

Minimal lepton flavor violation implications of the $b \rightarrow s$ anomalies

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ABSTRACT: The latest measurements of rare $b \rightarrow s$ decays in the LHCb experiment have led to results in tension with the predictions of the standard model (SM), including a tentative indication of the violation of lepton flavor universality. Assuming that this situation will persist because of new physics, we explore some of the potential consequences in the context of the SM extended with the seesaw mechanism involving right-handed neutrinos plus effective dimension-six lepton-quark operators under the framework of minimal flavor violation. We focus on a couple of such operators which can accommodate the LHCb anomalies and conform to the minimal flavor violation hypothesis in both their lepton and quark parts. We examine specifically the lepton-flavor-violating decays $B \rightarrow K^{(*)}\ell\ell'$, $B_s \rightarrow \phi\ell\ell'$, $B \rightarrow (\pi, \rho)\ell\ell'$, and $B_{d,s} \rightarrow \ell\ell'$, as well as $K_L \rightarrow e\mu$ and $K \rightarrow \pi e\mu$, induced by such operators. The estimated branching fractions of some of these decay modes with $\mu\tau$ in the final states are allowed by the pertinent experimental constraints to reach a few times 10^{-7} if other operators do not yield competitive effects. We also look at the implications for $B \rightarrow K^{(*)}\nu\nu$ and $K \rightarrow \pi\nu\nu$, finding that their rates can be a few times larger than their SM values. These results are testable in future experiments.

KEYWORDS: Rare Decays, Beyond Standard Model, B-Physics, Global Symmetries

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1 Introduction

The recently acquired data on a number of observables in $b \rightarrow s$ decays have revealed some intriguing tensions with the expectations of the standard model (SM). Specifically, last year the LHCb Collaboration [1] determined the ratio of branching fractions of the decays $B^+ \rightarrow K^+ \mu^+ \mu^-$ and $B^+ \rightarrow K^+ e^+ e^-$ to be $R_K = 0.745_{-0.074}^{+0.090}(\text{stat}) \pm 0.036(\text{syst})$ for the dilepton invariant mass squared range of 1-6 GeV². This result diverges from the lepton universality in the SM by 2.6σ . Moreover, earlier LHCb [2] reported a local discrepancy at the 3.7σ level from the SM prediction for one of the angular observables in the decay $B^0 \rightarrow K^{*0} \mu^+ \mu^-$. This disagreement has persisted after an updated analysis was done using the full LHCb Run I dataset [3]. In addition, the latest measurements by LHCb [4, 5] of the branching fractions of several rare $b \rightarrow s$ decays favor values less than those estimated in the SM.

Although the statistical significance of these anomalies is still too low for a definite conclusion, they may be hinting at the presence of physics beyond the SM. Subsequent model-independent theoretical works have in fact shown that new physics (NP) could resolve the tensions [6–18].¹ In particular, NP contributing via dimension-six operators of the form [14–16]

$$\mathcal{L}_{\text{SM+NP}} = \frac{\alpha_e G_F V_{ts}^* V_{tb}}{\sqrt{2}\pi} \bar{s} \gamma^\beta P_L b \bar{\ell} \gamma_\beta (C_9^\ell + C_{10}^\ell \gamma_5) \ell + \text{H.c.} \tag{1.1}$$

can produce one of the best fits to the $b \rightarrow s$ data if the Wilson coefficients $C_i^\ell = C_i^{\text{SM}} + C_i^{\ell, \text{NP}}$ contain NP effects mainly in $C_{9,10}^{\mu, \text{NP}}$ which satisfy the condition $C_9^{\mu, \text{NP}} = -C_{10}^{\mu, \text{NP}} \sim -0.5$. In this Lagrangian, α_e and G_F denote the usual fine structure and Fermi constants, $V_{ts, tb}$ are elements of the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix, $P_L = (1 - \gamma_5)/2$, and at the m_b scale $C_9^{\text{SM}} \simeq -C_{10}^{\text{SM}} \simeq 4.2$ universally for all charged leptons. In contrast,

¹The tensions may also be alleviated by including the effects of charm-anticharm resonances [19].

dimension-six lepton-quark operators with tensor structures are excluded by the measurements and (pseudo)scalar operators can explain them only with much fine-tuning [14], while the counterpart of $\mathcal{L}_{\text{SM}+\text{NP}}$ with right-handed quark chirality leads to much poorer fits to the data [15, 16].

In view of the lepton nonuniversal nature of $C_i^{\ell,\text{NP}}$ and the size of $C_i^{\mu,\text{NP}}$ relative to C_i^{SM} , if the tentative indications of NP are substantiated by upcoming experiments, one generally expects that there can be $b \rightarrow s$ transitions which violate lepton-flavor symmetry and have rates within reach of searches in the near future [20].² Such a possibility for lepton flavor violation (LFV), and other LFV phenomena that could have connections to the interactions of concern, have been examined further in the literature in the contexts of various NP scenarios [22–32].

In this paper, we also take these anomalies in $b \rightarrow s$ data to be due to NP and explore some of the potential consequences for a variety of rare meson decays with LFV. To do so, we adopt the framework of so-called minimal flavor violation (MFV), which is based on the hypothesis that Yukawa couplings are the only sources for the breaking of flavor and CP symmetries [33–38], as flavor-dependent quark interactions beyond the SM are empirically ruled out if they cause substantial flavor-changing neutral currents. Although the implementation of the MFV principle for quarks is straightforward, there is no unique way to extend it to the lepton sector, as the SM alone does not accommodate LFV and it is still unknown whether neutrinos are Dirac or Majorana particles. Since there is now compelling evidence for neutrino masses and mixing [39], it is interesting to formulate leptonic MFV by incorporating ingredients beyond the SM that can account for this observation [40]. Thus, here we consider the SM expanded with the addition of three heavy right-handed neutrinos as well as effective dimension-six quark-lepton operators with MFV built-in. The heavy neutrinos participate in the usual seesaw mechanism to endow light neutrinos with Majorana masses. We will focus on a couple of such operators which can bring about the NP contributions mentioned earlier and satisfy the MFV criterion in both the quark and lepton sectors.³ We will examine how these operators may contribute to a number of rare $b \rightarrow s$ and $b \rightarrow d$ decays with LFV as well as the rare kaon decays $K_L \rightarrow e\mu$, $K \rightarrow \pi e\mu$, and $K \rightarrow \pi\nu\nu$.

In the next section, after briefly reviewing the MFV framework, we introduce the NP operators of interest. In section 3 we write down the decay amplitudes and proceed with our numerical analysis. We provide our conclusions in section 4. Additional information and lengthy formulas are relegated to an appendix.

²The general arguments of [20] regarding lepton-flavor violation do not hold for certain models, such as those studied in [21].

³Various scenarios of leptonic MFV have been discussed in the literature [41–49].

2 Operators with minimal flavor violaton

In the SM supplemented with three right-handed Majorana neutrinos, the renormalizable Lagrangian for fermion masses can be written as

$$\begin{aligned} \mathcal{L}_m = & -(Y_u)_{kl} \bar{Q}_{k,L} U_{l,R} \tilde{H} - (Y_d)_{kl} \bar{Q}_{k,L} D_{l,R} H - (Y_\nu)_{kl} \bar{L}_{k,L} \nu_{l,R} \tilde{H} - (Y_e)_{kl} \bar{L}_{k,L} E_{l,R} H \\ & - \frac{1}{2} (M_\nu)_{kl} \overline{\nu_{k,R}^c} \nu_{l,R} + \text{H.c.}, \end{aligned} \quad (2.1)$$

where summation over $k, l = 1, 2, 3$ is implicit, $Y_{u,d,\nu,e}$ are matrices for the Yukawa couplings, $Q_{k,L}$ ($L_{k,L}$) denote left-handed quark (lepton) doublets, $U_{l,R}$ and $D_{l,R}$ ($\nu_{l,R}$ and $E_{l,R}$) represent right-handed up- and down-type quarks (neutrinos and charged leptons), respectively, H stands for the Higgs doublet, $\tilde{H} = i\tau_2 H^*$ with τ_2 being the second Pauli matrix, M_ν is a matrix for the Majorana masses of $\nu_{l,R}$, and $\nu_{k,R}^c \equiv (\nu_{k,R})^c$, the superscript referring to charge conjugation. With the nonzero elements of M_ν chosen to be much greater than those of $vY_\nu/\sqrt{2}$, the seesaw mechanism becomes operational [50–58], which leads to the light neutrinos' mass matrix $m_\nu = -(v^2/2)Y_\nu M_\nu^{-1} Y_\nu^T = U_{\text{PMNS}} \hat{m}_\nu U_{\text{PMNS}}^T$, where $v \simeq 246$ GeV is the Higgs's vacuum expectation value, U_{PMNS} denotes the Pontecorvo-Maki-Nakagawa-Sakata (PMNS [59, 60]) matrix, and $\hat{m}_\nu = \text{diag}(m_1, m_2, m_3)$ contains the light neutrinos' eigenmasses $m_{1,2,3}$. This allows one to pick the form [61]

$$Y_\nu = \frac{i\sqrt{2}}{v} U_{\text{PMNS}} \hat{m}_\nu^{1/2} O M_\nu^{1/2}, \quad (2.2)$$

where O is in general a complex matrix satisfying $OO^T = \mathbb{1}$, the right-hand side being a 3×3 unit matrix.

