

# Stable Bianchi III attractor in $U(1)_R$ gauged supergravity

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**ABSTRACT:** Bianchi attractors are homogeneous but anisotropic extremal black brane horizons. We study the  $AdS_3 \times \mathbb{H}^2$  solution which is a special case of Bianchi type III in a  $U(1)_R$  gauged supergravity. For a wide range of values for certain free parameters in gauged supergravity, there exist a large class of solutions that satisfy conditions for the attractor mechanism to hold. We investigate the response of the solution against linearized fluctuations of the scalar field. The sufficient conditions for the attractor mechanism ensure that there exist a solution for the scalar fluctuation which dies out at the horizon. Furthermore, we solve for the gauge field and metric fluctuations that are sourced by scalar fluctuations and show that they are well behaved near the horizon. Thus, we have an example of a stable Bianchi attractor in gauged supergravity. We also analyze the Killing spinor equations of gauged supergravity in the background of our solution. We find that a radial Killing spinor consistent with the Bianchi III symmetry breaks supersymmetry.

**KEYWORDS:** Black Holes in String Theory, Supergravity Models, Gauge-gravity correspondence, Holography and condensed matter physics (AdS/CMT)

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## Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>Bianchi III solution in Einstein-Maxwell theory</b>	<b>4</b>
<b>3</b>	<b>Gauged supergravity and generalised attractors</b>	<b>7</b>
3.1	Gauged supergravity	7
3.2	Generalised attractors	9
3.3	Bianchi III solution in $U(1)_R$ gauged supergravity	10
<b>4</b>	<b>Linearized fluctuations about attractor value</b>	<b>12</b>
4.1	Gauge field fluctuations	13
4.2	Scalar fluctuations	14
4.3	Metric fluctuations	15
<b>5</b>	<b>Supersymmetry analysis</b>	<b>17</b>
<b>6</b>	<b>Summary and conclusions</b>	<b>19</b>
<b>A</b>	<b>Notations and conventions</b>	<b>20</b>
<b>B</b>	<b>Linearized Einstein equations</b>	<b>21</b>
<b>C</b>	<b>Coefficients of the linearized fluctuations</b>	<b>22</b>

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## 1 Introduction

Recent progress in the studies of extremal black holes in Anti de-Sitter space have witnessed the beginning of a dialogue between gravity and condensed matter physics. In gauge-gravity duality [1], extremal solutions provide the dual gravity description of zero temperature ground states of strongly coupled field theories. Many condensed matter theories exhibit a wide variety of phases. In particular, systems at quantum criticality can be strongly coupled and display novel phase transitions due to quantum fluctuations at zero temperature [2]. The subject is an active area of research and we refer the reader to some of the review articles for references [3–5].

Given such a large number of phases in condensed matter systems, it is reasonable to expect that there is also a similar zoo of extremal solutions in the dual gravity side. Earlier studies focused on extremal systems with translational and rotational symmetry that exhibit Lifshitz scaling and hyperscaling violations [6–12]. In some cases, such solutions have been embedded in string theory [13–20]. Extremal black branes dual to field theories with reduced symmetries are also equally interesting and have been studied [21–29].

Recently, new classes of extremal solutions exhibiting reduced symmetries have been found [24, 25]. These metrics are homogeneous but anisotropic extremal black brane horizons in five dimensions. They have been classified using the Bianchi classification [30, 31], which is well known in cosmological context and are now known as the “Bianchi attractors”. These geometries arise as *exact* solutions to gravity coupled to simple matter in the presence of a cosmological constant. Recently, Bianchi type metrics satisfying reasonable energy conditions have been shown to numerically interpolate to Lifshitz or  $AdS_2 \times S^3$  from which they can be connected to  $AdS_5$  [32]. This provides some evidence towards the expectation that they are attractor geometries.

The attractor mechanism has been thoroughly studied for extremal black holes in supergravity theories [33, 34].<sup>1</sup> Originally studied for supersymmetric black holes, it was understood later that the attractor mechanism is a consequence of extremality rather than supersymmetry [37], and has been shown to work for extremal non-supersymmetric black holes [38, 39]. Recently much progress has been made towards the generalization of attractor mechanism for gauged supergravity theories [40–50]. The simplest Bianchi type I geometries such as Lifshitz geometries have already been embedded in gauged supergravity [51, 52].

A prescription fairly general enough to capture the essential features of homogeneous geometries as generalised attractor solutions of gauged supergravity was given in [44]. The generalised attractors are defined as solutions to equation of motion when all the fields and curvature tensors are constants in tangent space. These solutions are characterised by constant anholonomy coefficients and are regular by construction. Following this prescription some of the Bianchi type geometries were embedded in five dimensional gauged supergravity [50].

The generalised attractor solutions existed at critical points rather than an absolute minimum of the attractor potential. The stability of such solutions for small perturbations of the scalar fields about the attractor value were studied [49]. By stability, we mean an investigation on the response of a system subject to linearized perturbations of the fields about their fixed point values. If the perturbations are regular as opposed to being divergent when one approaches the fixed point, then it is a stable attractor. There is also the notion of stability as described by the B.F. bound [53, 54]. However, we do not discuss this here.

It was found in [49], that the stress energy tensor in gauged supergravity depends on linearized scalar fluctuations due to the interaction terms. Therefore, for back-reaction to be small as one approaches the attractor geometry, the scalar fluctuations are required to be regular near the horizon. For the solutions constructed in [49, 50], the scalar fluctuations about the critical values were regular near the horizon only when the Bianchi geometries factorized as  $AdS_2 \times M$ , where  $M$  is a homogeneous space of dimension three. The factorized geometries have the unphysical property that the entropy does not vanish as the temperature goes to zero.

In this work, we seek to study an interesting class of Bianchi type solutions which do

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<sup>1</sup>See [35, 36] for recent reviews on the subject.

not factorize and are stable under linearized scalar fluctuations. Our strategy is to rely on the conventional wisdom of the physics of stable attractor points for extremal black holes. Namely, there are two sufficient conditions for the attractor mechanism [39]. First, there must exist a critical point of the effective potential. Second, the Hessian of the effective potential evaluated at the solution must have positive eigenvalues. These two conditions are always met by supersymmetric solutions. For non-supersymmetric extremal black hole solutions the above two conditions are sufficient to guarantee a stable attractor.

Keeping the above strategy in mind, we study the  $AdS_3 \times \mathbb{H}^2$  solution which is a special case of Bianchi type III in gauged supergravity. Supersymmetric  $AdS_3 \times \mathbb{H}^2$  solutions have been studied earlier in  $U(1)^3$  gauged supergravity [58]. In the context of wrapped branes,  $AdS_3 \times \mathbb{H}^2$  solutions have been constructed in type IIB supergravity compactified on  $S^5$  [66]. We consider the  $U(1)_R$  gauged supergravity [63, 64] for our study. We find that there are a large class of type III solutions that exist at a critical point corresponding to a minimum of the attractor potential. We do a linearized fluctuation analysis of the scalar field about its attractor value. For the scalar fluctuations sufficient conditions for a stable attractor guarantees the existence of a solution which dies out at the horizon. We then determine the gauge field and metric fluctuations that are sourced by scalar fluctuations. We find that the simplicity of the solution causes the source term in the gauge field fluctuations to vanish. Hence there are no gauge field fluctuations sourced by the scalar fluctuations in this case. As a result the metric fluctuations are sourced purely by scalar fluctuations. We solve the equations for the metric fluctuations with the source terms and show that they vanish as one approaches the horizon.

The results of the stability analysis are as follows. The Bianchi type III metric

$$ds^2 = -\hat{r}^{2\beta_t} d\hat{t}^2 + \frac{d\hat{r}^2}{\hat{r}^2} + d\hat{x}^2 + e^{-2\hat{x}} d\hat{y}^2 + \hat{r}^{2\beta_t} d\hat{z}^2 \tag{1.1}$$

which has the scaling symmetries

$$\hat{t} \rightarrow \frac{\hat{t}}{\alpha^{\beta_t}}, \quad \hat{r} \rightarrow \alpha \hat{r}, \quad \hat{x} \rightarrow \hat{x}, \quad \hat{y} \rightarrow \hat{y}, \quad \hat{z} \rightarrow \frac{\hat{z}}{\alpha^{\beta_t}}, \tag{1.2}$$

is a generalised attractor solution in gauged supergravity. The solution exists at a critical point  $\phi_c$  such that

$$\left. \frac{\partial \mathcal{V}_{attr}}{\partial \phi} \right|_{\phi_c} = 0, \quad \left. \frac{\partial^2 \mathcal{V}_{attr}}{\partial \phi^2} \right|_{\phi_c} > 0, \tag{1.3}$$

where  $\mathcal{V}_{attr}$  is the attractor potential. The above conditions are expressed in terms of some free parameters in gauged supergravity that are not fixed by any symmetries and are met for a wide range of values. Thus a class of solutions exists at a minimum of the attractor potential and the Hessian has a positive eigenvalue. The scalar field fluctuations  $\delta\phi$  about the attractor values are of the form

$$\delta\phi \sim \hat{r}^\Delta, \quad \Delta > 0. \tag{1.4}$$

The scalar fluctuations are regular near the horizon  $\hat{r} \rightarrow 0$ . All the metric fluctuations  $\gamma_{\mu\nu}$  are of the form

$$\gamma_{\mu\nu} \sim g_{\mu\nu} \hat{r}^\Delta \tag{1.5}$$

and are regular near the horizon. Thus, we have a class of Bianchi III solutions which are stable with respect to linearized fluctuations of scalar, gauge field and metric fluctuations about the attractor value. The solution is an example of a stable Bianchi attractor in gauged supergravity.

Given that the solution is a stable Bianchi attractor, we also investigate its supersymmetry properties. The study of supersymmetry of Bianchi attractors is very interesting since it can lead to solutions such as domain walls interpolating between Bianchi attractors and AdS. Besides, supersymmetry equations are first order differential equations and are often easier to solve. Earlier studies on supersymmetry of Bianchi type metrics have focused on the Bianchi I class. The simplest of which is AdS space. In this case, there are two types of Killing spinors, one which is purely radial and the other which depends on all coordinates [55, 56]. The radial spinor generates the Poincaré supersymmetries while the other spinor generates the conformal supersymmetries. The earliest works were on supersymmetric black string solutions whose near horizon geometries take the form  $AdS_3 \times \mathbb{H}^2$  [57, 58]. The Supersymmetry of the Bianchi I metrics such as Lifshitz, have also been studied in four dimensional gauged supergravity [51, 52]. In five dimensional  $U(1)^3$  gauged supergravity Bianchi I types such as  $AdS_2 \times \mathbb{R}^3$ ,  $AdS_3 \times \mathbb{R}^2$  have been found to be supersymmetric [59]. In the above cases the geometries preserve 1/4 of the supersymmetry and the Killing spinor equations were solved for a spinor which depended only on the radial direction.

