

# Killing quantum entanglement by acceleration or a black hole

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**ABSTRACT:** We consider two entangled accelerating qubits coupled with real scalar fields, each described by the Unruh-Wald model. It is demonstrated that because of the Unruh effect of the fields, the bipartite entanglement between the two qubits suddenly dies when the acceleration of one or more qubits are large enough. We also consider three entangled accelerating qubits in GHZ state and in W state, with equal acceleration-frequency ratio, and found that in either state, the tripartite entanglement suddenly dies at a certain value of acceleration-frequency ratio. The equivalence between the Rindler metric and the Schwarzschild metric in the vicinity of the horizon of a black hole implies that for two entangled qubits outside a black hole, the entanglement suddenly dies when one or both of the qubits are close enough to the horizon, while the three entangled qubits in GHZ or W state, the tripartite entanglement suddenly dies when these qubits are close enough to the horizon.

**KEYWORDS:** Models of Quantum Gravity, Black Holes, Quantum Dissipative Systems

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**1 Introduction**

Recent two decades witnessed both intensive and extensive investigations on quantum entanglement, “the characteristic trait of quantum mechanics” in the words of Schrödinger [1]. More recently, this trend has been extended to the realms of high energy physics. The relativistic effects of acceleration and gravitation on quantum entanglement were investigated, shedding new light on the subject of quantum effects of gravity.

A particularly concerned subject is the Unruh effect, that is, the particle content of a field is observer-dependent, and thus an accelerating detector in the Minkowski vacuum of a field feels a thermal bath of particles of this field [2–8]. The consequences of the Unruh effect on various kinds of entanglement have been studied. For example, because of the Unruh effect, a state of two field modes that is maximally entangled in an inertial frame becomes less entangled when one of the modes is observed by an accelerating detector, and degrades with the increase of acceleration, towards zero at the limit of infinite acceleration [9]. Analogous entanglement degradation occurs for field modes observed by two detectors close to the horizon of a black hole, with one of them freely falling while the other barely escapes [9, 10]. This problem was then extended to the case of three entangled modes, and it was found that when one of the modes is observed by an accelerating detector, the tripartite entanglement, which cannot be reduced to all kinds of bipartite entanglement, does not approach zero in the infinite acceleration limit [11, 12]. Extension of such studies to Fermion fields was also made [13–16]. These results are consistent with the general feature that entanglement of a field state depends on the choice of single particle modes [17]. Entanglement between detectors have also been studied in the case that one of the two entangled detectors accelerates while the other moves uniformly, and was found to exhibit entanglement sudden death [18]. One approach is to model the detectors as harmonic

oscillators [19], for which calculations were also made on the physical details [20, 21]. Another approach is to model the detectors as qubits [22]. Yet another approach is to consider the detectors as an open quantum system [23–25]. Other quantum informational quantities such as discord [26, 27] and quantum Fisher information [28, 29] have also been studied. So far there was no work on the case that more than one of the entangled detectors accelerate.

In this paper, we consider two causally separated but quantum-entangled qubits, each of which independently accelerates and is coupled with a scalar field as described by the Unruh-Wald model. Our work originated in generalizing a previous work on the decoherence of one qubit due to acceleration [30]. We show that because of the Unruh effect of the fields, the entanglement between the qubits vanishes at finite values of acceleration instead of in the infinite limit, i.e. exhibit entanglement sudden death. Our result implies that outside of the black hole, the entanglement between the two entangled detecting qubits vanishes when one or both of the qubits are close enough to the horizon. We also report a result on the tripartite entanglement in three entangled accelerating qubits.

## 2 Formalism

Let us consider two qubits  $A$  and  $B$  far away from each other, that is, we assume there is no causal connection between the two qubits. For each qubit, we apply the model of the Unruh and Wald [6].

The Hamiltonian of each qubit  $q$  ( $q = A, B$ ) is

$$H_q = \Omega_q Q_q^\dagger Q_q, \tag{2.1}$$

where the creation operator  $Q_q^\dagger$  and annihilation operator  $Q_q$  are defined by  $Q_q|0\rangle_q = Q_q^\dagger|1\rangle_q = 0$ ,  $Q_q^\dagger|0\rangle_q = |1\rangle_q$  and  $Q_q|1\rangle_q = |0\rangle_q$ , with subscript  $q = a, b$ .  $\Omega_q$  gives the energy difference between eigenstates  $|1\rangle_q$  and  $|0\rangle_q$ .

Each qubit is locally coupled with a field  $\Phi_q$  within a small region around it, the interaction Hamiltonian being

$$H_{I_q}(t_q) = \epsilon_q(t_q) \int_{\Sigma_q} \Phi_q(\mathbf{x}) [\psi_q(\mathbf{x})Q_q + \psi_q^*(\mathbf{x})Q_q^\dagger] \sqrt{-g} d^3x, \tag{2.2}$$

where  $\mathbf{x}$  and  $t_q$  are spacetime coordinates in the comoving frame of the qubit, the integral is over the spacelike Cauchy surface  $\Sigma_q$  at given time  $t_q$ ,  $\epsilon_q(t_q)$  is the coupling constant with a finite duration of qubit-field interaction,  $\psi_q(\mathbf{x})$  is a smooth function nonvanishing within a small volume around the qubit [5]. The fields  $\Phi_A$  and  $\Phi_B$  could be the same or different.