We assume that the right-handed neutrinos are degenerate in mass,

$$M_\nu = \mathcal{M} \text{diag}(1, 1, 1).$$

The MFV hypothesis [38, 40] then implies that \mathcal{L}_m is formally invariant under the global flavor group $\mathcal{G}_f = G_q \times G_\ell$, where $G_q = \text{SU}(3)_Q \times \text{SU}(3)_U \times \text{SU}(3)_D$ and $G_\ell = \text{SU}(3)_L \times \text{O}(3)_\nu \times \text{SU}(3)_E$. This entails that $Q_{k,L}$, $U_{k,R}$, $D_{k,R}$, $L_{k,L}$, $\nu_{k,R}$, and $E_{k,R}$ belong to the fundamental representations of their respective flavor groups,

$$\begin{aligned} Q_L &\rightarrow V_Q Q_L, & U_R &\rightarrow V_U U_R, & D_R &\rightarrow V_D D_R, \\ L_L &\rightarrow V_L L_L, & \nu_R &\rightarrow \mathcal{O}_\nu \nu_R, & E_R &\rightarrow V_E E_R, \end{aligned} \quad (2.3)$$

where $V_{Q,U,D,L,E} \in \text{SU}(3)_{Q,U,D,L,E}$ and $\mathcal{O}_\nu \in \text{O}(3)_\nu$ is an orthogonal real matrix [38, 40, 41]. Furthermore, under \mathcal{G}_f the Yukawa couplings transform in the spurion sense according to

$$Y_u \rightarrow V_Q Y_u V_U^\dagger, \quad Y_d \rightarrow V_Q Y_d V_D^\dagger, \quad Y_\nu \rightarrow V_L Y_\nu \mathcal{O}_\nu^T, \quad Y_e \rightarrow V_L Y_e V_E^\dagger. \quad (2.4)$$

Taking advantage of the symmetry under \mathcal{G}_f , we can work in the basis where

$$Y_d = \frac{\sqrt{2}}{v} \text{diag}(m_d, m_s, m_b), \quad Y_e = \frac{\sqrt{2}}{v} \text{diag}(m_e, m_\mu, m_\tau) \quad (2.5)$$

and the fermion fields $U_k, D_k, \tilde{\nu}_{k,L}, \nu_{k,R}$, and E_k refer to the mass eigenstates. More explicitly, $(U_1, U_2, U_3) = (u, c, t)$, $(D_1, D_2, D_3) = (d, s, b)$, and $(E_1, E_2, E_3) = (e, \mu, \tau)$. We can then express $Q_{k,L}, L_{k,L}$, and Y_u in relation to the CKM matrix V_{CKM} and U_{PMNS} as

$$Q_{k,L} = \begin{pmatrix} (V_{\text{CKM}}^\dagger)_{kl} U_{l,L} \\ D_{k,L} \end{pmatrix}, \quad L_{k,L} = \begin{pmatrix} (U_{\text{PMNS}})_{kl} \tilde{\nu}_{l,L} \\ E_{k,L} \end{pmatrix}, \quad Y_u = \frac{\sqrt{2}}{v} V_{\text{CKM}}^\dagger \text{diag}(m_u, m_c, m_t). \quad (2.6)$$

To put together effective Lagrangians beyond the SM with MFV built-in, one inserts products of the Yukawa matrices among the relevant fields to construct \mathcal{G}_f -invariant operators that are singlet under the SM gauge group [38, 40]. Of interest here are the matrix products

$$A_q = Y_u Y_u^\dagger = V_{\text{CKM}}^\dagger \text{diag}(y_u^2, y_c^2, y_t^2) V_{\text{CKM}}, \quad B_q = Y_d Y_d^\dagger = \text{diag}(y_d^2, y_s^2, y_b^2), \quad (2.7)$$

$$A_\ell = Y_\nu Y_\nu^\dagger = \frac{2\mathcal{M}}{v^2} U_{\text{PMNS}} \hat{m}_\nu^{1/2} O O^\dagger \hat{m}_\nu^{1/2} U_{\text{PMNS}}^\dagger, \quad B_\ell = Y_e Y_e^\dagger = \text{diag}(y_e^2, y_\mu^2, y_\tau^2), \quad (2.8)$$

where $y_f = \sqrt{2}m_f/v$. Since the biggest eigenvalues of A_q and B_q are, respectively, $y_t^2 \sim 1$ and $y_b^2 \sim 3 \times 10^{-4}$ at the W -boson mass scale, for our purposes we can devise objects containing at most two powers of A_q and neglect contributions with B_q , as higher powers of A_q can be related to lower ones by means of the Cayley-Hamilton identity [62, 63]. As for A_ℓ , to maximize the NP effects we assume that the right-handed neutrinos' mass \mathcal{M} is sufficiently large to make the maximum eigenvalue of A_ℓ equal to unity, which fulfills the perturbativity requirement [48, 49, 62, 63]. Hence, as in the quark sector, we will keep terms up to order A_ℓ^2 and drop those with B_ℓ , whose elements are at most $y_\tau^2 \sim 1 \times 10^{-4}$. Accordingly, the pertinent building blocks are

$$\Delta_q = \zeta_0 \mathbb{1} + \zeta_1 A_q + \zeta_2 A_q^2, \quad \Delta_\ell = \xi_0 \mathbb{1} + \xi_1 A_\ell + \xi_2 A_\ell^2, \quad (2.9)$$

where in our model-independent approach $\zeta_{0,1,2}$ and $\xi_{0,1,2}$ are free parameters expected to be at most of $\mathcal{O}(1)$. Hence one or more of them may be suppressed or vanish, depending on the underlying theory. Since these parameters have negligible imaginary components [48, 49, 62, 63], we can make the approximations $\Delta_q^\dagger = \Delta_q$ and $\Delta_\ell^\dagger = \Delta_\ell$.

It follows that the \mathcal{G}_f -invariant dimension-six operators which are SM gauge singlet and of the type that can readily give rise to the NP terms in eq. (1.1) are

$$\mathcal{O}_1 = \bar{Q}_L \gamma_\eta \Delta_{q1} Q_L \bar{L}_L \gamma^\eta \Delta_{\ell1} L_L, \quad \mathcal{O}_2 = \bar{Q}_L \gamma_\eta \Delta_{q2} \tau_a Q_L \bar{L}_L \gamma^\eta \Delta_{\ell2} \tau_a L_L, \quad (2.10)$$

where the objects Δ_{qj} and $\Delta_{\ell j}$ are, respectively, of the same form as Δ_q and Δ_ℓ in eq. (2.9), but have their own independent coefficients ζ_{rj} and ξ_{rj} , and $a = 1, 2, 3$ is implicitly summed over. These generalize operators that were previously introduced under the assumption of MFV only in the quark [38] or lepton [40] part. The MFV effective Lagrangian of interest is then

$$\mathcal{L}_{\text{MFV}} = \frac{1}{\Lambda^2} (\mathcal{O}_1 + \mathcal{O}_2), \quad (2.11)$$

where the mass scale Λ characterizes the heavy NP underlying these interactions. In general, one could also consider dimension-six (pseudo)scalar or tensor operators, or operators

with a right-handed quark current, that fulfill the MFV requirements and contribute to $b \rightarrow s\ell\ell'$ transitions. However, since as discussed in section 1 the current data do not favor such types of operators, their contributions can be neglected.

Before going into further details, it is instructive to go over the flavor structures of \mathcal{O}_1 and \mathcal{O}_2 . Expanding them in terms of the upper and lower components of the left-handed doublets Q_k and L_k and suppressing the gamma matrices, we arrive at

$$\begin{aligned} \mathcal{O}_1 &= (\hat{\Delta}_{q1})_{mn} (\Delta_{\ell 1})_{kl} (\bar{U}_m U_n \bar{\nu}_k \nu_l + \bar{U}_m U_n \bar{E}_k E_l) + (\Delta_{q1})_{mn} (\Delta_{\ell 1})_{kl} (\bar{D}_m D_n \bar{\nu}_k \nu_l + \bar{D}_m D_n \bar{E}_k E_l), \\ \mathcal{O}_2 &= (\hat{\Delta}_{q2})_{mn} (\Delta_{\ell 2})_{kl} (\bar{U}_m U_n \bar{\nu}_k \nu_l - \bar{U}_m U_n \bar{E}_k E_l) - (\Delta_{q2})_{mn} (\Delta_{\ell 2})_{kl} (\bar{D}_m D_n \bar{\nu}_k \nu_l - \bar{D}_m D_n \bar{E}_k E_l) \\ &\quad + 2(V_{\text{CKM}}^\dagger \hat{\Delta}_{q2})_{mn} (\Delta_{\ell 2})_{kl} \bar{D}_m U_n \bar{\nu}_k E_l + 2(\hat{\Delta}_{q2} V_{\text{CKM}})_{mn} (\Delta_{\ell 2})_{kl} \bar{U}_m D_n \bar{E}_k \nu_l, \end{aligned} \quad (2.12)$$

where $\hat{\Delta}_{qj} = V_{\text{CKM}} \Delta_{qj} V_{\text{CKM}}^\dagger = \zeta_{0j} \mathbb{1} + \zeta_{1j} \text{diag}(0, 0, y_t^2) + \zeta_{2j} \text{diag}(0, 0, y_t^4)$ with the $y_{u,c}$ terms having been dropped, summation over $k, l, m, n = 1, 2, 3$ is implicit, and $\nu_k = (U_{\text{PMNS}})_{kn} \tilde{\nu}_n$ represents a flavor eigenstate. Hence, in the approximations we have made, \mathcal{L}_{MFV} does not cause flavor-changing transitions between up-type quarks. In contrast, flavor changes among down-type quarks can occur with either charged leptons or neutrinos being emitted, but according to eq. (2.12) the two operators contribute differently to the two types of processes, which will be treated in more detail below. We will especially deal with the exclusive decays of $b \rightarrow q\ell\ell'$ and $b \rightarrow q\nu\nu'$ for $q = s, d$ and $\ell \neq \ell'$, as well as $s \rightarrow d\ell\ell'$ and $s \rightarrow d\nu\nu'$.

3 Decay amplitudes and numerical analysis

In the presence of \mathcal{L}_{MFV} , one can generalize eq. (1.1) to

$$\mathcal{L}_{bq\ell\ell'} = \frac{\sqrt{2}\alpha_e \lambda_{qb} G_F}{\pi} C_{\ell\ell'} \bar{q} \gamma^\eta P_L b \bar{\ell} \gamma_\eta P_L \ell' + \text{H.c.}, \quad (3.1)$$

where $\lambda_{qb} = V_{tq}^* V_{tb}$ is the CKM factor, $q = s, d$, and

$$C_{\ell\ell'} = \delta_{\ell\ell'} C_9^{\text{SM}} + c_{\ell\ell'}, \quad (3.2)$$

having set $C_{10}^{\text{SM}} = -C_9^{\text{SM}}$. Accordingly, for $\ell\ell' = E_k E_l$ one can derive from \mathcal{L}_{MFV}

$$c_{E_k E_l} = \tilde{c}_{E_k E_l}^{(1)} + \tilde{c}_{E_k E_l}^{(2)} = \tilde{\xi}_0^+ \delta_{kl} + \tilde{\xi}_1^+ (A_\ell)_{kl} + \tilde{\xi}_2^+ (A_\ell^2)_{kl}, \quad (3.3)$$

where $\tilde{c}_{E_k E_l}^{(j)}$ belongs to \mathcal{O}_j and $\tilde{\xi}_{0,1,2}^+$ are given by

$$\tilde{\xi}_r^+ = \tilde{\xi}_{r1} + \tilde{\xi}_{r2}, \quad r = 0, 1, 2, \quad \tilde{\xi}_{rj} = \frac{\pi(\zeta_{1j} y_t^2 + \zeta_{2j} y_t^4) \xi_{rj}}{\sqrt{2} \alpha_e \Lambda^2 G_F}, \quad j = 1, 2, \quad (3.4)$$

the terms with $y_{u,c}$ in $\tilde{\xi}_{rj}$ having been dropped. It follows that $|C_{\ell\ell'}| = |C_{\ell'\ell}|$. Based on eq. (3.1), one can write down the corresponding Lagrangian $\mathcal{L}_{sd\ell\ell'}$ for $s \rightarrow d\ell^-\ell'^+$.

