

# Viscosity of holographic fluid in the presence of dark matter sector

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**ABSTRACT:** Based on the gauge/gravity correspondence, the hydrodynamic response coefficients, shear and Hall viscosities, have been studied. The holographic model of Einstein-Maxwell-AdS (3 + 1)-dimensional system additionally coupled with the another gauge field mimicking the *dark matter* sector, as well as, gravitational Chern-Simons term bounded with a dynamical scalar field, were taken into account. Condensation of the scalar field in the presence of the deformation chemical potential for the *dark matter* gauge field provide the parity violating terms. Both shear and Hall viscosities have been calculated and their dependence on  $\alpha$  — the coupling constant between matter and *dark matter* sectors has been studied. To the lowest order in the derivative expansion and perturbation in  $\alpha$ , the shear viscosity is not influenced by the *dark matter*, while the Hall component linearly depends on  $\alpha$ .

**KEYWORDS:** AdS-CFT Correspondence, Gauge-gravity correspondence, Holography and condensed matter physics (AdS/CMT), Black Holes

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**Contents**

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>Holographic model</b>	<b>3</b>
<b>3</b>	<b>Equations of motion for the perturbed system</b>	<b>7</b>
3.1	Zeroth order equations	9
<b>4</b>	<b>The viscosities in the presence of dark matter sector</b>	<b>13</b>
4.1	Dependence of Hall viscosity on condensation value	14
<b>5</b>	<b>Summary and conclusions</b>	<b>16</b>

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**1 Introduction**

The exploit of the gauge/gravity correspondence [1–3] in studying strongly correlated systems resulted, among others, in establishing the lower bound  $\hbar/4\pi$  on the ratio of the shear viscosity  $\eta_s$  to entropy density  $s$  in holographic fluid [4]. This interesting result has contributed to the deeper understanding of the state of strongly interacting quark-gluon plasma obtained at RHIC [5–7]. Related studies based on the gauge/gravity duality [8] have also triggered the shear viscosity measurements in the ultracold Fermi gases [9], and more recently in the condensed matter systems such as graphene [10, 11] and strongly correlated oxide [12]. The comprehensive discussion of this novel set of experiments is given in [13].

The desire to understand the long wave-length particle behavior *via* hydrodynamic analogy has a long history. It goes back to the Madelung’s hydrodynamic formulation of quantum mechanics [14], which later has been applied to many different condensed matter systems including quantum wires [15], two-dimensional weakly [16] and strongly [17] interacting electron gas in magnetic field and even nanoscale conductors [18]. The early proposal in the field of elementary particle physics [19, 20] due to Landau, suggesting such description of hadronic fireballs also has to be mentioned in this context.

Recently, there has been a great resurgence of interest in the description of viscosity components in fluids by means of quantum theory [21–23] or gauge/gravity duality [24–41, 43–51]. The problem was elaborated from various points of view including the universality of the conjectured lower bound on the ratio  $\eta_s/s$  and its measurements in an interacting gas of fermions near the unitarity limit [9]. Various hydrodynamic response functions have been probed using AdS/CFT approach [52]–[54]. Among them the antisymmetric part of the viscosity tensor, the so-called Hall viscosity analogous to Hall conductivity has attracted a lot of attention. The Hall viscosity being non-dissipative viscosity coefficient does not

contribute to the entropy production of the fluid. In quantum fluids, at zero temperature, the dissipative shear and bulk viscosities disappear. On the contrary, non-dissipative Hall viscosity can remain nonzero in systems with broken parity or time-reversal symmetry [43]. There is, however a problem with this component of the viscosity related to the the question of how to measure it in condensed matter systems [55].

The gauge/gravity correspondence offers quite a deep insight into the problem of understanding the dynamics of strongly interacting systems. However, one has to remember that in the hydrodynamic description the universal ratio [27–35], strictly speaking, is valid for all gauge theories with Einstein gravity in the limit  $N \rightarrow \infty$  and  $\lambda \rightarrow \infty$ , where  $N$  is the numbers of colors, while  $\lambda$  stands for t’Hooft coupling. The division of the  $\eta_s$  by entropy density allows to get rid of the number of degrees of freedom and obtain the universal bound. On the other hand, it turns out that in higher derivatives theories the aforementioned bound is not universal [36]. The anisotropic theories also allow the bound violation [38–40]. The violation of the viscosity bound has also been predicted in massive [41, 42] and in the quadratic Gauss-Bonnet gravity [37]. In the later work it has been noted that the field excitations in the dual field theory enable the superluminal propagation velocities for the Gauss-Bonnet coupling constant greater than 9/100.

The techniques developed in the AdS/CFT correspondence enable studies of parity violating effects in hydrodynamical systems at strong coupling [43–51]. In general the systems with parity violations acquire additional response parameter/transport coefficient in the long wave-length limit. As it is well known from classic physics the asymmetric component of the viscosity appears at the same order as the shear viscosity in the hydrodynamical derivative expansion. It is subject to the parity or time reversal violation. In condensed matter physics it is called Hall viscosity [21] and we adopt this name in the following.

The holographic model of Hall viscosity in (2+1)-dimensional system was given in [43], where the dynamical Chern-Simons term was used in the calculations. The further generalizations, both analytical and numerical, in the model with Chern-Simons and Maxwell terms [44], as well as, the Born-Infeld black branes [45] were presented. The Hall and shear viscosities, in the model with dynamical Chern-Simons terms, were elaborated in [46, 47]. The spontaneously generated angular momentum in models with gauge and Chern-Simons terms were studied in [48, 50]. Moreover, recently it was reported that the ratio between Hall viscosity and angular momentum density is a constant, at least near the critical regime [51]. It was envisaged in the holographic  $p_x + ip_y$  model, which is different from the Chern-Simons one studied before. The ground state breaks spatial parity by locking it to non-Abelian gauge parity, which in turn is broken by the appearance of non-Abelian gauge connection, the only source of the Hall viscosity, angular momentum density and Hall conductivity.

The studies of hydrodynamic response via gauge/gravity duality is of interest *per se*. The additional motivation behind our work here is related to the widely debated issue of the invisible component of the matter in the Universe, the so called *dark matter* and its experimental detection. In the previous studies [56–60] we have analyzed the properties of holographic superconductors and vortices with the hope to find such modifications of their properties which will allow the detection of the dark matter. The effect of *dark matter* on the properties of superconductors has also been discussed in [61, 62].

Here we extend the previous work to the analysis of a fluid response. To this end we adopt the simple holographic realization of  $(2 + 1)$ -dimensional isotropic holographic fluid with spontaneously broken parity and additionally supplemented by the *dark matter* sector. The *dark matter* sector will be represented by the  $U(1)$ -gauge field which is coupled to the ordinary Maxwell one.