We presume that the distance between the two qubits is so large that there is no physical coupling or influence between the neighboring fields of the two qubits during the interaction times. The total Hamiltonian is simply  $H_A + H_{\Phi_A} + H_{I_A} + H_B + H_{\Phi_B} + H_{I_B}$ , where  $H_{\Phi_q}$  is the Klein-Gordon Hamiltonian for  $\Phi_q$ . In the Minkowski spacetime, each qubit is confined in its own Rindler wedge and possesses boost Killing fields which are timelike. The only extra constraint on the trajectories of the two qubits is that the time

interval of the interaction between each qubit and its neighboring field multiplied by the speed of light is smaller than the shortest distance between the interaction regions of the two qubits.

Therefore after a time duration longer than the interacting times  $T_q \gg 1/\Omega_q$ , the state of the whole system in the interaction picture is transformed by

$$U_A \otimes U_B,$$

where  $U_q$  is the unitary transformation acting on qubit  $q$  and the field  $\Phi_q$  in its neighboring region, as given by the Unruh-Wald model. To the first order [6],

$$U_q \approx 1 - i \int \Phi_{q0}(t', \mathbf{x}) \epsilon_q(t') [Q_{q0} e^{-i\Omega_q t'} \psi_q(\mathbf{x}) + Q_{q0}^\dagger e^{i\Omega_q t'} \psi_q^*(\mathbf{x})] \sqrt{-g'} d^3x dt', \quad (2.3)$$

where  $\Phi_{q0}$  and  $Q_{q0}$  are  $\Phi_q$  and  $Q_q$  for  $\epsilon_q = 0$ , i.e. when the qubit-field coupling is turned off. It can be obtained that

$$U_q \approx 1 + iQ_{q0} a^\dagger(\Gamma_q^*) - iQ_{q0}^\dagger a(\Gamma_q^*), \quad (2.4)$$

where  $a(\Gamma_q^*)$  and  $a^\dagger(\Gamma_q^*)$  are the annihilation and the creation operators of  $\Gamma_q^*$ , with

$$\Gamma_q(x) \equiv -2i \int [G_R(x; x') - G_A(x; x')] \epsilon_q(t') e^{i\Omega_q t'} \psi_q^*(\mathbf{x}') \sqrt{-g'} d^4x', \quad (2.5)$$

$G_{Rq}$  and  $G_{Aq}$  being the retarded and advanced Green functions of the field  $\Phi_q$ , respectively.

For any mode  $\chi_q$ , we can write

$$a(\Gamma_q^*) = \langle \Gamma_q^*, \chi_q \rangle a(\chi_q) + \langle \Gamma_q^*, \chi'_q \rangle a(\chi'_q), \quad (2.6)$$

$$a^\dagger(\Gamma_q^*) = \langle \Gamma_q^*, \chi_q \rangle^* a^\dagger(\chi_q) + \langle \Gamma_q^*, \chi'_q \rangle^* a^\dagger(\chi'_q), \quad (2.7)$$

where  $\chi'_q$  is some mode orthogonal to  $\chi_q$ .  $\langle \Gamma_q^*, \chi_q \rangle = \frac{i}{2} \int_{\Sigma_q} [\Gamma_q \partial_\mu \chi_q - (\partial_\mu \Gamma_q) \chi_q] dS^\mu$ , where  $\Sigma_q$  is some Cauchy surface. This inner product can be assumed to be negligible unless  $\chi_q$  is at a frequency  $\approx \Omega_q$ . Therefore, each qubit  $q$  is only coupled with the field mode  $\chi(\Omega_q)$  with frequency  $\Omega_q$ , which is further assumed to be nondegenerate. Hence we only need to consider  $\chi_{\Omega_A}$  and  $\chi_{\Omega_B}$  in studying the qubits  $A$  and  $B$ . All the other modes are decoupled with the qubits.

Now we consider the Fock state  $|n\rangle_{\Omega_q}$ , containing  $n$  particles in the mode  $\chi(\Omega_q)$  of the field  $\Phi_q$ , as observed in the Rindler wedge confining qubit  $q$ ,

$$\begin{aligned} a(\Gamma_q^*) |n\rangle_{\chi_q} &= \mu_q a(\Omega_q) |n\rangle_{\Omega_q} = \mu_q \sqrt{n} |n-1\rangle_{\Omega_q}, \\ a^\dagger(\Gamma_q^*) |n\rangle_{\Omega_q} &= \mu_q^* a^\dagger(\Omega_q) |n\rangle_{\Omega_q} = \mu_q^* \sqrt{n+1} |n+1\rangle_{\Omega_q}, \end{aligned} \quad (2.8)$$

where  $\mu_q \equiv \langle \Gamma_q^*, \chi_{\Omega_q} \rangle = \int \epsilon_q(t) e^{i\Omega_q t} \psi_q^*(\mathbf{x}) \chi_{\Omega_q}(t, \mathbf{x}) \sqrt{-g} d^4x$ .