From the foregoing, we can derive the contributions of \mathcal{L}_{MFV} to the amplitudes for a number of $b \rightarrow s\ell^-\ell'^+$ transitions with $\ell \neq \ell'$. Thus, with $C_{\ell\ell'} = c_{\ell\ell'}$, for $\bar{B} \rightarrow \bar{K} \ell^-\ell'^+$ we obtain

$$\mathcal{M}_{\bar{B} \rightarrow \bar{K} \ell\ell'} = \frac{-\alpha_e \lambda_{sb} G_F c_{\ell\ell'}}{\sqrt{2} \pi} \left[(p_B^\eta + p_K^\eta) F_1 + \frac{m_B^2 - m_K^2}{s} (F_0 - F_1) \dot{p}^\eta \right] \bar{\ell} \gamma_\eta P_L \ell', \quad (3.5)$$

where $p_{B,K}$ and $\dot{p} = p_\ell + p_{\ell'}$ denote the four-momenta of the mesons and dilepton, respectively, and $F_{0,1}$ stand for the form factors in the hadronic matrix element $\langle \bar{K} | \bar{s} \gamma^\eta (1 - \gamma_5) b | \bar{B} \rangle$, which are described in appendix A and depend on the Lorentz-invariant $\dot{s} = \dot{p}^2$. For $\bar{B} \rightarrow \bar{K}^* \ell^- \ell'^+$ we arrive at

$$\mathcal{M}_{\bar{B} \rightarrow \bar{K}^* \ell \bar{\ell}} = \frac{-\alpha_e \lambda_{sb} G_F c_{\ell\ell'}}{\sqrt{2}\pi} \left[A \epsilon_{\eta\varsigma\chi\omega} \epsilon^{*\varsigma} p_B^\chi p_{K^*}^\omega - i\mathbb{C} \epsilon_\eta^* + i\mathbb{D} \epsilon^* \cdot \dot{p} (p_B + p_{K^*})_\eta + i\mathbb{H} \epsilon^* \cdot \dot{p} \dot{p}_\eta \right] \bar{\ell} \gamma^\eta P_L \ell', \quad (3.6)$$

$$\begin{aligned} A &= 2V/(m_B + m_{K^*}), & \mathbb{C} &= A_1(m_B + m_{K^*}), & \mathbb{D} &= A_2/(m_B + m_{K^*}), \\ \dot{s}\mathbb{H} &= \mathbb{C} - \mathbb{D}(m_B^2 - m_{K^*}^2) - 2A_0 m_{K^*}, \end{aligned} \quad (3.7)$$

where V and $A_{0,1,2}$ are the form factors for $\langle \bar{K}^* | \bar{s} \gamma_\eta (1 - \gamma_5) b | \bar{B} \rangle$, which are defined in appendix A. We look at $\bar{B}_s \rightarrow \phi \ell^- \ell'^+$ as well, which has an amplitude $\mathcal{M}_{\bar{B}_s \rightarrow \phi \ell \bar{\ell}}$ analogous to $\mathcal{M}_{\bar{B} \rightarrow \bar{K}^* \ell \bar{\ell}}$. For $\bar{B}_s \rightarrow \ell^- \ell'^+$ we find

$$\mathcal{M}_{\bar{B}_s \rightarrow \ell \bar{\ell}} = \frac{i\alpha_e \lambda_{sb} f_{B_s} G_F c_{\ell\ell'}}{2\sqrt{2}\pi} \bar{\ell} [m_{\ell'} - m_\ell + (m_{\ell'} + m_\ell) \gamma_5] \ell', \quad (3.8)$$

where f_{B_s} is the B_s decay constant, which is also defined in appendix A.

The MFV Lagrangian in eq. (2.11) generates lepton-flavor-violating $b \rightarrow d \ell^- \ell'^+$ processes with the same coefficient $c_{\ell\ell'}$ in eq. (3.2), but a different CKM factor, λ_{db} . This makes it of interest to include them in our study, particularly $\bar{B} \rightarrow \pi \ell^- \ell'^+$, $\bar{B} \rightarrow \rho \ell^- \ell'^+$, and $\bar{B}_d \rightarrow \ell^- \ell'^+$. Generally, there could be other dimension-six MFV operators that also contribute to $b \rightarrow d \ell^- \ell'^+$ and therefore can enhance or reduce the impact of $c_{\ell\ell'}$. Hereafter, we focus on the possibility that the effects of such operators are unimportant. Under these assumptions, the rates of these decay channels have expressions similar in form to those for $\bar{B} \rightarrow K \ell^- \ell'^+$, $\bar{B} \rightarrow \bar{K}^* \ell^- \ell'^+$, and $\bar{B}_s \rightarrow \ell^- \ell'^+$, respectively, but are comparatively smaller because $|\lambda_{db}/\lambda_{sb}|^2 \sim 1/22$.

Before starting our numerical calculation, we need to specify our choices further. Since the A_ℓ matrix in eq. (2.8) can be realized in many different ways, we concentrate on the least complicated possibility that O is a real orthogonal matrix, in which case

$$A_\ell = \frac{2\mathcal{M}}{v^2} U_{\text{PMNS}} \hat{m}_\nu U_{\text{PMNS}}^\dagger. \quad (3.9)$$

For U_{PMNS} , we employ the standard parametrization [39], with its elements being determined from the results of a recent fit to global neutrino data in ref. [64], which depend on whether neutrino masses have a normal hierarchy (NH), $m_1 < m_2 < m_3$, or an inverted one (IH), $m_3 < m_1 < m_2$. Since the empirical information on the absolute scale of $m_{1,2,3}$ is still far from precise [39], for definiteness we pick $m_1 = 0$ ($m_3 = 0$) in the NH (IH) case. Requiring the biggest eigenvalue of A_ℓ to be unity then implies $\mathcal{M} \simeq 6.1 \times 10^{14}$ GeV. For the elements of V_{CKM} , we adopt the results of the latest fit performed in ref. [65, 66]. With these numbers, we can evaluate the branching fractions of the decay modes discussed earlier using the rate formulas collected in appendix A, which also describes our choices for the relevant hadronic form factors and decay constants.

We now attempt to attain the largest branching fractions of the $b \rightarrow q\ell\bar{\ell}'$ decays of interest, for $\ell \neq \ell'$, under our MFV framework by scanning the space of the $\tilde{\xi}_r^+$ parameters, which enter the NP term, $c_{\ell\ell'}$, in the Wilson coefficient $C_{\ell\ell'}$ according to eqs. (3.2)–(3.4). This amounts to maximizing $|c_{\ell\ell'}|$. Simultaneously we need to impose the pertinent restrictions from the existing b -meson data. Thus, based on the results of an analysis of the latest $b \rightarrow s$ measurements including the LHCb anomalies [15, 16], we require

$$c_{ee} = 0, \quad -0.71 < c_{\mu\mu} < -0.35, \quad (3.10)$$

which can lead to one of the best fits to the data [16]. Nevertheless, it is possible to let c_{ee} have some nonvanishing value as well [15]. Accordingly, we alternatively impose

$$0 < c_{ee} < 0.3, \quad -0.65 < c_{\mu\mu} < -0.45, \quad (3.11)$$

which is well within the allowed 1σ best-fit region in the second plot of figure 6 in ref. [15].

In addition, since $c_{\ell\ell'}$ for a specific pair of ℓ and $\ell' \neq \ell$ affects also $s \rightarrow d\ell\bar{\ell}'$ processes, we need to take into account the available experimental bounds, which may imply complementary limitations on $c_{\ell\ell'}$. Indeed, if like before other dimension-six MFV operators do not give rise to competitive effects on $s \rightarrow d\ell\bar{\ell}'$, we find in kaon data that the branching-fraction limit [39] $\mathcal{B}(K_L \rightarrow e^\pm\mu^\mp)_{\text{exp}} = (\mathcal{B}(K_L \rightarrow e^+\mu^-) + \mathcal{B}(K_L \rightarrow e^-\mu^+))_{\text{exp}} < 4.7 \times 10^{-12}$ can translate into the strongest restriction on $c_{\ell\ell'}$ among lepton-flavor-violating meson decays. Thus, applying eq. (A.8) in the appendix, with the central values of the input parameters, we then extract

$$|c_{e\mu}|^2 < 0.16, \quad (3.12)$$

which we will also impose.

In our scans of the $\tilde{\xi}_r^+$ parameter space to maximize $|c_{\ell\ell'}|$, we find that the bound in eq. (3.12) is always reached. This implies that it can already be used to estimate the largest branching fractions of various $\bar{b} \rightarrow \bar{q}e\mu$ decays and $K \rightarrow \pi e\mu$ within our MFV scenario with $\mathcal{O}_{1,2}$ taken to be the main operators responsible. We display the results in the third column of table 1 and compare them with the corresponding experimental limits if available.⁴ One observes that the predicted $\mathcal{B}(B \rightarrow Ke^\pm\mu^\mp)$ is only 4 times below its measured bound and, therefore, may be probed in near-future searches. For the other modes, the predictions are lower than their experimental counterparts by more than an order of magnitude.

For the decay channels with $\ell\ell' = e\tau$ or $\mu\tau$, one can entertain many different possibilities. For several of them, we present the results listed in table 2, where we have separated those obtained under the constraint in either eq. (3.10) or eq. (3.11), besides the requirement in eq. (3.12). Furthermore, to get the numbers without (within) parentheses in the table we have employed the central values of the neutrino mixing parameters from ref. [64] associated with the normal (inverted) hierarchy of light neutrino masses, except for the

⁴In conformity to the experimental reports [67–69], the $B \rightarrow K^{(*)}e\mu$ prediction in this table is the simple average over the B^+ and B^0 channels, namely $\mathcal{B}(B \rightarrow K^{(*)}e^\pm\mu^\mp) = (\mathcal{B}(B^+ \rightarrow K^{(*)+}e^\pm\mu^\mp) + \mathcal{B}(B^0 \rightarrow K^{(*)0}e^\pm\mu^\mp))/2$, whereas the $B \rightarrow \pi e\mu$ prediction is from $\mathcal{B}(B \rightarrow \pi e^\pm\mu^\mp) = (\mathcal{B}(B^+ \rightarrow \pi^+e^\pm\mu^\mp) + 2\mathcal{B}(B^0 \rightarrow \pi^0e^\pm\mu^\mp))/2$ and similarly for $B \rightarrow \rho e^\pm\mu^\mp$.