As was mentioned, the motivation standing behind our studies is to elucidate the imprint of the *dark matter* on physical phenomena, which detailed analysis would in turn allow to detect dark matter and thus answer one of the most tantalizing questions of the contemporary physics. The present paper is a continuation and extension of the efforts aiming at elucidation of the effect of dark matter on the properties of condensed systems [56–62]. The existing literature on the subject reports numerous theoretical and observational evidences supporting the existence of dark matter and its role as a possible source of the observed anomalies [63, 64, 67–79]. Here we are interested in its influence on the viscosity of holographic fluids. In view of the laboratory experiments [9–12] which outcomes seem to agree with the predictions of theories based on gauge/gravity duality we hope that future observations will find imprints of *dark matter* on the properties of quantum fluids.

The paper is organized as follows. In section 2 we describe the main features of the model under inspection with two  $U(1)$ -gauge fields coupled together. As already mentioned one of them is the ordinary Maxwell field and the other is responsible for the *dark matter* sector. General setup and the equations of motion are derived in section 3, while their solution to lowest order are presented in section 3.1. The effect of dark matter sector on shear and Hall viscosities is analyzed in section 4, while the subsection 4.1 is devoted to the analysis of the temperature dependence of the Hall viscosity. We end up the paper with summary and conclusions (section 5).

## 2 Holographic model

The gravitational background for the holographic model of viscosities constitutes the four-dimensional deformation of the general relativity, the so-called Chern-Simons gravity, in the formulation proposed in [81]. Chern-Simons modified gravity authorizes the effective extension of Einstein theory taking into account gravitational parity violation. The aforementioned extension is motivated by anomaly cancelation in string theory and particle physics. There were proposed some astrophysical tests of Chern-Simons modified gravity including Solar system, binary pulsars, galactic rotation curves and gravitational wave experiments, as well as, the possible explanation of cosmological matter-anti matter asymmetry (for the contemporary review of the various aspects of the theory see [82]). Moreover, it turns out that the static solution of equation of motion in dynamical Chern-Simons gravity with  $U(1)$ -gauge field is diffeomorphic to an open set of Reissner-Nordström non-extremal solution with electric charge [83].

The gravitational action in  $(3+1)$  dimensions is taken in the form

$$S_g = \int \sqrt{-g} d^4x \left( R - 2 \frac{\Lambda}{L^2} - \frac{1}{2} \nabla_\mu \theta \nabla^\mu \theta - V(\theta) - \frac{\lambda}{4} \theta * R R \right), \quad (2.1)$$

where  $\theta$  is the pseudo scalar field,  $\Lambda = -3$  stands for the cosmological constant,  $L$  the radius of the AdS space-time, which from now on is taken as  $L=1$ . The Pontryagin density term and dual Riemann tensor are provided by

$$*R R = *R^\alpha{}_\beta{}^{\gamma\delta} R^\beta{}_{\alpha\gamma\delta}, \quad *R^\alpha{}_\beta{}^{\gamma\delta} = \frac{1}{2}\epsilon^{\gamma\delta\eta\psi} R^\alpha{}_{\beta\eta\psi}. \quad (2.2)$$

$\lambda$  is a coupling constant. The field  $\theta$ , sometimes called Chern-Simons coupling field, being a function of spacetime coordinates, serves as a deformation function. One can observe that when  $\theta = 0$  and  $V(\theta) = 0$ , the above modification of gravity reduces to the Einstein theory. As was mentioned we treat  $\theta$  as a pseudo scalar [44], so the gravitational Chern-Simons term does not break the parity. But the pseudo scalar term violates parity spontaneously, as well as, enables to receive a pseudo scalar condensate at the boundary.

The potential  $V(\theta)$  assumes standard Ginzburg-Landau form of the Mexican hat type for  $m^2 < 0$  and  $c > 0$ ,

$$V(\theta) = \frac{1}{2} m^2 \theta^2 + \frac{1}{4} c \theta^4. \quad (2.3)$$

As was found in [84, 85] scalar field can develop an instability if its mass square violates the near horizon AdS Breitenlohner-Freedman bound. When the adequate condition is received, the scalar field will be stable at infinity but will condense near the event horizon of black brane. It happens that the condensed solution will comprise nontrivial radial profile for scalar field. Namely, at low temperatures the field in question, explores extreme values of its potential. Therefore the non-linearities in the potential connected with scalar field will be of a great importance.

At zero temperature, one expects that the  $\theta$  field condenses until the value of it at the black brane event horizon reaches some point near the bottom of the Mexican hat potential. At the aforementioned point the effective AdS-mass will fulfill the AdS Breitenlohner-Freedman bound and the condensation will stop. The condensation is expected to persist up to some finite temperature  $T_c$ , below which  $\theta$  condensates.  $m^2$  is the effective mass near the point  $\theta = 0$ . It ought to satisfy the Breitenlohner-Freedman condition.

The matter field is composed of the Abelian-Higgs sector coupled to the second U(1)-gauge field which in our theory describes the *dark matter* sector [86]. The action incorporating *dark matter* is provided by

$$S_m = \int \sqrt{-g} d^4x \left( -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{\alpha}{4} F_{\mu\nu} B^{\mu\nu} \right), \quad (2.4)$$

where  $F_{\mu\nu} = 2\nabla_{[\mu} A_{\nu]}$  stands for the ordinary Maxwell field strength tensor, while the second U(1)-gauge field  $B_{\mu\nu}$  is given by  $B_{\mu\nu} = 2\nabla_{[\mu} B_{\nu]}$ .

It can be observed that the bulk action  $S_g + S_m$  conserves parity, so a pseudo scalar  $\theta$ , in the last term of the equation (2.1), is of a key importance to introduce the parity violation in the boundary theory via  $\theta$ -condensation.

Let us comment on the motivation for introduction a *dark matter* sector in the form of equation (2.4). In our previous works dedicated to the subject of the *dark matter* influence on holographic s-wave and p-wave superconductors we have established that the  $\alpha$ -coupling constant of Maxwell and U(1)-gauge *dark matter* fields influence the various characteristics

of the superconductors. If one treats the AdS/CFT correspondence as a kind of method enabling us insight into the properties of strongly correlated systems, these changes may be treated as the guideline in future experiments detecting *dark matter*.

Of course, the action (2.4) can be rewritten in the form

$$S_m = \int d^4x \sqrt{-g} \left( -\frac{1}{4} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} - \frac{1}{4} B'_{\mu\nu} B'^{\mu\nu} \right), \quad (2.5)$$

where we defined

$$A_\mu = \tilde{A}_\mu - \frac{\alpha}{2} B_\mu, \quad (2.6)$$

$$B'_{\mu\nu} = \sqrt{1 - \frac{\alpha^2}{4}} B_{\mu\nu}, \quad (2.7)$$

$$\tilde{F}_{\mu\nu} = \nabla_{[\mu} \tilde{A}_{\nu]}. \quad (2.8)$$

The other form of the action in question can be obtained when the ordinary Maxwell field is multiplied by a coefficient. It implies

$$S_m = \int d^4x \sqrt{-g} \left( -\frac{1}{4} F'_{\mu\nu} F'^{\mu\nu} - \frac{1}{4} \tilde{B}_{\mu\nu} \tilde{B}^{\mu\nu} \right), \quad (2.9)$$

where one defines the following:

$$B_\mu = \tilde{B}_\mu - \frac{\alpha}{2} A_\mu, \quad (2.10)$$

$$F'_{\mu\nu} = \sqrt{1 - \frac{\alpha^2}{4}} F_{\mu\nu}, \quad (2.11)$$

$$\tilde{B}_{\mu\nu} = \nabla_{[\mu} \tilde{B}_{\nu]}. \quad (2.12)$$

It turns out in section 4, that in order to envisage the influence of *dark matter* sector on the Hall viscosity, we should take into account the effects of *dark matter* backreaction on the metric. To do this we expand all the adequate quantities in series in  $\alpha$ -coupling constant and calculate the backreaction up to the linear order.