Hence  $U_q$  evolves only the qubit  $q$  and the mode  $\chi_{\Omega_q}$ , while the other modes of  $\Phi_q$  are not affected and can be ignored,

$$U_{\Omega_q} |0\rangle_q |n\rangle_{\Omega_q} = |0\rangle_q |n\rangle_{\Omega_q} - i\sqrt{n} \mu_q |1\rangle_q |n-1\rangle_{\Omega_q}, \quad (2.9)$$

$$U_{\Omega_q} |1\rangle_q |n\rangle_{\Omega_q} = |1\rangle_q |n\rangle_{\Omega_q} + i\sqrt{n+1} \mu_q^* |0\rangle_q |n+1\rangle_{\Omega_q}. \quad (2.10)$$

### 3 Entangled states of the two detecting qubits

We now suppose the initial state of the two qubits to be

$$|\Psi_i\rangle = \alpha|0\rangle_A|1\rangle_B + \beta|1\rangle_A|0\rangle_B, \quad (3.1)$$

where  $\alpha$  and  $\beta$  are superposition coefficients satisfying  $|\alpha|^2 + |\beta|^2 = 1$ . The results for the initial state of the form of  $\alpha|0\rangle_A|0\rangle_B + \beta|1\rangle_A|1\rangle_B$  are similar. Without causal connection between the two qubits or between the fields, each qubit detects a thermal bath of the Unruh particles determined by its own acceleration. With each qubit in its own Rindler wedge, the initial state of the whole system, as observed by the observers comoving with the qubits, is described by the density matrix

$$\rho_i = |\Psi_i\rangle\langle\Psi_i| \otimes \rho_{\Omega_A} \otimes \rho_{\Omega_B} \otimes \rho', \quad (3.2)$$

where

$$\rho_{\Omega_q} = C_q \sum_{n_q} e^{-2\pi n_q \Omega_q / a_q} |n_q\rangle_{\Omega_q} \langle n_q|, \quad (3.3)$$

$a_q$  is the acceleration of qubit  $q$ ,  $C_q \equiv \sqrt{1 - e^{-2\pi\Omega_q/a_q}}$ ,  $\rho'$  is the state of the other decoupled modes and is ignored henceforth.

The final state of the system in the interaction picture is

$$\rho_f = U_B^\dagger U_A^\dagger \rho_i U_A U_B, \quad (3.4)$$

which can be evaluated by substituting eqs. (2.9) and (2.10). Subsequently by tracing out the fields, we obtain the reduced density matrix of the two qubits, with respect to the comoving observers

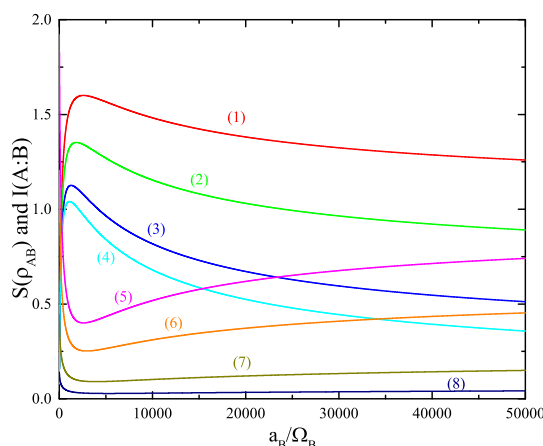
$$\begin{aligned} \rho_{AB} = & C_A^2 C_B^2 \sum_{n_A, n_B} \frac{e^{-2\pi(n_A \Omega_A / a_A + n_B \Omega_B / a_B)}}{Z_{n_A n_B}} \\ & \times \{ [|\alpha|^2(n_B + 1)|\mu_B|^2 + |\beta|^2(n_A + 1)|\mu_A|^2] |00\rangle\langle 00| \\ & + \alpha\beta^* |01\rangle\langle 10| + \beta\alpha^* |10\rangle\langle 01| \\ & + [|\alpha|^2 + |\beta|^2 n_B(n_A + 1)|\mu_B|^2 |\mu_A|^2] |01\rangle\langle 01| \\ & + [|\alpha|^2 n_A(n_B + 1)|\mu_A|^2 |\mu_B|^2 + |\beta|^2] |10\rangle\langle 10| \\ & + [|\alpha|^2 n_A |\mu_A|^2 + |\beta|^2 n_B |\mu_B|^2] |11\rangle\langle 11| \}, \end{aligned}$$

where  $Z_{n_A n_B} \equiv 1 + n_A |\mu_A|^2 + n_B |\mu_B|^2 + n_A n_B |\mu_A|^2 |\mu_B|^2 + |\alpha|^2 (|\mu_B|^2 + n_A |\mu_A|^2 |\mu_B|^2) + |\beta|^2 (|\mu_A|^2 + n_B |\mu_B|^2 |\mu_A|^2)$ .