In this spirit, we study the Killing spinor equations of  $\mathcal{N} = 2, U(1)_R$  gauged supergravity in the background of the Bianchi type III metric. We choose the radial ansatz for the Killing spinor, since it preserves the time translation symmetries and homogeneous symmetries of the type III metric. However, we find that the radial ansatz breaks all the supersymmetries. This suggests that the stable type III solution that we have constructed may be a non-supersymmetric attractor.

The paper is organised as follows. In section 2 we construct a magnetic Bianchi type III solution in Einstein-Maxwell theory with massless gauge fields. Following that, we provide some background in  $U(1)_R$  gauged supergravity and generalised attractors in section 3.1 and section 3.2. In the next subsection section 3.3 we embed the Bianchi type III solution in the  $U(1)_R$  gauged supergravity. We discuss the linearized fluctuation analysis of the gauge field, scalar field and metric in section 4. We analyze the Killing spinor equation in gauged supergravity with the background Bianchi type III metric in section 5. We conclude and summarize our results in section 6. We summarize some of the notations and conventions in section A. We provide some details regarding the linearized Einstein equations in section B and list the coefficients that appear in the metric fluctuations in section C.

## 2 Bianchi III solution in Einstein-Maxwell theory

We begin with a quick review of some elements of the Bianchi III symmetry. The Bianchi classification of real Lie algebras in three dimensions is well known in the literature [30, 31]. There are nine types of such algebras. In three dimensional Euclidean space, Killing vectors

that generate homogeneous symmetries close to form Lie algebras that are isomorphic to the Bianchi classification.

The Bianchi III algebra is generated by the Killing vectors  $X_i$

$$X_1 = \partial_{\hat{y}}, \quad X_2 = \partial_{\hat{z}}, \quad X_3 = \partial_{\hat{x}} + \hat{y}\partial_{\hat{y}}, \tag{2.1}$$

$$[X_1, X_3] = X_1. \tag{2.2}$$

The only non trivial Killing vector is the translation in the  $\hat{x}$  direction that is accompanied by a unit weight scaling in the  $\hat{y}$  direction. To write a metric which is manifestly invariant under this symmetry, one identifies the vector fields  $\tilde{e}_i$  that commute with the Killing vectors

$$[\tilde{e}_i, X_j] = 0. \tag{2.3}$$

The invariant vector fields for the type III case are

$$\tilde{e}_1 = e^{\hat{x}}\partial_{\hat{y}}, \quad \tilde{e}_2 = \partial_{\hat{z}}, \quad \tilde{e}_3 = \partial_{\hat{x}}, \tag{2.4}$$

$$[\tilde{e}_1, \tilde{e}_3] = -\tilde{e}_1, [\tilde{e}_1, \tilde{e}_2] = 0, [\tilde{e}_2, \tilde{e}_3] = 0. \tag{2.5}$$

Note that  $\tilde{e}_1$  and  $\tilde{e}_3$  form a sub-algebra. This sub-algebra is generated by the Killing vectors of the hyperbolic space  $\mathbb{H}^2$  in two dimensions. The two dimensional analogue of the Bianchi classification consists of two distinct algebras. One is a trivial algebra with commuting generators corresponding to  $\mathbb{R}^2$  and the other is the algebra that corresponds to  $\mathbb{H}^2$  [30].

The duals of the  $\tilde{e}_i$  are one forms  $\omega^i$

$$\omega^1 = e^{-\hat{x}}d\hat{y}, \quad \omega^2 = d\hat{z}, \quad \omega^3 = d\hat{x}, \tag{2.6}$$

that are invariant under the type III homogeneous symmetry. The invariant one forms satisfy the relation

$$d\omega^1 = \omega^1 \wedge \omega^3. \tag{2.7}$$

The metric written in terms of the invariant one forms

$$ds_3^2 = (\omega^1)^2 + (\omega^2)^2 + (\omega^3)^2 \tag{2.8}$$

is manifestly invariant under the homogeneous type III symmetries.

We are interested in five dimensional black brane horizons with homogeneous symmetries in the spatial directions. These geometries are obtained from gravity coupled to simple matter in the presence of a cosmological constant and are known as the Bianchi attractors [24, 25]. For the purposes of this article, we construct a simple type III solution in Einstein-Maxwell theory sourced by a single massless gauge field and a cosmological constant. We take the type III metric to be of the form

$$ds^2 = -\hat{r}^{2\beta_t} d\hat{t}^2 + \frac{d\hat{r}^2}{\hat{r}^2} + (\omega^3)^2 + (\omega^1)^2 + \hat{r}^{2\beta_2} (\omega^2)^2, \tag{2.9}$$

where  $\beta_t, \beta_2$  are positive exponents. For the case  $\beta_t = \beta_2$ , the metric becomes  $AdS_3 \times EAdS_2$ . To see this we substitute for the invariant one forms from (2.6) and make the coordinate transformation  $\hat{x} = \ln \hat{\rho}$  to get,

$$ds^2 = (-\hat{r}^{2\beta_t} dt^2 + \frac{d\hat{r}^2}{\hat{r}^2} + \hat{r}^{2\beta_t} d\hat{z}^2) + \left( \frac{d\hat{y}^2 + d\hat{\rho}^2}{\hat{\rho}^2} \right). \quad (2.10)$$

When one performs a Kaluza-Klein reduction of the above solution one gets the  $AdS_2 \times EAdS_2$  solution in four dimensions with hyper scale violation [25].

We now construct the Type III solution (2.9) in Einstein-Maxwell theory. The action is of the form

$$S = \int d^5x \sqrt{-g} (R - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \Lambda), \quad (2.11)$$

where  $\Lambda > 0$  corresponds to Anti de-Sitter space in our conventions. We are interested in a magnetic solution and we choose the gauge field to have components along the  $\omega^1$  direction

$$A = A_3 \omega^1, \quad (2.12)$$

where  $A_3$  is a constant.<sup>2</sup> The gauge field equations are automatically satisfied with this ansatz and the independent trace reversed Einstein equations are

$$\begin{aligned} A_3^2 - 6\beta_t(\beta_2 + \beta_t) + 2\Lambda &= 0, \\ A_3^2 - 6(\beta_2^2 + \beta_t^2) + 2\Lambda &= 0, \\ -A_3^2 - 3 + \Lambda &= 0, \\ A_3^2 - 6\beta_2(\beta_2 + \beta_t) + 2\Lambda &= 0. \end{aligned} \quad (2.13)$$

The  $\hat{t}\hat{t}$  and  $\hat{z}\hat{z}$  equations imply

$$\beta_2 = \beta_t \quad (2.14)$$

and the rest of the equations give the solution

$$\Lambda = 1 + 4\beta_t^2, \quad A_3 = \sqrt{-2 + 4\beta_t^2}. \quad (2.15)$$

Thus we have a magnetic type III solution sourced by a massless gauge field and parametrized by  $\beta_t$ , which satisfies the condition

$$\beta_t^2 > \frac{1}{2}, \quad (2.16)$$

such that  $A_3$  is real. In the following section, we construct a similar solution in  $U(1)_R$  gauged supergravity.

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<sup>2</sup>The notation  $A_3$  is just chosen for convenience.

### 3 Gauged supergravity and generalised attractors

#### 3.1 Gauged supergravity

In this section, we review essential material in  $\mathcal{N} = 2, d = 5$  gauged supergravity relevant for our purpose. The general supergravity coupled to vector, tensor, hyper multiplets with a gauging of the symmetries of the scalar manifold and R symmetry is discussed in [60]. We work with the  $\mathcal{N} = 2, d = 5$  gauged supergravity coupled to a single vector multiplet and a gauging of the  $U(1)_R$  symmetry [61–64].

The gravity multiplet consists of two gravitinos  $\psi_\mu^i$ ,  $i = 1, 2$ , and a graviphoton. The vector multiplet consists of a vector  $A_\mu$ , a real scalar  $\phi$  and the gaugini  $\lambda_i$ . The vector in the vector multiplet and the graviphoton are collectively represented by  $A_\mu^I$ ,  $I = 0, 1$ .

The scalars in the theory parametrize a very special manifold described by the cubic surface (see for eg [65])

$$N \equiv C_{IJK} h^I h^J h^K = 1, \quad h^I \equiv h^I(\phi). \quad (3.1)$$

The constants  $C_{IJK}$  are real and symmetric. The condition (3.1) is solved by going to a basis [61, 62], with  $h^I = \sqrt{\frac{2}{3}} \xi^I|_{N=1}$  such that,

$$N(\xi) = \sqrt{2} \xi^0 (\xi^1)^2 = 1, \quad (3.2)$$

where,

$$\xi^0 = \frac{1}{\sqrt{2} \phi^2}, \quad \xi^1 = \phi. \quad (3.3)$$

From the definition of the basis, we find that the  $h^I$  are related to the scalars  $\phi$  in the Lagrangian through

$$h^0 = \frac{1}{\sqrt{3} \phi^2}, \quad h^1 = \sqrt{\frac{2}{3}} \phi. \quad (3.4)$$

It is clear from the scalar parametrization that the only non-zero coefficients for  $C_{IJK}$  are  $C_{011} = \sqrt{3}/2$  and its permutations.

The ambient metric used to raise and lower the index  $I$  is defined through

$$a_{IJ} = -\frac{1}{2} \frac{\partial}{\partial h^I} \frac{\partial}{\partial h^J} \ln N|_{N=1}, \quad (3.5)$$

and takes the form

$$a_{IJ} = \begin{bmatrix} \phi^4 & 0 \\ 0 & \frac{1}{\phi^2} \end{bmatrix}. \quad (3.6)$$

The metric on the scalar manifold is obtained from the ambient metric (3.5) through

$$g_{xy} = h_x^I h_y^J a_{IJ}, \quad h_x^I = -\frac{\sqrt{3}}{2} \frac{\partial h^I}{\partial \phi^x}. \quad (3.7)$$

Since we only have a single scalar field, using the equations (3.4) and (3.5) we obtain

$$g(\phi) = \frac{3}{\phi^2}. \quad (3.8)$$



The field content and the various definitions above are identical to the ungauged theory. The difference in the gauged theory is the presence of a scalar potential. The process of gauging converts some of the global symmetries of the Lagrangian into local symmetries. One of the global symmetries enjoyed by the fermions in a  $\mathcal{N} = 2$  theory is the  $SU(2)_R$  symmetry. For the case of interest, we consider the gauging of the abelian  $U(1)_R \subset SU(2)_R$ . The R symmetry is gauged by replacing the usual Lorentz covariant derivative acting on the fermions with  $U(1)_R$  gauge covariant derivative as follows

$$\begin{aligned} \nabla_\mu \lambda^i &\rightarrow \nabla_\mu \lambda^i + g_R A_\mu(U(1)_R) \delta^{ij} \lambda_j, \\ \nabla_\mu \psi_\nu^i &\rightarrow \nabla_\mu \psi_\nu^i + g_R A_\mu(U(1)_R) \delta^{ij} \psi_{\nu j}. \end{aligned} \tag{3.9}$$

We refer the reader to section A for conventions on raising and lowering of the  $SU(2)$  indices. The  $\delta_{ij}$  in the covariant derivatives are the usual Kronecker delta symbols and  $g_R$  is the  $U(1)_R$  gauge coupling constant. The  $U(1)_R$  gauge field is a linear combination of the gauge fields in the theory

$$A_\mu(U(1)_R) = V_I A_\mu^I, \tag{3.10}$$

where the parameters  $V_I \in \mathbb{R}$  are free.<sup>3</sup>

The  $U(1)_R$  covariantization breaks the supersymmetry and therefore compensating terms are added to the Lagrangian for supersymmetric closure [64]. These terms result in the form of a potential for the scalar fields,

$$\mathcal{V}(\phi) = -2g_R^2 V_1 \left[ \frac{2\sqrt{2}V_0}{\phi} + \phi^2 V_1 \right]. \tag{3.11}$$

The potential has a critical point at

$$\phi_* = \left( \sqrt{2} \frac{V_0}{V_1} \right)^{1/3}. \tag{3.12}$$

The vacuum solution at this critical point is a supersymmetric Anti de-Sitter space with a cosmological constant  $\mathcal{V}(\phi_*) = -6g_R^2 V_1^2 \phi_*^2$ .