Therefore, in what follows we shall use the action (2.4), where one has the explicit dependence on  $\alpha$ -coupling constant. The equations of motion obtained from the variation of the action  $S = S_g + S_m$  with respect to the metric, the scalar field and gauge fields are given by

$$G_{\mu\nu} + \Lambda g_{\mu\nu} - \lambda C_{\mu\nu} = T_{\mu\nu}(\theta) + T_{\mu\nu}(F) + T_{\mu\nu}(B) + \alpha T_{\mu\nu}(F, B), \quad (2.13)$$

$$\nabla_\mu \nabla^\mu \theta - \frac{\partial V}{\partial \theta} = \frac{\lambda}{4} *R R, \quad (2.14)$$

$$\nabla_\mu F^{\mu\nu} + \frac{\alpha}{2} \nabla_\mu B^{\mu\nu} = 0, \quad (2.15)$$

$$\nabla_\mu B^{\mu\nu} + \frac{\alpha}{2} \nabla_\mu F^{\mu\nu} = 0. \quad (2.16)$$

The contributions to the energy momentum tensors are given by

$$T_{\mu\nu}(\theta) = \frac{1}{2} \nabla_\mu \theta \nabla_\nu \theta - \frac{1}{4} g_{\mu\nu} \nabla_\delta \theta \nabla^\delta \theta - \frac{1}{2} g_{\mu\nu} V(\theta), \quad (2.17)$$

$$T_{\mu\nu}(F) = \frac{1}{2} F_{\mu\delta} F_\nu^\delta - \frac{1}{8} g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}, \quad (2.18)$$

$$T_{\mu\nu}(B) = \frac{1}{2} B_{\mu\delta} B_\nu^\delta - \frac{1}{8} g_{\mu\nu} B_{\alpha\beta} B^{\alpha\beta}, \quad (2.19)$$

$$T_{\mu\nu}(F, B) = \frac{1}{2} F_{\mu\delta} B_\nu^\delta - \frac{1}{8} g_{\mu\nu} F_{\alpha\beta} B^{\alpha\beta}. \quad (2.20)$$

The above system of equations can be reduced to the following relations:

$$R_{\mu\nu} + 3 g_{\mu\nu} - \lambda C_{\mu\nu} = t_{\mu\nu}(\theta) + T_{\mu\nu}(F) + T_{\mu\nu}(B) + \alpha T_{\mu\nu}(F, B), \quad (2.21)$$

where we have denoted

$$t_{\mu\nu}(\theta) = \frac{1}{2} \nabla_\mu \theta \nabla_\nu \theta + \frac{1}{2} g_{\mu\nu} V(\theta), \quad (2.22)$$

$$C^{\mu\nu} = \nabla_\gamma \theta \epsilon^{\gamma\kappa\delta(\mu} \nabla_\delta R_{\kappa}^{\nu)} + \nabla_\gamma \nabla_\delta \theta * R^{\delta(\mu\nu)\gamma}. \quad (2.23)$$

In order to find the Hall viscosity we have to compute its contribution to the hydrodynamics flow of the boundary field theory. Therefore, it is important to write the stress tensor connected with the action of the considered theory, as we have to keep the fields fixed at the boundary. The general procedure is similar to that of finding the boundary Gibbons-Hawking term in general relativity. Namely, the variation of the boundary term spoils the principle leading to Einstein equations because of the fact that it contains the contribution proportional to the extrinsic curvature of the boundary. Thus, the variation will constitute two terms, a bulk piece that vanishes when the equations of motion are fulfilled and a boundary term. The situation can be cured by adding to the action a counter term canceling the boundary one.

In the case of Chern-Simons theory, it can be shown that if  $\theta$  pseudo scalar field vanishes asymptotically for any solutions of the equations of motion derived from the action in the theory in question, the stress energy tensor is the same as of an asymptotically AdS<sub>4</sub> spacetime. Namely, taking variations of the Chern-Simons gravity action we obtain [43, 87, 88]

$$\delta S_{\text{CS}} = -\frac{\lambda}{4} \int d^4x \sqrt{-g} \theta * R R = -\lambda \delta S_1 - \lambda \delta S_2 + \lambda \delta S_3, \quad (2.24)$$

where the explicit forms are given by

$$\delta S_1 = \int d^4x \sqrt{-g} \delta g_{\alpha\beta} \nabla_\lambda \nabla_\gamma (\theta * R^{\lambda\alpha\gamma\beta}), \quad (2.25)$$

$$\delta S_2 = \int d^4x \sqrt{-g} \nabla_\alpha (\theta * R^\beta{}_\xi{}^{\gamma\lambda} \delta \Gamma^\xi_{\beta\lambda}), \quad (2.26)$$

$$\delta S_3 = \int d^4x \sqrt{-g} \nabla_\beta \left[ \delta g_{\lambda\gamma} \nabla_\xi (\theta * R^{\beta\lambda\xi\gamma}) \right]. \quad (2.27)$$

It can be proved that

$$\delta S_1 = \int d^4x \sqrt{-g} \delta g_{\alpha\beta} C^{\alpha\beta}, \quad (2.28)$$

whereas the term  $\delta S_2$  can be cast in the form of a variation of the extrinsic curvature plus the term containing dual curvature term multiplied by the variation of the second kind of Christofel symbol. Using the Gaussian normal coordinates

$$ds^2 = d\eta^2 + g_{ij} dx^i dx^j, \tag{2.29}$$

and the Codazzi equation, as well as having in mind the fact that in the spacetime with negative cosmological constant, solutions of Einstein equations admit the expansion (the so-called Fefferman-Graham expansion [89]) provided by

$$g_{ij} = e^{2\eta} g_{ij}^{(0)} + g_{ij}^{(2)} + e^{-2\eta} g_{ij}^{(4)} + \dots \tag{2.30}$$

one may find that the term under consideration, i.e.  $\delta S_2$ , vanishes as  $\int_{\partial M} d^3x \theta \sim 0$ . We can think about the boundary as being situated at  $\eta \rightarrow \infty$ , with the metric tensor conformal to  $g_{ij}^{(0)}$ .

