi_{c2}}\rangle = 5\langle\mathcal{O}^{\chi_{c0}}\rangle$ and $\langle v^2\rangle^{\chi_c} \equiv \langle v^2\rangle^{\chi_{c0}} = \langle v^2\rangle^{\chi_{c2}}$. If we take $a_8 = a_F = 0.1$ then $\langle v^2\rangle^{\chi_c} = 0.32 \pm 0.04$ is obtained. If we ignore the contributions of the a_8 and a_F terms, then $\langle v^2\rangle^{\chi_c} = 0.21 \pm 0.03$. Therefore $\langle v^2\rangle^{\chi_c} = 0.2 \pm 0.1$ is compatible to these results. In this study, we select $\alpha_s = 0.23 \pm 0.03$ and the di-photon decay width for χ_{c0} and χ_{c2} are $(2.23 \pm 0.17) \times 10^{-4}$ MeV/c and $(2.59 \pm 0.16) \times 10^{-4}$ MeV/c, respectively, cited from PDG [80].

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	$1S\,(\mathrm{GeV}^3)$	$2S\,(\mathrm{GeV}^3)$	$3S\,(\mathrm{GeV}^3)$	$1P\,(\mathrm{GeV}^5)$	$2P\,(\mathrm{GeV}^5)$
Cornell	1.454	0.927	0.791	0.131	0.186
B-T	0.81	0.529	0.455	0.075	0.102
Re-est	1.132 ± 0.322	0.728 ± 0.199	0.623 ± 0.168	0.103 ± 0.028	0.144 ± 0.042

Table 4. The wave functions at the origin [88]. The two sets represent the results from the Cornell potential and the B-T potential. "Re-est" are averaged from the two sets of functions with the uncertainties.

and

$$\begin{aligned}\langle 0|\mathcal{O}^{\chi_{cJ}(mP)}|0\rangle &= (2J+1)\big(1+\mathcal{O}(v^2)\big)\langle 0|\mathcal{O}^{\chi_{c0}(mP)}|0\rangle \\ &\approx (2J+1)\langle 0|\mathcal{O}^{\chi_{c0}(mP)}|0\rangle\,. \end{aligned} \tag{4.5}$$

In the last step, we ignore the $\mathcal{O}(v^2)$ term to simplify the input parameters. The results markedly depend on the selections of the wave functions at the origin. Studies in refs. [69, 73] have adopted two sets of wave functions at the origin with large gaps. We re-estimate the wave functions at the origin by averaging the two sets of wave functions with the uncertainties in table 4. The wave functions at the origin for $4S$ and $3P$ states are estimated like ref. [72] as

$$\begin{aligned} R_{4S} &= 2R_{3S} - R_{2S} = 0.518 \pm 0.391\,\mathrm{GeV}^3, \\ R'_{3P} &= (R'_{1P} + R'_{2P})/2 = 0.124 \pm 0.025\,\mathrm{GeV}^5. \end{aligned} \tag{4.6}$$

In the BESIII energy region, the contributions from the phase space are significant for the cross sections. However, the contributions are hard to be determined because of the non-perturbative effects. In the previous works, two different strategies are used to remedy the non-perturbative effects from the phase space integrand, a extra factor is introduced in ref. [73] and the charm quark mass is set to half of the meson mass in ref. [72]. Furthermore, as stated in refs. [32, 89, 90], the v^2 corrections from the phase space, which are related to the terms in the short-distance cross section expansion different with those in the sub-amplitude expansion, could be resummed to all orders in v^2 by the 'shape functions' method. In this paper, we calculate the contributions of the phase space just by a simplified expansion by eq. (3.2). Therefore, we analyze the cross sections without (set $v^2 = 0$ in eq. (3.2)) and with the phase space contributions for comparative and referential purposes.

Table 5 presents the total cross sections up-to $\alpha_s v^2$ order with the uncertainties for η_c process. And figure 6 presents the corresponding cross sections in the BESIII energy region. The uncertainties for the total cross sections originate from the uncertainties of m_c, α_s, $\langle v^2\rangle$, and the wave functions at the origin. The phase space reduces the numerical results by a factor of 25%$\sim$10% and enhances the uncertainties by a factor of 35%$\sim$25% in the BESIII energy region $4 \sim 5\,\mathrm{GeV}$. The $\mathcal{O}(\alpha_s v^2)$ corrections negligibly contribute to the total cross sections for η_c process. Numerical simulations reveal that these corrections are approximately one-eighth and one-tenth of the $\mathcal{O}(\alpha_s)$ contributions in the energy regions of the B-factories and BESIII, respectively.

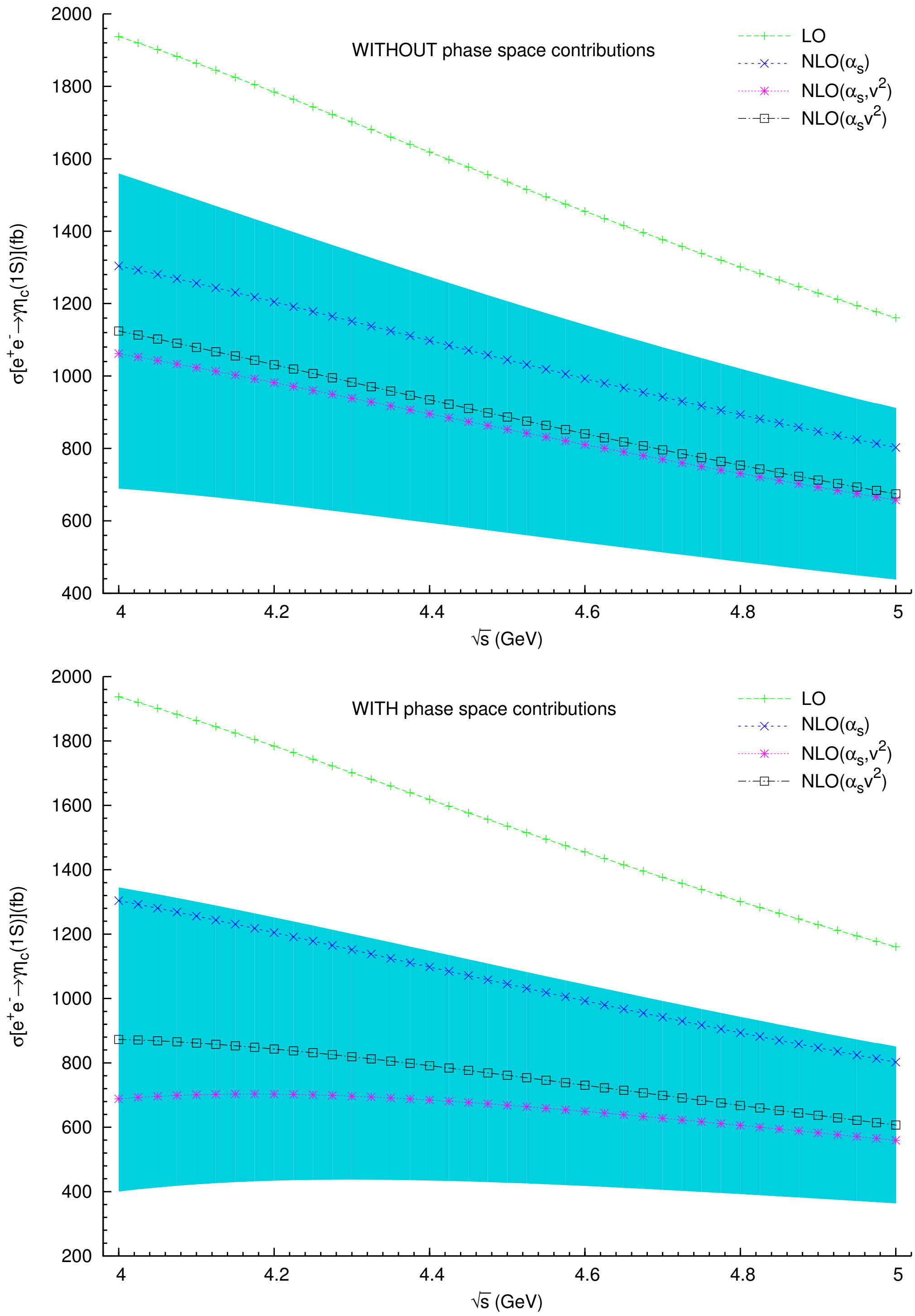

Figure 6. $\sigma[e^+e^- \to \eta_c + \gamma]$ in the BESIII energy region. The uncertainties for the total cross sections originate from the uncertainties of m_c, α_s, $\langle v^2 \rangle$, and the wave functions at the origin.