The bosonic part of the Lagrangian is

$$\begin{aligned} \hat{e}^{-1} \mathcal{L} &= -\frac{1}{2} R - \frac{1}{4} a_{IJ} F_{\mu\nu}^I F^{J\mu\nu} - \frac{1}{2} g(\phi) \partial_\mu \phi \partial^\mu \phi \\ &\quad - \mathcal{V}(\phi) + \frac{\hat{e}^{-1}}{6\sqrt{6}} C_{IJK} \epsilon^{\mu\nu\rho\sigma\tau} F_{\mu\nu}^I F_{\rho\sigma}^J A_\tau^K, \end{aligned} \tag{3.13}$$

where  $\hat{e} = \sqrt{-\det g_{\mu\nu}}$  and  $C_{IJK}$  are the constant symmetric coefficients that appeared in the definition of the scalar manifold (3.1).

We also list the various field equations for reference. The gauge field equations are

$$\partial_\mu (\hat{e} a_{IJ} F^{J\mu\nu}) = -\frac{1}{2\sqrt{6}} \epsilon^{\nu\lambda\rho\sigma\tau} F_{\lambda\rho}^J F_{\sigma\tau}^K. \tag{3.14}$$

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<sup>3</sup>When the gauging of R symmetry is accompanied by gauging of a non-abelian symmetry group  $K$  of the scalar manifold, the  $V_I$  are constrained by  $f_{JK}^I V_I = 0$ , where  $f_{JK}^I$  are structure constants of  $K$ .

The scalar field equations are

$$\frac{1}{\hat{e}} \partial_\mu (\hat{e} g(\phi) \partial^\mu \phi) - \frac{1}{2} \frac{\partial g(\phi)}{\partial \phi} \partial_\mu \phi \partial^\mu \phi - \frac{\partial}{\partial \phi} \left[ \frac{1}{4} a_{IJ} F_{\mu\nu}^I F^{J\mu\nu} + \mathcal{V}(\phi) \right] = 0 \quad (3.15)$$

and the Einstein equations are

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = T_{\mu\nu}, \quad (3.16)$$

where the stress energy tensor is

$$T_{\mu\nu} = g_{\mu\nu} \left[ \frac{1}{4} a_{IJ} F_{\mu\nu}^I F^{J\mu\nu} + \mathcal{V}(\phi) + \frac{1}{2} g(\phi) \partial_\mu \phi \partial^\mu \phi \right] - [a_{IJ} F_{\mu\lambda}^I F_\nu^{J\lambda} + g(\phi) \partial_\mu \phi \partial_\nu \phi]. \quad (3.17)$$

### 3.2 Generalised attractors

We now outline a brief discussion on a class of solutions to the field equations known as generalised attractors [44]. For a  $\mathcal{N} = 2, d = 5$  gauged supergravity with generic gauging of scalar manifolds and in the presence of hyper/tensor multiplets, the generalised attractor equations were shown to be algebraic in [50]. The  $U(1)_R$  gauged supergravity discussed in section 3.1 is a special case of the general gauged theory. The relevant field equations which follow from (3.13) can be simply obtained by setting the tensors, hyperscalars and the coupling constant associated with gauging of the scalar manifold to zero in the field equations derived in [50].

Generalised attractors are defined as solutions to equations of motion that reduce to algebraic equations when all the fields and Riemann tensor components are constants in tangent space

$$\phi = const, \quad A_a^I = const, \quad c_{ab}^c = const, \quad (3.18)$$

where  $a = 0, 1, \dots, 4$ , are tangent space indices. The  $c_{ab}^c$ , referred to as anholonomy coefficients are structure constants that appear in the Lie bracket of the vielbeins

$$[e_a, e_b] = c_{ab}^c e_c, \quad e_a \equiv e_a^\mu \partial_\mu. \quad (3.19)$$

In the absence of torsion, the spin connections are expressed in terms of the anholonomy coefficients

$$\omega_{abc} = \frac{1}{2} (c_{abc} - c_{acb} - c_{bca}), \quad (3.20)$$

which are constants.<sup>4</sup> Thus the curvature tensor components expressed in terms of the spin connections as

$$R_{abc}{}^d = -\omega_{ac}{}^e \omega_{be}{}^d + \omega_{bc}{}^e \omega_{ae}{}^d - c_{ab}{}^e \omega_{ec}{}^d \quad (3.21)$$

are constants in tangent space. Hence, the generalised attractor solutions characterised by constant anholonomy coefficients and are regular.

At the attractor points defined by (3.18) the scalar field equation (3.15) reduces to the condition

$$\frac{\partial \mathcal{V}_{attr}(\phi, A)}{\partial \phi} = 0 \quad (3.22)$$

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<sup>4</sup>The antisymmetry properties of the spin connection and anholonomy coefficients are  $\omega_a{}^{bc} = -\omega_a{}^{cb}$  and  $c_{ab}{}^c = -c_{ba}{}^c$  respectively.

on an attractor potential

$$\mathcal{V}_{attr}(\phi, A) = \frac{1}{4} a_{IJ} F_{\mu\nu}^I F^{J\mu\nu} + \mathcal{V}(\phi) . \tag{3.23}$$

Solving (3.22) gives the critical value of the scalar  $\phi_c$  in terms of the charges  $A$ . The critical point is a minimum when the Hessian has positive eigenvalues, which is also the condition for a stable attractor solution [39].

We also list the tangent space generalised attractor equations for the gauge and Einstein equations for reference. The gauge field equations are

$$a_{IJ}(\omega_a^a F^{Jbc} + \omega_a^b F^{Jac}) = 0 , \tag{3.24}$$

where the the field strength is

$$F_{ab}^I \equiv e_b^\mu e_a^\nu (\partial_\mu e_\nu^c - \partial_\nu e_\mu^c) A_c^I = c_{ab}^c A_c^I , \tag{3.25}$$

and the Chern-Simons term vanishes for the Bianchi attractors [50]. The Einstein equations are

$$R_{ab} - \frac{1}{2} R \eta_{ab} = T_{ab}^{attr} , \tag{3.26}$$

where

$$T_{ab}^{attr} = \mathcal{V}_{attr}(\phi, A) \eta_{ab} - a_{IJ} F_{ac}^I F_b^{Jc} . \tag{3.27}$$

In the following section we solve the algebraic attractor equations and find a Bianchi type III solution.

### 3.3 Bianchi III solution in $U(1)_R$ gauged supergravity

We choose the Bianchi type III ansatz as before in eq.(2.9). The gauge field ansatz is also same as before,

$$A_{\hat{y}}^I = e^{-x} A_3^I , \quad A_3^0 \equiv A_3 , \tag{3.28}$$

where we have turned on only the graviphoton  $I = 0$  for simplicity. Similar to the Einstein-Maxwell case studied in section 2 earlier, the gauge field equations (3.24) are trivially satisfied in the  $U(1)_R$  gauged supergravity as expected.

At the attractor point the scalars are constant. Hence the scalar equations reduce to extremization of the attractor potential (3.22). The attractor potential has the form

$$\mathcal{V}_{attr}(\phi, A) = \frac{1}{2\phi} (A_3^2 \phi^5 - 4g_R^2 V_1 (2\sqrt{2}V_0 + V_1 \phi^3)) . \tag{3.29}$$

The second term is the contribution of the potential (3.11). We would like to briefly contrast the nature of the possible critical points possible from (3.29) as compared to some of the earlier works [49, 50]. The Bianchi attractors constructed in gauged supergravity were attractor solutions such that the critical points of the attractor potential coincided with the critical points of the scalar potential (3.11). This was a simplification which was possible because the attractor potential had additional terms due to gauging of the scalar manifold or with multiple field strengths in the absence of such gauging. For the  $U(1)_R$

case with just one gauge field considered here, the attractor potential (3.29) does not allow such critical points for non-trivial gauge fields. It is also important to note that in [50], the Bianchi III solution could not be obtained from the Bianchi VI<sub>h</sub> solution by taking the limit  $h \rightarrow 0$  since it resulted in a singular gauge field.<sup>5</sup>

The scalar field equation then reduces to,

$$\frac{\partial \mathcal{V}_{attr}(\phi, A)}{\partial \phi} = \frac{2}{\phi^2} (A_3^2 \phi^5 + 4g_R^2 V_1 (\sqrt{2}V_0 - V_1 \phi^3)) = 0. \quad (3.30)$$

In principle, one can solve for  $\phi$  from the above equation. In practice, it is much easier to solve the scalar equation simultaneously with the Einstein equation to get nice compact expressions.