In the case of  $\delta S_3$ , with a help of Bianchi identity, it can be envisaged that its boundary behavior is of the form  $\epsilon^{\alpha\beta\gamma\lambda} \nabla_\alpha \theta \nabla_\gamma K_\lambda^\xi \delta g_{\xi\beta} \rightarrow 0$ . Thus, the only relevant term to the variation of the action is the conventional Gibbons-Hawking one. On the other hand, the only counter term will be a boundary cosmological constant renormalization, because of the fact that the boundary under consideration is flat. Then, the stress tensor will be provided by

$$\delta S = \frac{1}{2} \int d^3x \sqrt{g_{ij}^{(0)}} T^{ij} \delta g_{ij}^{(0)}, \tag{2.31}$$

where  $g_{ij}^{(0)}$  is the metric on the conformal boundary.

### 3 Equations of motion for the perturbed system

In this section we shall consider the boosted black brane solution in Einstein-Maxwell-*dark matter* gravity in AdS four-dimensional spacetime. To derive the hydrodynamic equations and response parameters we have to perturb the velocity field, black brane temperature and charge in the bulk. Such perturbations back-react on the metric and thus change the background. We shall calculate the back-reaction perturbatively by expanding the relevant functions up to the linear order in derivatives. In the present theory these functions additionally depend on the coupling to the dark matter. To capture this dependence analytically we shall assume small value of  $\alpha$  and expand all relevant functions with respect to it. Thus we are dealing with two expansions and in this paper we limit the calculations to the linear order in the coupling to the *dark matter* sector and the perturbing fields.

In order to solve the equations of motion perturbatively, order by order, we write the line element expanded up to the first order in the boundary derivatives around the coordinates origin,  $x_\nu = 0$ . Having obtained the metric solution near the origin one can extend it to the whole manifold iteratively [90, 91].



It turns out that the line element satisfying the equations of motion can be written as

$$ds^2 = -2H(r, T, Q)u_\alpha dx^\alpha dr - r^2 F(r, T, Q)u_\alpha u_\beta dx^\alpha dx^\beta + r^2(\eta_{\alpha\beta} + u_\alpha u_\beta)dx^\alpha dx^\beta, \quad (3.1)$$

$$A = A(r, T, Q)u_\alpha dx^\alpha, \quad (3.2)$$

$$B = B(r, T, Q)u_\alpha dx^\alpha, \quad (3.3)$$

$$\theta = \theta(r, T, Q). \quad (3.4)$$

By  $T$  and  $Q$ , we have denoted the Hawking temperature and the charge of the boosted black brane. This background geometry describes hydrodynamics in  $(2+1)$ -dimensional spacetime at thermal equilibrium at the boundary. The velocity is given by

$$u^\nu = \frac{1}{\sqrt{1-\beta^2}}(1, \beta^i). \quad (3.5)$$

If one allows the constant quantities like  $T$ ,  $Q$ ,  $A_\mu$ ,  $B_\mu$ ,  $\theta$  to be slowly varying functions of the boundary coordinates than the expansions near the coordinate origin, given to the first order, provided by

$$F(r, T, Q) = F(r) + \frac{\partial F}{\partial T}x^\mu \partial_\mu T + \frac{\partial F}{\partial Q}x^\mu \partial_\mu Q = F(r) + \delta F, \quad (3.6)$$

$$H(r, T, Q) = Q(r) + \frac{\partial H}{\partial T}x^\mu \partial_\mu T + \frac{\partial H}{\partial Q}x^\mu \partial_\mu Q = H(r) + \delta H, \quad (3.7)$$

$$A(r, Q, T) = A(r) + \frac{\partial A}{\partial T}x^\mu \partial_\mu T + \frac{\partial A}{\partial Q}x^\mu \partial_\mu Q = A(r) + \delta A, \quad (3.8)$$

$$B(r, Q, T) = B(r) + \frac{\partial B}{\partial T}x^\mu \partial_\mu T + \frac{\partial B}{\partial Q}x^\mu \partial_\mu Q = B(r) + \delta B, \quad (3.9)$$

$$\theta(r, Q, T) = \theta(r) + \frac{\partial \theta}{\partial T}x^\mu \partial_\mu T + \frac{\partial \theta}{\partial Q}x^\mu \partial_\mu Q = \theta(r) + \delta \theta, \quad (3.10)$$

$$u^\alpha = (1, x^\gamma \partial_\gamma \beta^i), \quad (3.11)$$

$$T = T_0 + x^\mu \partial_\mu T, \quad (3.12)$$

$$Q = Q_0 + x^\mu \partial_\mu Q, \quad (3.13)$$

are sufficient to calculate the viscosities. As we have already mentioned, the above functions do depend on  $\alpha$ , but this will be discussed later on. In agreement with previous studies [36, 43–45] we perform the entire analysis in the comoving frame, where the fluid velocity equals zero at the boundary. The resulting inhomogenous background line element and the gauge fields yield

$$ds^2 = 2 H(r) dv dr - r^2 F(r) dv^2 + r^2 dx^\mu dx_\mu + \quad (3.14)$$

$$+ \epsilon \left[ 2 \delta H dv dr - 2 H(r) x^\alpha \partial_{\alpha\beta\gamma} dx^\gamma dr - r^2 \delta F dv^2 + 2 r^2 (F(r) - 1) x^\mu \partial_\mu \beta_\gamma dv dx^\gamma \right],$$

$$\theta = \theta(r) + \epsilon \delta \theta, \quad (3.15)$$

$$A = -A(r) dv + \epsilon \left[ A(r) x^\gamma \partial_\gamma \beta_\zeta dx^\zeta \right], \quad (3.16)$$

$$B = -B(r) dv + \epsilon \left[ B(r) x^\gamma \partial_\gamma \beta_\zeta dx^\zeta \right], \quad (3.17)$$

where the small parameter  $\epsilon$  serves as bookkeeping device, which power denotes the number of the derivatives taken into account along the boundary.

It is important to notice, that with parameters depending on the boundary coordinates, the ansatz (3.14) of the background line element does not satisfy the equations of motion for the underlying theory. Thus the next step is to correct the line element order by order in  $\epsilon$ , to make the metric tensor, the gauge fields and the pseudo scalar  $\theta$  fulfill equations (2.13)–(2.16), order by order in the perturbation series. The correction to the line element is taken in the following form

$$ds_{\text{corr}}^{(1)2} = \epsilon \left[ \frac{k(r)}{r^2} dv^2 + 2 p(r) dv dr - r^2 p(r) dx_i dx^i + \frac{2}{r} w_i(r) dv dx^i + r^2 \alpha_{ij}(r) dx^i dx^j \right], \quad (3.18)$$

$$\theta_{\text{corr}} = \epsilon \theta, \quad (3.19)$$

$$A_{\text{corr}} = \epsilon (\tilde{a}_v(r) dv + \tilde{a}_m(r) dx^m), \quad (3.20)$$

$$B_{\text{corr}} = \epsilon (\tilde{b}_v(r) dv + \tilde{b}_m(r) dx^m), \quad (3.21)$$

where  $\alpha_{ij}$  is symmetric and traceless.

