OPE methods for the holomorphic Higgs portal

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ABSTRACT: We develop a systematic and general approach to study the effective Higgs Lagrangian in a supersymmetric framework in which the Higgs fields in the visible sector couple weakly to another sector. The extra sector may be strongly coupled in general. It is assumed to be superconformal in the ultraviolet, but develop a mass-gap with supersymmetry breaking in the infrared. The main technique used in our approach is that of the operator product expansion (OPE). By using OPE methods we are able to compute the parameters in the Higgs Lagrangian to quadratic order and make general statements that are applicable to many classes of models. Not only does this approach allow us to understand the traditional problems plaguing simple models from a different perspective, it also reveals new possibilities for solutions of these problems. The methods and results of our work should be useful in constructing a viable and natural model of physics beyond the Standard Model.

KEYWORDS: Supersymmetry Breaking, Conformal and W Symmetry

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1 Introduction

With the recent discovery of the Higgs boson at the LHC, it is safe to assume that the electroweak symmetry is broken by the Higgs mechanism. However, the observed mass of the Higgs boson near 126 GeV, and the null evidence (so far) for any beyond-the-Standard-Model (BSM) physics, has begun to pose important questions about the notion of “electroweak naturalness.” For example, within the context of low-scale supersymmetry, minimal realizations such as the (R-parity conserving) minimal supersymmetric standard model (MSSM) are becoming increasingly fine-tuned from an electroweak-scale point of view, at least in the technical sense of ’t Hooft. One approach to this situation is to keep the

BSM model minimal (like the MSSM, for example) and accept some fine-tuning as a part of nature [1–3]. An alternative is to try to come up with not-so-minimal (but hopefully well-motivated) models that are either fully electroweak-natural, or at least alleviate the fine-tuning in the Higgs potential.

For the latter approach it is desirable to develop a unified treatment for both perturbative and strongly-coupled models, both to guide the intuition as well as for computational ease. In this work we take some steps in this direction. We consider a framework in which the Higgs fields in the MSSM couple directly to another sector responsible for supersymmetry breaking and its mediation, i.e. the messenger and/or supersymmetry-breaking sector, via the superpotential

$$W = \lambda_u \mathcal{H}_u \mathcal{O}_u + \lambda_d \mathcal{H}_d \mathcal{O}_d. \quad (1.1)$$

Here $\mathcal{O}_u$ and $\mathcal{O}_d$ are in general composite operators that belong to an SM representation conjugate to that of $\mathcal{H}_u$ and $\mathcal{H}_d$ respectively. Models with such terms have been studied previously in the literature in various contexts. A well-studied situation is that $\mathcal{O}_u$ and $\mathcal{O}_d$ are composed of vector-like matter fields in a weakly-coupled hidden sector [4–8]. Examples of models where the hidden sector is strongly-coupled include [9–13]. The presence of additional sectors with these couplings is also well-motivated from a top-down (string theory) point of view, as in [14–16]. Finally, such scenarios can be studied very generally by expressing observable parameters in terms of hidden sector correlators [17–19]. It is clear that the range of hidden-sector models included in (1.1) can run the gamut from weakly- to strongly-coupled. In general, the dynamics of such a sector and its coupling to the Higgs can have important effects on various terms in the Higgs potential as well as Higgs couplings. For example, the quadratic terms in the Higgs potential, which are determined by μ and the supersymmetry-breaking parameters in the Higgs sector, as well as the quartic terms, which determine the physical Higgs boson masses and mixings, are affected in general.

In this paper we focus on the computation of the quadratic terms in the Higgs potential. The emphasis of our work is to develop techniques that are applicable to a large class of models, even those in which the additional sector is strongly coupled. The primary tool that we will use in this regard is the operator product expansion (OPE). On general grounds, since local physics should be captured by local operators, it is expected that the product of two nearby operators can be replaced by a linear combination of local operators,

$$\mathcal{O}_i(x) \mathcal{O}_j(0) = \sum_k c_{ij}^k(x) \mathcal{O}_k(0), \quad (1.2)$$

with c_{ij}^k referred to as OPE or Wilson coefficients. All x -dependence of the operator product $\mathcal{O}_i(x) \mathcal{O}_j(0)$ is included in c_{ij}^k . In conformal theories (1.2) is a convergent expansion inside correlation functions where no other operators are within a distance x from the origin, while in nonconformal theories (1.2) is in general an asymptotic expansion valid in the limit $x \rightarrow 0$. In such cases the OPE is a very powerful way of separating high- from low-energy physics, for the Wilson coefficients c_{ij}^k are determined by UV physics, while the expectation values of operators on the right-hand side are determined by IR

physics. Therefore, if the UV physics is under control, the OPE can be used to gain an understanding of the dynamics even in the IR.

One possibility is that the UV physics is asymptotically free, as in the case of QCD, where OPE techniques have been extensively used to determine important constraints on the low energy theory, via QCD sum rules [20]. OPE methods and dispersion relations have also been used in computing the cross-section of $e^+e^- \rightarrow \text{hadrons}$. There, the electromagnetic current-current correlation function $\langle J_{EM}(x)J_{EM}(0) \rangle$ can be computed by replacing it with the OPE and then using dispersion relations to obtain it in the physical region.

Another possibility is that the UV physics for the additional sector has a large symmetry group, such as the superconformal group. Of course, the IR physics is neither conformal (the states in the additional sector ultimately acquire a mass) nor supersymmetric (the multiplets in the additional sector have mass splittings $\propto \sqrt{F}$ if they couple to supersymmetry breaking). But it is still possible to apply OPE techniques if the symmetries are regarded as spontaneously broken, since then they are restored in the UV. Just like in QCD, then, OPE methods can be used to describe low-energy observables. This was demonstrated in [21], utilizing results of [17, 22, 23], for the SM gauge current-current correlation functions $\langle J_a(x)J_a(0) \rangle$, $a = 1, 2, 3$, from which gaugino and sfermion masses could be obtained.

In this work we will use OPE techniques to compute parameters in the Higgs Lagrangian when the Higgs fields couple to an additional sector via (1.1), and in which the additional sector has (at least an approximate) superconformal symmetry in the UV. Starting from the expressions of [18], where soft parameters are expressed in terms of two-point functions of hidden-sector operators, we will show, utilizing the general formalism of [24], that the approximate superconformal symmetry provides powerful constraints on the form of such terms. Our strategy is to use the (kinematically constrained) form of three-point functions in $\mathcal{N} = 1$ superconformal theories to extract the OPE of the first two operators with the third. In doing that, we can identify classes of hidden-sector operators that contribute to the two-point functions of [18], and consequently the parameters in the Higgs Lagrangian.

Our treatment brings a new perspective on the μ/B_μ problem [4] and the A/m_H^2 problem [8] in models of gauge-mediated supersymmetry breaking, and also opens up new possibilities for viable electroweak symmetry breaking (EWSB). As a nontrivial consistency check of our methods we reproduce the well-known results for the Higgs soft terms in the weakly-coupled toy model of [4]. Finally, we make some comments on the computation of the quartic terms in the Higgs potential via OPE techniques, but a detailed computation is left for the future.