The independent Einstein equations (3.16) are

$$\begin{aligned} 2(1 + \beta_2^2)\phi + A_3^2 \phi^5 - 4g_R^2 V_1 (2\sqrt{2}V_0 + V_1 \phi^3) &= 0, \\ 2(1 + \beta_2 \beta_t)\phi + A_3^2 \phi^5 - 4g_R^2 V_1 (2\sqrt{2}V_0 + V_1 \phi^3) &= 0, \\ 2(\beta_2^2 + \beta_2 \beta_t + \beta_t^2)\phi - A_3^2 \phi^5 - 4g_R^2 V_1 (2\sqrt{2}V_0 + V_1 \phi^3) &= 0, \\ 2(1 + \beta_t^2)\phi + A_3^2 \phi^5 - 4g_R^2 V_1 (2\sqrt{2}V_0 + V_1 \phi^3) &= 0. \end{aligned} \quad (3.31)$$

From the  $\hat{t}\hat{t}$  and the  $\hat{z}\hat{z}$  equations we get

$$\beta_2 = \beta_t. \quad (3.32)$$

The equations now simplify to

$$\begin{aligned} 2(1 + \beta_t^2)\phi + A_3^2 \phi^5 - 4g_R^2 V_1 (2\sqrt{2}V_0 + V_1 \phi^3) &= 0, \\ 6\beta_t^2 \phi - A_3^2 \phi^5 - 4g_R^2 V_1 (2\sqrt{2}V_0 + V_1 \phi^3) &= 0. \end{aligned} \quad (3.33)$$

We solve for  $A_3$  from the above equations to obtain

$$A_3 = \frac{\sqrt{-1 + 2\beta_t^2}}{\phi^2}, \quad (3.34)$$

and

$$(1 + 4\beta_2^2)\phi - 4g_R^2 V_1 (2\sqrt{2}V_0 + V_1 \phi^3) = 0. \quad (3.35)$$

This equation can be solved together with the scalar equation (3.30) to determine the critical point

$$\phi_c = 4\sqrt{2}g_R^2 V_0 V_1, \quad \beta_t = \frac{1}{2} \sqrt{1 + 128g_R^6 V_0^2 V_1^4} \quad (3.36)$$

For the gauge field to be real we require

$$\beta_t^2 > \frac{1}{2}. \quad (3.37)$$

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<sup>5</sup>The Bianchi VI<sub>h</sub> algebra has a free parameter  $h$ . The Bianchi V algebra is obtained in the limit  $h \rightarrow 1$ , while the Bianchi III algebra is obtained in the limit  $h \rightarrow 0$  [30, 31].

We note that the same condition was obtained for the Type III solution in Einstein-Maxwell theory (2.16). It is also clear from (3.36) that the condition is satisfied for arbitrary values of the gauged supergravity parameters  $g_R, V_0, V_1$ .

We now examine the nature of the critical point given by eqs.(3.36) and (3.34). The Hessian evaluated at the critical point

$$\left. \frac{\partial^2 \mathcal{V}_{attr}(\phi, A)}{\partial \phi^2} \right|_{\phi_c} = \frac{-7 + 8\beta_t^2}{\phi_c^2} \tag{3.38}$$

is positive provided we choose

$$\beta_t^2 > \frac{7}{8}. \tag{3.39}$$

We choose this condition for  $\beta_t^2$ , since above this bound we also satisfy the general condition for a stable attractor solution. In terms of the gauged supergravity parameters the condition on  $\beta_t^2$  translates to

$$g_R^6 V_0^2 V_1^4 > \frac{5}{256}, \tag{3.40}$$

which can be satisfied for a wide range of values for the parameters  $g_R, V_0, V_1$ , since none of them are constrained in anyway. Thus, for various values of  $g_R, V_0, V_1$  satisfying (3.40) we find a class of type III Bianchi metrics as generalised attractor solutions in  $U(1)_R$  gauged supergravity.

The attractor potential evaluated at the critical point given by (3.34) and (3.36) takes a remarkably simple form

$$\mathcal{V}_{attr}|_{\phi_c} = -(1 + \beta_t^2), \tag{3.41}$$

which will be useful later. To summarize, the type III solution is

$$\begin{aligned} ds^2 &= -\hat{r}^{2\beta_t} dt^2 + \frac{d\hat{r}^2}{\hat{r}^2} + (\omega^3)^2 + (\omega^1)^2 + \hat{r}^{2\beta_2} (\omega^2)^2, \\ A_3 &= \frac{\sqrt{-1 + 2\beta_t^2}}{\phi_c^2}, \quad \phi_c = 4\sqrt{2}g_R^2 V_0 V_1, \\ \beta_2 &= \beta_t, \quad \beta_t = \frac{1}{2}\sqrt{1 + 128g_R^6 V_0^2 V_1^4}, \quad \beta_t^2 > \frac{7}{8}. \end{aligned} \tag{3.42}$$

We have seen that the Hessian of the effective potential evaluated on this solution has a positive eigenvalue suggesting that it is a stable attractor. In the following section we provide more evidence by considering linearized fluctuations of the scalar, gauge and metric fields about their attractor values and showing that they are well behaved near the horizon.

#### 4 Linearized fluctuations about attractor value

In this section, we study the linearized fluctuations of the gauge field, scalar field and metric about their attractor values. For  $\mathcal{N} = 2, d = 5$  gauged supergravity coupled to vector multiplets with a generic gauging of the scalar manifold and gauging of R symmetry the linearized equations were derived in [49]. The corresponding equations for the  $U(1)_R$  case that follow from (3.13) can be simply obtained by setting the coupling constant associated with gauging of the scalar manifold to zero.

The linearized fluctuations about the attractor values are of the following form,

$$\begin{aligned} &\phi_c + \epsilon \delta\phi(\hat{r}), \\ &A_\mu + \epsilon \delta A_\mu(\hat{r}), \\ &g_{\mu\nu} + \epsilon \gamma_{\mu\nu}(\hat{r}), \end{aligned} \tag{4.1}$$

where  $\epsilon < 1$ . The attractor values of the scalar field and gauge field are  $\phi_c$ ,  $A_\mu$ , respectively. We take the near horizon metric  $g_{\mu\nu}$  as the type III Bianchi metric (3.42). We have chosen all the fluctuations to depend purely on the radial direction  $\hat{r}$ , since it is this behavior that is most interesting from the point of view of an RG flow. Also, this is the first thing to attempt before going to much complicated cases. The magnetic type III solution (3.42) offers lot of simplifications. In particular, we will see that the source term in the gauge field fluctuations vanishes and this simplifies the procedure of solving for the metric fluctuations later on.

#### 4.1 Gauge field fluctuations

The equation satisfied by the linearized gauge field fluctuations is

$$a_{IJ}|_{\phi_c} \nabla_\mu F_\delta^{\mu\nu J} = - \frac{\partial a_{IJ}}{\partial \phi} \Big|_{\phi_c} \nabla_\mu (F^{\mu\nu J} \delta\phi), \tag{4.2}$$

where

$$F_\delta^{\mu\nu J} = \partial^\mu \delta A^\nu - \partial^\nu \delta A^\mu, \tag{4.3}$$

and  $F^{\mu\nu J}$  is the field strength corresponding to the attractor solution. We can simplify (4.2) using the attractor equation for the gauge field (3.14), where the Chern-Simons term vanishes and the scalars are independent of spacetime coordinates at the attractor point. Thus we have

$$a_{IJ}|_{\phi_c} \nabla_\mu F_\delta^{\mu\nu J} = - \frac{\partial a_{IJ}}{\partial \phi} \Big|_{\phi_c} F^{\mu\nu J} \partial_\mu \delta\phi. \tag{4.4}$$

For the gauge field ansatz (3.28), the non-trivial field strength component is only along the  $F^{\hat{x}\hat{y}}$  direction. Since the scalar field fluctuation in (4.1) depends only on the radial direction, the right hand side of (4.4) vanishes. Hence, there are no gauge field fluctuations that are sourced by the scalar fluctuations in this case. Thus the linearized fluctuations of the gauge field about the attractor value satisfy the attractor equation

$$a_{IJ}|_{\phi_c} \nabla_\mu F_\delta^{\mu\nu J} = 0. \tag{4.5}$$

From the point of view of the attractor mechanism in supergravity [33, 34], it is the behavior of the scalar fields that is most relevant for our case. Hence, we do not consider any independent gauge field fluctuations here. Thus, we can drop the gauge field fluctuations for the rest of the analysis in the following sections.

In a general situation as opposed to the simple example considered here, the source term in (4.4) need not vanish. In such a case, however one may still be able to solve the problem in certain situations where the scalar fluctuation equations decouple from

gauge field fluctuations at linearized level [49]. So solving the linearized equation for scalar fluctuations determines the source term in the gauge field fluctuation, which can then in principle be solved. However, the situation becomes more complicated for the metric fluctuations since both the gauge field and scalar fluctuations will enter through the stress tensor.

Another notable simplification is that currently we are working with the  $U(1)_R$  gauged supergravity. When the gauging of the symmetries of scalar manifold is also considered there are additional terms in (4.2) and solving for the gauge field fluctuations is much harder in the presence of additional scalar source terms.<sup>6</sup>

## 4.2 Scalar fluctuations

We will now solve the linearized equations for the scalar fluctuations about the attractor value  $\phi_c$ . The linearized equation for the scalar field obtained from (3.13) takes a remarkably simple form,

$$g(\phi_c)\nabla_\mu\nabla^\mu\delta\phi - \left.\frac{\partial^2\mathcal{V}_{attr}}{\partial\phi^2}\right|_{\phi_c}\delta\phi = 0, \tag{4.6}$$

where  $g(\phi)$  and the attractor potential are defined in (3.8) and (3.29) respectively. Using (3.38), we define

$$\lambda = \left.\frac{1}{g(\phi_c)}\frac{\partial^2\mathcal{V}_{attr}}{\partial\phi^2}\right|_{\phi_c} = \frac{-7 + 8\beta_t^2}{3} \tag{4.7}$$

which is positive for the solution of interest, since  $\beta_t^2 > \frac{7}{8}$ . Using the expression for the metric (3.3), equation (4.6) can be simplified as

$$[\hat{r}^2\partial_{\hat{r}}^2 + (1 + 2\beta_t)\hat{r}\partial_{\hat{r}} - \lambda]\delta\phi = 0. \tag{4.8}$$

The general solution for this equation is of the form

$$\delta\phi = C_1\hat{r}\sqrt{\lambda+\beta_t^2-\beta_t} + C_2\hat{r}^{-\sqrt{\lambda+\beta_t^2-\beta_t}}. \tag{4.9}$$

The type III metric (2.9) is written in a coordinate system such that the horizon is located at  $\hat{r} = 0$ . We require the scalar fluctuations (4.1) to vanish as  $\hat{r}^\Delta$  for  $\Delta > 0$  such that the scalar field approaches its attractor value as  $\hat{r} \rightarrow 0$ . Therefore, we choose  $C_2 = 0$ . The other constant  $C_1$  cannot be fixed at this stage as the equation (4.6) is valid only near the horizon. However, we can choose  $C_1 = C_s \in \mathbb{R}$  since the scalar fields in five dimensional gauged supergravity are real. In addition, for non-trivial fluctuations  $C_s \neq 0$ . Thus the scalar fluctuations which are well behaved near the horizon are of the form

$$\delta\phi = C_s\hat{r}^\Delta, \quad \Delta = \sqrt{\lambda + \beta_t^2 - \beta_t}. \tag{4.10}$$

Note that, the condition obtained from (3.38) indeed ensures that the scalar fluctuations are well behaved as  $\hat{r} \rightarrow 0$  near the horizon.

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<sup>6</sup>See for example, eq. (3.5) of [49].

To fix the constants in the solution completely, one has to solve the scalar equation in the background of a solution which interpolates from Bianchi III to AdS with appropriate boundary conditions. Such interpolating metrics obeying reasonable energy conditions that interpolate to Lifshitz or  $AdS_2 \times S^3$  which can then be connected to AdS have been constructed numerically in [32]. However, they are not yet known to arise as solutions to Einstein gravity coupled to some simple matter theory.