In the case that qubit  $A$  moves uniformly while qubit  $B$  accelerates,  $\rho_{AB}$  is

$$\begin{aligned} \rho_{AB} = & C_B^2 \sum_{n_B} \frac{e^{-2\pi n_B \Omega_B / a_B}}{Z_{n_B}} \\ & \times \{ [|\alpha|^2(n_B + 1)|\mu_B|^2 + |\beta|^2 |\mu_A|^2] |00\rangle\langle 00| \\ & + \alpha\beta^* |01\rangle\langle 10| + \beta\alpha^* |10\rangle\langle 01| \\ & + [|\alpha|^2 + |\beta|^2 n_B |\mu_B|^2 |\mu_A|^2] |01\rangle\langle 01| \\ & + |\beta|^2 |10\rangle\langle 10| + |\beta|^2 n_B |\mu_B|^2 |11\rangle\langle 11| \}, \end{aligned}$$

where  $Z_{n_B} \equiv 1 + n_B |\mu_B|^2 + |\alpha|^2 |\mu_B|^2 + |\beta|^2 |\mu_A|^2 + |\beta|^2 n_B |\mu_A|^2 |\mu_B|^2$ .



**Figure 1.**  $S(\rho_{AB})$  [plots (1) to (4)] and mutual information  $I(A : B)$  [plots (5) to (8)] as functions of the acceleration-frequency ratio of qubit  $B$ , in the case that qubit  $A$  moves uniformly, for different initial states. (1,5):  $\alpha = 1/\sqrt{2}$ , (2,6):  $\alpha = 0.4$ , (3,7):  $\alpha = 0.2$ , (4,8):  $\alpha = 0.1$ .

We now study the correlation and entanglement in  $\rho_{AB}$ . Note that the entanglement and correlation are respectively the same in Schrödinger and interaction pictures.

#### 4 von Neumann entropy $S(\rho_{AB})$ and mutual information $I(A : B)$

The von Neumann entropy

$$S(\rho_{AB}) \equiv -\text{Tr} \rho_{AB} \log \rho_{AB} \tag{4.1}$$

is a measure of mixture of  $\rho_{AB}$ . On the other hand, in the Minkowski frame, the state of the whole system is a pure state, and  $S(\rho_{AB})$  quantifies the entanglement between the two qubits on one hand, and the fields on the other.

From  $S(\rho_{AB})$ , we also calculate the mutual information

$$I(A : B) \equiv S(\rho_A) + S(\rho_B) - S(\rho_{AB}), \tag{4.2}$$

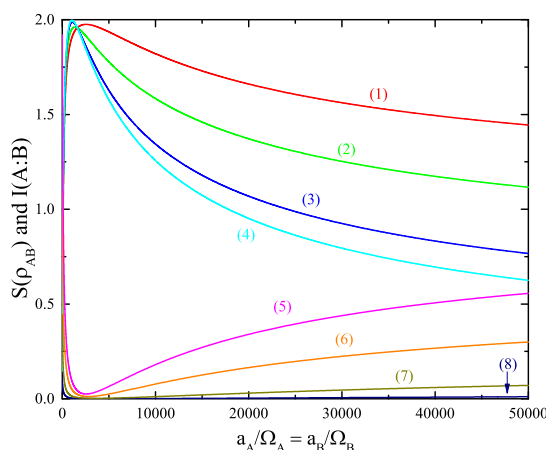
where

$$\rho_A = \text{Tr}_B \rho_{AB}, \tag{4.3}$$

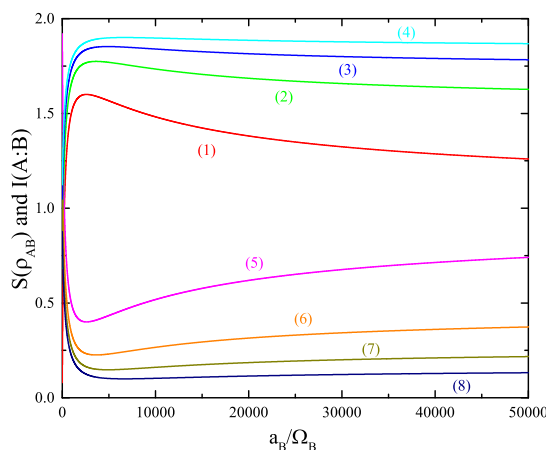
$$\rho_B = \text{Tr}_A \rho_{AB}. \tag{4.4}$$

$I(A : B)$  is the difference between the sum of the entropies of  $A$  and  $B$  as a whole on one hand, and the entropy of  $A$  plus  $B$  as a whole on the other, and is thus a quantification of the total correlation contributed by both entanglement and classical correlation. In the numerical calculations throughout this paper, the bases of the logarithms are chosen to be 2, and the parameters  $\mu_A, \mu_B$  are both set to be 0.1.

$S(\rho_{AB})$  and  $I(A : B)$  are depicted together in figures 1 to 3 for three cases. The result for the case of qubit  $A$  uniformly moving while qubit  $B$  accelerating is shown in figure 1.

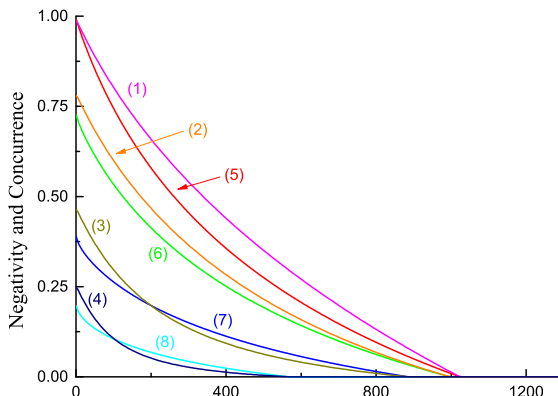


**Figure 2.**  $S(\rho_{AB})$  [plots (1) to (4)] and mutual information  $I(A : B)$  [plots (5) to (8)] as functions of the acceleration-frequency ratio, which is assumed to be the same for the two qubits, for different initial states. (1,5):  $\alpha = 1/\sqrt{2}$ , (2,6):  $\alpha = 0.4$ , (3,7):  $\alpha = 0.2$ , (4,8):  $\alpha = 0.1$ . The extreme values are insensitive to the initial state.