The paper is organized as follows. In section 2 we lay out our setup and describe our assumptions in detail, followed by a brief description of the OPE formalism and the constraints from superconformal symmetry. Section 3 forms the technical meat of the paper, in which the computations of the soft terms and μ by the OPE method are outlined. In section 4 we reproduce the results for the weakly coupled model of [4] for the Higgs soft terms using the OPE, and we elucidate some subtle features of perturbative computations with the OPE. Section 5 is devoted to a broad discussion of various phe-

nomenological implications of the results obtained. Finally, in section 6 we summarize our main results, and make comments about future directions. Some technical details of computations regarding the projection of the two-point function of $\mathcal{N} = 1$ superconformal primary operators to two-point functions of their conformal primary components are given in appendix A. In appendix B we give details on the projection of superconformal three-point functions to conformal primary ones, and we construct the associated OPEs of conformal primary operators.

2 The Higgs effective lagrangian

In this section we describe the setup in some detail. We are interested in a framework in which the Higgs fields in the MSSM couple to operators $\mathcal{O}_u$ and $\mathcal{O}_d$ in another sector via the superpotential

$$W = \lambda_u \mathcal{H}_u \mathcal{O}_u + \lambda_d \mathcal{H}_d \mathcal{O}_d = \lambda_u \epsilon^{ij} (\mathcal{H}_u)_i (\mathcal{O}_u)_j + \lambda_d \epsilon_{ij} (\mathcal{H}_d)^i (\mathcal{O}_d)^j.$$

Here $\mathcal{O}_u$ and $\mathcal{O}_d$ are SU(2) doublet operators of dimensions $\Delta_{\mathcal{O}_u}$ and $\Delta_{\mathcal{O}_d}$, with hypercharges opposite to that of $\mathcal{H}_u$ and $\mathcal{H}_d$ respectively (i, j are SU(2) indices). The couplings λ_u and λ_d are assumed to be perturbative at the mass scale M characterizing the additional sector.¹ Note that this still allows the sector in which $\mathcal{O}_u$ and $\mathcal{O}_d$ belong to be strongly coupled. We will consider the situation in which $\mathcal{O}_u$ and $\mathcal{O}_d$ belong to a sector responsible for supersymmetry breaking and its mediation to the visible sector. As such, these operators could just be part of the supersymmetry-breaking sector, or they could comprise a “messenger” sector distinct from the supersymmetry-breaking sector but coupled to it by a weak coupling (say κ). The latter case gives rise to a framework which has been called “general messenger Higgs mediation (GMHM)” in [19]. In this case, a double expansion in $\lambda_{u,d}$ and κ is possible. In this work, however, we will focus on the more general case where only an expansion in $\lambda_{u,d}$ is available, although we will make some comments about the more special case with a coupling κ .

Now, by integrating out the dynamics of the sector containing $\mathcal{O}_u$ and $\mathcal{O}_d$, one generates various terms in the Higgs Lagrangian. The terms in the Higgs Lagrangian at quadratic order and zero momentum are

$$\begin{aligned} \mathcal{L} &= \mathcal{Z}_u F_{H_u}^\dagger F_{H_u} + \mathcal{Z}_d F_{H_d}^\dagger F_{H_d} + \left(\mu \int d^2\theta \mathcal{H}_u \mathcal{H}_d + \text{c.c.} \right) - V_{\text{Higgs}}^{(\text{soft})} - V_{\text{Higgs}}^{(\text{other})}, \\ V_{\text{Higgs}}^{(\text{soft})} &= m_{H_u}^2 H_u^\dagger H_u + m_{H_d}^2 H_d^\dagger H_d + (B_\mu H_u H_d + \text{c.c.}) + (A_u H_u F_{\mathcal{H}_u}^\dagger + A_d H_d F_{\mathcal{H}_d}^\dagger + \text{c.c.}), \\ V_{\text{Higgs}}^{(\text{other})} &= (a'_u H_u F_{\mathcal{H}_d} + \text{c.c.}) + (a'_d F_{\mathcal{H}_u} H_d + \text{c.c.}) + (\gamma F_{\mathcal{H}_u} F_{\mathcal{H}_d} + \text{c.c.}). \end{aligned} \quad (2.1)$$

Some comments are in order. Here we have assumed that there is no bare μ term in the superpotential — it is only generated after integrating out the additional sector. Also, in addition to the usual soft supersymmetry breaking terms in the Higgs potential $V_{\text{Higgs}}^{(\text{soft})}$, there are additional terms, which we collectively denote as $V_{\text{Higgs}}^{(\text{other})}$.

¹Concretely, the dimensionless renormalized couplings $\lambda_i(\mu) \equiv \lambda_i \mu^{2-\Delta_{\mathcal{O}_i}}$ should satisfy $\lambda_i(M) \ll 1$.

In a supersymmetry-breaking vacuum in a globally supersymmetric theory (as considered here), the vacuum energy is strictly positive. In the typical case where some operator in the additional sector breaks supersymmetry with its F-term vacuum expectation value (vev), F , it can be shown that the vacuum energy density is equal to $|F|^2$. If $\sqrt{F}$ is smaller than the typical mass scale M of operators in the additional sector, then all observables can be expanded in powers of F/M^2 . In many models the terms in $V_{\text{Higgs}}^{(\text{other})}$ are typically suppressed by powers of F/M^2 relative to the terms in $V_{\text{Higgs}}^{(\text{soft})}$ and the μ parameter, and can thus be neglected. We will show, however, that, at least in principle, some of the terms in $V_{\text{Higgs}}^{(\text{other})}$ may not be suppressed relative to the μ parameter. We will also explain why such suppressions are so common in models found in the literature.

The parameters appearing in the Higgs Lagrangian can be computed in terms of the zero-momentum limit of two-point correlation functions involving components of $\mathcal{O}_u$ and $\mathcal{O}_d$. This is easy to see from an effective-field theory point of view, and can be explicitly worked out by expanding $\exp[i \int d^4x (\int d^2\theta (\lambda_u \mathcal{H}_u \mathcal{O}_u + \lambda_d \mathcal{H}_d \mathcal{O}_d) + \text{c.c.})]$ to quadratic order and matching with the effective Lagrangian (2.1). This has already been done in [18]. The soft parameters and the μ term generated at the scale M due to the superpotential (1.1) at leading order in $\lambda_{u,d}$ are given by²

$$\begin{aligned}\mu &= \frac{i}{8} \lambda_u \lambda_d \left\langle \int d^4x e^{-ip \cdot x} Q^\alpha(O_u(x)) Q_\alpha(O_d(0)) \right\rangle \Big|_{p \rightarrow 0}, \\ B_\mu &= \frac{i}{2^5} \lambda_u \lambda_d \left\langle \int d^4x e^{-ip \cdot x} Q^2(O_u(x)) Q^2(O_d(0)) \right\rangle \Big|_{p \rightarrow 0}, \\ \delta A_{u,d} &= -\frac{i}{8} |\lambda_{u,d}|^2 \left\langle \int d^4x e^{-ip \cdot x} Q^2(O_{u,d}(x)) O_{u,d}^\dagger(0) \right\rangle \Big|_{p \rightarrow 0}, \\ \delta m_{H_{u,d}}^2 &= -\frac{i}{2^5} |\lambda_{u,d}|^2 \left\langle \int d^4x e^{-ip \cdot x} Q^2 \bar{Q}^2(O_{u,d}(x)) O_{u,d}^\dagger(0) \right\rangle \Big|_{p \rightarrow 0}.\end{aligned}\quad (2.2)$$