### 4.3 Metric fluctuations

In this section, we solve the linearized metric fluctuations about the type III metric, that are sourced by scalar fluctuations (4.10). The linearized fluctuation equations of the metric have the form [49],

$$\begin{aligned} \nabla^\alpha \nabla_\alpha \bar{\gamma}_{\mu\nu} + 2R_{(\mu \nu)}^{\alpha \beta} \bar{\gamma}_{\beta\alpha} - 2R_{(\mu}^{\beta} \bar{\gamma}_{\nu)\beta} + g_{\mu\nu} \left( R_{\alpha\beta} \bar{\gamma}^{\alpha\beta} - \frac{2}{3} R \bar{\gamma} \right) + R \bar{\gamma}_{\mu\nu} \\ + 2(\dot{T}_{\mu\nu}^{attr}(g_{\alpha\beta} + \epsilon\gamma_{\alpha\beta})|_{\epsilon=0} + \dot{T}_{\mu\nu}^{attr}(\phi_c + \epsilon\delta\phi)|_{\epsilon=0}) = 0, \end{aligned} \quad (4.11)$$

where

$$\bar{\gamma}_{\mu\nu} = \gamma_{\mu\nu} - \frac{1}{2} \gamma g_{\mu\nu}, \quad \gamma = g^{\mu\nu} \gamma_{\mu\nu}, \quad \bar{\gamma} = -\frac{3}{2} \gamma. \quad (4.12)$$

The dots indicate derivatives with respect to  $\epsilon$ . The covariant derivatives, raising and lowering are with respect to the near horizon metric  $g_{\mu\nu}$ . The Riemann tensor, Ricci tensor and curvature that appear in (4.11) are also with respect to  $g_{\mu\nu}$ .

The contribution of the linearized metric fluctuations from the stress energy tensor are

$$\begin{aligned} \dot{T}_{\mu\nu}^{attr}(g_{\alpha\beta} + \epsilon\gamma_{\alpha\beta})|_{\epsilon=0} = \mathcal{V}_{attr}|_{\phi_c} \left( \bar{\gamma}_{\mu\nu} - \frac{2\bar{\gamma}}{3} g_{\mu\nu} \right) \\ - (\bar{\gamma}_{\lambda\sigma} - \frac{\bar{\gamma}}{3} g_{\lambda\sigma}) \left( \frac{1}{2} T_{attr}^{\lambda\sigma}|_{\phi_c} g_{\mu\nu} + a_{IJ}|_{\phi_c} F_\mu^I{}^\lambda F_\nu^J{}^\sigma \right). \end{aligned} \quad (4.13)$$

where

$$T_{\mu\nu}^{attr} = \mathcal{V}_{attr}|_{\phi_c} g_{\mu\nu} - a_{IJ}|_{\phi_c} F_{\mu\lambda}^I F_\nu^{\lambda J} \quad (4.14)$$

and  $\mathcal{V}_{attr}|_{\phi_c}$  is defined by (3.41). The contribution of the linearized scalar fluctuations from the stress energy tensor are

$$\dot{T}_{\mu\nu}^{attr}(\phi_c + \epsilon\delta\phi)|_{\epsilon=0} = \frac{\partial \mathcal{V}_{attr}}{\partial \phi} \Big|_{\phi_c} g_{\mu\nu} \delta\phi - \frac{\partial a_{IJ}}{\partial \phi} \Big|_{\phi_c} F_{\mu\lambda}^I F_\nu^{\lambda J} \delta\phi, \quad (4.15)$$

which can be further simplified using the attractor equation (3.22) to get

$$\dot{T}_{\mu\nu}^{attr}(\phi_c + \epsilon\delta\phi)|_{\epsilon=0} = -\frac{\partial a_{IJ}}{\partial \phi} \Big|_{\phi_c} F_{\mu\lambda}^I F_\nu^{\lambda J} \delta\phi. \quad (4.16)$$

We can now solve for the metric fluctuations by plugging in the scalar fluctuations (4.10). First, let us simplify the form of (4.11) by making a few observations. We note that the type III metric in its explicit form

$$ds^2 = -\hat{r}^{2\beta_t} dt^2 + \frac{d\hat{r}^2}{\hat{r}^2} + d\hat{x}^2 + e^{-2\hat{x}} d\hat{y}^2 + \hat{r}^{2\beta_t} d\hat{z}^2 \quad (4.17)$$



is diagonal. Therefore, It is reasonable to expect fluctuations only along the diagonal directions. Hence we can choose the fluctuations  $\gamma_{\mu\nu}$  to be symmetric. As a result the antisymmetrized terms in (4.11) vanish, as can be checked explicitly. Thus we have

$$\begin{aligned} \nabla^\alpha \nabla_\alpha \bar{\gamma}_{\mu\nu} + g_{\mu\nu} \left( R_{\alpha\beta} \bar{\gamma}^{\alpha\beta} - \frac{2}{3} R \bar{\gamma} \right) + R \bar{\gamma}_{\mu\nu} + 2(\dot{T}_{\mu\nu}^{attr}(g_{\alpha\beta} + \epsilon\gamma_{\alpha\beta})|_{\epsilon=0} \\ + \dot{T}_{\mu\nu}^{attr}(\phi_c + \epsilon\delta\phi)|_{\epsilon=0}) = 0, \end{aligned} \quad (4.18)$$

with the contributions from the stress energy tensor corresponding to metric and scalar fluctuations as given by (4.13) and (4.16) respectively.

We choose the fluctuation terms of the metric in  $g_{\mu\nu} + \epsilon\gamma_{\mu\nu}(\hat{r})$  to be of the form

$$\begin{aligned} \gamma_{\hat{t}\hat{t}} &= C_{\hat{t}} \hat{r}^{2\beta_t} \tilde{\gamma}_{\hat{t}\hat{t}}(\hat{r}), \\ \gamma_{\hat{r}\hat{r}} &= C_{\hat{r}} \frac{1}{\hat{r}^2} \tilde{\gamma}_{\hat{r}\hat{r}}(\hat{r}), \\ \gamma_{\hat{x}\hat{x}} &= C_{\hat{x}} \tilde{\gamma}_{\hat{x}\hat{x}}(\hat{r}), \\ \gamma_{\hat{y}\hat{y}} &= C_{\hat{y}} e^{-2\hat{x}} \tilde{\gamma}_{\hat{y}\hat{y}}(\hat{r}), \\ \gamma_{\hat{z}\hat{z}} &= C_{\hat{z}} \hat{r}^{2\beta_t} \tilde{\gamma}_{\hat{z}\hat{z}}(\hat{r}), \end{aligned} \quad (4.19)$$

where  $C_{\hat{t}}, C_{\hat{r}}, C_{\hat{x}}, C_{\hat{y}}, C_{\hat{z}}$  are constants which are to be determined in terms of the gauged supergravity parameters  $g_R, V_0, V_1$ , and the coefficient  $C_s$  in the scalar fluctuation (4.10).

Because of the way the perturbations have been chosen in (4.19), one can contract the Einstein equations with the vielbeins and write the final expressions in terms of the  $\tilde{\gamma}_{\mu\nu}(\hat{r})$ . We also observe that the source term from the scalar fluctuation (4.16) appears only in the  $\hat{x}\hat{x}$  and  $\hat{y}\hat{y}$  directions. While the source goes like  $\hat{r}^\Delta$ , the Einstein equations will contain terms like  $\hat{r}^2 \partial_{\hat{r}}^2 \tilde{\gamma}_{\mu\nu}$ ,  $\hat{r} \partial_{\hat{r}} \tilde{\gamma}_{\mu\nu}$ ,  $\tilde{\gamma}_{\mu\nu}$ . Hence one expects the fluctuations  $\tilde{\gamma}_{\mu\nu}$  to also go like  $\hat{r}^\Delta$ . This can be checked by observing the explicit equations, which are rather messy. We refer the reader to the appendix section B for more details. Thus all the metric fluctuations should have the behavior

$$\tilde{\gamma}_{\hat{t}\hat{t}} = \tilde{\gamma}_{\hat{r}\hat{r}} = \tilde{\gamma}_{\hat{x}\hat{x}} = \tilde{\gamma}_{\hat{y}\hat{y}} = \tilde{\gamma}_{\hat{z}\hat{z}} = \hat{r}^\Delta. \quad (4.20)$$

We now substitute (4.20) in eqs. (4.18) and reduce them to an algebraic system,

$$\begin{aligned} 4(\beta_t^2(3C_{\hat{r}} + 3C_{\hat{t}} + C_{\hat{x}} + C_{\hat{y}} + 3C_{\hat{z}}) + 2C_{\hat{t}} + C_{\hat{x}} + C_{\hat{y}}) \\ + 6\beta_t \Delta(C_{\hat{r}} - C_{\hat{t}} + C_{\hat{x}} + C_{\hat{y}} + C_{\hat{z}}) + \Delta^2(C_{\hat{r}} - C_{\hat{t}} + C_{\hat{x}} + C_{\hat{y}} + C_{\hat{z}}) = 0, \end{aligned}$$

$$\begin{aligned} C_{\hat{r}}(-4(5\beta_t^2 + \beta_t + 1) + 2(\beta_t - 2)\Delta + \Delta^2) - 2(\beta_t - 2)\Delta(C_{\hat{t}} + C_{\hat{x}} + C_{\hat{y}} + C_{\hat{z}}) \\ + 4\beta_t(\beta_t(-C_{\hat{t}} + C_{\hat{x}} + C_{\hat{y}} - C_{\hat{z}}) + C_{\hat{t}} + C_{\hat{x}} + C_{\hat{y}} + C_{\hat{z}}) \\ + \Delta^2(-(C_{\hat{t}} + C_{\hat{x}} + C_{\hat{y}} + C_{\hat{z}})) - 4(C_{\hat{t}} + 2(C_{\hat{x}} + C_{\hat{y}}) + C_{\hat{z}}) = 0, \end{aligned}$$

$$\begin{aligned} (16 - 32\beta_t^2)C_s - \phi_c((4\beta_t^2 + 2\beta_t\Delta + \Delta^2)(C_{\hat{r}} + C_{\hat{t}} + C_{\hat{y}} + C_{\hat{z}}) \\ + C_{\hat{x}}(12\beta_t^2 - 2\beta_t\Delta - \Delta^2 + 12)) = 0, \end{aligned}$$