Decay mode	Branching fractions	
	Measured upper limit at 90% CL [39, 70, 71]	Prediction
$B \rightarrow Ke^\pm\mu^\mp$	3.8×10^{-8}	9.7×10^{-9}
$B \rightarrow K^*e^\pm\mu^\mp$	5.1×10^{-7}	2.4×10^{-8}
$B_s \rightarrow \phi e^\pm\mu^\mp$	—	2.4×10^{-8}
$B_s \rightarrow e^\pm\mu^\mp$	1.1×10^{-8}	2.9×10^{-11}
$B \rightarrow \pi e^\pm\mu^\mp$	9.2×10^{-8}	4.1×10^{-10}
$B \rightarrow \rho e^\pm\mu^\mp$	3.2×10^{-6}	1.1×10^{-9}
$B^0 \rightarrow e^\pm\mu^\mp$	2.8×10^{-9}	8.9×10^{-13}
$K^+ \rightarrow \pi^+ e^- \mu^+$	1.3×10^{-11}	3.6×10^{-14}
$K_L \rightarrow \pi^0 e^\pm \mu^\mp$	7.6×10^{-11}	4.5×10^{-14}

Table 1. Predicted upper limits on the branching fractions of exclusive meson decays involving $e\mu$ in the final states, calculated with $|c_{e\mu}|$ from the empirical limit $\mathcal{B}(K_L \rightarrow e^\pm\mu^\mp)_{\text{exp}} < 4.7 \times 10^{-12}$ at 90% confidence level [39], under the assumption that the effects of operators $\mathcal{O}_{1,2}$ dominate these processes. For comparison, the experimental counterparts are also displayed if available.

Dirac CP -violation angle δ which has a greater uncertainty than the other parameters. To reflect this uncertainty, the left and right numbers inside the square brackets have been computed with the minimum and maximum, respectively, of $\delta/\text{degree} = 306_{-70}^{+39}(254_{-62}^{+63})$ from ref. [64] in the NH (IH) case.

We remark that in table 2 each vertical set of results for the $e\tau$ ($\mu\tau$) channels comes from the same maximized value of $|c_{e\tau}|$ ($|c_{\mu\tau}|$). However, for the $e\tau$ and $\mu\tau$ numbers in the same column, the respective sets of $\tilde{\xi}_r^+$ values which maximize $|c_{e\tau}|$ and $|c_{\mu\tau}|$ are not always the same. Moreover, for the $e\tau$ ($\mu\tau$) results in different columns, the $\tilde{\xi}_r^+$ sets which maximize $|c_{e\tau}|$ ($|c_{\mu\tau}|$) are generally also different, as we are interested in attaining the biggest branching fractions in the different scenarios. We note, in addition, that the results in the NH (IH) case follow from $|c_{e\tau}| \lesssim 0.9$ (0.8) and $|c_{\mu\tau}| \lesssim 1.9$ (1.4), with $|\tilde{\xi}_0^+| \sim 0.2\text{-}0.7$ (1-2) and $|\tilde{\xi}_{1,2}^+| \sim 4\text{-}17$ (40-120), implying that $\Lambda/|(\zeta_{1j}y_t^2 + \zeta_{2j}y_t^4)\tilde{\xi}_{rj}|^{1/2} > 1.7$ (0.65) TeV if $\tilde{\xi}_{r1} \sim \tilde{\xi}_{r2}$.

It is evident from table 2 that for each decay mode the results in the different cases are roughly of similar size and differ from each other within only factors of 4 or less. More interestingly, we notice that a few of the predicted branching fractions of the $\mu\tau$ channels can be as high as a few times 10^{-7} . Although they are still at least about two orders of magnitude below the existing empirical limits, which are presently not many, upcoming experiments will expectedly offer ample opportunities to look for these decays and improve the data situation.

Since the restraint in eq. (3.12) would lessen if there were other operators having destructive interference with $c_{e\mu}$ in the K_L decay amplitude, their presence would bring

Decay mode	Branching fractions		
	Measured upper limit at 90% CL [70, 71]	Predictions	
		(I)	(II)
$B^+ \rightarrow K^+ e^\pm \tau^\mp$	3.0×10^{-5}	$[0.9, 2.7] ([1.5, 2.2]) \times 10^{-8}$	$[1.1, 3.5] ([2.0, 2.6]) \times 10^{-8}$
$B^+ \rightarrow K^{*+} e^\pm \tau^\mp$	—	$[1.7, 5.3] ([3.0, 4.3]) \times 10^{-8}$	$[2.2, 6.9] ([3.9, 5.1]) \times 10^{-8}$
$B_s \rightarrow \phi e^\pm \tau^\mp$	—	$[1.7, 5.0] ([2.8, 4.1]) \times 10^{-8}$	$[2.1, 6.6] ([3.7, 4.8]) \times 10^{-8}$
$B_s \rightarrow e^\pm \tau^\mp$	—	$[0.9, 2.6] ([1.5, 2.1]) \times 10^{-8}$	$[1.1, 3.5] ([1.9, 2.5]) \times 10^{-8}$
$B^+ \rightarrow \pi^+ e^- \tau^+$	2.0×10^{-5}	$[0.2, 0.7] ([0.4, 0.5]) \times 10^{-9}$	$[0.3, 0.9] ([0.5, 0.6]) \times 10^{-9}$
$B^+ \rightarrow \rho^+ e^\pm \tau^\mp$	—	$[0.8, 2.4] ([1.4, 2.0]) \times 10^{-9}$	$[1.0, 3.2] ([1.8, 2.3]) \times 10^{-9}$
$B^0 \rightarrow e^\pm \tau^\mp$	2.8×10^{-5}	$[0.3, 0.8] ([0.5, 0.7]) \times 10^{-9}$	$[0.3, 1.1] ([0.6, 0.8]) \times 10^{-9}$
$B^+ \rightarrow K^+ \mu^\pm \tau^\mp$	4.8×10^{-5}	$[0.6, 0.9] ([0.4, 0.5]) \times 10^{-7}$	$[0.8, 1.4] ([0.5, 0.8]) \times 10^{-7}$
$B^+ \rightarrow K^{*+} \mu^\pm \tau^\mp$	—	$[1.1, 1.8] ([0.7, 1.0]) \times 10^{-7}$	$[1.5, 2.8] ([1.0, 1.5]) \times 10^{-7}$
$B_s \rightarrow \phi \mu^\pm \tau^\mp$	—	$[1.1, 1.7] ([0.7, 1.0]) \times 10^{-7}$	$[1.5, 2.6] ([0.9, 1.4]) \times 10^{-7}$
$B_s \rightarrow \mu^\pm \tau^\mp$	—	$[0.6, 0.9] ([0.4, 0.5]) \times 10^{-7}$	$[0.8, 1.4] ([0.5, 0.8]) \times 10^{-7}$
$B^+ \rightarrow \pi^+ \mu^\pm \tau^\mp$	7.2×10^{-5}	$[2.8, 4.6] ([1.8, 2.6]) \times 10^{-9}$	$[3.9, 6.9] ([2.4, 3.8]) \times 10^{-9}$
$B^+ \rightarrow \rho^+ \mu^\pm \tau^\mp$	—	$[5.1, 8.4] ([3.2, 4.8]) \times 10^{-9}$	$[7.1, 13] ([4.4, 7.0]) \times 10^{-9}$
$B^0 \rightarrow \mu^\pm \tau^\mp$	2.2×10^{-5}	$[1.7, 2.8] ([1.1, 1.6]) \times 10^{-9}$	$[2.4, 4.3] ([1.5, 2.4]) \times 10^{-9}$

Table 2. Predicted upper limits on the branching fractions of exclusive b -meson decays involving $(e, \mu)\tau$ in the final states, computed with the maximal $|c_{(e,\mu)\tau}|$ determined under the imposed constraint set (I) $c_{ee} = 0$, $-0.71 < c_{\mu\mu} < -0.35$, and $|c_{e\mu}| < 0.4$ or (II) $0 < c_{ee} < 0.3$, $-0.65 < c_{\mu\mu} < -0.45$, and $|c_{e\mu}| < 0.4$, as discussed in the text, under the assumption that the effects of operators $\mathcal{O}_{1,2}$ dominate these lepton-flavor-violating processes. The numbers without (within) parentheses correspond to neutrino mixing parameters belonging to the normal (inverted) hierarchy of neutrino masses, whereas the left and right numbers inside square brackets reflect the minimum and maximum empirical values of the Dirac phase δ in U_{PMNS} . For comparison, the data are also displayed if available.

about a different set of predictions for the processes listed in tables 1 and 2. Changes in the predictions would also occur if other operators significantly modified $b \rightarrow d\ell\bar{\ell}'$. Thus, our scenario in which $\mathcal{O}_{1,2}$ dominate these LFV transitions will be tested when one or more of them are discovered and the acquired data compared to the predictions.

It is worth commenting, in addition, that the parameter values which are responsible for the predictions above also translate into reductions in the rates of the lepton-flavor-conserving $b \rightarrow (s, d)\tau^+\tau^-$ decays by up to a few tens percent with respect to their SM estimates. This implies that future observations of these processes with good precision will serve as important complementary tests on our NP scenario.

Now, as indicated in the preceding section, \mathcal{L}_{MFV} in eq. (2.11) also contributes to transitions with neutrinos in the final states. Specifically for $b \rightarrow q\nu\nu'$ decays the effective

Lagrangian is given by

$$\mathcal{L}_{bq\nu\nu'} = \frac{\sqrt{2}\alpha_e\lambda_{qb}G_F}{\pi}C_{\nu\nu'}\bar{q}\gamma^\eta P_L b\bar{\nu}\gamma_\eta P_L\nu' + \text{H.c.}, \quad C_{\nu\nu'} = \delta_{\nu\nu'}C_L^{\text{SM}} + c_{\nu\nu'}, \quad (3.13)$$

where $C_L^{\text{SM}} \simeq -6.4$ is the SM prediction [22] and $c_{\nu\nu'}$ arises from NP. From \mathcal{L}_{MFV} , one then gets for $\nu\nu' = \nu_k\nu_l$

$$c_{\nu_k\nu_l} = \tilde{c}_{E_k E_l}^{(1)} - \tilde{c}_{E_k E_l}^{(2)} = \tilde{\xi}_0^- \delta_{kl} + \tilde{\xi}_1^- (A_\ell)_{kl} + \tilde{\xi}_2^- (A_\ell^2)_{kl}, \quad (3.14)$$

where $\tilde{c}_{E_k E_l}^{(1,2)}$ also enter $c_{E_k E_l}$ in eq. (3.3), but with the opposite relative sign, and $\tilde{\xi}_r^- = \tilde{\xi}_{r1} - \tilde{\xi}_{r2}$. Therefore, $c_{\nu\nu'}$ and $c_{\ell\ell'}$ are generally independent of each other [22].