We will consider the contributions solely generated from the superpotential (1.1). Clearly, μ and B_μ arise from a *chiral-chiral* two-point function while $\delta A_{u,d}$ and $\delta m_{H_{u,d}}^2$ arise from a *chiral-antichiral* one.³ Since

$$Q^2(Q^\alpha(O_u(x)) Q_\alpha(O_d(0))) = -Q^2(O_u(x)) Q^2(O_d(0)),$$

we see that there is a relation between μ and B_μ , similar to the relation between $\delta A_{u,d}$ and $\delta m_{H_{u,d}}^2$. Finally, note that B_μ , $\delta A_{u,d}$, and $\delta m_{H_{u,d}}^2$ can be written as $\langle Q(\cdot) \rangle$, while μ cannot. Thus, the soft parameters are generated only when supersymmetry is broken (as they must), i.e. when $\langle 0|Q \neq 0$, while μ may receive contributions consistent with supersymmetry.

²We are following the conventions of Wess & Bagger [25], taking care of factors of i and minus signs as explained in the appendix of [23]. The action of operators is always the adjoint action, i.e. $Q(O) \equiv [Q, O]$. Note that there is an extra factor of $\frac{1}{2}$ compared to [18] from taking into account the SU(2) gauge indices.

³Note that the μ -term in (2.1) actually gives rise to three terms in the Lagrangian— $\mu(-\psi_{\mathcal{H}_u} \psi_{\mathcal{H}_d} + H_u F_{\mathcal{H}_d} + F_{\mathcal{H}_u} H_d) + \text{c.c.}$

Similarly, the terms in $V_{\text{Higgs}}^{(\text{other})}$ are given by

$$\begin{aligned} a'_{u,d} &= \frac{i}{8} \lambda_u \lambda_d \left\langle \int d^4x e^{-ip \cdot x} Q^\alpha(O_{u,d}(x) Q_\alpha(O_{d,u}(0))) \right\rangle \Big|_{p \rightarrow 0}, \\ \gamma &= \frac{i}{2} \lambda_u \lambda_d \left\langle \int d^4x e^{-ip \cdot x} O_u(x) O_d(0) \right\rangle \Big|_{p \rightarrow 0}, \\ \delta \mathcal{Z}_{u,d} &= \frac{i}{2} |\lambda_{u,d}|^2 \left\langle \int d^4x e^{-ip \cdot x} O_{u,d}(x) O_{u,d}^\dagger(0) \right\rangle \Big|_{p \rightarrow 0}. \end{aligned} \quad (2.3)$$

Again, $a'_{u,d}$ and γ arise from a *chiral-chiral* correlation function while $\delta \mathcal{Z}_{u,d}$ arises from a *chiral-antichiral* one. As explained in [18], $\delta \mathcal{Z}_{u,d}$ corresponds to the contribution to the (supersymmetric) wave-function renormalization of the Higgs fields due to (1.1), and affects the overall normalization of the physical observables. In terms of the expansion in $\lambda_{u,d}$, these corrections are negligible; hence we will not study $\delta \mathcal{Z}_{u,d}$ in detail. On the other hand, it can be seen from (2.1) and (2.3) that $a'_{u,d}$ arises from effects which cannot be captured by the μ term in the superpotential alone. Finally, γ gives rise to corrections to Higgs parameters as well as interactions between four MSSM sfermions (after eliminating F_{H_u} and F_{H_d}). We will show using our methods that γ must be suppressed in phenomenologically viable models, but $a'_{u,d}$ may be generated at the same order as μ in general.

2.1 OPE methods and approximations

Given (2.2) and (2.3), one would like to compute, at least approximately, the relevant (momentum-space) correlation functions in a manner applicable to many cases. The OPE is very useful in this regard: we can apply the OPE to the two-point functions in (2.2) and (2.3), and identify operators that can obtain nonzero vevs consistent with Poincaré and gauge invariance.⁴ Of course, we also need to compute the necessary OPE coefficients. Those are computable in the regime of large space-like or Euclidean (unphysical) momenta, but the answer can be straightforwardly analytically continued to the physical region (large time-like momenta). The critical observation, then, is that if the two-point function $A(s)$ (where $s = -p^2$) has a branch cut starting at some threshold s_0 and no other singularities in the physical sheet, we can use the dispersion relation (see figure 1)

$$A(s) = \frac{1}{2\pi i} \int_{s_0}^{\infty} ds' \frac{\text{Disc } A(s')}{s' - s} = \frac{1}{\pi} \int_{s_0}^{\infty} ds' \frac{\text{Im } A(s')}{s' - s}. \quad (2.4)$$

To summarize, our strategy is the following: we approximate $A(s)$ by going to large s , applying the OPE, keeping only the first few terms in the $1/s$ expansion, and finally using the dispersion relation (2.4) to obtain an approximation to $A(s)$ even at $s = 0$, as needed in (2.2) and (2.3).

Note that, beyond the truncation of the OPE (which can be avoided in some tractable cases), there are two more approximations being used here. First, although the OPE is

⁴From the general properties of any CFT, such operators $\mathcal{O}$ must be scalar conformal primaries, i.e. such that $K_\mu(\mathcal{O}(0)) = 0$. Such operators can only appear as the descendants of superconformal primaries with Lorentz representation $(j, \bar{j}) = \{(0, 0), (0, \frac{1}{2}), (\frac{1}{2}, 0), (\frac{1}{2}, \frac{1}{2})\}$.

Figure 1. The dashed contour can be deformed to the solid contour giving (2.4).

strictly valid for large s , we apply it in the entire region from s_0 to ∞ . In our case we take $s_0 = (2M)^2$, where, as mentioned earlier, M is the typical mass scale in the additional sector. This is a good approximation as long as M is at least somewhat higher than the electroweak scale. The other approximation comes about when we neglect the fact that the position s_0 of the branch point technically depends on the masses of the states in the additional sector, which are *not* all at M due to the presence of supersymmetry breaking. However, this is a good approximation at least at small F/M^2 , since the OPE is insensitive to the precise positions of these branch points.