### 3.1 Zeroth order equations

From the equation (3.14) we read off the zeroth order line element which is provided by

$$ds^2 = 2 H(r) dv dr - r^2 F(r) dv^2 + r^2 (dx^2 + dy^2), \quad (3.22)$$

and find the following equations of motion:

$$F''(r) + \left( \frac{6}{r} - \frac{H'(r)}{H(r)} \right) F'(r) + \frac{2}{r} \left( \frac{3}{r} - \frac{H'(r)}{H(r)} \right) F(r) - \frac{1}{r^2} (6 - V(\theta)) H^2(r) = \frac{A'(r)^2}{2 r^2} + \frac{B'(r)^2}{2 r^2} + \alpha \frac{A'(r) B'(r)}{r^2}, \quad (3.23)$$

$$F'(r) + \left( \frac{3}{r} - \frac{H'(r)}{H(r)} \right) F(r) - \frac{1}{2r} [6 - V(\theta)] H^2(r) + \frac{A'(r)^2}{4 r} + \frac{B'(r)^2}{4 r} + \alpha \frac{A'(r) B'(r)}{4 r} = 0, \quad (3.24)$$

$$\frac{H'(r)}{H(r)} = \frac{r}{4} \theta'(r)^2, \quad (3.25)$$

$$A''(r) + A'(r) \left( \frac{2}{r} - \frac{H'(r)}{H(r)} \right) + \frac{\alpha}{2} \left[ B'(r) \left( \frac{2}{r} - \frac{H'(r)}{H(r)} \right) + B''(r) \right] = 0, \quad (3.26)$$

$$B''(r) + B'(r) \left( \frac{2}{r} - \frac{H'(r)}{H(r)} \right) + \frac{\alpha}{2} \left[ A'(r) \left( \frac{2}{r} - \frac{H'(r)}{H(r)} \right) + A''(r) \right] = 0, \quad (3.27)$$

$$\theta''(r) + \theta'(r) \left( \frac{4}{r} + \frac{F'(r)}{F(r)} - \frac{H'(r)}{H(r)} \right) - \frac{H^2(r)}{r^2 F(r)} \frac{\partial V}{\partial \theta} = 0, \quad (3.28)$$

where the prime denotes derivation with respect to  $r$ -coordinate.

Having in mind equation for  $\theta$  field, one remarks that its asymptotic behavior is of the form

$$\theta = \frac{\mathcal{O}_-}{r^{\Delta_-}} + \frac{\mathcal{O}_+}{r^{\Delta_+}} + \dots, \quad (3.29)$$

where  $\Delta_{\pm} = 3/2 \pm \sqrt{9/4 + m^2}$ . In the following, we turn off the mode  $\mathcal{O}_-$  and the mode  $\mathcal{O}_+$  is identified with the condensate at the boundary. It turns out that [92] for  $-9/4 < m^2 < -5/2$ , both  $\mathcal{O}_-$  and  $\mathcal{O}_+$  are renormalizable and one can point either one as a source and the other as a condensate.

In the neutral Hall viscosity case [43],  $\mathcal{O}_-$  can be turned off only if  $c < -\frac{3}{4}$ , which in turn violates the positive energy condition [93] and makes the solution in question unstable. In the considered case of  $V(\theta)$  being a Mexican hat type potential, the  $\theta^4$  term engenders that the solution is regular at the event horizon of charged black brane.

To proceed further let us note that the symmetry of equations (3.26) and (3.27) is such that both  $A(r)$  and  $B(r)$  independently of each other have to fulfill the equations (3.31) and (3.32) below. For future convenience we rewrite the whole set of equations as

$$F'(r) + F(r) C(r) + D(r) + E(\alpha, r) = 0, \quad (3.30)$$

$$A''(r) + A'(r) \left( \frac{2}{r} - \frac{r}{4} \theta'(r)^2 \right) = 0, \quad (3.31)$$

$$B''(r) + B'(r) \left( \frac{2}{r} - \frac{r}{4} \theta'(r)^2 \right) = 0, \quad (3.32)$$

$$\theta''(r) + \theta'(r) \left( \frac{4}{r} + \frac{F'(r)}{F(r)} \right) - \frac{r}{4} \theta'(r)^3 - \frac{H^2(r)}{r^2 F(r)} \frac{\partial V}{\partial \theta} = 0, \quad (3.33)$$

$$\frac{H'(r)}{H(r)} = \frac{r}{4} \theta'(r)^2, \quad (3.34)$$

where we have set

$$C(r) = \frac{3}{r} - \frac{r}{4} \theta'(r)^2, \quad (3.35)$$

$$D(r) = -3 \frac{H^2(r)}{r} + \frac{V(\theta) H^2(r)}{2r} + \frac{A'(r)^2}{4r} + \frac{B'(r)^2}{4r}, \quad (3.36)$$

$$E(\alpha, r) = \alpha \frac{A'(r) B'(r)}{4r}. \quad (3.37)$$

We are interested in the effect of dark matter on the viscosities. Even though  $\alpha$  enters equations only *via*  $E(\alpha, r)$  above, other functions implicitly depend on it. To access this dependence in an analytic form we expand all functions to the lowest, linear order

$$F(r) = F^{(0)}(r) + \alpha f(r) + \mathcal{O}(\alpha^{n \geq 2}), \quad (3.38)$$

$$H(r) = H^{(0)}(r) + \alpha h(r) + \mathcal{O}(\alpha^{n \geq 2}), \quad (3.39)$$

$$A(r) = A^{(0)}(r) + \alpha a(r) + \mathcal{O}(\alpha^{n \geq 2}), \quad (3.40)$$

$$B(r) = B^{(0)}(r) + \alpha b(r) + \mathcal{O}(\alpha^{n \geq 2}), \quad (3.41)$$

$$\theta(r) = \theta^{(0)}(r) + \alpha \zeta(r) + \mathcal{O}(\alpha^{n \geq 2}). \quad (3.42)$$

We note in passing that to expect all functions to depend in a linear manner on  $\alpha$  is not obvious, but it is natural to expect linear dependence due to the fact that only the first power of  $\alpha$  enters the system of equations to be solved.