$$\begin{aligned}
& (16 - 32\beta_t^2)C_s - \phi_c(4\beta_t^2(C_{\hat{r}} + C_{\hat{t}} + C_{\hat{x}} + 3C_{\hat{y}} + C_{\hat{z}}) + 2\beta_t\Delta(C_{\hat{r}} + C_{\hat{t}} + C_{\hat{x}} - C_{\hat{y}} + C_{\hat{z}}) \\
& \quad + \Delta^2(C_{\hat{r}} + C_{\hat{t}} + C_{\hat{x}} - C_{\hat{y}} + C_{\hat{z}}) + 6(C_{\hat{r}} + C_{\hat{t}} + C_{\hat{x}} + C_{\hat{y}} + C_{\hat{z}})) = 0, \\
& -4\beta_t^2(3C_{\hat{r}} + 3C_{\hat{t}} + C_{\hat{x}} + C_{\hat{y}} + 3C_{\hat{z}}) - 6\beta_t\Delta(C_{\hat{r}} + C_{\hat{t}} + C_{\hat{x}} + C_{\hat{y}} - C_{\hat{z}}) \\
& \quad - \Delta^2(C_{\hat{r}} + C_{\hat{t}} + C_{\hat{x}} + C_{\hat{y}} - C_{\hat{z}}) - 4(C_{\hat{x}} + C_{\hat{y}} + 2C_{\hat{z}}) = 0, \quad (4.21)
\end{aligned}$$

which can be solved to determine the coefficients. Note that the other parameters  $\phi_c, \Delta, \beta_t$  that enter the equations are all expressible in terms of the gauged supergravity parameters  $g_R, V_0, V_1$  from eqs (3.36) and (4.10). However, we will express everything in terms of  $\beta_t$  for convenience. Thus the solution for the coefficients are,

$$\begin{aligned}
C_{\hat{t}} &= \frac{C_s}{\phi_c} F_0(\beta_t), \\
C_{\hat{r}} &= \frac{C_s}{\phi_c} F_1(\beta_t), \\
C_{\hat{x}} &= \frac{C_s}{\phi_c} F_2(\beta_t), \\
C_{\hat{y}} &= \frac{C_s}{\phi_c} F_3(\beta_t), \\
C_{\hat{z}} &= \frac{C_s}{\phi_c} F_4(\beta_t). \quad (4.22)
\end{aligned}$$

where  $F_i(\beta_t), i = 0, \dots, 4$  are complicated functions of  $\beta_t$  which are given in section C. Note that all the coefficients are proportional to the coefficient  $C_s$ . This is a consistency check that the metric fluctuations considered in the analysis are sourced by the scalar fluctuations.

Thus the full metric along with the fluctuations is

$$\begin{aligned}
ds^2 &= -\left(1 + C_{\hat{t}}\hat{r}^\Delta\right)\hat{r}^{2\beta_t}d\hat{t}^2 + \left(1 + C_{\hat{r}}\hat{r}^\Delta\right)\frac{d\hat{r}^2}{\hat{r}^2} + \left(1 + C_{\hat{x}}\hat{r}^\Delta\right)d\hat{x}^2 \\
& \quad + \left(1 + C_{\hat{y}}\hat{r}^\Delta\right)e^{-2\hat{x}}d\hat{y}^2 + \left(1 + C_{\hat{z}}\hat{r}^\Delta\right)\hat{r}^{2\beta_t}d\hat{z}^2. \quad (4.23)
\end{aligned}$$

From eq (4.7) and eq (4.10), we see that positivity of  $\lambda$  implies  $\Delta$  is positive for the solution (3.42). Hence, all the metric fluctuations are well behaved and the metric approaches the type III attractor metric as one approaches the horizon  $\hat{r} \rightarrow 0$ . The reader may worry that the perturbation in  $\hat{r}$  is well behaved only if  $\Delta > 2$ . However there is no need to put any additional condition, since the behavior at  $\hat{r} \rightarrow 0$  is dictated by the  $\frac{1}{\hat{r}^2}$  term owing to  $\Delta$  being positive. Thus we have constructed a stable Bianchi III attractor solution in gauged supergravity. In the following section, we investigate the supersymmetry of this solution.

## 5 Supersymmetry analysis

In this section, we analyze the Killing spinor equations for the  $U(1)_R$  gauged supergravity with the Bianchi type III solution (3.42) as the background. The Killing spinor equation

is obtained by setting the supersymmetric variation of the gravitino to zero. For the  $\mathcal{N} = 2, U(1)_R$  gauged supergravity the gravitino variation is [63],

$$\delta\psi_{\mu i} = \nabla_{\mu}(\omega)\epsilon_i + \frac{i}{4\sqrt{6}}h_I(\gamma_{\mu\nu\rho} - 4g_{\mu\nu}\gamma_{\rho})F^{I\nu\rho}\epsilon_i + \delta'\psi_{\mu i} . \quad (5.1)$$

Our notations and conventions are summarized in section A. The indices  $I$  label the number of vectors and the scalars  $h_I$  are as defined in section 3.1. Although we have only one gauge field for the solution (3.42), we will keep the  $I$  indices for the gauge fields to avoid introducing the explicit form of  $h_I$  in the equations. The term  $\delta'\psi_{\mu i}$  is the modification in the supersymmetry variations as a result of the  $U(1)_R$  gauging. Explicitly it takes the form,

$$\delta'\psi_{\mu i} = -\frac{i}{\sqrt{6}}g_R h^I V_I \gamma_{\mu} \delta_{ij} \epsilon^j , \quad (5.2)$$

where  $V_I$  are the parameters that appear in the  $U(1)_R$  gauging. Note that the  $\delta_{ij}$  is not used to raise or lower the  $SU(2)$  index.

We now proceed to analyze the Killing spinor equations. The vielbeins and spin connections of the metric (3.42) are

$$\begin{aligned} e_{\hat{t}}^0 &= r^{\beta_t} , \quad e_{\hat{r}}^1 = \frac{1}{\hat{r}} , \quad e_{\hat{x}}^2 = 1 , \quad e_{\hat{y}}^3 = e^{-\hat{x}} , \quad e_{\hat{z}}^4 = \hat{r}^{\beta_t} , \\ \omega_{\hat{t}}^{01} &= \beta_t \hat{r}^{\beta_t} , \quad \omega_{\hat{y}}^{32} = -e^{-\hat{x}} , \quad \omega_{\hat{z}}^{41} = \beta_t \hat{r}^{\beta_t} . \end{aligned} \quad (5.3)$$

Substituting the above in (5.1), the Killing spinor equations can be written as

$$\begin{aligned} \gamma_0 \hat{r}^{-\beta_t} \partial_{\hat{t}} \epsilon_i - \frac{\beta_t}{2} \gamma_1 \epsilon_i + \frac{i}{2\sqrt{6}} A_3^I h_I \gamma_{23} \epsilon_i + \frac{i}{\sqrt{6}} g_R h^I V_I \delta_{ij} \epsilon^j &= 0 , \\ \gamma_1 \hat{r} \partial_{\hat{r}} \epsilon_i - \frac{i}{2\sqrt{6}} A_3^I h_I \gamma_{23} \epsilon_i - \frac{i}{\sqrt{6}} g_R h^I V_I \delta_{ij} \epsilon^j &= 0 , \\ \gamma_2 \partial_{\hat{x}} \epsilon_i + \frac{i}{\sqrt{6}} A_3^I h_I \gamma_{23} \epsilon_i - \frac{i}{\sqrt{6}} g_R h^I V_I \delta_{ij} \epsilon^j &= 0 , \\ \gamma_3 e^{\hat{x}} \partial_{\hat{y}} \epsilon_i - \frac{1}{2} \gamma_2 \epsilon_i + \frac{i}{\sqrt{6}} A_3^I h_I \gamma_{23} \epsilon_i - \frac{i}{\sqrt{6}} g_R h^I V_I \delta_{ij} \epsilon^j &= 0 , \\ \gamma_4 \hat{r}^{-\beta_t} \partial_{\hat{z}} \epsilon_i + \frac{\beta_t}{2} \gamma_1 \epsilon_i - \frac{i}{2\sqrt{6}} A_3^I h_I \gamma_{23} \epsilon_i - \frac{i}{\sqrt{6}} g_R h^I V_I \delta_{ij} \epsilon^j &= 0 . \end{aligned} \quad (5.4)$$

The  $\gamma_a$  matrices that appear in the above set of equations are in tangent space.

We choose a radial profile for the Killing spinor. This is motivated by the fact that the radial spinor preserves the time translation and homogeneous symmetries of the type III metric (2.9). Moreover, it is well known that the radially dependent spinor generates the Poincaré supersymmetries in AdS [55, 56]. Furthermore, some of the Bianchi type I solutions such as the Lifshitz and  $AdS_3 \times \mathbb{R}^2$  solutions in gauged supergravity preserve 1/4 of the supersymmetries for the radial spinor [51, 52, 59].

We choose the spinor ansatz

$$\epsilon_i = f(\hat{r}) \chi_i , \quad (5.5)$$

where  $\chi_i$  is a constant symplectic majorana spinor. Substituting (5.5) in the Killing spinor equation (5.4), we see that  $\hat{t}, \hat{z}$  equations become identical. Adding the  $\hat{t}$  equation and the radial equation we get

$$\hat{r}\partial_{\hat{r}}f(\hat{r}) - \frac{\beta_t}{2}f(\hat{r}) = 0, \tag{5.6}$$

which is solved by

$$f(\hat{r}) = \hat{r}^{\frac{\beta_t}{2}}. \tag{5.7}$$

Using the above in (5.5) and substituting it in the Killing spinor equation (5.4) we get,

$$\begin{aligned} \frac{\beta_t}{2}\gamma_1\chi_i - \frac{i}{2\sqrt{6}}A_3^I h_I \gamma_{23}\chi_i - \frac{i}{\sqrt{6}}g_R h^I V_I \delta_{ij}\chi^j &= 0, \\ \frac{i}{\sqrt{6}}A_3^I h_I \gamma_{23}\chi_i - \frac{i}{\sqrt{6}}g_R h^I V_I \delta_{ij}\chi^j &= 0, \\ \frac{1}{2}\gamma_2\chi_i - \frac{i}{\sqrt{6}}A_3^I h_I \gamma_{23}\chi_i + \frac{i}{\sqrt{6}}g_R h^I V_I \delta_{ij}\chi^j &= 0. \end{aligned} \tag{5.8}$$

From the last two of the above equations, it follows that

$$\gamma_2\chi_i = 0. \tag{5.9}$$

This condition breaks all of the supersymmetry. The origin of the  $\gamma_2$  term is the spin connection term due to the  $EAdS_2$  (2.10) part of the type III metric. Thus, a naive radial spinor does not preserve supersymmetry in this case. This suggests that the stable Bianchi III metric we have constructed may be a non-supersymmetric attractor. However, it is possible that there may be a more general ansatz similar to the one studied in [58] for a black string solution that interpolates between  $AdS_3 \times \mathbb{H}^2$  and  $AdS_5$  in a  $U(1)^3$  gauged supergravity. We hope to explore this in detail in future works.

## 6 Summary and conclusions

We studied the  $AdS_3 \times \mathbb{H}^2$  solution which is a special case of the Bianchi type III class in  $U(1)_R$  gauged supergravity. We found that there exist a class of such solutions parametrized by  $g_R, V_0, V_1$  that satisfied the two sufficient requirements for the attractor mechanism, namely the existence of a critical point of the attractor potential and that the Hessian of the attractor potential should have a positive eigenvalue.