As we already mentioned, in manipulations leading to (2.4) we assume that there are no poles in $A(s)$ below the threshold s_0 . This is a good approximation in models where the operators in $A(s)$ belong to some weakly-coupled sector, or perhaps certain classes of strongly coupled sectors. Nevertheless, possible poles, either from bound states or fundamental one-particle states, can make (2.4) inaccurate. This is because the Wilson coefficients in the OPE used in the evaluation of $A(s)$ will get extra contributions from poles, beyond those captured by the branch cuts in (2.4). While it would be interesting to do a more general analysis including possible contributions from such poles, in this work we will focus on the simplest analytical structure, describing contributions to soft parameters which should be present in any model with a threshold s_0 . Moreover, it is important to stress that although the computation of the Wilson coefficients is affected by the analytic structure of the two-point function, the form of the OPE is completely general and is not affected by the possible presence of poles, i.e. poles do not result in extra operators in the right-hand side of the OPE.

3 Computations of Higgs parameters

As mentioned above, we are interested in computing the chiral-chiral OPE of superfields $\mathcal{O}_u$ and $\mathcal{O}_d$, and the chiral-antichiral OPE of superfields $\mathcal{O}_{u,d}$ and $\bar{\mathcal{O}}_{u,d}$. These OPEs can be written as a sum over superconformal primary operators (those annihilated, at the origin, by the S^α and $\bar{S}^{\dot{\alpha}}$ generators of the superconformal algebra) and their descendants (obtained by acting with Q_α and $\bar{Q}_{\dot{\alpha}}$). The OPE $\Phi_1 \times \Phi_2$ of two superconformal primary superfields Φ_1 and Φ_2 will contain a superconformal primary superfield $\mathcal{O}$ only if the

superspace three-point function $\langle \Phi_1 \Phi_2 \mathcal{O}^\dagger \rangle$ is nonvanishing. Therefore, for the case at hand, one has to study three-point functions of the type $\langle \mathcal{O}_u \mathcal{O}_d \bar{\mathcal{O}}^I \rangle$ and $\langle \mathcal{O}_{u,d} \bar{\mathcal{O}}_{u,d} \bar{\mathcal{O}}^I \rangle$, where $\mathcal{O}^I$ is a general superconformal primary superfield with Lorentz index I . The Lorentz index I can be labeled by spins $(j, \bar{j})$ according to the representation of $\text{SO}(4) \simeq \text{SU}(2) \times \text{SU}(2)$. It is also customary to label the spin for traceless symmetric tensor representations by $j = \bar{j} = \frac{\ell}{2}$.

3.1 Superconformal two-point functions

In an $\mathcal{N} = 1$ superconformal theory the two-point function of superconformal primary operators is determined up to a constant. Moreover, operators of different scaling dimensions have vanishing two-point functions with each other. The two-point function of an operator $\mathcal{O}^I$ with its Hermitian conjugate $\bar{\mathcal{O}}^I$ (bars and daggers are both used in this work to denote Hermitian conjugation) is given by

$$\langle \mathcal{O}^I(z_1) \bar{\mathcal{O}}^I(z_2) \rangle = C_{\mathcal{O}} \frac{\mathcal{I}^{I\bar{I}}(x_{1\bar{2}}, x_{\bar{1}2})}{x_{1\bar{2}}^{2q} x_{\bar{1}2}^{2\bar{q}}}, \quad (3.1)$$

where $C_{\mathcal{O}}$ is positive coefficient⁵ in a unitary theory, and $\mathcal{I}^{I\bar{I}}$ is an appropriate tensor that accounts for the Lorentz structure of the left-hand side. For example, for a vector operator $\mathcal{O}^\mu$ we can take $\mathcal{I}^{\mu\nu} = \text{Tr}(\bar{\sigma}^\mu x_{1\bar{2}} \bar{\sigma}^\nu x_{\bar{1}2}) / 2\sqrt{x_{1\bar{2}}^2 x_{\bar{1}2}^2}$. The charges q and $\bar{q}$ are such that the scaling dimension and the R-charge of $\mathcal{O}^I$ are given by $\Delta = q + \bar{q}$ and $R = \frac{2}{3}(q - \bar{q})$ respectively. For more details the reader is referred to [24].

3.2 Three-point functions for the chiral-chiral OPE

A formalism for describing the three-point functions of $\mathcal{N} = 1$ superconformal primary operators was presented in [24] in full generality. The case of two scalar chiral superfields with a general third superfield was worked out in [26, 27]. The general form of the three-point function in this case is

$$\langle \Phi_1(z_{1+}) \Phi_2(z_{2+}) \bar{\mathcal{O}}^I(z_3) \rangle = \frac{\lambda_{12\mathcal{O}}}{x_{31}^{2\Delta_1} x_{32}^{2\Delta_2}} \bar{t}^I(\bar{X}_3, \Theta_3, \bar{\Theta}_3), \quad (3.2)$$

where $\lambda_{12\mathcal{O}}$ is a complex coefficient in general, z represents general superspace coordinates $\{x, \theta, \bar{\theta}\}$, and z_+ represents the chiral superspace coordinates $\{y, \theta\}$, with $y = x + i\theta\sigma\bar{\theta}$. The supersymmetric interval between points x_i and x_j is given by

$$x_{ij} = -x_{j\bar{i}} \equiv \bar{y}_i - y_j + 2i\theta_j \sigma \bar{\theta}_i = x_{ij} - i\theta_i \sigma \bar{\theta}_i - i\theta_j \sigma \bar{\theta}_j + 2i\theta_j \sigma \bar{\theta}_i,$$

and so

$$\begin{aligned} x_{ij}^2 = & x_{ij}^2 - 2i\theta_i x_{ij} \cdot \sigma \bar{\theta}_i - 2i\theta_j x_{ij} \cdot \sigma \bar{\theta}_j + 4i\theta_j x_{ij} \cdot \sigma \bar{\theta}_i \\ & + 2\theta_i^2 \bar{\theta}_i^2 + 2\theta_j^2 \bar{\theta}_j^2 - 8\theta_i \theta_j \bar{\theta}_i^2 - 8\theta_j^2 \bar{\theta}_i \bar{\theta}_j + 8\theta_j^2 \bar{\theta}_i^2, \end{aligned}$$

where $x_{ij} = x_i - x_j$. The quantities $\Theta_3, \bar{\Theta}_3$ and $X_3, \bar{X}_3$ are given by

$$\bar{\Theta}_3 = i \left(\frac{1}{x_{31}^2} \theta_{31} x_{1\bar{3}} - \frac{1}{x_{32}^2} \theta_{32} x_{2\bar{3}} \right) = \Theta_3^\dagger, \quad \bar{X}_3^\mu = \frac{1}{2} \frac{x_{3\bar{2}} x_{2\bar{1}} x_{1\bar{3}}}{x_{23}^2 x_{31}^2} \text{Tr}(\bar{\sigma}^\mu \sigma^\nu \bar{\sigma}^\rho \sigma^\sigma) = (X_3^\mu)^\dagger.$$

⁵A basis of operators can always be chosen so that all $C_{\mathcal{O}}$'s are equal to one, but in this work we will keep the $C_{\mathcal{O}}$'s explicit.