$\sqrt{s}$ (GeV)	4.25	4.50	4.75
$1S$ OP	$1007{\pm}286{\pm}68{\pm}114{\pm}198$	$887{\pm}252{\pm}60{\pm}105{\pm}155$	$775{\pm}220{\pm}52{\pm}95{\pm}123$
$1S$ WP	$832{\pm}237{\pm}57{\pm}231{\pm}209$	$762{\pm}217{\pm}52{\pm}189{\pm}162$	$683{\pm}194{\pm}46{\pm}156{\pm}129$
$2S$ OP	$282 \pm 77 \pm 17 \pm 26$	$284 \pm 77 \pm 18 \pm 29$	$269 \pm 74 \pm 18 \pm 30$
$2S$ WP	$99 \pm 27 \pm 5 \pm 117$	$155 \pm 42 \pm 10 \pm 93$	$176 \pm 48 \pm 12 \pm 76$
$3S$ OP	$101 \pm 27 \pm 5 \pm 7$	$142 \pm 38 \pm 8 \pm 12$	$153 \pm 41 \pm 9 \pm 14$
$3S$ WP	$-74 \pm 20 \pm 5 \pm 95$	$19 \pm 5 \pm 0.4 \pm 73$	$65 \pm 18 \pm 4 \pm 58$
$4S$ OP		$58 \pm 44 \pm 3 \pm 4$	$81 \pm 64 \pm 5 \pm 7$
$4S$ WP		$-52 \pm 39 \pm 4 \pm 59$	$6 \pm 4 \pm 0.1 \pm 46$
$\sqrt{s}$ (GeV)	5.00	10.6	11.2
$1S$ OP	$674{\pm}192{\pm}45{\pm}85{\pm}100$	$55{\pm}16{\pm}2{\pm}8{\pm}5$	$45{\pm}13{\pm}2{\pm}7{\pm}4$
$1S$ WP	$607{\pm}173{\pm}41{\pm}130{\pm}103$	$54{\pm}15{\pm}2{\pm}9{\pm}5$	$44{\pm}13{\pm}2{\pm}7{\pm}4$
$2S$ OP	$247 \pm 68 \pm 17 \pm 29$	$25 \pm 7 \pm 1 \pm 4$	$20 \pm 6 \pm 1 \pm 3$
$2S$ WP	$179 \pm 49 \pm 13 \pm 63$	$24 \pm 7 \pm 1 \pm 4$	$20 \pm 5 \pm 1 \pm 4$
$3S$ OP	$151 \pm 41 \pm 10 \pm 16$	$18 \pm 5 \pm 1 \pm 3$	$15 \pm 4 \pm 1 \pm 2$
$3S$ WP	$87 \pm 23 \pm 6 \pm 48$	$18 \pm 5 \pm 1 \pm 3$	$15 \pm 4 \pm 1 \pm 3$
$4S$ OP	$93 \pm 70 \pm 6 \pm 9$	$14 \pm 11 \pm 1 \pm 2$	$12 \pm 9 \pm 1 \pm 2$
$4S$ WP	$36 \pm 27 \pm 2 \pm 37$	$13 \pm 10 \pm 1 \pm 3$	$11 \pm 8 \pm 1 \pm 2$

Table 5. The total cross sections in fb up-to $\alpha_s v^2$ order of $e^+e^- \to \eta_c(nS)+\gamma$ with $n = 1, 2, 3, 4$ in the BESIII and B-factories energy region. 'WP' and 'OP' indicate considering or ignoring the phase space contributions, respectively. The uncertainties in each cell originate from the uncertainties of the wave functions at the origin, α_s, $\langle v^2\rangle$, and charm quark mass m_c in turns. For the excited states, we select the charm quark mass as the half of the meson mass in the calculations, therefore there are no m_c uncertainties. The masses of $\eta_c(nS)$ are selected as 3.639 GeV, 3.994 GeV, and 4.250 GeV for $n = 2, 3, 4$ respectively [74, 80].

Table 6 presents the total cross sections up-to $\alpha_s v^2$ order for χ_{c0} process with the uncertainties. The positive $\mathcal{O}(\alpha_s)$ corrections and negative $\mathcal{O}(v^2)$ corrections cancel to each other in the BESIII energy region [72], but $\mathcal{O}(\alpha_s v^2)$ parts also contribute negative corrections which decrease the LO cross sections significantly even to a negative values. And the uncertainties are too large compared with the central values to give a reliable prediction for χ_{c0} processes in the BESIII energy region.

Table 7 and table 8 present the total cross sections up-to the $\alpha_s v^2$ order for χ_{c1} and χ_{c2} processes, respectively, with the uncertainties. In the BESIII energy region, they exhibit similar tends. In addition, figure 7 and figure 8 show that the total cross sections for χ_{c2} decrease slightly faster than those for χ_{c1} as the energy increasing. The $\mathcal{O}(\alpha_s v^2)$ contributions are about half of the $\mathcal{O}(v^2)$ ones in this region as discussed in section 3. The phase space contribution reduces the total cross sections by a factor of 10% $\sim$ 20% in the BESIII energy region for both χ_{c0} and χ_{c1} processes. The corresponding uncertainties markedly decrease. From the tables, χ_{c1} and χ_{c2} states will be found in the BESIII energy region even if the lower bound of the numerical values is adopted for the cross sections.

For higher $\eta_c(ns)$ and $\chi_{cJ}(nP)$ states, the masses of these states extremely approximate the BESIII beam energy. NRQCD factorization will be broken down near the endpoint.