**Figure 3.**  $S(\rho_{AB})$  [plots (1) to (4)] and mutual information  $I(A : B)$  [plots (5) to (8)] as functions of the acceleration-frequency ratio of qubit  $B$ , for different given values of acceleration-frequency ratio of qubit  $A$ : (1,5):  $a_A/\Omega_A = 0$ , (2,6):  $a_A/\Omega_A = 100$ , (3,7):  $a_A/\Omega_A = 200$ , (4,8):  $a_A/\Omega_A = 300$ . It is set that  $\alpha = \beta = \frac{1}{\sqrt{2}}$ .

The special case of two maximally entangled qubits with one of them uniformly moving was previously studied by using a different approach and assuming no coupling between the uniformly moving qubit and the field [7]. In our studies, both qubits always couple with the fields. Figure 2 depicts the result for the case that the acceleration-frequency ratios of the two qubits are always equal. Figure 3 gives the results for various given values of  $a_A/\Omega_A$ . As shown in these figures, when both  $a_A/\Omega_A$  and  $a_B/\Omega_B$  are close to 0,  $\rho_{AB}$  is close to the pure state  $|\Psi_i\rangle$ , hence  $\rho_A$  is close to  $|\alpha|^2|0\rangle\langle 0| + |\beta|^2|1\rangle\langle 1|$  while  $\rho_B$  is close to  $|\beta|^2|0\rangle\langle 0| + |\alpha|^2|1\rangle\langle 1|$ , therefore  $S(\rho_{AB})$  is close to 0 while  $I(A : B)$  is close to  $-2|\alpha|^2 \log |\alpha|^2 - 2|\beta|^2 \log |\beta|^2$ . With the increase of one or both of the acceleration-frequency ratios,  $S(\rho_{AB})$  quickly increases up



**Figure 4.** Logarithmic negativity [plots (1) to (4)] and concurrence [plots (5) to (8)] as functions of the acceleration-frequency ratio of qubit  $B$ , in the case that qubit  $A$  moves uniformly, for different initial state. (1,5):  $\alpha = 1/\sqrt{2}$ , (2,6):  $\alpha = 0.4$ , (3,7):  $\alpha = 0.2$ , (4,8):  $\alpha = 0.1$ .

to a maximum while  $I(A : B)$  quickly decreases down to a minimum. The actual extreme values depend on the details of the dynamics, but we can make the following estimation. If  $\rho_{AB}$  is maximally mixed,  $S(\rho_{AB})$  reaches the absolute maximum 2 while  $S(\rho_A)$  and  $S(\rho_B)$  reach the absolute maximum 1, because  $\rho_{AB}$  is 4-dimensional while  $\rho_A$  and  $\rho_B$  are 2-dimensional. Therefore the minimal value of  $I(A : B)$  is near 0. When the acceleration-frequencies further increase,  $S(\rho_{AB})$  slowly decreases while  $I(A : B)$  slowly increases. In the limit of  $a_B/\Omega_B \rightarrow \infty$  while  $a_A/\Omega_A = 0$ ,  $\rho_{AB} \rightarrow |\alpha|^2|00\rangle\langle 00| + |\beta|^2|11\rangle\langle 11|$ . In the limit of both  $a_A/\Omega_A$  and  $a_B/\Omega_B$  approach  $\infty$ ,  $\rho_{AB} \rightarrow |\alpha|^2|10\rangle\langle 10| + |\beta|^2|01\rangle\langle 01|$ . In both of these two limits,  $S(\rho_{AB})$ ,  $S(\rho_A)$ ,  $S(\rho_B)$  and  $I(A : B)$  all approach  $-|\alpha|^2 \log |\alpha|^2 - |\beta|^2 \log |\beta|^2$ .

### 5 Entanglement between qubits $A$ and $B$

Now we turn to the entanglement between qubits  $A$  and  $B$ , which are in the mixed state  $\rho_{AB}$ . For the two-qubit mixed state  $\rho_{AB}$ , a measure of the entanglement is the logarithmic negativity

$$\log \|\rho_{AB}^{T_A}\|, \tag{5.1}$$

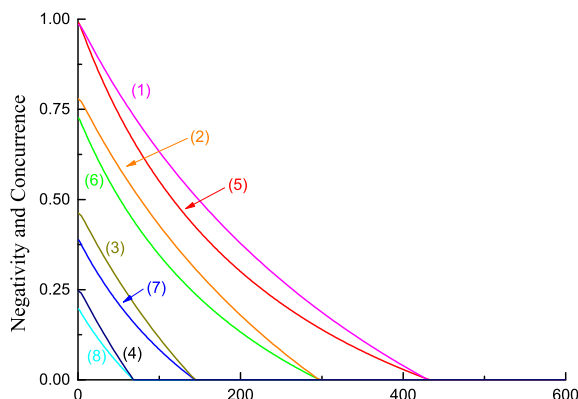
where  $\|\rho_{AB}^{T_A}\|$  is the trace norm of  $\rho_{AB}^{T_A}$ , which is the partial transpose of  $\rho_{AB}$  [31, 32]. Another entanglement measure is the concurrence