In the following we define upright quantities X by exchanging a Lorentz vector index with a pair of dotted-undotted spinor indices with the use of the Pauli matrices, i.e. $X_{\alpha\dot{\alpha}} = \sigma_{\alpha\dot{\alpha}}^{\mu} X_{\mu}$. For example,

$$\bar{X}_3 = -\frac{x_{3\bar{2}}x_{21}x_{1\bar{3}}}{x_{\bar{2}3}^2 x_{\bar{3}1}^2} = X_3^{\dagger}.$$

The chirality of $\Phi_{1,2}$ implies that the function $\bar{t}^{\bar{I}}$ can only depend on $\bar{X}_3, \bar{\Theta}_3$ and must satisfy a Ward identity. These constraints are obeyed by the three families of solutions listed below.

Solution (I):

$$\begin{aligned} \bar{t}_1(\bar{X}_3, \bar{\Theta}_3) &= 1, \\ (\Delta_{\mathcal{O}} &= \Delta_1 + \Delta_2, R_{\mathcal{O}} = R_1 + R_2), \end{aligned}$$

Solution (II):

$$\begin{aligned} \bar{t}_{2\alpha_2\ldots\alpha_{\ell}}^{\dot{\alpha}_1\ldots\dot{\alpha}_{\ell}}(\bar{X}_3, \bar{\Theta}_3) &= \bar{\Theta}_3^{(\dot{\alpha}_1} \bar{X}_{3\alpha_2}^{\dot{\alpha}_2} \cdots \bar{X}_{3\alpha_{\ell}}^{\dot{\alpha}_{\ell})}, \\ \left(\Delta_{\mathcal{O}} &= \Delta_1 + \Delta_2 + \ell - \frac{1}{2}, R_{\mathcal{O}} = R_1 + R_2 - 1 \right), \end{aligned}$$

Solution (III):

$$\begin{aligned} \bar{t}_3^{\mu_1\ldots\mu_{\ell}}(\bar{X}_3, \bar{\Theta}_3) &= \bar{\Theta}_3^2 \frac{\bar{X}_3^{\mu_1} \cdots \bar{X}_3^{\mu_{\ell}}}{\bar{X}_3^{\Delta_1 + \Delta_2 - \Delta_{\mathcal{O}} + \ell + 1}}, \\ (\Delta_{\mathcal{O}} &\geq |\Delta_1 + \Delta_2 - 3| + \ell + 2, R_{\mathcal{O}} = R_1 + R_2 - 2). \end{aligned}$$

Here $\Delta_{\mathcal{O}}$ and $R_{\mathcal{O}}$ are the scaling dimension and superconformal R-charge of the operator $\mathcal{O}$ respectively (similar notation for others). We will denote the operators for solution n and spin ℓ by $\mathcal{O}_{n,\ell}$.

Among the various operators in the OPE of the component fields of $\Phi_{1,2}$ we are only interested in scalar conformal primaries (annihilated by K_{μ} at the origin) that could develop nonzero vevs without breaking Poincaré invariance and SM gauge invariance. Note that such scalar conformal primaries can either be superconformal primaries or superconformal descendants. Now, in order to contain a scalar component, an $\mathcal{N} = 1$ superfield must have $j + \bar{j} = 0, \frac{1}{2}, 1$. Thus, the corresponding three-point functions must be given by the $\ell = 0$ case of [Solution \(I\)](#), the $\ell = 1$ case of [Solution \(II\)](#), or the $\ell = 0, 1$ cases of [Solution \(III\)](#). In the notation described above, the contributing operators are then $\mathcal{O}_{1,0}$, $\mathcal{O}_{2,1}^{\alpha}$, $\mathcal{O}_{3,0}$, and $\mathcal{O}_{3,1}^{\mu}$, which will have component expansions of the form

$$\begin{aligned} \mathcal{O} &= O + i\theta QO + i\bar{\theta}\bar{Q}O + \frac{1}{4}\theta^2 Q^2 O + \frac{1}{4}\bar{\theta}^2 \bar{Q}^2 O + \cdots, \\ \mathcal{O}_{\alpha} &= O_{\alpha} - \frac{i}{2}\theta_{\alpha} Q^{\beta} O_{\beta} + \cdots, \\ \mathcal{O}^{\mu} &= O^{\mu} + i\theta QO^{\mu} + i\bar{\theta}\bar{Q}O^{\mu} + \frac{1}{8}\theta\sigma^{\mu}\bar{\theta}(Q\sigma^{\nu}\bar{Q}O_{\nu}) + \cdots. \end{aligned}$$

Note that in the θ -expansion of a superfield one gets component fields that are not necessarily conformal primaries. As explained in appendix A, this happens at order $\theta\bar{\theta}$ and higher. In this work, when it is necessary, we will indicate that an operator is conformal primary by writing it as $[\cdot]_p$.

3.3 Three point functions for the chiral-antichiral OPE

The superconformal three-point function of a chiral superfield, an antichiral superfield and a general third operator was worked out in [26], with the result

$$\langle \Phi(z_{1+}) \bar{\Phi}(z_{2-}) \bar{\mathcal{O}}^{\mu_1 \dots \mu_\ell}(z_3) \rangle = \frac{\lambda_{12} \mathcal{O}^{\mu_1 \dots \mu_\ell}}{x_{23}^{2\Delta_\Phi} x_{31}^{2\Delta_\Phi}} \bar{X}_3^{\Delta_\Phi - 2\Delta_\Phi - \ell} \bar{X}_3^{\mu_1} \dots \bar{X}_3^{\mu_\ell}, \quad (3.3)$$

where

$$\bar{X}_3^2 = \frac{x_{21}^2}{x_{23}^2 x_{31}^2}.$$

As opposed to the chiral-chiral case, where there were three classes of structures in the right-hand side of (3.2), we see here that the solution of the superconformal constraints results in a unique class of structures for the chiral-antichiral three-point function.

3.4 Results

Armed with the above superfield three-point functions, we can expand these in $\theta, \bar{\theta}$ to compute component three-point functions. Using these, we can then extract the terms in the right-hand side of the relevant chiral-chiral and chiral-antichiral OPEs and compute contributions to the parameters in the Higgs Lagrangian. We do this below, starting with parameters which are determined by the chiral-chiral OPE. For details on the derivation of the various expressions below the reader is referred to the appendices.