We investigated the stability of the solution in gauged supergravity by studying the linearized fluctuations of the gauge field, scalar field, metric about their attractor values. The stress energy tensor in gauged supergravity depends on linearized fluctuations of scalars and gauge fields [49]. In order to avoid backreaction and deviation from the attractor geometry, all the fluctuations have to be well behaved as one approaches the horizon.

For the solution (3.42), we showed that the source term in the gauge field fluctuations vanishes. Thus there are no gauge field fluctuations sourced by scalar fluctuations. The metric fluctuation equations are sourced completely by the scalar perturbations. We showed that for the solution satisfying the sufficient conditions for the attractor mechanism, the scalar fluctuations are well behaved near the horizon. We also solved the metric fluctuations

and showed that all the fluctuations are regular. Since all the linearized fluctuations are well behaved near the horizon, we infer that the type III Bianchi solution is a stable attractor solution at the linearized level.

One of the simplifications that aided us in the stability analysis was that there were no gauge field fluctuations which are sourced by scalar fluctuations. As we commented before in section 4.1, this need not happen in general. For more complicated situations we expect that as long as the solution satisfies the sufficient conditions for the attractor mechanism [39], the Bianchi type geometries might be stable with respect to linearized fluctuations about the attractor values. We hope to explore these aspects and look for more interesting solutions in future.

In the long run, we hope our stability analysis will provide motivation to explore the possibility of construction of analytic black brane solutions which interpolate between IR and UV attractor geometries. In particular, it will be very interesting to construct solutions that are asymptotically AdS. Such interpolating solutions will be helpful to explore the holographic duals of Bianchi attractors. Recent progress in this direction include numerical solutions which interpolate between Bianchi types and Lifshitz or  $AdS_2 \times S^3$  from where they can be connected to anti de-Sitter space [32]. It will be valuable to construct analytic interpolating solutions in a simple theory of gravity coupled to suitable matter.

In this paper, we also investigated the supersymmetry of the Bianchi type III solution. We studied the Killing spinor equations of  $\mathcal{N} = 2, U(1)_R$  gauged supergravity with the background metric (3.42). We chose a radial profile for the Killing spinor since it preserves the time translations and homogeneous symmetries of the metric. However, we found that the naive radial spinor which gives supersymmetric Bianchi I spaces such as AdS and Lifshitz fails for this case. This suggests that the stable type III solution we obtained may be a non-supersymmetric attractor. It would be interesting to construct supersymmetric Bianchi attractors in gauged supergravity along the lines of the  $AdS_3 \times \mathbb{H}^2$  solution in [58]. In a related exploration, it would be worthwhile to construct Bianchi attractors from wrapped branes [66] in supergravity. We hope to report these in future works.

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## A Notations and conventions

In this section, we summarize our notations and conventions on tangent space and spinors. We use greek indices for spacetime and roman for tangent space. Our conventions for the

flat tangent space metric is  $\eta_{ab} = (-, +, +, +, +)$ . The tangent space indices are denoted by  $a, b = 0, 1, 2, 3, 4$ .

The tangent space matrices satisfy the usual Clifford algebra

$$\{\gamma_a, \gamma_b\} = 2\eta_{ab} . \tag{A.1}$$

Antisymmetrization is done with the following convention,

$$\gamma_{a_1 a_2 \dots a_n} = \gamma_{[a_1 a_2 \dots a_n]} = \frac{1}{n!} \sum_{\sigma \in P_n} \text{Sign}(\sigma) \gamma_{a_{\sigma(1)}} \gamma_{a_{\sigma(2)}} \dots \gamma_{a_{\sigma(n)}} . \tag{A.2}$$

In  $d = 5$  only  $I, \gamma_a, \gamma_{ab}$  form an independent set, other matrices are related by the general identity for  $d = 2k + 3$ ,

$$\gamma^{\mu_1 \mu_2 \dots \mu_s} = \frac{-i^{-k+s(s-1)}}{(d-s)!} \epsilon^{\mu_1 \mu_2 \dots \mu_s} \gamma_{\mu_{s+1} \dots \mu_d} . \tag{A.3}$$

We also recollect that the spinors in five dimensions satisfy the symplectic majorana condition

$$\bar{\epsilon}^i \equiv (\epsilon_i^*)^t \gamma^0 = (\epsilon^i)^t C , \tag{A.4}$$

where  $C$  is the charge conjugation matrix which obeys  $C^t = C^{-1} = -C$ .

Unlike the case in four dimensions, the  $SU(2)$  indices are not raised and lowered by complex conjugation. Instead they are raised and lowered by the  $SU(2)$  covariant tensor with the conventions  $\varepsilon_{12} = \varepsilon^{12} = 1$ . Note that the  $SU(2)$  indices are always raised or lowered in the NW-SE direction

$$\epsilon^i = \varepsilon^{ij} \epsilon_j , \quad \epsilon_i = \epsilon^j \varepsilon_{ji} . \tag{A.5}$$

The covariant derivative acting on  $\epsilon_i$  is with respect to the Lorentz covariant spin connection  $\omega_\mu^{ab}$  defined as

$$\nabla_\mu(\omega) \epsilon_i = \partial_\mu \epsilon_i + \frac{1}{4} \omega_\mu^{ab} \gamma_{ab} \epsilon_i \tag{A.6}$$

## B Linearized Einstein equations

In this section, we provide the explicit form of the linearized equations that follow from (4.18). We substitute the expressions for the attractor potential (3.41), the scalar fluctuations (4.10), the terms from the stress energy tensor (4.13), (4.16) and the metric fluctuations (4.19) into the linearized Einstein equation (4.18). We then contract it with the vielbeins  $e_a^\mu$  to obtain the following equations. The  $\hat{t}\hat{t}$  equation is

$$\begin{aligned} & \hat{r}^2 \tilde{\gamma}_{\hat{r}\hat{r}}'' - \hat{r}^2 \tilde{\gamma}_{\hat{t}\hat{t}}'' + \hat{r}^2 \tilde{\gamma}_{\hat{x}\hat{x}}'' + \hat{r}^2 \tilde{\gamma}_{\hat{y}\hat{y}}'' + \hat{r}^2 \tilde{\gamma}_{\hat{z}\hat{z}}'' + 12\beta_t^2 \tilde{\gamma}_{\hat{r}\hat{r}} + 4(3\beta_t^2 + 2) \tilde{\gamma}_{\hat{t}\hat{t}} + 4\beta_t^2 \tilde{\gamma}_{\hat{x}\hat{x}} + 4\beta_t^2 \tilde{\gamma}_{\hat{y}\hat{y}} \\ & + 12\beta_t^2 \tilde{\gamma}_{\hat{z}\hat{z}} + 6\beta_t \hat{r} \tilde{\gamma}'_{\hat{r}\hat{r}} - 6\beta_t \hat{r} \tilde{\gamma}'_{\hat{t}\hat{t}} + 6\beta_t \hat{r} \tilde{\gamma}'_{\hat{x}\hat{x}} + 6\beta_t \hat{r} \tilde{\gamma}'_{\hat{y}\hat{y}} + 6\beta_t \hat{r} \tilde{\gamma}'_{\hat{z}\hat{z}} \\ & + \hat{r} \tilde{\gamma}'_{\hat{r}\hat{r}} - \hat{r} \tilde{\gamma}'_{\hat{t}\hat{t}} + \hat{r} \tilde{\gamma}'_{\hat{x}\hat{x}} + 4(\tilde{\gamma}_{\hat{x}\hat{x}} + \tilde{\gamma}_{\hat{y}\hat{y}}) + \hat{r} \tilde{\gamma}'_{\hat{y}\hat{y}} + \hat{r} \tilde{\gamma}'_{\hat{z}\hat{z}} = 0 . \end{aligned} \tag{B.1}$$

The  $\hat{r}\hat{r}$  equation is

$$\begin{aligned}
 & \hat{r}^2 \tilde{\gamma}_{\hat{r}\hat{r}}'' - \hat{r}^2 \tilde{\gamma}_{\hat{t}\hat{t}}'' - \hat{r}^2 \tilde{\gamma}_{\hat{x}\hat{x}}'' - \hat{r}^2 \tilde{\gamma}_{\hat{y}\hat{y}}'' - \hat{r}^2 \tilde{\gamma}_{\hat{z}\hat{z}}'' - 4(5\beta_t^2 + \beta_t + 1)\tilde{\gamma}_{\hat{r}\hat{r}} + 4\beta_t^2 \tilde{\gamma}_{\hat{x}\hat{x}} + 4\beta_t^2 \tilde{\gamma}_{\hat{y}\hat{y}} \\
 & - 4\beta_t^2 \tilde{\gamma}_{\hat{z}\hat{z}} + 2\beta_t \hat{r} \tilde{\gamma}'_{\hat{r}\hat{r}} - 2\beta_t \hat{r} \tilde{\gamma}'_{\hat{t}\hat{t}} - 4(\beta_t - 1)\beta_t \tilde{\gamma}_{\hat{t}\hat{t}} - 2\beta_t \hat{r} \tilde{\gamma}'_{\hat{x}\hat{x}} + 4\beta_t \tilde{\gamma}_{\hat{x}\hat{x}} \\
 & - 2\beta_t \hat{r} \tilde{\gamma}'_{\hat{y}\hat{y}} + 4\beta_t \tilde{\gamma}_{\hat{y}\hat{y}} - 2\beta_t \hat{r} \tilde{\gamma}'_{\hat{z}\hat{z}} + 4\beta_t \tilde{\gamma}_{\hat{z}\hat{z}} - 3\hat{r} \tilde{\gamma}'_{\hat{r}\hat{r}} + 3\hat{r} \tilde{\gamma}'_{\hat{t}\hat{t}} \\
 & - 4(\tilde{\gamma}_{\hat{t}\hat{t}} + 2(\tilde{\gamma}_{\hat{x}\hat{x}} + \tilde{\gamma}_{\hat{y}\hat{y}}) + \tilde{\gamma}_{\hat{z}\hat{z}}) + 3\hat{r} \tilde{\gamma}'_{\hat{x}\hat{x}} + 3\hat{r} \tilde{\gamma}'_{\hat{y}\hat{y}} + 3\hat{r} \tilde{\gamma}'_{\hat{z}\hat{z}} = 0. \quad (\text{B.2})
 \end{aligned}$$