$$C(\rho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}, \tag{5.2}$$

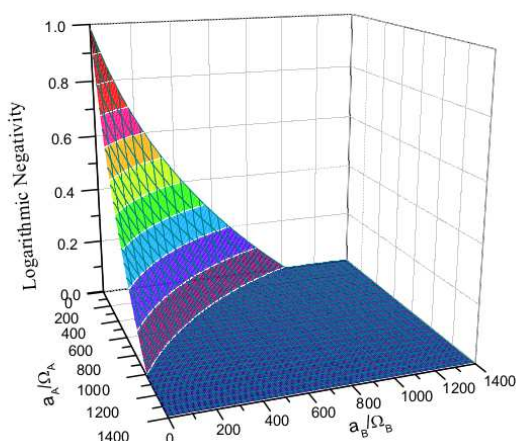
where  $\lambda_i$  ( $i = 1, 2, 3, 4$ ) are decreasingly ordered eigenvalues of the matrix  $\sqrt{\sqrt{\rho_{AB}}\rho_{\tilde{A}B}\sqrt{\rho_{AB}}}$ , with  $\rho_{\tilde{A}B} = (\sigma_y \otimes \sigma_y)\rho_{AB}^*(\sigma_y \otimes \sigma_y)$ ,  $\sigma_y$  being the  $y$ -component Pauli matrix [33].

As shown in figures 4 to 7, the logarithmic negativity and concurrence decrease with the increase of the acceleration-frequency ratio of each qubit and, especially, suddenly dies at a finite value of the acceleration-frequency ratio. As can be seen in these figures, the acceleration-frequency ratio of one qubit at which the entanglement suddenly dies decreases with the increase of that of the other qubit. When one of them is zero, the other must be





**Figure 5.** Logarithmic negativity [plots (1) to (4)] and concurrence [plots (5) to (8)] as functions of the acceleration-frequency ratios of  $A$  and  $B$ , which are assumed to be the same, for different initial state. (1,5):  $\alpha = 1/\sqrt{2}$ , (2,6):  $\alpha = 0.4$ , (3,7):  $\alpha = 0.2$ , (4,8):  $\alpha = 0.1$ .



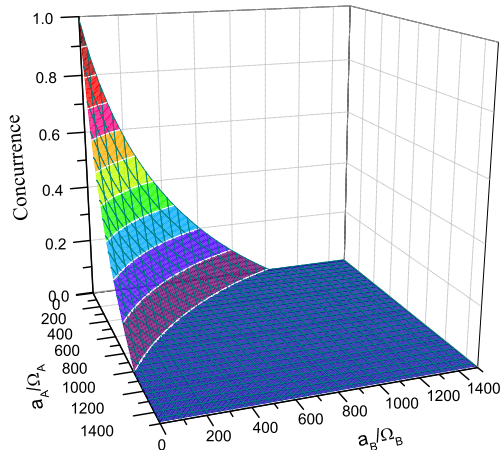
**Figure 6.** Logarithmic negativity as a function of  $a_A/\Omega_A$  and  $a_B/\Omega_B$ .  $\alpha = \beta = \frac{1}{\sqrt{2}}$ .

larger than some finite value. In figures 6 and 7, the 3D plots of logarithmic negativity and concurrence are symmetric with respect to the plane  $a_A/\Omega_A = a_B/\Omega_B$ , as can be seen in the the expression of  $\rho_{AB}$ . These 3D plots also indicate that entanglement sudden death occurs on a curve of  $a_A/\Omega_A$  and  $a_B/\Omega_B$ .

## 6 Tripartite entanglement in three entangled accelerating qubits

We have also extended our study to three accelerating qubits  $A$ ,  $B$  and  $C$ , by using the formalism similar to the two-qubit case above. It is well known that there are two types of 3-qubit states, GHZ state

$$|\text{GHZ}\rangle_{ABC} = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle), \quad (6.1)$$



**Figure 7.** Concurrence as a function of the acceleration-frequency ratios of  $A$  and  $B$ .  $\alpha = \beta = \frac{1}{\sqrt{2}}$ .

and  $W$  state

$$|W\rangle_{ABC} = \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle), \quad (6.2)$$

each representing a different type of tripartite entanglement [34].