$\sqrt{s}$ (GeV)	4.25	4.50	4.75
$1P$ OP	17.4±4.7±15.8±10.8±39.2	−5.1±1.4±7.2±2.2±13.9	−8.8±2.4±3.3±1.0±4.1
$1P$ WP	18.3±5.0±13.9±11.3±35.8	−3.6±1.0±6.5±3.0±13.6	−8.0±2.2±3.0±0.5±4.5
$2P$ OP	$2558 \pm 746 \pm 308 \pm 933$	$552 \pm 161 \pm 83 \pm 177$	$170 \pm 49 \pm 32 \pm 51$
$2P$ WP	$1174 \pm 723 \pm 202 \pm 540$	$428 \pm 174 \pm 60 \pm 114$	$141 \pm 58 \pm 25 \pm 37$
$3P$ OP		$1331 \pm 268 \pm 163 \pm 479$	$320 \pm 64 \pm 48 \pm 102$
$3P$ WP		$932 \pm 188 \pm 108 \pm 280$	$248 \pm 50 \pm 35 \pm 66$
$\sqrt{s}$ (GeV)	5.00	10.6	11.2
$1P$ OP	−7.3±2.0±1.4±2.2±0.	1.6±0.4±0.±0.5±0.4	1.4±0.4±0.±0.4±0.3
$1P$ WP	−6.9±1.9±1.4±2.0±0.5	1.6±0.4±0.±0.5±0.4	1.3±0.4±0.±0.4±0.3
$2P$ OP	$58 \pm 17 \pm 15 \pm 18$	$0.7 \pm 0.2 \pm 0. \pm 0.3$	$0.6 \pm 0.2 \pm 0. \pm 0.2$
$2P$ WP	$51 \pm 21 \pm 12 \pm 15$	$0.6 \pm 0.3 \pm 0. \pm 0.3$	$0.6 \pm 0.2 \pm 0. \pm 0.2$
$3P$ OP	$104 \pm 21 \pm 20 \pm 32$	$0.4 \pm 0.1 \pm 0. \pm 0.2$	$0.4 \pm 0.1 \pm 0. \pm 0.2$
$3P$ WP	$87 \pm 17 \pm 15 \pm 23$	$0.4 \pm 0.1 \pm 0. \pm 0.2$	$0.4 \pm 0.1 \pm 0. \pm 0.2$

Table 6. The total cross sections in fb up-to $\alpha_s v^2$ order of $e^+e^- \to \chi_{c0}(nP) + \gamma$ with $n = 1, 2, 3$ in the BESIII and B-factories energy region. 'WP' and 'OP' indicate considering or ignoring the phase space contributions, respectively. The uncertainties in each cell originate from the uncertainties of the wave functions at the origin, α_s, $\langle v^2 \rangle$, and charm quark mass m_c in turns. For the excited states, we select the charm quark mass as the half of the meson mass in the calculations, therefore there are no m_c uncertainties. The mass of $\chi_{c0}(nP)$ is selected as 3.918 GeV and 4.131 GeV for $n = 2, 3$ respectively [74, 80].

$\sqrt{s}$ (GeV)	4.25	4.50	4.75
$1P$ OP	1716±466±185±151±86	1127±306±117±64±0.3	783±213±79±24±27
$1P$ WP	1435±390±156±11±25	967±263±102±16±26	685±186±70±25±39
$2P$ OP	$10456 \pm 3050 \pm 1099 \pm 3524$	$3374 \pm 984 \pm 374 \pm 881$	$1603 \pm 468 \pm 180 \pm 326$
$2P$ WP	$6809 \pm 1986 \pm 700 \pm 1077$	$2399 \pm 700 \pm 264 \pm 393$	$1209 \pm 353 \pm 136 \pm 129$
$3P$ OP		$7586 \pm 1529 \pm 784 \pm 2690$	$2290 \pm 462 \pm 251 \pm 638$
$3P$ WP		$4831 \pm 974 \pm 486 \pm 1313$	$1597 \pm 322 \pm 173 \pm 292$
$\sqrt{s}$ (GeV)	5.00	10.6	11.2
$1P$ OP	568±154±55±5±34	15.0±4.1±0.8±2.1±2.8	11.9±3.2±0.6±1.8±2.3
$1P$ WP	505±137±49±26±40	14.7±4.0±0.8±2.3±2.7	11.7±3.2±0.6±1.9±2.3
$2P$ OP	$915 \pm 267 \pm 102 \pm 145$	$10.4 \pm 3.0 \pm 0.7 \pm 1.2$	$8.2 \pm 2.4 \pm 0.5 \pm 1.0$
$2P$ WP	$720 \pm 210 \pm 81 \pm 47$	$10.0 \pm 2.9 \pm 0.7 \pm 1.4$	$7.9 \pm 2.3 \pm 0.5 \pm 1.1$
$3P$ OP	$1061 \pm 214 \pm 119 \pm 234$	$7.6 \pm 1.5 \pm 0.5 \pm 0.8$	$5.9 \pm 1.2 \pm 0.4 \pm 0.6$
$3P$ WP	$786 \pm 159 \pm 88 \pm 97$	$7.3 \pm 1.5 \pm 0.5 \pm 0.9$	$5.7 \pm 1.1 \pm 0.4 \pm 0.8$

Table 7. The total cross sections in fb up-to $\alpha_s v^2$ order of $e^+e^- \to \chi_{c1}(nP) + \gamma$ with $n = 1, 2, 3$ in the BESIII and B-factories energy region. 'WP' and 'OP' indicate considering or ignoring the phase space contributions, respectively. The uncertainties in each cell originate from the uncertainties of the wave functions at the origin, α_s, $\langle v^2 \rangle$, and charm quark mass m_c in turns. For the excited states, we select the charm quark mass as the half of the meson mass in the calculations, therefore there are no m_c uncertainties. The mass of $\chi_{c1}(nP)$ is selected as 3.901 GeV and 4.178 GeV for $n = 2, 3$ respectively [74, 80].

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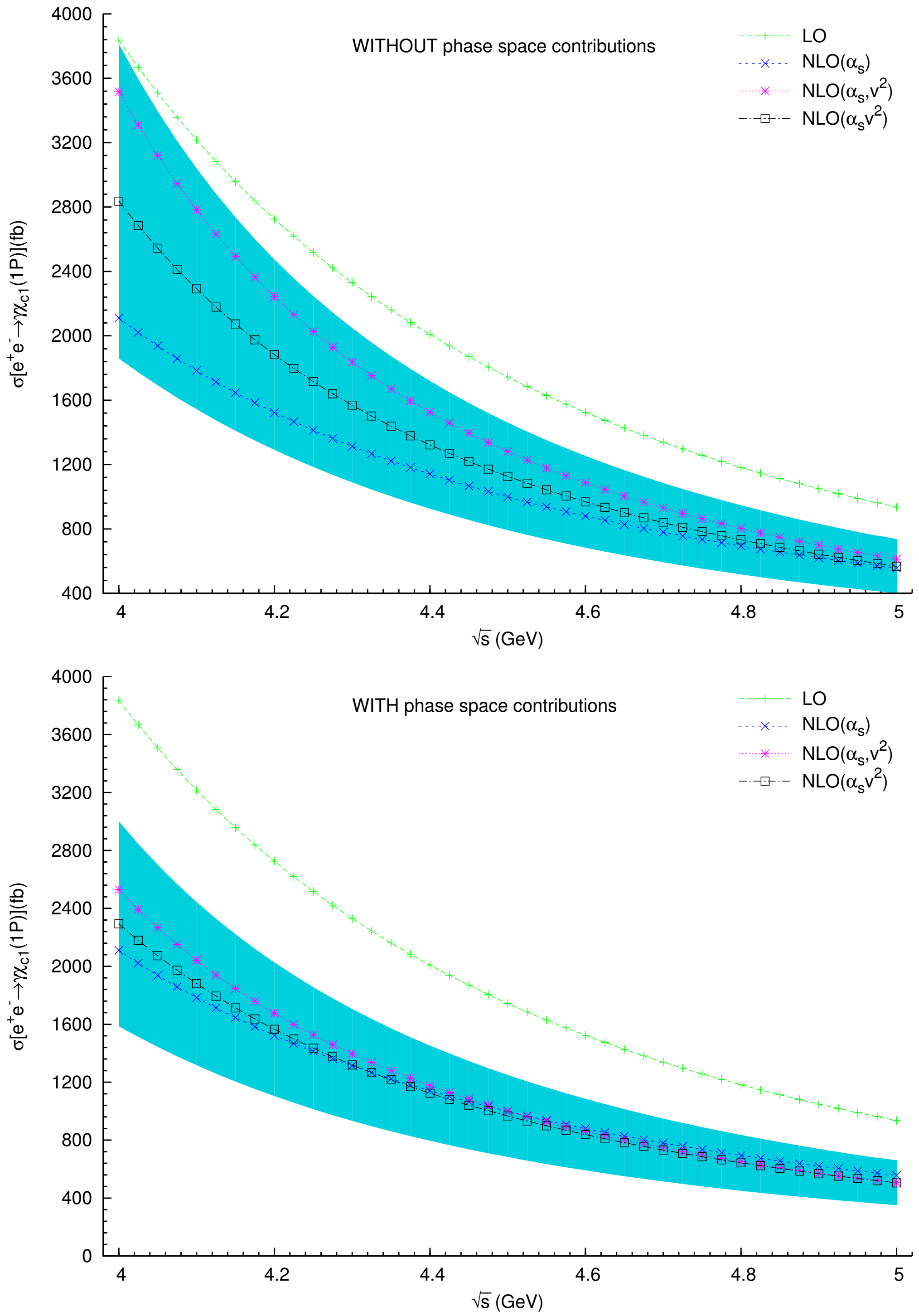