Since only the contributions of $c_{\nu_k\nu_l}$ with $k = l = 1, 2, 3$ can interfere with the SM contribution to $b \rightarrow q\nu\nu'$, and since the neutrinos are not detected, it is straightforward to derive the ratio of branching fractions

$$r_{B \rightarrow K^{(*)}\nu\nu'} = \frac{\mathcal{B}(\bar{B} \rightarrow \bar{K}^{(*)}\nu\nu')}{\mathcal{B}(\bar{B} \rightarrow \bar{K}^{(*)}\nu\nu')_{\text{SM}}} = \frac{1}{3} \sum_k \left(\left| 1 + \frac{c_{\nu_k\nu_k}}{C_L^{\text{SM}}} \right|^2 + \sum_{l \neq k} \left| \frac{c_{\nu_k\nu_l}}{C_L^{\text{SM}}} \right|^2 \right). \quad (3.15)$$

These channels are not yet observed, but there are experimental limits on their branching fractions [70, 71]. Here the relevant bound is [22] $\mathcal{B}(\bar{B} \rightarrow \bar{K}\nu\nu)_{\text{exp}} < 4.3\mathcal{B}(\bar{B} \rightarrow \bar{K}\nu\nu)_{\text{SM}}$, and so we can impose

$$r_{B \rightarrow K^{(*)}\nu\nu'} < 4.3. \quad (3.16)$$

Their counterparts in the kaon sector, $K^+ \rightarrow \pi^+\nu\nu'$ and $K_L \rightarrow \pi^0\nu\nu'$, are similarly affected by $c_{\nu\nu'}$. In its presence, the effective Lagrangian for $sd\nu\nu'$ interactions is given by

$$\mathcal{L}_{ds\nu\nu'} = -\frac{\sqrt{2}\alpha_e\lambda_t G_F}{\pi s_w^2} X_{\ell\ell'} \bar{s}\gamma^\eta P_L d\bar{\nu}\gamma_\eta P_L\nu + \text{H.c.}, \quad X_{\nu\nu'} = \delta_{\nu\nu'} X_{\text{SM}} + x_{\nu\nu'}, \quad (3.17)$$

where $\lambda_t = \lambda_{sd}$, $s_w^2 = \sin^2\theta_W = 0.231$, the SM term [72, 73] $X_{\text{SM}} = X + |V_{us}|^4 \text{Re}(V_{cs}^* V_{cd}) P_c / \lambda_t$ consisting of 2 contributions from top- and charm-loop diagrams, respectively, with $X = -C_L^{\text{SM}} s_w^2$ and $|V_{us}|^4 \text{Re}(V_{cs}^* V_{cd}) = \lambda^7/2 - \lambda^5$ in the Wolfenstein parametrization, and $x_{\nu\nu'} = -c_{\nu\nu'} s_w^2$ due to NP. The latest predictions of the SM are [74] $\mathcal{B}(K^+ \rightarrow \pi^+\nu\nu)_{\text{SM}} = (9.11 \pm 0.72) \times 10^{-11}$ and $\mathcal{B}(K_L \rightarrow \pi^0\nu\nu)_{\text{SM}} = (3.0 \pm 0.3) \times 10^{-11}$. Generalizing the expressions for these branching fractions to include the \mathcal{L}_{MFV} effects, which modify the X part, we obtain

$$\mathcal{B}(K^+ \rightarrow \pi^+\nu\nu) = \frac{\kappa_+'}{3\lambda^{10}} \sum_k \left\{ [\text{Re}\lambda_t (X - c_{\nu_k\nu_k} s_w^2) + (\lambda^7/2 - \lambda^5) P_c]^2 + (\text{Im}\lambda_t)^2 (X - c_{\nu_k\nu_k} s_w^2)^2 + |\lambda_t|^2 s_w^4 \sum_{l \neq k} |c_{\nu_k\nu_l}|^2 \right\}, \quad (3.18)$$

$$\mathcal{B}(K_L \rightarrow \pi^0\nu\nu) = \frac{\kappa_L (\text{Im}\lambda_t)^2}{3\lambda^{10}} \sum_k \left[(X - c_{\nu_k\nu_k} s_w^2)^2 + s_w^4 \sum_{l \neq k} |c_{\nu_k\nu_l}|^2 \right], \quad (3.19)$$

where $\kappa'_+ = 0.997 \times 5.173 \times 10^{-11}$, $X = 1.481$, $P_c = 0.404$, and $\kappa_L = 2.231 \times 10^{-10}$ are the central values from ref. [74]. In view of the data [75] $\mathcal{B}(K^+ \rightarrow \pi^+ \nu \nu)_{\text{exp}} = (17.3^{+11.5}_{-10.5}) \times 10^{-11}$ and [76] $\mathcal{B}(K_L \rightarrow \pi^0 \nu \nu)_{\text{exp}} < 2.6 \times 10^{-8}$, we may then demand only

$$0.7 < r_{K^+ \rightarrow \pi^+ \nu \nu} = \frac{\mathcal{B}(K^+ \rightarrow \pi^+ \nu \nu)}{\mathcal{B}(K^+ \rightarrow \pi^+ \nu \nu)_{\text{SM}}} < 3.2. \quad (3.20)$$

In addition, comparing eqs. (3.15) and (3.19), one infers that $r_{B \rightarrow K^{(*)} \nu \nu}$ is equal to its $K_L \rightarrow \pi^0 \nu \nu$ counterpart, $r_{K_L \rightarrow \pi^0 \nu \nu}$.

For the parameter values that yield the examples in table 2, if $c_{\ell\ell'}^{(2)} = 0$ in eqs. (3.3) and (3.14), we find that $r_{B \rightarrow K^{(*)} \nu \nu} = r_{K_L \rightarrow \pi^0 \nu \nu}$ can be as large as 1.22 (1.15) in the NH (IH) case, whereas $r_{K^+ \rightarrow \pi^+ \nu \nu}$ can reach 1.15 (1.11). If $c_{\ell\ell'}^{(1)} = 0$ instead, the results above for the decay modes with charged leptons are unchanged, but now the branching fractions of the modes with neutrinos tend to be reduced by up to 14% with respect to their SM expectations. All of these numbers are well within their corresponding restrictions in eqs. (3.16) and (3.20).

If both $c_{\ell\ell'}^{(1)}$ and $c_{\ell\ell'}^{(2)}$ are nonzero, then in general $c_{\ell\ell'}$ and $c_{\nu\nu'}$ are not connected and, consequently, they can be maximized independently. In that case, our results in table 2 are still the same, but for the channels with neutrinos, after scanning the $\tilde{\xi}_r^-$ parameter space subject to eqs. (3.16) and (3.20), we obtain a maximum value of $r_{B \rightarrow K^{(*)} \nu \nu} = r_{K_L \rightarrow \pi^0 \nu \nu}$ that saturates its limit of 4.3 and $r_{K^+ \rightarrow \pi^+ \nu \nu}$ that reaches ~ 3.1 , with $\tilde{\xi}_0^- \sim -7$ and $|\tilde{\xi}_{1,2}^-| \sim 0$. Thus future measurements can offer significant checks on these predictions.

Finally, we mention that, in the approximations we made, \mathcal{L}_{MFV} produces vanishing effects on the SM-dominated transitions $b \rightarrow (u, c) \nu \ell$, $s \rightarrow u \nu \ell$, and $c \rightarrow (d, s) \nu \bar{\ell}$. Although its contribution to $t \rightarrow b \nu \bar{\ell}$ is nonzero,

$$\mathcal{M}_{t \rightarrow b \nu \bar{\ell}}^{\text{MFV}} = \frac{-2V_{tb}^* (\zeta_{02} + \zeta_{12} y_t^2 + \zeta_{22} y_t^4) (\Delta_{\ell 2})_{kl}}{\Lambda^2} \bar{b} \gamma^\eta P_L t \bar{\nu}_k \gamma_\eta P_L E_l, \quad (3.21)$$

we estimate that its branching fraction is only under 10^{-3} for optimistic choices of the parameter values, and so it is much smaller than $\mathcal{B}(t \rightarrow b W^+ \rightarrow b \nu \ell^+)_{\text{SM}} \sim 0.1$ which is consistent with the data [39].

4 Conclusions

We have entertained the possibility that the anomalies recently detected in the measurements of rare $b \rightarrow s$ processes are NP signals and explored some of the potential implications for a number of exclusive lepton-flavor-violating meson decays. Adopting the effective theory framework of MFV based on the SM plus 3 heavy right-handed neutrinos participating in the seesaw mechanism, we concentrate on a couple of dimension-six 4-fermion operators which accommodate the interactions that can yield one of the best fits to the $b \rightarrow s$ data. Assuming that these operators conform to the MFV hypothesis in both their quark and lepton parts, we evaluate their effects on various meson decays that violate lepton-flavor symmetry, subject to additional relevant b -meson and kaon constraints. For simplification,

we further assume that other dimension-six MFV operators do not induce competitive contributions to these transitions. This scenario can be tested when they are discovered and the resulting predictions confronted with the acquired data.

With the preceding premises, our numerical work shows that for decays with charged leptons in the final states $c_{e\mu}$ is the most restricted among the Wilson coefficients $c_{\ell\ell'}$ for $\ell \neq \ell'$, its strictest bound being supplied by the experimental limit for $K_L \rightarrow e\mu$. Interestingly, among the other modes considered, the resulting prediction for $\mathcal{B}(B \rightarrow Ke\mu)$ is the closest to its empirical bound, being only 4 times smaller, and therefore may be probed by forthcoming searches. Moreover, the predicted branching fractions of the exclusive $b \rightarrow s\mu\tau$ decays can be as large as a few times 10^{-7} and hence may be examined as well in near-future experiments. For decay channels with neutrinos in the final states, $B \rightarrow K^{(*)}\nu\nu$ and $K \rightarrow \pi\nu\nu$, we find that they are comparatively far less restricted and that the impact of the MFV operators can enhance their branching fractions by up to a few times with respect to the SM expectations. Thus, planned measurements on these processes involving neutrinos will provide important checks on the enhancements.

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A Decay rates

In what follows we assume that the charged leptons ℓ and ℓ' in the final states of the decays are different in flavor. Furthermore, the decays are induced by left-handed current-current operators of the type in eq. (3.1).