We apply the above formalism to three entangled qubits  $A$ ,  $B$  and  $C$ , as described by eq. (2.1), each of which is locally coupled with a field  $\Phi_q$  ( $q = A, B, C$ ). For the reason given above, only the mode  $\chi_{\Omega_q}$  needs to be considered. In this way, for GHZ state, we obtain the density matrix of the qubits,

$$\begin{aligned} \rho_{ABC}(\text{GHZ}) = & \frac{1}{2} C_A^2 C_B^2 C_C^2 \sum_{n_A, n_B, n_C} \frac{e^{-2\pi(n_A \Omega_A / a_A + n_B \Omega_B / a_B + n_C \Omega_C / a_C)}}{Z_{n_A n_B n_C}} \\ & [ (1 + (n_A + 1)(n_B + 1)(n_C + 1) |\mu_A|^2 |\mu_B|^2 |\mu_C|^2) |000\rangle \langle 000| \\ & + (1 + n_A n_B n_C |\mu_A|^2 |\mu_B|^2 |\mu_C|^2) |111\rangle \langle 111| + |111\rangle \langle 000| \\ & + |000\rangle \langle 111| + (n_A |\mu_A|^2 + (n_B + 1)(n_C + 1) |\mu_B|^2 |\mu_C|^2) |100\rangle \langle 100| \\ & + (n_B |\mu_B|^2 + (n_A + 1)(n_C + 1) |\mu_A|^2 |\mu_C|^2) |010\rangle \langle 010| \\ & + (n_C |\mu_C|^2 + (n_A + 1)(n_B + 1) |\mu_A|^2 |\mu_B|^2) |001\rangle \langle 001| \\ & + (n_A n_B |\mu_A|^2 |\mu_B|^2 + (n_C + 1) |\mu_C|^2) |110\rangle \langle 110| \\ & + (n_A n_C |\mu_A|^2 |\mu_C|^2 + (n_B + 1) |\mu_B|^2) |101\rangle \langle 101| \\ & + (n_B n_C |\mu_B|^2 |\mu_C|^2 + (n_A + 1) |\mu_A|^2) |011\rangle \langle 011| ], \end{aligned} \quad (6.3)$$

where

$$\begin{aligned} Z_{n_A n_B n_C} = & 2 + (2n_A + 1) |\mu_A|^2 + (2n_B + 1) |\mu_B|^2 + (2n_C + 1) |\mu_C|^2 \\ & + (2n_A n_B + n_A + n_B + 1) |\mu_A|^2 |\mu_B|^2 + (2n_A n_C + n_A + n_C + 1) |\mu_A|^2 |\mu_C|^2 \\ & + (2n_B n_C + n_B + n_C + 1) |\mu_B|^2 |\mu_C|^2 \\ & + (2n_A n_B n_C + n_A n_B + n_B n_C + n_A n_C + n_A + n_B + n_C + 1) |\mu_A|^2 |\mu_B|^2 |\mu_C|^2. \end{aligned} \quad (6.4)$$

If the three qubits are in W state, their density matrix can be obtained as

$$\begin{aligned}
 \rho_{ABC}(W) = & C_A^2 C_B^2 C_C^2 \sum_{n_A, n_B, n_C} \frac{e^{-2\pi(n_A \Omega_A/a_A + n_B \Omega_B/a_B + n_C \Omega_C/a_C)}}{Z_{n_A n_B n_C}} [ |001\rangle\langle 010| \\
 & + |001\rangle\langle 100| + |100\rangle\langle 001| + |100\rangle\langle 010| + |010\rangle\langle 001| + |010\rangle\langle 100| \\
 & + n_B |\mu_B|^2 |011\rangle\langle 110| + n_A |\mu_A|^2 |101\rangle\langle 110| + n_C |\mu_C|^2 |011\rangle\langle 101| \\
 & + n_A |\mu_A|^2 |110\rangle\langle 101| + n_C |\mu_C|^2 |101\rangle\langle 011| + n_B |\mu_B|^2 |110\rangle\langle 011| \\
 & + ((n_A + 1)|\mu_A|^2 + (n_B + 1)|\mu_B|^2 + (n_C + 1)|\mu_C|^2) |000\rangle\langle 000| \\
 & + (1 + (n_A + 1)n_C |\mu_A|^2 |\mu_C|^2 + (n_B + 1)n_C |\mu_B|^2 |\mu_C|^2) |001\rangle\langle 001| \\
 & + (1 + (n_A + 1)n_B |\mu_A|^2 |\mu_B|^2 + n_B(n_C + 1)|\mu_B|^2 |\mu_C|^2) |010\rangle\langle 010| \\
 & + ((n_A + 1)n_B n_C |\mu_A|^2 |\mu_B|^2 |\mu_C|^2 + n_B |\mu_B|^2 + n_C |\mu_C|^2) |011\rangle\langle 011| \\
 & + (1 + n_A(n_B + 1)|\mu_A|^2 |\mu_B|^2 + n_A(n_C + 1)|\mu_A|^2 |\mu_C|^2) |100\rangle\langle 100| \\
 & + (n_A(n_B + 1)n_C |\mu_A|^2 |\mu_B|^2 |\mu_C|^2 + n_A |\mu_A|^2 + n_C |\mu_C|^2) |101\rangle\langle 101| \\
 & + (n_A |\mu_A|^2 + n_B |\mu_B|^2 + n_A n_B(n_C + 1)|\mu_A|^2 |\mu_B|^2 |\mu_C|^2) |110\rangle\langle 110| \\
 & + (n_A n_B |\mu_A|^2 |\mu_B|^2 + n_A n_C |\mu_A|^2 |\mu_C|^2 + n_B n_C |\mu_B|^2 |\mu_C|^2) |111\rangle\langle 111| ],
 \end{aligned} \tag{6.5}$$

where

$$\begin{aligned}
 Z_{n_A n_B n_C} = & 3 + (3n_A + 1)|\mu_A|^2 + (3n_B + 1)|\mu_B|^2 + (3n_C + 1)|\mu_C|^2 \\
 & + (3n_A n_B + n_A + n_B)|\mu_A|^2 |\mu_B|^2 + (3n_B n_C + n_B + n_C)|\mu_B|^2 |\mu_C|^2 \\
 & + (3n_A n_C + n_A + n_C)|\mu_A|^2 |\mu_C|^2 \\
 & + (3n_A n_B n_C + n_A n_B + n_B n_C + n_A n_C)|\mu_A|^2 |\mu_B|^2 |\mu_C|^2.
 \end{aligned} \tag{6.6}$$