The  $\alpha^0$ -order equations are given by

$$F^{(0)'}(r) + F^{(0)}(r) C^{(0)}(r) + D^{(0)}(r) = 0, \quad (3.43)$$

$$\frac{H^{(0)'}(r)}{H^{(0)}(r)} = \frac{r}{4} \theta^{(0)'}{}^2(r), \quad (3.44)$$

$$A^{(0)''}(r) + A^{(0)'}(r) \left( \frac{2}{r} - \frac{r}{4} \theta^{(0)'}{}^2(r) \right) = 0, \quad (3.45)$$

$$B^{(0)''}(r) + B^{(0)'}(r) \left( \frac{2}{r} - \frac{r}{4} \theta^{(0)'}{}^2(r) \right) = 0, \quad (3.46)$$

$$\theta^{(0)''}(r) + \theta^{(0)'}(r) \left( \frac{4}{r} + \frac{F^{(0)'}(r)}{F^{(0)}(r)} \right) - \frac{r}{4} \theta^{(0)'}{}^3(r) + \quad (3.47)$$

$$- \left( m^2 \theta^{(0)}(r) + c \theta^{(0)3} \right) \frac{H^{(0)2}(r)}{r^2 F^{(0)}(r)} = 0, \quad (3.48)$$

The inspection of the above relations enables us to solve them as the first order differential equation. In zeroth order we obtain the following:

$$H^{(0)}(r) = h_1 \exp \left[ \int_r^\infty ds \frac{s}{4} \theta^{(0)'}{}^2(s) \right], \quad (3.49)$$

$$F^{(0)}(r) = \left[ - \int_r^\infty dr D^{(0)}(r) \exp \left( \int_x^\infty dx C^{(0)}(x) \right) + C_2 \right] \times \quad (3.50)$$

$$\times \exp \left[ - \int_r^\infty ds C^{(0)}(s) \right],$$

where  $C_2$  and  $h_1$  are constants.

In the next step we find the equations in  $\alpha^1$ -order. They are given by

$$f'(r) + f(r) \left( \frac{3}{r} - \frac{r}{4} \theta'^{(0)2}(r) \right) + Q(r) = 0, \quad (3.51)$$

$$a''(r) + a'(r) \left( \frac{2}{r} - \frac{r}{4} \theta'^{(0)2}(r) \right) - \frac{r}{2} \zeta'(r) \theta'^{(0)}(r) A^{(0)}(r) = 0, \quad (3.52)$$

$$b''(r) + b'(r) \left( \frac{2}{r} - \frac{r}{4} \theta'^{(0)2}(r) \right) - \frac{r}{2} \zeta'(r) \theta'^{(0)}(r) B^{(0)}(r) = 0, \quad (3.53)$$

$$h'(r) - \frac{H'^{(0)}(r)}{H^{(0)}(r)} h(r) - r \zeta'(r) \theta'^{(0)}(r) H^{(0)}(r) = 0, \quad (3.54)$$

$$\zeta''(r) + \zeta'(r) \left( -r \theta'^{(0)2}(r) + \frac{4}{r} + \frac{F'^{(0)}(r)}{F^{(0)}(r)} \right) + \zeta(r) M(r) + K(r) = 0, \quad (3.55)$$

where  $Q(r)$ ,  $M(r)$  and  $K(r)$  are denoted by

$$\begin{aligned}
 Q(r) = & -\frac{r}{2}F^{(0)}(r)\zeta'(r)\theta'^{(0)}(r) - \frac{6}{r}h(r)H^{(0)}(r) + \frac{a'(r)A'^{(0)}(r)}{2r} + \frac{b'(r)B'^{(0)}(r)}{2r} \\
 & + \frac{m^2}{2}\left[2\theta^{(0)2}(r)h(r)H^{(0)}(r) + \frac{\zeta(r)\theta^{(0)}(r)H^{(0)2}(r)}{r}\right] + \frac{A'^{(0)}(r)B'^{(0)}(r)}{4r} \\
 & + \frac{c}{2}\left[\frac{\theta^{(0)3}(r)\zeta(r)}{r} + 2h(r)H^{(0)}(r)\theta^{(0)4}\right],
 \end{aligned} \tag{3.56}$$

$$M(r) = -\frac{r}{2}\theta^{(0)2}(r) - \left(m^2 + \theta^{(0)2}(r)\right) \frac{H^{(0)2}(r)}{r^2 F^{(0)}(r)}, \tag{3.57}$$

$$\begin{aligned}
 K(r) = & -\left(m^2\theta^{(0)}(r) + c\theta^{(0)3}\right)\left(\frac{2H^{(0)}(r)h(r)}{r^2 F^{(0)}(r)} - \frac{f(r)H^{(0)2}(r)}{r^2 F^{(0)2}(r)}\right) \\
 & + \left(\frac{f'(r)}{F^{(0)}(r)} - \frac{F'^{(0)}(r)f(r)}{F^{(0)}(r)}\right)\theta^{(0)'}.
 \end{aligned} \tag{3.58}$$

As in the previous zeroth-order case, we have obtained two linear differential equations bounded with the fluctuations of the underlying line element. The exact forms of the  $\alpha^1$ -corrections of  $F(r)$  and  $H(r)$  are given by the relations

$$f(r) = \left[C - \int_r^\infty dr Q(r) \exp\left[\int_{r'}^\infty dr' \left(\frac{3}{r'} - \frac{r'}{4} \theta'^{(0)2}(r')\right)\right]\right] \times \tag{3.59}$$

$$\times \exp\left[-\int_x^\infty dx \left(\frac{3}{x} - \frac{x}{4} \theta'^{(0)2}(x)\right)\right], \tag{3.60}$$

$$h(r) = \left[D + \int_r^\infty dr (r \zeta'(r) \theta'^{(0)}(r) H^{(0)}(r)) \exp\left[-\int_x^\infty dx \frac{H'^{(0)}(x)}{h^{(0)}(x)}\right]\right] \times \tag{3.61}$$

$$\times \exp\left[-\int_s^\infty ds \frac{H'^{(0)}(s)}{h^{(0)}(s)}\right].$$

The important question concerns the validity of  $\alpha$ -coupling constant expansion. At the first step let us consider the situation when  $\alpha = 0$ .<sup>1</sup> From the relation (3.27) one gets that  $B$  is equal to zero if this *dark matter* field does not carry the chemical potential. On the other hand, the scalar field  $\theta$  can be non-zero due to the fact that it is responsible for the instabilities.

The other case to consider is connected with the fact when  $\alpha$ -coupling constant is small, equations (3.51)–(3.58). As *dark matter* gauge field component is equal to zero for the background field, then the leading correction for the other quantities in question can be set to zero. In the process of this, one receives the homogeneous system of differential equations (it seems that  $Q(r)$  destroys homogeneity but  $B^{(0)} = 0$  and it is equal to zero). The conclusion one can draw is that nothing sources the perturbative correction. Moreover, by similar arguments applied to the exact set of equations (3.30)–(3.37) one can show that corrections to all orders in  $\alpha$  vanish unless  $\mu_D \neq 0$ .

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<sup>1</sup>We are grateful to the referee for asking the question, which contributed to the clarification of this important point.

Therefore in what follows, in order to obtain the expansions described by the relations (3.38)–(3.42), we assume that the *dark matter* gauge field carries the deformation chemical potential, i.e.,  $B(r \rightarrow \infty) = \mu_D$ . This is a quite new situation comparing to the description of Hall viscosity in the AdS Einstein-Maxwell case [44].