The hadronic matrix elements pertinent to the decay $\bar{B} \rightarrow \bar{K}\ell\ell'$ are then

$$\langle \bar{K} | \bar{s}\gamma^\mu b | \bar{B} \rangle = \frac{m_B^2 - m_K^2}{\hat{s}} \hat{p}^\mu F_0 + \left(p_B^\mu + p_K^\mu - \frac{m_B^2 - m_K^2}{\hat{s}} \hat{p}^\mu \right) F_1 \quad (\text{A.1})$$

and $\langle \bar{K} | \bar{s}\gamma^\mu \gamma_5 b | \bar{B} \rangle = 0$, where m_B (m_K) and p_B (p_K) are the \bar{B} (\bar{K}) mass and four-momentum, respectively, $\hat{p} = p_B - p_K$, and the form factors $F_{0,1}$ depend on the Lorentz-invariant $\hat{s} = \hat{p}^2$ according to [77]

$$F_0 = a_{00} + a_{01}z + a_{02}z^2 + a_{03}z^3, \quad F_1 = \frac{a_{10} + a_{11}z + a_{12}z^2 - \frac{1}{3}(a_{11} - 2a_{12})z^3}{1 - \hat{s}/(m_B + 0.04578 \text{ GeV})^2},$$

$$z = \frac{\sqrt{t_+ - \hat{s}} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - \hat{s}} + \sqrt{t_+ - t_0}}, \quad t_\pm = (m_{B^\pm} \pm m_{K^\pm})^2, \quad t_0 = \left(1 - \sqrt{1 - t_-/t_+}\right)t_+, \quad (\text{A.2})$$

the a 's being constants. Numerically, we adopt $(a_{00}, a_{01}, a_{02}, a_{03}) = (0.54, -1.91, 1.83, -0.02)$ and $(a_{10}, a_{11}, a_{12}) = (0.43, -0.67, -1.12)$, which are their central values from ref. [15]. For the $\bar{B} \rightarrow \pi$ form-factors, which are relevant to $\bar{B} \rightarrow \pi\ell\ell'$ and defined analogously to $F_{0,1}$ in eq. (A.1), we make use of the parametrization choice

preferred in ref. [78] as well as the relation $\langle \pi^- | \bar{d} \gamma^\eta b | B^- \rangle = \sqrt{2} \langle \pi^0 | \bar{d} \gamma^\eta b | \bar{B}^0 \rangle$ based on isospin symmetry.

Similarly, for $\bar{B} \rightarrow \bar{K}^* \ell \bar{\ell}'$ the requisite matrix elements are

$$\begin{aligned} \langle \bar{K}^* | \bar{s} \gamma_\eta b | \bar{B} \rangle &= \frac{2V}{m_B + m_{K^*}} \epsilon_{\eta\rho\sigma\tau} \varepsilon^{*\rho} p_B^\sigma p_{K^*}^\tau, \\ \langle \bar{K}^* | \bar{s} \gamma^\eta \gamma_5 b | \bar{B} \rangle &= 2iA_0 m_{K^*} \frac{\varepsilon^* \cdot \tilde{p}}{\tilde{s}} \tilde{p}^\eta + iA_1 (m_B + m_{K^*}) \left(\varepsilon^{*\eta} - \frac{\varepsilon^* \cdot \tilde{p}}{\tilde{s}} \tilde{p}^\eta \right) \\ &\quad - \frac{iA_2 \varepsilon^* \cdot \tilde{p}}{m_B + m_{K^*}} \left(p_B^\eta + p_{K^*}^\eta - \frac{m_B^2 - m_{K^*}^2}{\tilde{s}} \tilde{p}^\eta \right), \end{aligned} \quad (\text{A.3})$$

where $\tilde{p} = p_B - p_{K^*}$ and the form factors V and $A_{0,1,2}$ are functions of $\tilde{s} = \tilde{p}^2$. For $\bar{B}_s \rightarrow \phi \ell \bar{\ell}'$, the hadronic matrix elements have expressions similar to those for $\bar{B} \rightarrow \bar{K}^* \ell \bar{\ell}'$. In numerical work, we adopt the $\bar{B} \rightarrow \bar{K}^*$ and $\bar{B}_s \rightarrow \phi$ form-factors available in ref. [79] from combined fits to light-cone-sum-rule and lattice results. For the $b \rightarrow d$ transition $\bar{B} \rightarrow \rho \ell \bar{\ell}'$, the form factors are defined analogously to those in eq. (A.3), we again utilize the parametrization provided in ref. [79], and as in the $\bar{B} \rightarrow \pi$ case we have $\langle \rho^- | \bar{d} \gamma^\eta (1 - \gamma_5) b | \bar{B}^0 \rangle = \sqrt{2} \langle \rho^0 | \bar{d} \gamma^\eta (1 - \gamma_5) b | \bar{B}^0 \rangle$.

The formulas in the last two paragraphs lead us to the amplitudes in eqs. (3.5) and (3.6), respectively. Subsequently, we arrive at the differential rates

$$\frac{d\Gamma_{\bar{B} \rightarrow \bar{K} \ell \bar{\ell}'}}{d\dot{s}} = \frac{|\alpha_e \lambda_{qb} c_{\ell \ell'} G_F|^2 \sqrt{f_1 \tilde{g}}}{1536 \pi^5 m_B^3 \dot{s}^3} \left[F_1^2 f_2 \hat{g} + F_0^2 f_3 (m_B^2 - m_K^2)^2 \right], \quad (\text{A.4})$$

$$\begin{aligned} \frac{d\Gamma_{\bar{B} \rightarrow \bar{K}^* \ell \bar{\ell}'}}{d\dot{s}} &= \frac{|\alpha_e \lambda_{qb} c_{\ell \ell'} G_F|^2 \sqrt{f_1 \tilde{g}}}{6144 \pi^5 m_B^3 m_{K^*}^2 \dot{s}^3} \left\{ 2 \left[(\mathbb{A}^2 \tilde{g} s + 6\mathbb{C}^2 s + \mathbb{C} \mathbb{D} \tilde{g}) f_2 + 2A_0^2 f_3 \tilde{g} \right] m_{K^*}^2 \right. \\ &\quad \left. + [\mathbb{C}^2 + 2\mathbb{C} \mathbb{D} (s - m_B^2) + \mathbb{D}^2 \tilde{g}] f_2 \tilde{g} \right\}, \end{aligned} \quad (\text{A.5})$$

where $\dot{s} = (p_\ell + p_{\ell'})^2$,

$$\begin{aligned} f_1 &= \mathcal{K}(\dot{s}, m_\ell^2, m_{\ell'}^2), & f_2 &= 2\dot{s}^2 - (m_\ell^2 + m_{\ell'}^2)\dot{s} - (m_\ell^2 - m_{\ell'}^2)^2, \\ \hat{g} &= \mathcal{K}(m_B^2, m_K^2, \dot{s}), & f_3 &= 3(m_\ell^2 + m_{\ell'}^2)\dot{s} - 3(m_\ell^2 - m_{\ell'}^2)^2, \\ \tilde{g} &= \mathcal{K}(m_B^2, m_{K^*}^2, \dot{s}), & \mathcal{K}(x, y, z) &= x^2 + y^2 + z^2 - 2(xy + yz + xz). \end{aligned} \quad (\text{A.6})$$

In computing the branching fractions, we integrate eqs. (A.4) and (A.5) over the whole kinematical ranges $(m_\ell + m_{\ell'})^2 \leq \dot{s} \leq (m_B - m_K)^2$ and $(m_\ell + m_{\ell'})^2 \leq \dot{s} \leq (m_B - m_{K^*})^2$, respectively. We do likewise for $\bar{B}_s \rightarrow \phi \ell \bar{\ell}'$ and $\bar{B} \rightarrow (\pi, \rho) \ell \bar{\ell}'$.

To examine the purely leptonic decay $\bar{B}_q \rightarrow \ell \bar{\ell}'$, the hadronic matrix elements we need are $\langle 0 | \bar{q} \gamma^\eta b | \bar{B}_q \rangle = 0$ and $\langle 0 | \bar{q} \gamma^\eta \gamma_5 b | \bar{B}_q \rangle = -i f_{B_q} p_B^\eta$, where f_{B_q} is the decay constant. The amplitude in eq. (3.8) then follows, leading to the decay rate

$$\Gamma_{\bar{B}_q \rightarrow \ell \bar{\ell}'} = \frac{|\alpha_e \lambda_{qb} c_{\ell \ell'} f_{B_q} G_F|^2 \mathcal{K}^{1/2}(m_B^2, m_\ell^2, m_{\ell'}^2)}{32 \pi^3 m_B} \left[m_\ell^2 + m_{\ell'}^2 - \frac{(m_\ell^2 - m_{\ell'}^2)^2}{m_B^2} \right]. \quad (\text{A.7})$$

The kaon reaction $K_L \rightarrow e^- \mu^+$ proceeds from the components of $K_L \simeq (K^0 + \bar{K}^0)/\sqrt{2}$ both decaying into $e^- \mu^+$. Analogously to $B_q \rightarrow \ell \bar{\ell}'$, the necessary hadronic matrix elements

are $\langle 0|\bar{s}\gamma^n d|K^0\rangle = \langle 0|\bar{d}\gamma^n s|\bar{K}^0\rangle = 0$ and $\langle 0|\bar{s}\gamma^n\gamma_5 d|K^0\rangle = \langle 0|\bar{d}\gamma^n\gamma_5 s|\bar{K}^0\rangle = -if_K p_K^\eta$, where f_K is the kaon decay constant. Neglecting m_e , with the aid of eq. (A.7) we then find

$$\Gamma_{K_L \rightarrow e\bar{\mu}} = \frac{|\alpha_e c_{e\mu} f_K G_F \text{Re}\lambda_{sd}|^2 m_{K^0} m_\mu^2}{16\pi^3} \left(1 - \frac{m_\mu^2}{m_{K^0}^2}\right)^2. \quad (\text{A.8})$$

The amplitude for $K^+ \rightarrow \pi^+ e^- \mu^+$, also containing $c_{e\mu}$, is similar to that for $K^+ \rightarrow \pi^0 \nu \mu^+$ which arises mainly from SM interactions described by $\mathcal{L}_{su\ell}^{\text{SM}} = -\sqrt{8}G_F V_{us}^* \bar{s}\gamma^\eta P_L u \bar{\nu}\gamma_\eta P_L \mu + \text{H.c.}$ It follows that one can conveniently express the branching fraction $\mathcal{B}(K^+ \rightarrow \pi^+ e^- \mu^+)$ in relation to the well-measured $\mathcal{B}(K^+ \rightarrow \pi^0 \nu \mu^+)_{\text{exp}} = (3.353 \pm 0.034) \times 10^{-2}$, upon assuming isospin symmetry and neglecting m_e , without having to know the $K \rightarrow \pi$ form-factors in great detail. Thus, since $\langle \pi^+|\bar{s}\gamma^\eta d|K^+\rangle = \sqrt{2}\langle \pi^0|\bar{s}\gamma^\eta u|K^+\rangle$, we arrive at

$$\mathcal{B}(K^+ \rightarrow \pi^+ e^- \mu^+) \simeq \frac{|\alpha_e \lambda_{sd} c_{e\mu}|^2}{2\pi^2 |V_{us}|^2} \mathcal{B}(K^+ \rightarrow \pi^0 \nu \mu^+)_{\text{exp}}. \quad (\text{A.9})$$

Similarly, for $K_L \rightarrow \pi^0 e^- \mu^+$, since $\langle \pi^0|\bar{d}\gamma^\eta s|\bar{K}^0\rangle = -\langle \pi^0|\bar{s}\gamma^\eta d|K^0\rangle = \langle \pi^0|\bar{s}\gamma^\eta u|K^+\rangle$, we get

$$\mathcal{B}(K_L \rightarrow \pi^0 e^\pm \mu^\mp) = 2\mathcal{B}(K_L \rightarrow \pi^0 e^- \mu^+) \simeq \frac{\tau_{K_L} |\alpha_e c_{e\mu} \text{Im}\lambda_{sd}|^2}{\pi^2 \tau_{K^+} |V_{us}|^2} \mathcal{B}(K^+ \rightarrow \pi^0 \nu \mu^+)_{\text{exp}}. \quad (\text{A.10})$$

In numerical applications of these rate formulas, we employ the B^+ and $B_{s,d}$ (K_L and K^+) lifetimes from ref. [70, 71] ([39]), $\alpha_e = 1/133$, and $G_F = 1.1664 \times 10^{-5} \text{ GeV}^2$. For the decay constants, we adopt the central values of $f_{B_d} = (190.5 \pm 4.2) \text{ MeV}$, $f_{B_s} = (227.7 \pm 4.5) \text{ MeV}$, and $f_K = (156.3 \pm 0.9) \text{ MeV}$ from ref. [80].