The tripartite entanglement is the genuine three-party entanglement that cannot be reduced to any bipartite entanglement. We use the negativity three-tangle as the measure of the tripartite entanglement [35], which is defined as

$$\pi \equiv \frac{1}{3}(\pi_A + \pi_B + \pi_C), \tag{6.7}$$

where

$$\pi_A \equiv \mathcal{N}_{A(BC)}^2 - \mathcal{N}_{AB}^2 - \mathcal{N}_{AC}^2, \tag{6.8}$$

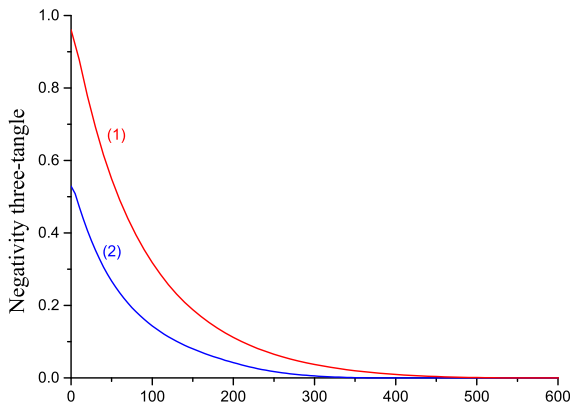
with

$$\mathcal{N}_{A(BC)} \equiv \|\rho_{ABC}^{T_A}\| - 1, \tag{6.9}$$

$$\mathcal{N}_{AB} \equiv \|\rho_{AB}^{T_A}\| - 1. \tag{6.10}$$

Here  $\rho_{ABC}$  is the density matrix of the three qubits,  $\rho_{AB}$  is the reduced density matrix of  $A$  and  $B$ , other quantities are similarly defined.

For simplicity, here we only present the result for the case that the acceleration-frequency ratios of the three qubits are the same. As shown in figure 8. For either GHZ or W state, the negativity three-tangle decreases with the increase of the acceleration-frequency ratio, and suddenly dies at a certain value. We have also found that entanglement sudden death generally occurs when at least two qubits accelerate, no matter whether the accelerations are equal. More details will be discussed elsewhere.



**Figure 8.** Negativity three-tangle as a function of the acceleration-frequency ratio of the three qubits in GHZ state (1) and in W state (2).

## 7 Discussion and summary

Now we come to the black hole. The spacetime outside its horizon is described by the Schwarzschild metric

$$ds^2 = \left(1 - \frac{2m}{r}\right) dt^2 - \left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2, \quad (7.1)$$

where the notations are standard. The proper acceleration of a static observer at  $r$  is

$$a = \frac{m}{r^2} \left(1 - \frac{2m}{r}\right)^{-1/2}. \quad (7.2)$$

It is well known that near the horizon  $r = 2m$ , the Schwarzschild metric can be approximated as the Rindler metric [10, 36]. The closer  $r$  is to the horizon, the larger the acceleration. The uniform movement corresponds to free falling into the black hole.

Consider the entangled states studied above. Suppose at most one of them freely fall into the black hole, while the other is near the horizon  $r = 2m$ . According to the calculation above, we know that the entanglement between the qubits suddenly dies when one or more accelerating qubits are close enough to the horizon. For each qubit near the horizon, we have

$$r \approx 2m \left[1 - \frac{1}{(4ma)^2}\right]^{-1}, \quad (7.3)$$

from which the location of the entanglement sudden death can be determined.

Finally, one may wonder the reason of the entanglement sudden death as studied here. We think the Bosonic fields act as a drain of the entanglement originally exists between the qubits, because there are infinite number of Fock states  $|n\rangle$  for each Bosonic field mode. In contrast, we conjecture that there is no entanglement sudden death if the fields are Fermionic as there are only two Fock states  $|0\rangle$  and  $|1\rangle$  for each Fermionic field mode. The absence of entanglement sudden death was recently noted in entanglement between Unruh modes [16]. Our work implicates that entanglement sudden death of the qubits can act

as a probe of the nature of ambient fields coupled with the qubits and the existence of acceleration or gravity.

To summarize, we have studied the entanglement of two accelerating qubits coupled with scalar fields, and demonstrate the occurrence of its sudden death. We also found the entanglement sudden death of tripartite entanglement of three accelerating qubits in GHZ and W states. These results imply the entanglement sudden death of field-coupled qubits near the horizon of a black hole. This work might be useful to the issue of black hole firewall [37] or energetic curtain [38].

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