Figure 7. $\sigma[e^+e^- \to \chi_{c1} + \gamma]$ in the BESIII energy region. The uncertainties for the total cross sections originate from the uncertainties of m_c, α_s, $\langle v^2 \rangle$, and the wave functions at the origin.

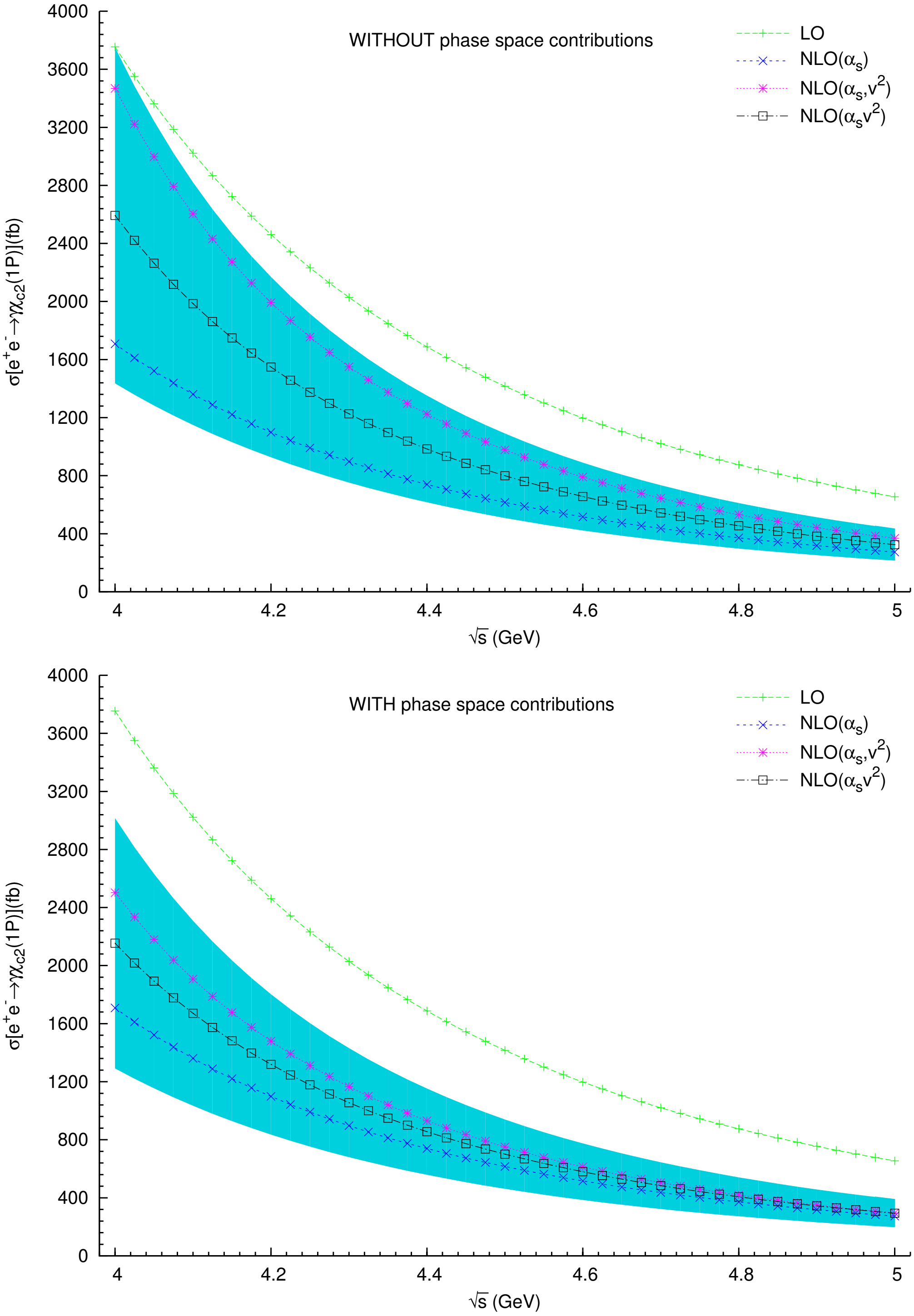

Figure 8. $\sigma[e^+e^- \to \chi_{c2} + \gamma]$ in the BESIII energy region. The uncertainties for the total cross sections originate from the uncertainties of m_c, α_s, $\langle v^2 \rangle$, and the wave functions at the origin.

$\sqrt{s}$ (GeV)	4.25	4.50	4.75
$1P$ OP	1375±374±211±192±267	799±217±128±92±112	497±135±82±47±50
$1P$ WP	1178±320±179±94±193	701±191±111±43±82	443±121±73±21±36
$2P$ OP	17037±4969±1898±6041	4564±1331±568±1305	1878±548±252±444
$2P$ WP	11250±3281±1205±3147	3316±967±402±681	1451±423±190±230
$3P$ OP		13164±2654±1424±4895	3253±656±395±983
$3P$ WP		8465±1707±878±2546	2314±466±273±513
$\sqrt{s}$ (GeV)	5.00	10.6	11.2
$1P$ OP	325±88±55±25±23	3.1±0.8±0.8±0.2±0.4	2.3±0.6±0.6±0.2±0.3
$1P$ WP	294±80±50±10±16	3.0±0.8±0.8±0.2±0.4	2.3±0.6±0.6±0.2±0.3
$2P$ OP	$945 \pm 275 \pm 133 \pm 188$	$2.7 \pm 0.8 \pm 0.6 \pm 0.1$	$2.0 \pm 0.6 \pm 0.5 \pm 0.1$
$2P$ WP	$761 \pm 222 \pm 106 \pm 96$	$2.6 \pm 0.8 \pm 0.6 \pm 0.1$	$1.9 \pm 0.6 \pm 0.5 \pm 0.1$
$3P$ OP	$1305 \pm 263 \pm 171 \pm 328$	$2.2 \pm 0.4 \pm 0.5 \pm 0.$	$1.6 \pm 0.3 \pm 0.4 \pm 0.$
$3P$ WP	$990 \pm 200 \pm 127 \pm 171$	$2.1 \pm 0.4 \pm 0.5 \pm 0.1$	$1.5 \pm 0.3 \pm 0.3 \pm 0.1$

Table 8. The total cross sections in fb up-to $\alpha_s v^2$ order of $e^+e^- \to \chi_{c2}(nP) + \gamma$ with $n = 1, 2, 3$ in the BESIII and B-factories energy region. 'WP' and 'OP' indicate considering or ignoring the phase space contributions, respectively. The uncertainties in each cell originate from the uncertainties of the wave functions at the origin, α_s, $\langle v^2 \rangle$, and charm quark mass m_c in turns. For the excited states, we select the charm quark mass as the half of the meson mass in the calculations, therefore there are no m_c uncertainties. he mass of $\chi_{c2}(nP)$ is selected as 3.927 GeV and 4.208 GeV for $n = 2, 3$ respectively [74, 80].