3.4.1 μ

From (2.2), we see that μ is determined by $\langle Q^\alpha(O_u(x)) Q_\alpha(O_d(0)) \rangle$. The OPE $Q^\alpha(O_u(x)) \times Q_\alpha(O_d(0))$ can be computed as follows. One picks out the $\theta_1 \theta_2$ term in the expansion of (3.2), as this corresponds to the three-point function between $Q^\alpha \mathcal{O}_u$, $Q_\alpha \mathcal{O}_d$, and a third operator. From this, one can extract the terms in the OPE $Q^\alpha(O_u(x)) \times Q_\alpha(O_d(0))$. The result is

$$\begin{aligned} Q^\alpha(O_u(x)) Q_\alpha(O_d(0)) &= c_{\mu;1} Q^2 O_{1,0}(0) + c_{\mu;2} Q^\alpha O_{2,1\alpha}(0) \\ &+ \sum_i (c_{\mu;3}^i O_{3,0;i}(0) + c_{\mu;4}^i [Q^2 \bar{Q}^2 O_{3,0;i}]_p(0)) \\ &+ c_{\mu;5}^i [Q \sigma_\mu \bar{Q} O_{3,1;i}^\mu]_p(0) + \dots, \end{aligned} \quad (3.4)$$

where i is a counting index, with

$$\begin{aligned}
 c_{\mu;1} &= \frac{\lambda_{\mathcal{O}_u \mathcal{O}_d \mathcal{O}_{1,0}}}{C_{\mathcal{O}_{1,0}}} \frac{\Delta_{\mathcal{O}_u} \Delta_{\mathcal{O}_d}}{(\Delta_{\mathcal{O}_u} + \Delta_{\mathcal{O}_d})(\Delta_{\mathcal{O}_u} + \Delta_{\mathcal{O}_d} - 1)}, \\
 c_{\mu;2} &= \frac{\lambda_{\mathcal{O}_u \mathcal{O}_d \mathcal{O}_{2,1}}}{C_{\mathcal{O}_{2,1}}} \frac{\Delta_{\mathcal{O}_u} - \Delta_{\mathcal{O}_d}}{\Delta_{\mathcal{O}_u} + \Delta_{\mathcal{O}_d} - 2}, \\
 c_{\mu;3}^i &= \check{c}_{\mu;3}^i x^{\Delta_{\mathcal{O}_{3,0;i}} - \Delta_{\mathcal{O}_u} - \Delta_{\mathcal{O}_d} - 1}, \\
 c_{\mu;4}^i &= \check{c}_{\mu;4}^i x^{\Delta_{\mathcal{O}_{3,0;i}} - \Delta_{\mathcal{O}_u} - \Delta_{\mathcal{O}_d} + 1}, \\
 c_{\mu;5}^i &= \check{c}_{\mu;5}^i x^{\Delta_{\mathcal{O}_{3,1;i}^\mu} - \Delta_{\mathcal{O}_u} - \Delta_{\mathcal{O}_d}},
 \end{aligned}$$

where

$$\begin{aligned}
 \check{c}_{\mu;3}^i &= 4 \frac{\lambda_{\mathcal{O}_u \mathcal{O}_d \mathcal{O}_{3,0;i}}}{C_{\mathcal{O}_{3,0;i}}}, \\
 \check{c}_{\mu;4}^i &= \frac{1}{2^4} \frac{\lambda_{\mathcal{O}_u \mathcal{O}_d \mathcal{O}_{3,0;i}}}{C_{\mathcal{O}_{3,0;i}}} \frac{(\Delta_{\mathcal{O}_u} - \Delta_{\mathcal{O}_d} - \Delta_{\mathcal{O}_{3,0;i}} - 1)(\Delta_{\mathcal{O}_u} - \Delta_{\mathcal{O}_d} + \Delta_{\mathcal{O}_{3,0;i}} + 1)}{\Delta_{\mathcal{O}_{3,0;i}}(\Delta_{\mathcal{O}_{3,0;i}} + 1)(\Delta_{\mathcal{O}_u} + \Delta_{\mathcal{O}_d} - \Delta_{\mathcal{O}_{3,0;i}} - 1)(\Delta_{\mathcal{O}_u} + \Delta_{\mathcal{O}_d} - \Delta_{\mathcal{O}_{3,0;i}} - 3)}, \\
 \check{c}_{\mu;5}^i &= \frac{i}{2} \frac{\lambda_{\mathcal{O}_u \mathcal{O}_d \mathcal{O}_{3,1;i}^\mu}}{C_{\mathcal{O}_{3,1;i}^\mu}} \frac{\Delta_{\mathcal{O}_u} - \Delta_{\mathcal{O}_d}}{(\Delta_{\mathcal{O}_{3,1;i}^\mu} - 2)(\Delta_{\mathcal{O}_{3,1;i}^\mu} - \Delta_{\mathcal{O}_u} - \Delta_{\mathcal{O}_d})}.
 \end{aligned}$$

For a flavor of what is involved in the derivation of the c_μ^i 's see appendix B. Note that the three-point function coefficients $\lambda_{\mathcal{O}_u \mathcal{O}_d \mathcal{O}_{2,1}}$ and $\lambda_{\mathcal{O}_u \mathcal{O}_d \mathcal{O}_{3,1;i}^\mu}$ are antisymmetric under $\mathcal{O}_u \leftrightarrow \mathcal{O}_d$.

Some comments are in order. We see that all three solutions in section 3.2 contribute to the OPE. From (3.4) we see that a supersymmetric contribution to μ can only arise from $\mathcal{O}_{3,0}$, since all other contributions arise as $Q(\cdot)$. However, $\mathcal{O}_{3,0}$ can also get contributions from supersymmetry-breaking. If the theory contains spurions, then operators constructed with them can get a vev. Such operators correspond to $\mathcal{O}_{3,0}$ in (3.4). In a given model one has to identify candidate operators that can appear in the right-hand side of (3.4) and obtain an expectation value consistent with Poincaré and gauge invariance. Those are generally not unique operators, so (3.4) contains a sum over all appropriate operators in the right-hand side. By substituting the above result in (2.2) and using dispersion relations, it is possible to compute μ . We will do this explicitly for a model in section 4.

Before we proceed, though, let us examine the first two terms of (3.4) more carefully. Those terms have no x -dependence, which is a reflection of the fact that the scaling dimension of the operators $Q^2 \mathcal{O}_{1,0}$ and $Q^\alpha \mathcal{O}_{2,1\alpha}$ is determined by the scaling dimensions of the operators in the left-hand side of (3.4). Now, from (2.2) we see that we have to Fourier-transform (3.4) and take the zero-momentum limit. If we use the OPE to express the operator product in the two-point function, though, the zero-momentum limit becomes problematic, for the Wilson coefficient cannot be evaluated directly in that limit. However, we can use the dispersion relation (2.4) to argue that the terms in the first line of (3.4) actually *do not* contribute to μ . Indeed, in order to regulate the Fourier transform let us use

$$i \int d^4x e^{-ip \cdot x} \frac{1}{(x^2)^\epsilon} = \pi^2 \frac{\Gamma(2 - \epsilon)}{2^{2\epsilon - 4} \Gamma(\epsilon)} \frac{1}{(p^2)^{2 - \epsilon}}, \quad (3.5)$$

and take the limit $\epsilon \rightarrow 0$ at the end of the computation. Expanding the right-hand side of (3.5) we see that all terms involving $\ln(-s)$ are multiplied with at least one power of ϵ , and thus applying (2.4) gives a vanishing result as $\epsilon \rightarrow 0$. Thus, the first two terms in (3.4) do not give rise to contributions to μ .