#### 4 The viscosities in the presence of dark matter sector

In this section we shall compute the shear and Hall viscosities in the theory under consideration. As was justified in [90, 91] viscosity components can be found by the inspection of the spatial components of the energy-momentum tensor like  $T_{xy}$  or  $T_{xx} - T_{yy}$ . To commence with let us consider the following:

$$R_{xy}^{(1)} + 3 g_{xy}^{(1)} - \lambda C_{xy}^{(1)} = t_{xy}^{(1)}(\theta) + T_{xy}^{(1)}(F) + T_{xy}^{(1)}(B) + \alpha T_{xy}^{(1)}(F, B), \quad (4.1)$$

where we have denoted by the superscript (1) the fluctuations connected with the leading order  $\mathcal{O}(\epsilon)$  in the derivative expansion. The exact form of the above relation is provided by

$$\begin{aligned} & \frac{1}{H(r)} \frac{d}{dr} \left[ -\frac{1}{2} \frac{r^4 F(r)}{H(r)} \frac{d\alpha_{xy}}{dr} \right] + \left( \frac{r^3 H'(r) F(r)}{H^3(r)} - \frac{r^3 F'(r)}{H^2(r)} - 3 \frac{r^2 F(r)}{H^2(r)} \right. \\ & \left. + 3r^2 - \frac{r^2}{2} V(\theta) - \frac{r^2 A'(r)^2}{4 H^2(r)} - \frac{r^2 B'(r)^2}{4 H^2(r)} - \alpha \frac{r^2 A'(r) B'(r)}{4 H^2(r)} \right) \alpha_{xy} \\ & = \frac{r}{H(r)} (\partial_x \beta_y + \partial_y \beta_x) + \frac{\lambda}{4 H(r)} \frac{d}{dr} \left( \frac{r^4 F'(r) \theta'(r)}{H^2(r)} \right) (\partial_x \beta_y - \partial_y \beta_x). \end{aligned} \quad (4.2)$$

One can see that having in mind equation (3.24), the second term in the relation (4.2) is exactly equal to  $-r^3/H^2(r)$  multiplied by (3.24) and hence equal to zero. It implies that for  $\alpha_{xy}$  we obtain

$$\alpha_{xy} = \int_r^\infty dl \frac{2 H(l)}{l^4 F(l)} \int_{r_H}^r ds \left[ s (\partial_x \beta_y + \partial_y \beta_x) + \frac{\lambda}{4} \frac{d}{ds} \left( \frac{s^4 F'(s) \theta'(s)}{H^2(s)} \right) (\partial_x \beta_x - \partial_y \beta_y) \right]. \quad (4.3)$$

In order to compute the asymptotical form of  $\alpha_{xy}$  we shall implement the formula [43] which is valid as  $r \rightarrow \infty$ , namely

$$r^n \alpha_{xy}(r) \rightarrow -\frac{r^{n+1}}{n} \frac{d}{dr} \alpha_{xy}(r). \quad (4.4)$$

Using the Graham-Fefferman coordinate system it was shown [95] that up to six dimensions of the spacetime, the expectation value for the stress-energy tensor of the dual theory is provided by

$$\langle T_{ij} \rangle = \frac{n}{16\pi G_N} g_{ij(n)} + X_{ij}(g_{(n)}), \quad (4.5)$$

where  $n$  denotes the spacetime dimension and  $X_{ij}(g_{(n)})$  is a function of metric tensor components

$$g_{ij}(x_\mu, r) = g_{ij(0)} + \frac{1}{r^2} g_{ij(2)} + \dots + \frac{1}{r^n} g_{ij(n)} + \dots \quad (4.6)$$

It can be seen that the exact form of  $\langle T_{ij} \rangle$  depends on the dimensionality of the spacetime in question and indicates the conformal anomalies of the boundary conformal field theory.

In the odd dimensional spacetime, when the gravitational conformal anomalies are equal to zero, it reduces to

$$\langle T_{ij} \rangle = \frac{n}{16\pi G_N} g_{ij(n)}. \quad (4.7)$$

One remarks also that when  $r \rightarrow \infty$  the values of the functions  $F(r)$  and  $H(r)$  tend to 1. All the above reveal that in the case under consideration we get

$$\begin{aligned} \langle T_{xy} \rangle = \frac{3}{16\pi G_N} \alpha_{xy(3)} = & -\frac{1}{16\pi G_N} (\partial_x \beta_y + \partial_y \beta_x) \\ & - \frac{1}{8\pi G_N} \left[ (\partial_x \beta_y - \partial_y \beta_x) \frac{\lambda}{4} \frac{r^4 F'(r) \theta'(r)}{H^2(r)} \right]_{r=r_H}. \end{aligned} \quad (4.8)$$

The first term is the usual shear mode with the shear viscosity  $\eta_s$  provided by

$$\eta_s = \frac{1}{16\pi G_N}. \quad (4.9)$$

It can be seen that  $\eta_s$  is not corrected by the  $\alpha$ -coupling constant, at the leading order in it, while the second one is proportional to the Hall viscosity

$$\eta_H = -\frac{1}{8\pi G_N} \frac{\lambda}{4} \frac{r^4 F'(r) \theta'(r)}{H^2(r)} \Big|_{r=r_H}. \quad (4.10)$$

When one divides it by the entropy density, we receive the Hall viscosity/ entropy ratio complement to Einstein-Maxwell *dark matter* Chern-Simons theory in AdS<sub>4</sub> spacetime subject to the backreaction effects. It depends linearly on the coupling  $\alpha$  do dark matter and reads

$$\frac{\eta_H}{s} = \frac{\eta_H^{(0)}}{s} \left( 1 + \alpha \Sigma \right) + \mathcal{O}(\alpha^{n \geq 2}), \quad (4.11)$$

where we set for  $\eta_H^{(0)}/s$  and  $\Sigma$  the following relations:

$$\frac{\eta_H^{(0)}}{s} = -\frac{\lambda}{2\pi} \left( \frac{r^4 F'^{(0)}(r) \theta'^{(0)}(r)}{H^{2(0)}(r)} \right) \Big|_{r=r_H}, \quad (4.12)$$

$$\Sigma = \left( \frac{f'(r)}{F'^{(0)}(r)} + \frac{\zeta'(r)}{\theta'^{(0)}(r)} - \frac{2 h(r)}{H^{(0)}(r)} \right) \Big|_{r=r_H}. \quad (4.13)$$

The term  $\Sigma$  is a direct consequence of the presence of the backreaction of the matter on the metric. The sign of the correction is not uniquely determined.

#### 4.1 Dependence of Hall viscosity on condensation value

As noted earlier the shear viscosity  $\eta_s$  takes on universal value not modified by the presence of *dark matter* or condensation value. On the contrary, the condensation of the pseudo scalar field  $\theta$  is essential to get the parity breaking [44] and consequently non-zero value of the Hall viscosity. Here we analyze the dependence of the Hall viscosity on the condensation value and temperature close to the critical one,  $T_c$ .