In our previous works by ref. [72], the charm quark mass is set to the half of the meson. But in refs. [69, 73], different strategies are used to remedy the phase space integrand near the threshold, an additional unitary factor is introduced, and the charm quark mass is set to about 1.5 GeV. Unfortunately, they obtain significantly different cross sections for the production of these near-threshold particles especially for the excited P-wave states. We remain the strategy in our previous work to set the quark mass to half of the meson mass. The results are shown in table 5, 6, 7 8 and in figure 9, 10, 11, 12. The numerical cross sections for $\eta_c(2S)$ states positively increase compared with those for $\mathcal{O}(\alpha_s + v^2)$. However, for $\eta_c(3S)$ state, the numerical values are still assigned to BESIII to determine the states. For excited P-wave states, the cross sections come down compared with the previous $\mathcal{O}(\alpha_s + v^2)$ results. But the numerical values are still referred for BESIII to find these states.

As discussed in our previous works, the results of $\eta_c(mS)$ and $\chi_{cJ}(nP)$ states are helpfull charifiying the nature of XYZ particles with the even charge conjugation, such as $X(3872)$, $X(3940)$, $X(4160)$ and $X(4350)$. Taking $X(3872)$ state for an example, we consider it as the mixture with $\chi_{c1}(2P)$ component [72], therefore, the cross sections for $X(3872)$ are determined by

$$d\sigma[e^+e^- \to \gamma X(3872) \to \gamma J/\psi\pi^+\pi^-] = d\sigma[e^+e^- \to \gamma\chi_{c1}(2P)] \times k\,, \tag{4.7}$$

where $k = Z_{c\bar{c}}^{X(3872)} \times \mathrm{Br}[X(3872) \to J/\psi\pi^+\pi^-]$. $\mathrm{Br}[X(3872) \to J/\psi\pi^+\pi^-]$ is the branching fraction for $X(3872)$ decay to $J/\psi\pi^+\pi^-$. $Z_{c\bar{c}}^{X(3872)}$ is the probability of the $\chi_{c1}(2P)$

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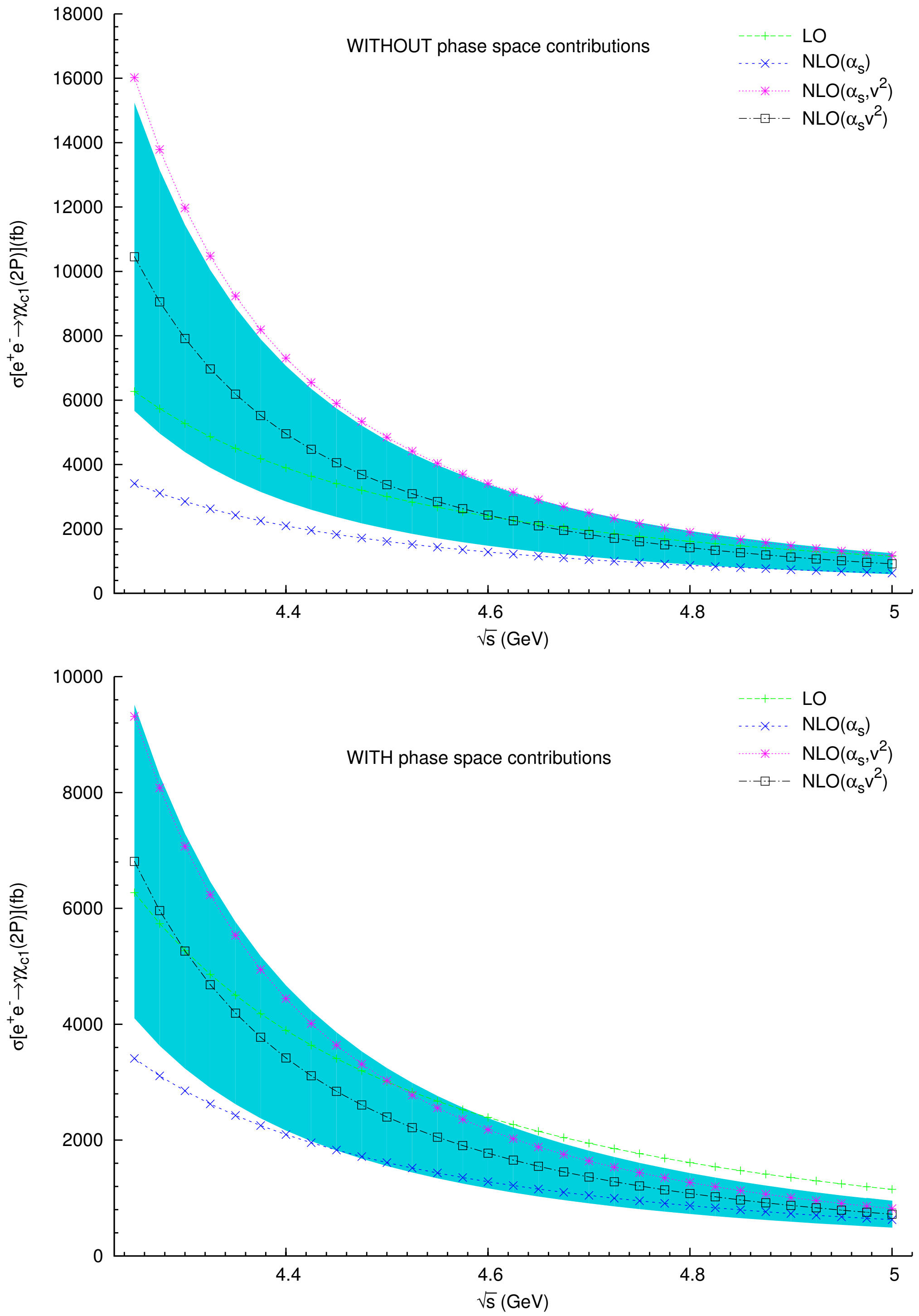

Figure 9. $\sigma[e^+e^- \to \chi_{c1}(2P)+\gamma]$ in the BESIII energy region. The uncertainties for the total cross sections originate from the uncertainties of α_s, $\langle v^2\rangle$ and the wave functions at the origin.