It is also possible to understand this result intuitively. The fact that the Wilson coefficients of the operators $Q^2 O_{1,0}$ and $Q^\alpha O_{2,1\alpha}$ have no x -dependence is inconsistent with the presence of a threshold at the scale M . Indeed, if there is no x -dependence one can take x to be very large (such as $x \gg \frac{1}{M}$), and still expect a non-vanishing contribution to μ . However, from general considerations one expects that the presence of a threshold at a scale M implies that a general two-point function should factorize, i.e. $\langle \mathcal{O}_1(x) \mathcal{O}_2(0) \rangle \sim \langle \mathcal{O}_1(x) \rangle \langle \mathcal{O}_2(0) \rangle$ for $x \gg \frac{1}{M}$. In the example above, the relevant operators are $Q^\alpha O_u$ and $Q_\alpha O_d$, which are assumed to not get vevs, since that would break the visible-sector gauge group. Thus, the two-point function must die away for $x \gg \frac{1}{M}$. This implies that the only consistent possibility is that these terms do not contribute, as can be verified from the computation with (3.5) above.

Consequently, we can use (2.2) to write

$$\mu = \lambda_u \lambda_d \sum_i (\tilde{c}_{\mu;3}^i \langle O_{3,0;i} \rangle + \tilde{c}_{\mu;4}^i \langle Q^2 \bar{Q}^2 O_{3,0;i} \rangle + \tilde{c}_{\mu;5}^i \langle Q \sigma_\mu \bar{Q} O_{3,1;i}^\mu \rangle), \quad (3.6)$$

with

$$\begin{aligned} \tilde{c}_{\mu;3}^i &= \frac{i}{8} \tilde{c}_{\mu;3}^i \int d^4x e^{-ip \cdot x} x^{\Delta_{\mathcal{O}_{3,0;i}} - \Delta_{\mathcal{O}_u} - \Delta_{\mathcal{O}_d} - 1} \Big|_{p \rightarrow 0}, \\ \tilde{c}_{\mu;4}^i &= \frac{i}{8} \tilde{c}_{\mu;4}^i \int d^4x e^{-ip \cdot x} x^{\Delta_{\mathcal{O}_{3,0;i}} - \Delta_{\mathcal{O}_u} - \Delta_{\mathcal{O}_d} + 1} \Big|_{p \rightarrow 0}, \\ \tilde{c}_{\mu;5}^i &= \frac{i}{8} \tilde{c}_{\mu;5}^i \int d^4x e^{-ip \cdot x} x^{\Delta_{\mathcal{O}_{3,1;i}^\mu} - \Delta_{\mathcal{O}_u} - \Delta_{\mathcal{O}_d}} \Big|_{p \rightarrow 0}. \end{aligned} \quad (3.7)$$

As stated in the introduction, the two-point functions in (2.2) and (2.3) are assumed to have a branch cut, starting from a threshold $s_0 = 4M^2$ in momentum space, and no other singularities. Each Wilson coefficient in (3.4) has a branch cut in the UV determined by superconformal symmetry. Its branch point in general differs from s_0 . After resumming the OPE, branch cuts from Wilson coefficients should combine into that of the two-point function. In cases where such complete resummation is not practical, the first few terms in the OPE may still provide a reasonable estimate, if their branch cuts are assumed to start from the threshold. This further approximation provides an IR regulator for evaluating (3.7). Fourier transforming a Wilson coefficient with (3.5), we may obtain its value at $p^2 \rightarrow 0$ by integrating around its branch cut that starts from s_0 . The result is

$$\lim_{p^2 \rightarrow 0} i \int d^4x e^{-ip \cdot x} x^\alpha \rightarrow \frac{\alpha + 2}{\alpha + 4} \Gamma^2 \left(1 + \frac{\alpha}{2} \right) \sin^2 \left(\frac{\alpha \pi}{2} \right) \frac{1}{M^{\alpha+4}}. \quad (3.8)$$

Applying this result to (3.7) we get

$$\begin{aligned}
 \hat{c}_{\mu;3}^i &= \frac{\lambda_{\mathcal{O}_u \mathcal{O}_d \mathcal{O}_{3,0;i}}}{C_{\mathcal{O}_{3,0;i}}} \frac{\eta_0 + 1}{\eta_0 + 3} \Gamma^2\left(\frac{\eta_0 + 1}{2}\right) \sin^2\left(\frac{\eta_0 \pi}{2}\right) \frac{1}{M^{\eta_0+3}}, \\
 \hat{c}_{\mu;4}^i &= \frac{1}{2^9} \frac{\lambda_{\mathcal{O}_u \mathcal{O}_d \mathcal{O}_{3,0;i}}}{C_{\mathcal{O}_{3,0;i}}} \frac{(\Delta_{\mathcal{O}_u} - \Delta_{\mathcal{O}_d} - \Delta_{\mathcal{O}_{3,0;i}} - 1)(\Delta_{\mathcal{O}_u} - \Delta_{\mathcal{O}_d} + \Delta_{\mathcal{O}_{3,0;i}} + 1)}{\Delta_{\mathcal{O}_{3,0;i}}(\Delta_{\mathcal{O}_{3,0;i}} + 1)} \\
 &\quad \times \frac{\eta_0 + 1}{\eta_0 + 5} \Gamma^2\left(\frac{\eta_0 + 1}{2}\right) \cos^2\left(\frac{\eta_0 \pi}{2}\right) \frac{1}{M^{\eta_0+5}}, \\
 \hat{c}_{\mu;5}^i &= -\frac{i}{2^4} \frac{\lambda_{\mathcal{O}_u \mathcal{O}_d \mathcal{O}_{3,1;i}}}{C_{\mathcal{O}_{3,1;i}}} \frac{\Delta_{\mathcal{O}_u} - \Delta_{\mathcal{O}_d}}{\Delta_{\mathcal{O}_{3,1;i}}^\mu - 2} \frac{\eta_1 + 2}{\eta_1(\eta_1 + 4)} \Gamma^2\left(\frac{\eta_1}{2} + 1\right) \sin^2\left(\frac{\eta_1 \pi}{2}\right) \frac{1}{M^{\eta_1+4}}, \quad (3.9)
 \end{aligned}$$

where

$$\eta_0 \equiv \Delta_{\mathcal{O}_{3,0;i}} - \Delta_{\mathcal{O}_u} - \Delta_{\mathcal{O}_d}, \quad \eta_1 \equiv \Delta_{\mathcal{O}_{3,1;i}}^\mu - \Delta_{\mathcal{O}_u} - \Delta_{\mathcal{O}_d}. \quad (3.10)$$

Equivalently, the calculation of the necessary OPE coefficients can be done directly in momentum space. We will see an explicit example of the latter approach in section 4.