The  $\hat{x}\hat{x}$  equation is

$$\begin{aligned}
 & -\frac{(2\beta_t^2 - 1)(8C_s \hat{r}^\Delta + \phi_c \tilde{\gamma}_{\hat{y}\hat{y}})}{\phi_c} - 2\beta_t^2 (\tilde{\gamma}_{\hat{r}\hat{r}} + \tilde{\gamma}_{\hat{t}\hat{t}} + 3\tilde{\gamma}_{\hat{x}\hat{x}} + \tilde{\gamma}_{\hat{z}\hat{z}}) - \frac{1}{2} \hat{r} ((2\beta_t + 1)\tilde{\gamma}'_{\hat{r}\hat{r}} \\
 & + 2\beta_t (\tilde{\gamma}'_{\hat{t}\hat{t}} - \tilde{\gamma}'_{\hat{x}\hat{x}} + \tilde{\gamma}'_{\hat{y}\hat{y}} + \tilde{\gamma}'_{\hat{z}\hat{z}}) + \hat{r} (\tilde{\gamma}''_{\hat{r}\hat{r}} + \tilde{\gamma}''_{\hat{t}\hat{t}} - \tilde{\gamma}''_{\hat{x}\hat{x}} + \tilde{\gamma}''_{\hat{y}\hat{y}} + \tilde{\gamma}''_{\hat{z}\hat{z}}) \\
 & + \tilde{\gamma}'_{\hat{t}\hat{t}} - \tilde{\gamma}'_{\hat{x}\hat{x}} + \tilde{\gamma}'_{\hat{y}\hat{y}} + \tilde{\gamma}'_{\hat{z}\hat{z}}) - 6\tilde{\gamma}_{\hat{x}\hat{x}} - \tilde{\gamma}_{\hat{y}\hat{y}} = 0. \quad (\text{B.3})
 \end{aligned}$$

The  $\hat{y}\hat{y}$  equation is

$$\begin{aligned}
 & -\frac{16(2\beta_t^2 - 1)C_s \hat{r}^\Delta}{\phi_c} + 2(-2\beta_t^2 (\tilde{\gamma}_{\hat{r}\hat{r}} + \tilde{\gamma}_{\hat{t}\hat{t}} + 3\tilde{\gamma}_{\hat{y}\hat{y}} + \tilde{\gamma}_{\hat{z}\hat{z}}) - \tilde{\gamma}_{\hat{x}\hat{x}} - 3\tilde{\gamma}_{\hat{y}\hat{y}}) + 2(1 - 2\beta_t^2) \tilde{\gamma}_{\hat{x}\hat{x}} \\
 & - \hat{r} ((2\beta_t + 1)\tilde{\gamma}'_{\hat{r}\hat{r}} + 2\beta_t (\tilde{\gamma}'_{\hat{t}\hat{t}} + \tilde{\gamma}'_{\hat{x}\hat{x}} - \tilde{\gamma}'_{\hat{y}\hat{y}} + \tilde{\gamma}'_{\hat{z}\hat{z}}) + \hat{r} (\tilde{\gamma}''_{\hat{r}\hat{r}} + \tilde{\gamma}''_{\hat{t}\hat{t}} + \tilde{\gamma}''_{\hat{x}\hat{x}} \\
 & - \tilde{\gamma}''_{\hat{y}\hat{y}} + \tilde{\gamma}''_{\hat{z}\hat{z}}) + \tilde{\gamma}'_{\hat{t}\hat{t}} + \tilde{\gamma}'_{\hat{x}\hat{x}} - \tilde{\gamma}'_{\hat{y}\hat{y}} + \tilde{\gamma}'_{\hat{z}\hat{z}}) - 6\tilde{\gamma}_{\hat{r}\hat{r}} - 6\tilde{\gamma}_{\hat{t}\hat{t}} - 6\tilde{\gamma}_{\hat{x}\hat{x}} - 6\tilde{\gamma}_{\hat{z}\hat{z}} = 0. \quad (\text{B.4})
 \end{aligned}$$

The  $\hat{z}\hat{z}$  equation is

$$\begin{aligned}
 & \hat{r}^2 (-\tilde{\gamma}''_{\hat{r}\hat{r}}) - \hat{r}^2 \tilde{\gamma}''_{\hat{t}\hat{t}} - \hat{r}^2 \tilde{\gamma}''_{\hat{x}\hat{x}} - \hat{r}^2 \tilde{\gamma}''_{\hat{y}\hat{y}} + \hat{r}^2 \tilde{\gamma}''_{\hat{z}\hat{z}} - 12\beta_t^2 \tilde{\gamma}_{\hat{r}\hat{r}} - 12\beta_t^2 \tilde{\gamma}_{\hat{t}\hat{t}} - 4\beta_t^2 \tilde{\gamma}_{\hat{x}\hat{x}} - 4\beta_t^2 \tilde{\gamma}_{\hat{y}\hat{y}} \\
 & - 12\beta_t^2 \tilde{\gamma}_{\hat{z}\hat{z}} - 6\beta_t \hat{r} \tilde{\gamma}'_{\hat{r}\hat{r}} - 6\beta_t \hat{r} \tilde{\gamma}'_{\hat{t}\hat{t}} - 6\beta_t \hat{r} \tilde{\gamma}'_{\hat{x}\hat{x}} - 6\beta_t \hat{r} \tilde{\gamma}'_{\hat{y}\hat{y}} + 6\beta_t \hat{r} \tilde{\gamma}'_{\hat{z}\hat{z}} - \hat{r} \tilde{\gamma}'_{\hat{r}\hat{r}} \\
 & - \hat{r} \tilde{\gamma}'_{\hat{t}\hat{t}} - \hat{r} \tilde{\gamma}'_{\hat{x}\hat{x}} - 4(\tilde{\gamma}_{\hat{x}\hat{x}} + \tilde{\gamma}_{\hat{y}\hat{y}} + 2\tilde{\gamma}_{\hat{z}\hat{z}}) - \hat{r} \tilde{\gamma}'_{\hat{y}\hat{y}} + \hat{r} \tilde{\gamma}'_{\hat{z}\hat{z}} = 0. \quad (\text{B.5})
 \end{aligned}$$

In the above equations, the prime indicates derivative with respect to  $\hat{r}$ . We see that all the double derivatives are multiplied by  $\hat{r}^2$ , while the single derivatives are multiplied by  $\hat{r}$ . Now, the  $\hat{x}\hat{x}$  and  $\hat{y}\hat{y}$  equations contain the source term which goes like  $\hat{r}^\Delta$ . It is then clear that the metric fluctuations  $\tilde{\gamma}_{\mu\nu}$  all go like  $\hat{r}^\Delta$ .

## C Coefficients of the linearized fluctuations

The various functions that appear in the coefficients (4.22) are

$$F_0(\beta_t) = -64(\beta_t^2 + 4)(2\beta_t^2 - 1) \frac{N_t^1(\beta_t) + N_t^2(\beta_t)}{D_1(\beta_t) + D_2(\beta_t) + D_3(\beta_t) + D_4(\beta_t)}, \quad (\text{C.1})$$

$$F_1(\beta_t) = 64(\beta_t^2 + 4)(2\beta_t^2 - 1) \frac{N_{\hat{r}}^1(\beta_t) + N_{\hat{r}}^2(\beta_t)}{D_1(\beta_t) + D_2(\beta_t) + D_3(\beta_t) + D_4(\beta_t)}, \quad (\text{C.2})$$

$$F_2(\beta_t) = 8(2\beta_t^2 - 1) \frac{N_{\hat{x}}^1(\beta_t) + N_{\hat{x}}^2(\beta_t) + N_{\hat{x}}^3(\beta_t)}{D_1(\beta_t) + D_2(\beta_t) + D_3(\beta_t) + D_4(\beta_t)}, \quad (\text{C.3})$$

$$F_3(\beta_t) = 8(2\beta_t^2 - 1) \frac{N_{\hat{y}}^1(\beta_t) + N_{\hat{y}}^2(\beta_t) + N_{\hat{y}}^3(\beta_t)}{D_1(\beta_t) + D_2(\beta_t) + D_3(\beta_t) + D_4(\beta_t)}, \quad (\text{C.4})$$

$$F_4(\beta_t) = -64(\beta_t^2 + 4)(2\beta_t^2 - 1) \frac{N_{\hat{z}}^1(\beta_t) + N_{\hat{z}}^1(\beta_t)}{D_1(\beta_t) + D_2(\beta_t) + D_3(\beta_t) + D_4(\beta_t)}, \quad (\text{C.5})$$

where,

$$\begin{aligned}
N_t^1(\beta_t) &= 272\beta_t^4 + 80(f(\beta_t) - 1)\beta_t^2 + 4(f(\beta_t) - 84)\beta_t, \\
N_t^2(\beta_t) &= -4f(\beta_t) + 16(7f(\beta_t) + 33)\beta_t^3 + 107, \\
N_{\tilde{r}}^1(\beta_t) &= 304\beta_t^4 + 8(14f(\beta_t) - 53)\beta_t^2 + 4(5f(\beta_t) + 84)\beta_t, \\
N_{\tilde{r}}^2(\beta_t) &= 28f(\beta_t) + 16(5f(\beta_t) - 33)\beta_t^3 - 179, \\
N_{\tilde{x}}^1(\beta_t) &= 4928\beta_t^6 + 4(1000f(\beta_t) + 4821)\beta_t^2 - 4(53f(\beta_t) - 924)\beta_t, \\
N_{\tilde{x}}^2(\beta_t) &= 644f(\beta_t) - 64(5f(\beta_t) - 33)\beta_t^5 + 16(68f(\beta_t) + 1419)\beta_t^4, \\
N_{\tilde{x}}^3(\beta_t) &= -16(166f(\beta_t) + 447)\beta_t^3 + 671, \\
N_{\tilde{y}}^1(\beta_t) &= 4928\beta_t^6 + 4(1216f(\beta_t) + 6009)\beta_t^2 - 4(107f(\beta_t) + 3612)\beta_t, \\
N_{\tilde{y}}^2(\beta_t) &= -4f(\beta_t) - 64(5f(\beta_t) - 33)\beta_t^5 + 16(68f(\beta_t) + 1689)\beta_t^4, \\
N_{\tilde{y}}^3(\beta_t) &= (21360 - 64f(\beta_t))\beta_t^3 + 7745, \\
N_{\tilde{z}}^1(\beta_t) &= (272\beta_t^4 + 80(f(\beta_t) - 1)\beta_t^2 + 4(f(\beta_t) - 84)\beta_t), \\
N_{\tilde{z}}^2(\beta_t) &= -4f(\beta_t) + 16(7f(\beta_t) + 33)\beta_t^3 + 107, \\
D_1(\beta_t) &= -33024\beta_t^8 - 8(3910f(\beta_t) + 13839)\beta_t^2 + 4(367f(\beta_t) - 1428)\beta_t, \\
D_2(\beta_t) &= -3276f(\beta_t) + 256(25f(\beta_t) + 99)\beta_t^7 - 128(58f(\beta_t) + 1525)\beta_t^6, \\
D_3(\beta_t) &= 192(147f(\beta_t) + 400)\beta_t^5 - 32(1178f(\beta_t) + 8565)\beta_t^4, \\
D_4(\beta_t) &= 48(309f(\beta_t) - 1045)\beta_t^3 - 10445, \\
f(\beta_t) &= \sqrt{-21 + 33\beta_t^2}. \tag{C.6}
\end{aligned}$$

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