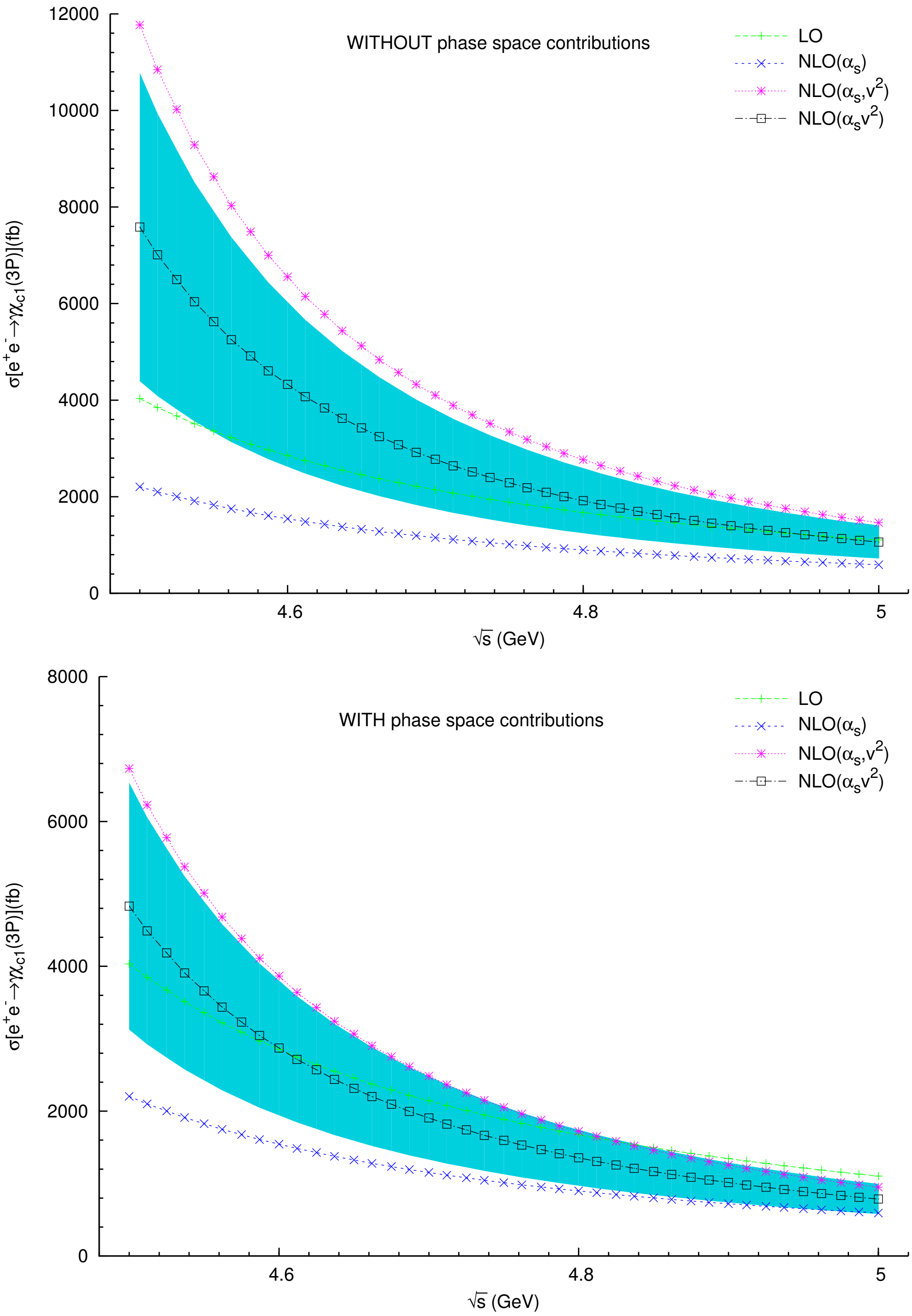

Figure 10. $\sigma[e^+e^- \to \chi_{c1}(3P) + \gamma]$ in the BESIII energy region. The uncertainties for the total cross sections originate from the uncertainties of α_s, $\langle v^2 \rangle$ and the wave functions at the origin.

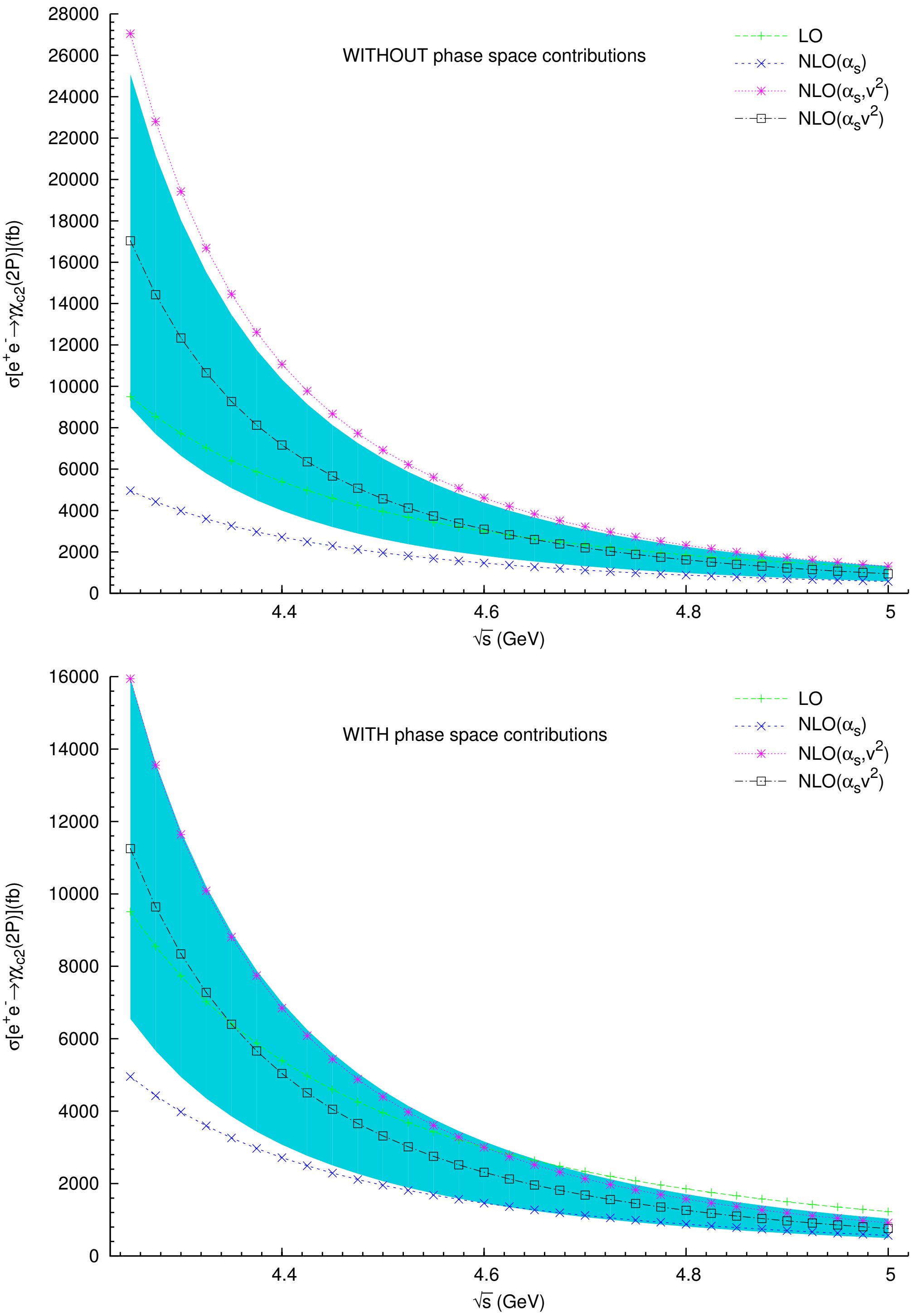

Figure 11. $\sigma[e^+e^- \to \chi_{c2}(2P) + \gamma]$ in the BESIII energy region. The uncertainties for the total cross sections originate from the uncertainties of α_s, $\langle v^2 \rangle$ and the wave functions at the origin.