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References

- [1] LHCb collaboration, *Test of lepton universality using $B^+ \rightarrow K^+ \ell^+ \ell^-$ decays*, *Phys. Rev. Lett.* **113** (2014) 151601 [[arXiv:1406.6482](https://arxiv.org/abs/1406.6482)] [[INSPIRE](https://inspirehep.net/literature/1247083)].
- [2] LHCb collaboration, *Measurement of Form-Factor-Independent Observables in the Decay $B^0 \rightarrow K^{*0} \mu^+ \mu^-$* , *Phys. Rev. Lett.* **111** (2013) 191801 [[arXiv:1308.1707](https://arxiv.org/abs/1308.1707)] [[INSPIRE](https://inspirehep.net/literature/118049)].
- [3] LHCb collaboration, *Angular analysis of the $B^0 \rightarrow K^{*0} \mu^+ \mu^-$ decay*, [LHCb-CONF-2015-002](https://arxiv.org/abs/1502.02890).
- [4] LHCb collaboration, *Differential branching fraction and angular analysis of the decay $B_s^0 \rightarrow \phi \mu^+ \mu^-$* , *JHEP* **07** (2013) 084 [[arXiv:1305.2168](https://arxiv.org/abs/1305.2168)] [[INSPIRE](https://inspirehep.net/literature/112188)].
- [5] LHCb collaboration, *Differential branching fractions and isospin asymmetries of $B \rightarrow K^{(*)} \mu^+ \mu^-$ decays*, *JHEP* **06** (2014) 133 [[arXiv:1403.8044](https://arxiv.org/abs/1403.8044)] [[INSPIRE](https://inspirehep.net/literature/122049)].
- [6] S. Descotes-Genon, J. Matias and J. Virto, *Understanding the $B \rightarrow K^{*} \mu^+ \mu^-$ Anomaly*, *Phys. Rev. D* **88** (2013) 074002 [[arXiv:1307.5683](https://arxiv.org/abs/1307.5683)] [[INSPIRE](https://inspirehep.net/literature/117001)].

- [7] W. Altmannshofer and D.M. Straub, *New physics in $B \rightarrow K^* \mu \mu$?*, *Eur. Phys. J. C* **73** (2013) 2646 [[arXiv:1308.1501](#)] [[INSPIRE](#)].
- [8] F. Beaujean, C. Bobeth and D. van Dyk, *Comprehensive Bayesian analysis of rare (semi)leptonic and radiative B decays*, *Eur. Phys. J. C* **74** (2014) 2897 [Erratum *ibid.* **C 74** (2014) 3179] [[arXiv:1310.2478](#)] [[INSPIRE](#)].
- [9] T. Hurth and F. Mahmoudi, *On the LHCb anomaly in $B \rightarrow K^* \ell^+ \ell^-$* , *JHEP* **04** (2014) 097 [[arXiv:1312.5267](#)] [[INSPIRE](#)].
- [10] R. Alonso, B. Grinstein and J. Martin Camalich, *SU(2) \times U(1) gauge invariance and the shape of new physics in rare B decays*, *Phys. Rev. Lett.* **113** (2014) 241802 [[arXiv:1407.7044](#)] [[INSPIRE](#)].
- [11] D. Ghosh, M. Nardecchia and S.A. Renner, *Hint of Lepton Flavour Non-Universality in B Meson Decays*, *JHEP* **12** (2014) 131 [[arXiv:1408.4097](#)] [[INSPIRE](#)].
- [12] T. Hurth, F. Mahmoudi and S. Neshatpour, *Global fits to $b \rightarrow s \ell \ell$ data and signs for lepton non-universality*, *JHEP* **12** (2014) 053 [[arXiv:1410.4545](#)] [[INSPIRE](#)].
- [13] S. Descotes-Genon, L. Hofer, J. Matias, and J. Virto, *Theoretical status of $B \rightarrow K^* \mu^+ \mu^-$: The path towards New Physics*, [arXiv:1503.03328](#) [[INSPIRE](#)].
- [14] G. Hiller and M. Schmaltz, *R_K and future $b \rightarrow s \ell \ell$ physics beyond the standard model opportunities*, *Phys. Rev. D* **90** (2014) 054014 [[arXiv:1408.1627](#)] [[INSPIRE](#)].
- [15] W. Altmannshofer and D.M. Straub, *New physics in $b \rightarrow s$ transitions after LHC run 1*, [arXiv:1411.3161](#) [[INSPIRE](#)].
- [16] W. Altmannshofer and D.M. Straub, *Implications of $b \rightarrow s$ measurements*, [arXiv:1503.06199](#) [[INSPIRE](#)].
- [17] P. Biancofiore, P. Colangelo and F. De Fazio, *Rare semileptonic $B \rightarrow K^* \ell^+ \ell^-$ decays in RS_c model*, *Phys. Rev. D* **89** (2014) 095018 [[arXiv:1403.2944](#)] [[INSPIRE](#)].
- [18] P. Biancofiore, P. Colangelo, F. De Fazio and E. Scrimieri, *Exclusive $b \rightarrow s \nu \bar{\nu}$ induced transitions in RS_c model*, *Eur. Phys. J. C* **75** (2015) 134 [[arXiv:1408.5614](#)] [[INSPIRE](#)].
- [19] J. Lyon and R. Zwicky, *Resonances gone topsy turvy – the charm of QCD or new physics in $b \rightarrow s \ell^+ \ell^-$?*, [arXiv:1406.0566](#) [[INSPIRE](#)].
- [20] S.L. Glashow, D. Guadagnoli and K. Lane, *Lepton Flavor Violation in B Decays?*, *Phys. Rev. Lett.* **114** (2015) 091801 [[arXiv:1411.0565](#)] [[INSPIRE](#)].
- [21] A. Celis, J. Fuentes-Martin, M. Jung and H. Serodio, *Family nonuniversal Z' models with protected flavor-changing interactions*, *Phys. Rev. D* **92** (2015) 015007 [[arXiv:1505.03079](#)] [[INSPIRE](#)].
- [22] A.J. Buras, J. Girrbach-Noe, C. Niehoff and D.M. Straub, *$B \rightarrow K^{(*)} \nu \bar{\nu}$ decays in the Standard Model and beyond*, *JHEP* **02** (2015) 184 [[arXiv:1409.4557](#)] [[INSPIRE](#)].
- [23] B. Gripaios, M. Nardecchia and S.A. Renner, *Composite leptoquarks and anomalies in B-meson decays*, *JHEP* **05** (2015) 006 [[arXiv:1412.1791](#)] [[INSPIRE](#)].
- [24] B. Bhattacharya, A. Datta, D. London and S. Shivashankara, *Simultaneous Explanation of the R_K and $R(D^{(*)})$ Puzzles*, *Phys. Lett. B* **742** (2015) 370 [[arXiv:1412.7164](#)] [[INSPIRE](#)].
- [25] A. Crivellin, G. D'Ambrosio and J. Heeck, *Explaining $h \rightarrow \mu^+ \tau^-$, $B \rightarrow K^* \mu^+ \mu^-$ and $B \rightarrow K \mu^+ \mu^- / B \rightarrow K e^+ e^-$ in a two-Higgs-doublet model with gauged $L_\mu - L_\tau$* , *Phys. Rev. Lett.* **114** (2015) 151801 [[arXiv:1501.00993](#)] [[INSPIRE](#)].

- [26] A. Crivellin, G. D'Ambrosio and J. Heeck, *Addressing the LHC flavor anomalies with horizontal gauge symmetries*, *Phys. Rev. D* **91** (2015) 075006 [[arXiv:1503.03477](#)] [[INSPIRE](#)].
- [27] S. Sahoo and R. Mohanta, *Scalar leptoquarks and the rare B meson decays*, *Phys. Rev. D* **91** (2015) 094019 [[arXiv:1501.05193](#)] [[INSPIRE](#)].
- [28] I. de Medeiros Varzielas and G. Hiller, *Clues for flavor from rare lepton and quark decays*, *JHEP* **06** (2015) 072 [[arXiv:1503.01084](#)] [[INSPIRE](#)].
- [29] D. Aristizabal Sierra, F. Staub and A. Vicente, *Shedding light on the $b \rightarrow s$ anomalies with a dark sector*, *Phys. Rev. D* **92** (2015) 015001 [[arXiv:1503.06077](#)] [[INSPIRE](#)].
- [30] S.M. Boucenna, J.W.F. Valle and A. Vicente, *Are the B decay anomalies related to neutrino oscillations?*, [arXiv:1503.07099](#) [[INSPIRE](#)].
- [31] D. Becirevic, S. Fajfer and N. Kosnik, *Lepton flavor nonuniversality in $b \rightarrow s\ell^+\ell^-$ processes*, *Phys. Rev. D* **92** (2015) 014016 [[arXiv:1503.09024](#)] [[INSPIRE](#)].
- [32] A. Crivellin, L. Hofer, J. Matias, U. Nierste, S. Pokorski and J. Rosiek, *Lepton-flavour violating B decays in generic Z' models*, [arXiv:1504.07928](#) [[INSPIRE](#)].
- [33] R.S. Chivukula and H. Georgi, *Composite Technicolor Standard Model*, *Phys. Lett. B* **188** (1987) 99 [[INSPIRE](#)].
- [34] L.J. Hall and L. Randall, *Weak scale effective supersymmetry*, *Phys. Rev. Lett.* **65** (1990) 2939 [[INSPIRE](#)].
- [35] A.J. Buras, P. Gambino, M. Gorbahn, S. Jager and L. Silvestrini, *Universal unitarity triangle and physics beyond the standard model*, *Phys. Lett. B* **500** (2001) 161 [[hep-ph/0007085](#)] [[INSPIRE](#)].
- [36] A.J. Buras, *Minimal flavor violation*, *Acta Phys. Polon. B* **34** (2003) 5615 [[hep-ph/0310208](#)] [[INSPIRE](#)].
- [37] A.L. Kagan, G. Perez, T. Volansky and J. Zupan, *General Minimal Flavor Violation*, *Phys. Rev. D* **80** (2009) 076002 [[arXiv:0903.1794](#)] [[INSPIRE](#)].
- [38] G. D'Ambrosio, G.F. Giudice, G. Isidori and A. Strumia, *Minimal flavor violation: An Effective field theory approach*, *Nucl. Phys. B* **645** (2002) 155 [[hep-ph/0207036](#)] [[INSPIRE](#)].
- [39] Particle Data Group collaboration, K.A. Olive et al., *Review of Particle Physics (RPP)*, *Chin. Phys. C* **38** (2014) 090001
- [40] V. Cirigliano, B. Grinstein, G. Isidori and M.B. Wise, *Minimal flavor violation in the lepton sector*, *Nucl. Phys. B* **728** (2005) 121 [[hep-ph/0507001](#)] [[INSPIRE](#)].
- [41] G.C. Branco, A.J. Buras, S. Jager, S. Uhlig and A. Weiler, *Another look at minimal lepton flavour violation, $l_i \rightarrow l_j \gamma$, leptogenesis and the ratio M_ν/Λ_{LFV}* , *JHEP* **09** (2007) 004 [[hep-ph/0609067](#)] [[INSPIRE](#)].
- [42] S. Davidson and F. Palorini, *Various definitions of Minimal Flavour Violation for Leptons*, *Phys. Lett. B* **642** (2006) 72 [[hep-ph/0607329](#)] [[INSPIRE](#)].
- [43] M.B. Gavela, T. Hambye, D. Hernandez and P. Hernández, *Minimal Flavour Seesaw Models*, *JHEP* **09** (2009) 038 [[arXiv:0906.1461](#)] [[INSPIRE](#)].
- [44] A.S. Joshipura, K.M. Patel and S.K. Vempati, *Type I seesaw mechanism for quasi degenerate neutrinos*, *Phys. Lett. B* **690** (2010) 289 [[arXiv:0911.5618](#)] [[INSPIRE](#)].