Let us consider the explicit form of the charged black brane line element [94]

$$ds^2 = 2 dv dr - r^2 F(r)dv^2 + r^2 (dx^2 + dy^2), \quad (4.14)$$

where the component of the metric tensor  $F(r)$  and the gauge field are provided by

$$F(r) = 1 - \frac{1 + 3\kappa}{r^3} + \frac{3\kappa}{r^4}, \quad A = 2 \sqrt{3} \kappa \left(1 - \frac{1}{r}\right) dv. \quad (4.15)$$

In what follows we set that the event horizon is situated at  $r_H = 1$ . In the chosen units the Hawking temperature and the chemical potential are given, respectively as

$$T_{BH} = \frac{3}{4\pi} (1 - \kappa), \quad \mu = 2 \sqrt{3} \kappa. \quad (4.16)$$

We shall use  $z = 1/r$  coordinates in which the equation of motion for  $\theta$  field implies

$$\theta'' + \theta' \left( \frac{F'(z)}{F(z)} - \frac{2}{z} \right) - \frac{m^2 \theta + c \theta^3}{z^2 F(z)} = 0. \quad (4.17)$$

To proceed further, let us expand  $\theta(z)$  near the black brane event horizon where  $z = 1$ . It yields

$$\theta(z) = \theta(1) + \theta'(1) (z - 1) + \frac{1}{2} \theta''(1) (z - 1)^2 + \dots, \quad (4.18)$$

with finite values of  $\theta'(1)$  and  $\theta''(1)$ . Note also that with our choice of units  $F(1) = 0$ . Calculating the limit  $z \rightarrow 1$  in equation (4.17) we get first the relation for  $\theta'(1)$

$$\theta'(1) = \frac{m^2 \theta(1) + c \theta^3(1)}{F'(1)}, \quad (4.19)$$

and  $\theta''(1)$

$$\theta''(1) = \frac{\theta'(1)}{2} \left[ \frac{(m^2 + 3 c \theta^2(1) - F''(1))}{F'(1)} \right]. \quad (4.20)$$

Using the above relations, we find the approximate form of  $\theta(z)$  near the black brane event horizon

$$\begin{aligned} \theta(z) = & \theta(1) + (z - 1) \frac{m^2 \theta(1) + c \theta^3(1)}{F'(1)} + \\ & + \frac{(z - 1)^2}{4} \left[ \frac{\theta'(1) (m^2 + 3 c \theta^2(1) - F''(1))}{F'(1)} \right] + \dots \end{aligned} \quad (4.21)$$

On the other hand, in the asymptotic AdS region, when  $z \rightarrow 0$ ,  $\theta$  behaves like  $\mathcal{O}_+ z^{\Delta_+}$ . In order to find  $\theta(1)$  and  $\mathcal{O}_+$ , we match smoothly the solution at the event horizon and in AdS region, in some intermediate point  $z_m$

$$\theta_H(z_m) = \theta_{\text{boundary}}(z_m), \quad \theta'_H(z_m) = \theta'_{\text{boundary}}(z_m). \quad (4.22)$$



Namely, one arrives at the following:

$$\mathcal{O}_+ z_m^{\Delta_+} = \theta(1) + (z_m - 1) \frac{m^2 \theta(1) + c \theta^3(1)}{F'(1)} + \frac{(z_m - 1)^2}{4} \left[ \frac{\theta'(1) (m^2 + 3 c \theta^2(1) - F''(1))}{F'(1)} \right], \quad (4.23)$$

$$\mathcal{O}_+ \Delta_+ z_m^{\Delta_+ - 1} = \frac{m^2 \theta(1) + c \theta^3(1)}{F'(1)} + \frac{(z_m - 1)}{2} \left[ \frac{\theta'(1) (m^2 + 3 c \theta^2(1) - F''(1))}{F'(1)} \right]. \quad (4.24)$$

We can draw a conclusion that  $\theta(1) \approx \mathcal{O}_+$ . On the other hand, as was revealed in [56–60], the condensation value is proportional to  $\sqrt{1 - \frac{T}{T_c}}$ . It suggests that at the critical temperature  $T_c$ , the Hall viscosity is independent on  $\alpha$ -coupling constant, i.e.,  $\eta_H$  does not depend on the *dark matter* sector, in the probe limit. As it has been shown in the preceding sections only the backreaction effects are responsible for revealing the aforementioned dependence.

## 5 Summary and conclusions

We have studied the holographic fluid in a model containing Maxwell gauge field and the other U(1) field describing the dark matter with the goal to analyze the influence of the latter on the fluid viscosities. To study two-dimensional flow we have used (3 + 1)-dimensional bulk and pseudo scalar field  $\theta$  coupled to the gravitational Chern-Simons term. The arena for our investigations is the anti de Sitter spacetime of charged black brane of finite temperature. The bulk pseudo scalar potential is composed of  $\theta^2$  and  $\theta^4$  terms which guarantee consistent solution at zero temperature. Contrary to the previous studies of Hall viscosity in AdS Einstein-Maxwell case [44], the spontaneously broken parity by the pseudo scalar hair, as well as, the *dark matter* gauge field deformation chemical potential, give rise to the emergence of a non-zero value of the Hall viscosity at the boundary.

We have solved the underlying equations of motion perturbatively up to the leading order in the  $\alpha$ -coupling constant and found that the shear viscosity mode is not corrected at the leading order by the presence of *dark matter*. On the contrary, the correction of the Hall viscosity to entropy ratio is modified linearly in  $\alpha$ . The leading term is the same as earlier derived in the model without dark matter [43, 44].

Parity or time reversal symmetry breaking is a necessary condition for the existence of Hall viscosity in the system with gauge fields. In the present approach the parity is broken spontaneously by the condensation of the  $\theta$  field below certain transition temperature  $T_c$ . We have found that the Hall viscosity depends on temperature as  $\eta_H \sim \sqrt{1 - \frac{T}{T_c}}$ . It is important to note that neither prefactor nor transition temperature itself in the above relation, depend on the coupling constant  $\alpha$  of the *dark matter*. This makes the condensation of  $\theta$  field completely different from earlier studied superconductors [56–60]. In the latter case both the condensation value and  $T_c$  showed marked dependence on  $\alpha$ . One crucial difference between the two systems is obvious. Condensing pseudo scalar field  $\theta$  is not charged under U(1)-gauge group.

The proper condensed matter interpretation of the field  $\theta$  is at present not clear, but the issue is of great interest, especially in view of the recent measurements of the *superfluid* Hall effect in an ultra cold gas of neutral atoms [96]. This together with close relation between Hall conductivity and Hall viscosity calls for further studies and condensed matter interpretation. The problem is also important from cosmological point of view as the measurements of the Hall viscosity can possibly be used for detection of the *dark matter*, by observing the time variations of  $\eta_H$ , i. e., in close analogy to the recent [97] proposal of the *dark matter* detection by the daily modulation of some parameters. There is however an important caveat. Despite great number of studies of the Hall viscosity there is no consensus how to measure this important parameter. Time will show if the very recent proposal [55, 98] to relate the Hall viscosity to the density response of the system turns out to be the feasible method to extract the former.

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