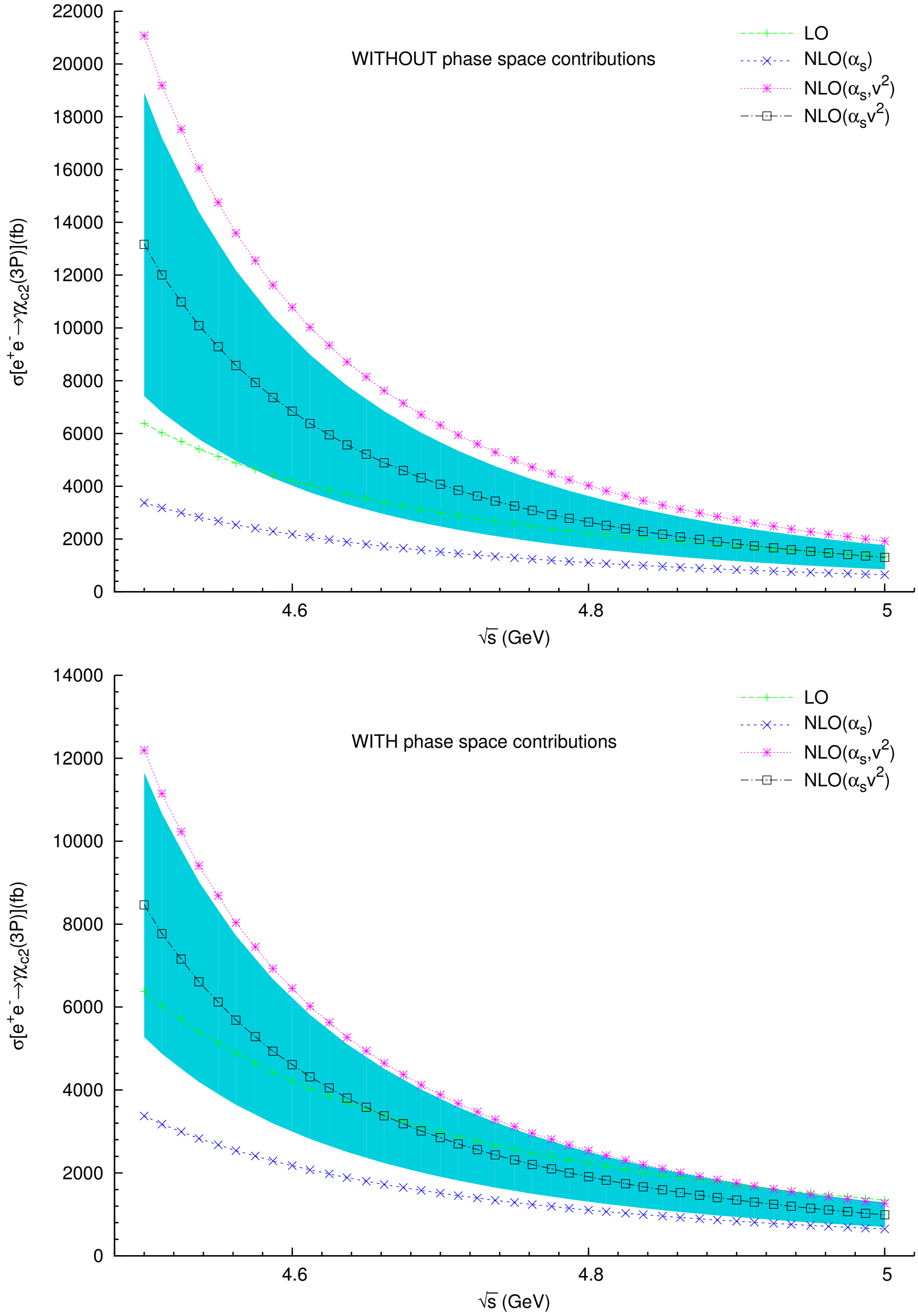

Figure 12. $\sigma[e^+e^- \to \chi_{c2}(3P) + \gamma]$ in the BESIII energy region. The uncertainties for the total cross sections originate from the uncertainties of α_s, $\langle v^2 \rangle$ and the wave functions at the origin.

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component in $X(3872)$. $k = 0.018 \pm 0.04$ [49, 78]. With the results up-to $\mathcal{O}(\alpha_s v^2)$, we revisit the cross sections for $X(3872)$ shown in figure 13. In the figure, we also give the total cross sections at the data points for the BESIII measurements including the contributions of the resonances ($\psi(4040)$ and $\psi(4160)$) which have been discussed in our previous paper and are listed here

$$\begin{aligned}&\left(\sigma_{\psi(4040)}[4.23] + \sigma_{\psi(4160)}[4.23]\right) \times k = (62 \pm 14)\,\mathrm{fb}\,,\\&\left(\sigma_{\psi(4040)}[4.26] + \sigma_{\psi(4160)}[4.26]\right) \times k = (37 \pm 8)\,\mathrm{fb}\,.\end{aligned} \tag{4.8}$$

From the figure, the cross sections for the predictions of $X(3872)$ may be smaller than the experiment data, but one still can not jump to conclusions for the nature of the $X(3872)$ and more data are required.

5 Summary

In this study, we extend our previous works on the production of charmonium with even charge conjugation in the processes $e^+e^- \to \eta_c(nS)\big(\chi_{cJ}(mP)\big) + \gamma$ up-to the $\mathcal{O}(\alpha_s v^2)$. The results indicate that these corrections exhibit a logarithmic singularity of $\ln(1-r)$, which is not observed in the $\mathcal{O}(\alpha_s)$ corrections near the threshold. The $\mathcal{O}(\alpha_s v^2)$ corrections also contribute to the total cross sections near the threshold and are important to the di-photon decay for χ_{c0} and χ_{c2} states. We revisit the numerical calculations to the cross-sections for $\eta_c(nS)$ and $\chi_{cJ}(mP)$ states using the results for the $\mathcal{O}(\alpha_s v^2)$ corrections.

Acknowledgments

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A The short-distance coefficients for $e^+e^- \to \gamma\eta_c$

In this section, we give the matching results of the short-distance coefficients for η_c process. The Lorentz invariance determines the amplitude should have the form of

$$A\epsilon^{\mu\nu\rho\tau}(\epsilon^*_Q)_\mu(\epsilon^*_k)_\nu k_\rho p_\tau\,. \tag{A.1}$$

Therefore the coefficients in $v^{(0)}$ must be like $A^{(0)}\epsilon_1$. The $\mathcal{O}(v^2)$ coefficients obtained in proceed of derivate the amplitude will be like $A^{(v^2)}\epsilon_1 + B^{(v^2)}\epsilon_2$. ϵ_1 and ϵ_2 have been defined as eq. (2.25). Therefore we write the $\mathcal{O}(v^2)$ short-distance coefficients into a plus of three parts as seen in the following results. The third term is introduced just to cancel the $\mathcal{O}(v^2)$ contributions of the relativistic normalization factor in eq. (2.12). And we omit the