- [45] R. Alonso, G. Isidori, L. Merlo, L.A. Muñoz and E. Nardi, *Minimal flavour violation extensions of the seesaw*, *JHEP* **06** (2011) 037 [[arXiv:1103.5461](#)] [[INSPIRE](#)].
- [46] D. Aristizabal Sierra, A. Degee and J.F. Kamenik, *Minimal Lepton Flavor Violating Realizations of Minimal Seesaw Models*, *JHEP* **07** (2012) 135 [[arXiv:1205.5547](#)] [[INSPIRE](#)].
- [47] X.-G. He, C.-J. Lee, J. Tandean and Y.-J. Zheng, *Seesaw Models with Minimal Flavor Violation*, *Phys. Rev. D* **91** (2015) 076008 [[arXiv:1411.6612](#)] [[INSPIRE](#)].
- [48] X.-G. He, C.-J. Lee, S.-F. Li and J. Tandean, *Large electron electric dipole moment in minimal flavor violation framework with Majorana neutrinos*, *Phys. Rev. D* **89** (2014) 091901 [[arXiv:1401.2615](#)] [[INSPIRE](#)].
- [49] X.-G. He, C.-J. Lee, S.-F. Li and J. Tandean, *Fermion EDMs with Minimal Flavor Violation*, *JHEP* **08** (2014) 019 [[arXiv:1404.4436](#)] [[INSPIRE](#)].
- [50] P. Minkowski, $\mu \rightarrow e\gamma$ at a Rate of One Out of 10^9 Muon Decays?, *Phys. Lett. B* **67** (1977) 421 [[INSPIRE](#)].
- [51] T. Yanagida, *Horizontal gauge symmetry and masses of neutrinos*, in *Proceedings of the Workshop on the Unified Theory and the Baryon Number in the Universe*, edited by O. Sawada and A. Sugamoto, KEK, Tsukuba (1979), p. 95.
- [52] T. Yanagida, *Horizontal Symmetry and Masses of Neutrinos*, *Prog. Theor. Phys.* **64** (1980) 1103 [[INSPIRE](#)].
- [53] M. Gell-Mann, P. Ramond and R. Slansky, *Complex Spinors And Unified Theories*, in *Supergravity*, edited by P. van Nieuwenhuizen and D. Freedman, North-Holland, Amsterdam (1979), p. 315.
- [54] P. Ramond, *The Family Group in Grand Unified Theories*, [hep-ph/9809459](#) [[INSPIRE](#)].
- [55] S.L. Glashow, in *Proceedings of the 1979 Cargese Summer Institute on Quarks and Leptons*, edited by M. Levy et al., Plenum Press, New York (1980), p. 687.
- [56] R.N. Mohapatra and G. Senjanović, *Neutrino Mass and Spontaneous Parity Violation*, *Phys. Rev. Lett.* **44** (1980) 912 [[INSPIRE](#)].
- [57] J. Schechter and J.W.F. Valle, *Neutrino Masses in $SU(2) \times U(1)$ Theories*, *Phys. Rev. D* **22** (1980) 2227 [[INSPIRE](#)].
- [58] J. Schechter and J.W.F. Valle, *Neutrino Decay and Spontaneous Violation of Lepton Number*, *Phys. Rev. D* **25** (1982) 774 [[INSPIRE](#)].
- [59] B. Pontecorvo, *Neutrino Experiments and the Problem of Conservation of Leptonic Charge*, *Sov. Phys. JETP* **26** (1968) 984 [[INSPIRE](#)].
- [60] Z. Maki, M. Nakagawa and S. Sakata, *Remarks on the unified model of elementary particles*, *Prog. Theor. Phys.* **28** (1962) 870 [[INSPIRE](#)].
- [61] J.A. Casas and A. Ibarra, *Oscillating neutrinos and $\mu \rightarrow e\gamma$* , *Nucl. Phys. B* **618** (2001) 171 [[hep-ph/0103065](#)] [[INSPIRE](#)].
- [62] G. Colangelo, E. Nikolidakis and C. Smith, *Supersymmetric models with minimal flavour violation and their running*, *Eur. Phys. J. C* **59** (2009) 75 [[arXiv:0807.0801](#)] [[INSPIRE](#)].
- [63] L. Mercolli and C. Smith, *EDM constraints on flavored CP-violating phases*, *Nucl. Phys. B* **817** (2009) 1 [[arXiv:0902.1949](#)] [[INSPIRE](#)].

- [64] M.C. Gonzalez-Garcia, M. Maltoni and T. Schwetz, *Updated fit to three neutrino mixing: status of leptonic CP-violation*, *JHEP* **11** (2014) 052 [[arXiv:1409.5439](#)] [[INSPIRE](#)].
- [65] J. Charles et al., *Current status of the Standard Model CKM fit and constraints on $\Delta F = 2$ New Physics*, *Phys. Rev. D* **91** (2015) 073007 [[arXiv:1501.05013](#)] [[INSPIRE](#)].
- [66] <http://ckmfitter.in2p3.fr>.
- [67] CLEO collaboration, K.W. Edwards et al., *Search for lepton flavor violating decays of B mesons*, *Phys. Rev. D* **65** (2002) 111102 [[hep-ex/0204017](#)] [[INSPIRE](#)].
- [68] BABAR collaboration, B. Aubert et al., *Measurements of branching fractions, rate asymmetries and angular distributions in the rare decays $B \rightarrow K\ell^+\ell^-$ and $B \rightarrow K^*\ell^+\ell^-$* , *Phys. Rev. D* **73** (2006) 092001 [[hep-ex/0604007](#)] [[INSPIRE](#)].
- [69] BABAR collaboration, B. Aubert et al., *Search for the rare decay $B \rightarrow p\ell^+\ell^-$* , *Phys. Rev. Lett.* **99** (2007) 051801 [[hep-ex/0703018](#)] [[INSPIRE](#)].
- [70] HEAVY FLAVOR AVERAGING GROUP (HFAG) collaboration, Y. Amhis et al., *Averages of b-hadron, c-hadron and τ -lepton properties as of summer 2014*, [arXiv:1412.7515](#) [[INSPIRE](#)].
- [71] <http://www.slac.stanford.edu/xorg/hfag>.
- [72] G. Buchalla, A.J. Buras and M.E. Lautenbacher, *Weak decays beyond leading logarithms*, *Rev. Mod. Phys.* **68** (1996) 1125 [[hep-ph/9512380](#)] [[INSPIRE](#)].
- [73] F. Mescia and C. Smith, *Improved estimates of rare K decay matrix-elements from $Kl3$ decays*, *Phys. Rev. D* **76** (2007) 034017 [[arXiv:0705.2025](#)] [[INSPIRE](#)].
- [74] A.J. Buras, D. Buttazzo, J. Girrbach-Noe and R. Knegjens, *$K^+ \rightarrow \pi^+\nu\bar{\nu}$ and $K_L \rightarrow \pi^0\nu\bar{\nu}$ in the Standard Model: Status and Perspectives*, [arXiv:1503.02693](#) [[INSPIRE](#)].
- [75] E949 collaboration, A.V. Artamonov et al., *New measurement of the $K^+ \rightarrow \pi^+\nu\bar{\nu}$ branching ratio*, *Phys. Rev. Lett.* **101** (2008) 191802 [[arXiv:0808.2459](#)] [[INSPIRE](#)].
- [76] E391A collaboration, J.K. Ahn et al., *Experimental study of the decay $K_L^0 \rightarrow \pi^0\nu\bar{\nu}$* , *Phys. Rev. D* **81** (2010) 072004 [[arXiv:0911.4789](#)] [[INSPIRE](#)].
- [77] HPQCD collaboration, C. Bouchard, G.P. Lepage, C. Monahan, H. Na and J. Shigemitsu, *Rare decay $B \rightarrow K\ell^+\ell^-$ form factors from lattice QCD*, *Phys. Rev. D* **88** (2013) 054509 [[arXiv:1306.2384](#)] [[INSPIRE](#)].
- [78] A. Ali, A.Y. Parkhomenko and A.V. Rusov, *Precise Calculation of the Dilepton Invariant-Mass Spectrum and the Decay Rate in $B^\pm \rightarrow \pi^\pm\mu^+\mu^-$ in the SM*, *Phys. Rev. D* **89** (2014) 094021 [[arXiv:1312.2523](#)] [[INSPIRE](#)].
- [79] A. Bharucha, D.M. Straub and R. Zwicky, *$B \rightarrow V\ell^+\ell^-$ in the Standard Model from Light-Cone Sum Rules*, [arXiv:1503.05534](#) [[INSPIRE](#)].
- [80] S. Aoki et al., *Review of lattice results concerning low-energy particle physics*, *Eur. Phys. J. C* **74** (2014) 2890 [[arXiv:1310.8555](#)] [[INSPIRE](#)].