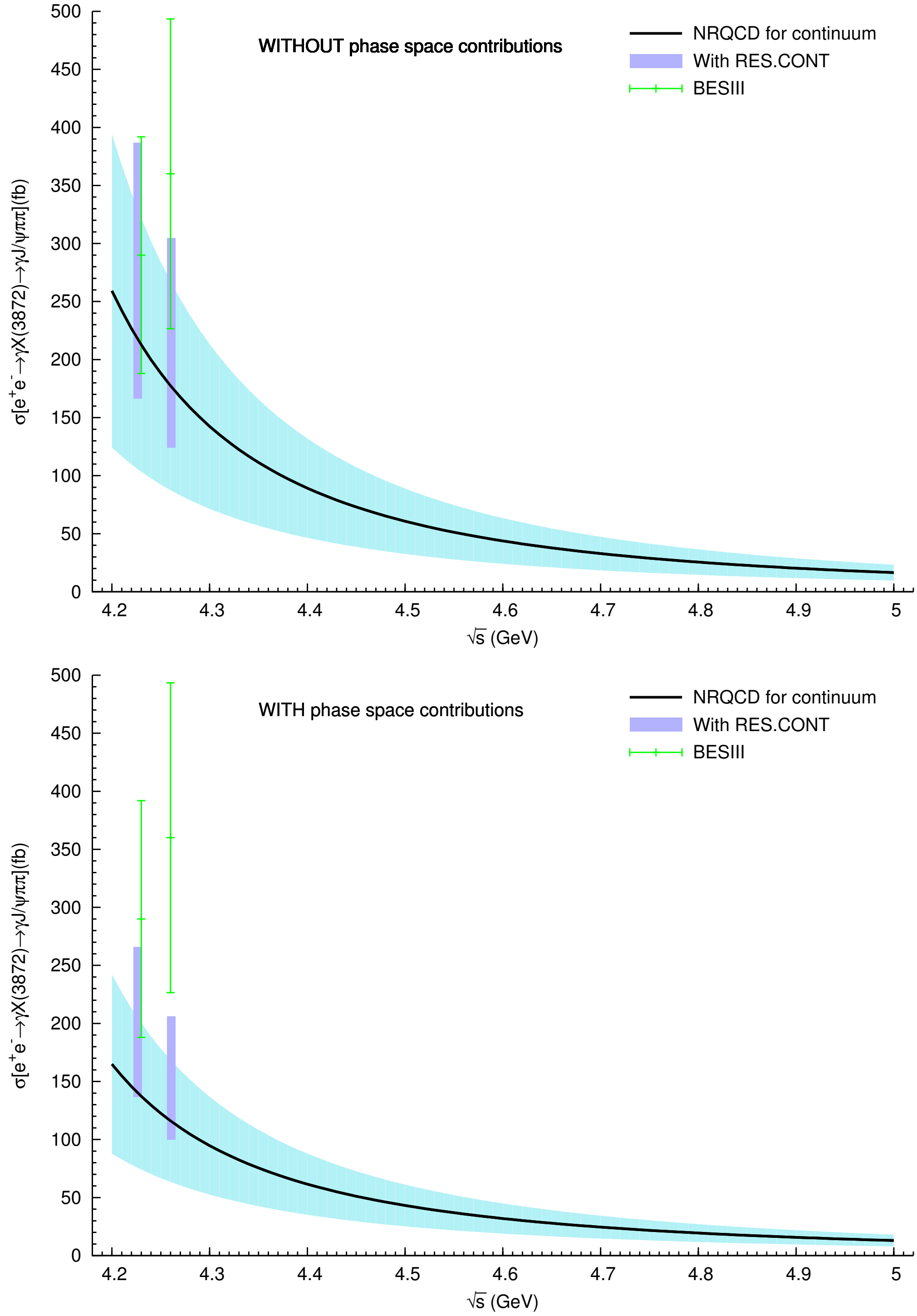

Figure 13. $\sigma[e^+e^- \to X(3872) + \gamma]$ in the BESIII energy region, where $X(3872)$ is considered as the mixture with $\chi_{c1}(2P)$ component [49, 72]. The uncertainties for the total cross sections originate from the uncertainties of m_c, α_s, $\langle v^2 \rangle$, and the wave functions at the origin. "With RES.CONT" means considering the contributions of both continuum and resonance.

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imaginary parts in the coefficients at $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(\alpha_s v^2)$, which don't contribute to cross sections at order of $\mathcal{O}(\alpha_s v^2)$.

$$d^{(0)} \equiv A^{(0)}\epsilon_1 = \frac{(4\pi\alpha)Q_c^2 r}{2m_c^3(1-r)}\epsilon_1 \,. \tag{A.2}$$

$$d^{(v^2)} = -\frac{(4\pi\alpha)Q_c^2 r(5-17r)}{24m_c^5(1-r)^2}\epsilon_1 + \frac{A^{(0)}}{m_c^2}\epsilon_2 - \frac{d^{(0)}}{4m_c^2} \,. \tag{A.3}$$

$$\begin{aligned} d^{(\alpha_s)} \equiv A^{(\alpha_s)}\epsilon_1 = &\frac{(4\pi\alpha)(4\pi\alpha_s)Q_c^2 C_A C_F r}{96\pi^2 m_c^3 N_c(2-r)^2(1-r)^2}\epsilon_1 \\ &\Big\{-6(1-r)[5r^2-r(19+\ln 16)+18+\ln 64]-\pi^2(1+r)(2-r)^2+6[r(r+2)-6]\ln r \\ &+18(2-r)^2[(1-\sqrt{1-r})\ln(1-\sqrt{1-r})+(1+\sqrt{1-r})\ln(1+\sqrt{1-r})] \\ &-12(1-r)(3-2r)\ln(1-r)+3(2-r)^2\Big[r\mathrm{Li}_2\Big(\frac{2}{r}-1\Big)+(2+r)\Big(\mathrm{Li}_2\Big(\frac{2}{1-\sqrt{1-r}}\Big) \\ &+\mathrm{Li}_2\Big(\frac{2}{1+\sqrt{1-r}}\Big)+\mathrm{Li}_2\Big(\frac{r}{2-r}\Big)-\mathrm{Li}_2\Big(\frac{2}{2-r}\Big)-\mathrm{Li}_2\Big(\frac{2}{r}\Big)\Big)\Big]\Big\} \,. \end{aligned} \tag{A.4}$$

$$\begin{aligned} d^{(\alpha_s v^2)} = &-\frac{(4\pi\alpha)(4\pi\alpha_s)Q_c^2 C_A C_F r}{1152\pi^2 m_c^5 N_c(2-r)^4(1-r)^3}\epsilon_1 \\ &\Big\{-2(1-r)[r^5(96\ln 2-211)-156r^4(3\ln 2-10)+23r^3(30\ln 2-181) \\ &+r^2(4598-606\ln 2)+36r(24\ln 2-37)-8(79+87\ln 2)]+\pi^2(21r^2+28r-5)(2-r)^4 \\ &-6(32r^6-251r^5+889r^4-1936r^3+2482r^2-1496r+216)\ln r \\ &-6(2-r)^4(63r+1)[(1-\sqrt{1-r})\ln(1-\sqrt{1-r})+(1+\sqrt{1-r})\ln(1+\sqrt{1-r})] \\ &-12(1-r)(16r^5-78r^4+115r^3-101r^2+144r-116)\ln(1-r) \\ &-3(2-r)^4\Big[3r(1+7r)\mathrm{Li}_2\Big(\frac{2}{r}-1\Big)+(21r^2+53r-10)\Big(\mathrm{Li}_2\Big(\frac{2}{1-\sqrt{1-r}}\Big) \\ &+\mathrm{Li}_2\Big(\frac{2}{1+\sqrt{1-r}}\Big)+\mathrm{Li}_2\Big(\frac{r}{2-r}\Big)-\mathrm{Li}_2\Big(\frac{2}{2-r}\Big)-\mathrm{Li}_2\Big(\frac{2}{r}\Big)\Big)\Big]\Big\}+\frac{A^{(\alpha_s)}}{m_c^2}\epsilon_2-\frac{d^{(\alpha_s)}}{4m_c^2} \,. \end{aligned} \tag{A.5}$$

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