3.4.2 B_μ

The parameter B_μ is determined by $\langle Q^2(\mathcal{O}_u(x))Q^2(\mathcal{O}_d(0)) \rangle$. Similar to the previous case, one has to now expand the three-point function (3.2) to order $\theta_1^2 \theta_2^2$, which corresponds to the three-point function between $\frac{1}{4}Q^2 \mathcal{O}_u$, $\frac{1}{4}Q^2 \mathcal{O}_d$, and a third operator. From this, one can extract the terms in the OPE $Q^2 \mathcal{O}_u(x) \times Q^2 \mathcal{O}_d(0)$.

It turns out that only [Solution \(III\)](#) with spin $\ell = 0$ contributes to the B_μ term. This can be understood as follows. For a potential [Solution \(I\)](#) contribution, every $\theta_{1,2}$ has to come with a $\bar{\theta}_3$. But since $\bar{\theta}_3^4$ vanishes, [Solution \(I\)](#) does not contribute. For a potential [Solution \(II\)](#) contribution with $\ell = 1$, one $\theta_{1,2}$ could come from $\bar{\Theta}_3$, but there are at least three $\theta_{1,2}$'s which need to be paired with $\bar{\theta}_3$. Again, since $\bar{\theta}_3^3$ vanishes, [Solution \(II\)](#) does not contribute as well. Finally, for [Solution \(III\)](#) with $\ell = 1$ the lowest scalar component field arises at order $\theta_3 \sigma_\mu \bar{\theta}_3$. So one needs at least order $\theta_1^2 \theta_2^2 \theta_3 \bar{\theta}_3^3$ because two θ 's could come from $\bar{\Theta}^2$. Again, since $\bar{\theta}_3^3$ vanishes, [Solution \(III\)](#) with $\ell = 1$ does not contribute. Of course these results can be understood from the relation between μ and B_μ we mentioned after (2.2).

The contribution from [Solution \(III\)](#) with $\ell = 0$ is

$$\begin{aligned}
 Q^2(\mathcal{O}_u(x))Q^2(\mathcal{O}_d(0)) &= \sum_i \hat{c}_{B_\mu}^i Q^2 \mathcal{O}_{3,0;i}(0), \\
 \hat{c}_{B_\mu}^i &= -4 \frac{\lambda_{\mathcal{O}_u \mathcal{O}_d \mathcal{O}_{3,0;i}}}{C_{\mathcal{O}_{3,0;i}}} x^{\Delta_{\mathcal{O}_{3,0;i}} - \Delta_{\mathcal{O}_u} - \Delta_{\mathcal{O}_d} - 1} = -\hat{c}_{\mu,3}^i, \quad (3.11)
 \end{aligned}$$

which allows us to compute B_μ using (2.2), with the result

$$B_\mu = \lambda_u \lambda_d \sum_i \hat{c}_{B_\mu}^i \langle Q^2 \mathcal{O}_{3,0;i} \rangle, \quad (3.12)$$

where

$$\hat{c}_{B_\mu}^i = -\frac{1}{4} \hat{c}_{\mu,3}^i. \quad (3.13)$$

To summarize, B_μ receives contributions only from $Q^2 O_{3,0}$. This is to be contrasted with μ , which receives contributions from $O_{3,0}$, $Q^2 \bar{Q}^2 O_{3,0}$, and $Q\sigma_\mu \bar{Q} O_{3,1}^\mu$. The implications of this result for solutions to the μ/B_μ problem in models of gauge mediation will be discussed more in section 6.

3.4.3 $\delta A_{u,d}$ and $\delta m_{H_{u,d}}^2$

We now study correlation functions which determine the chiral-antichiral OPE. From (2.2), we see that $\delta A_{u,d}$ and $\delta m_{H_{u,d}}^2$ are determined by $\langle Q^2(O_{u,d}(x)O_{u,d}^\dagger(0)) \rangle$ and $\langle Q^2 \bar{Q}^2(O_{u,d}(x)O_{u,d}^\dagger(0)) \rangle$ respectively. Thus, we carry out the OPE $O_{u,d}(x) \times O_{u,d}^\dagger(0)$, and then act with Q^2 and $Q^2 \bar{Q}^2$ to obtain $\delta A_{u,d}$ and $\delta m_{H_{u,d}}^2$ respectively. For this computation we can set $\theta_{1,2} = \bar{\theta}_{1,2} = 0$, in order to focus on the three-point function of $O_{u,d}$, $O_{u,d}^\dagger$ and a third operator. From this, one can extract terms in the OPE $O_{u,d}(x) \times O_{u,d}^\dagger(0)$ as before.

Now, it can be checked that the OPE $O_{u,d}(x) \times O_{u,d}^\dagger(0)$ receives contributions from various components of a general superfield $\mathcal{O}_0$ (the subscript stands for the spin $\ell = 0$). The possibilities are the scalar component O_0 , the $\theta\sigma^\mu\bar{\theta}$ component $V_{\mathcal{O}_0}^\mu$, and the $\theta^2\bar{\theta}^2$ component $D_{\mathcal{O}_0}$. However, we are ultimately interested in $\delta A_{u,d}$ and $\delta m_{H_{u,d}}^2$, which are obtained by acting on the above with Q^2 and $Q^2 \bar{Q}^2$ respectively and taking the vev. The only operator of interest after this is O_0 .

The superfield three-point function for the $\ell = 0$ case is

$$\langle \mathcal{O}_{u,d}(z_{1+}) \bar{\mathcal{O}}_{u,d}(z_{2-}) \bar{\mathcal{O}}_0(z_3) \rangle = \lambda_{\mathcal{O}_{u,d} \bar{\mathcal{O}}_{u,d} \mathcal{O}_0} \frac{x_{\bar{2}1}^{\Delta_{\mathcal{O}_0} - 2\Delta_{\mathcal{O}_{u,d}}}}{x_{\bar{2}3}^{\Delta_{\mathcal{O}_0}} x_{31}^{\Delta_{\mathcal{O}_0}}},$$

from which one can extract the OPE

$$O_{u,d}(x)O_{u,d}^\dagger(0) = c_{u,d}^i O_{0;i}^{u,d}(0) + \dots, \quad c_{u,d}^i = \check{c}_{u,d}^i x^{\Delta_{\mathcal{O}_{0;i}^{u,d}} - 2\Delta_{\mathcal{O}_{u,d}}}, \quad \check{c}_{u,d}^i = \frac{\lambda_{\mathcal{O}_{u,d} \bar{\mathcal{O}}_{u,d} \mathcal{O}_{0;i}^{u,d}}}{C_{\mathcal{O}_{0;i}^{u,d}}}. \quad (3.14)$$

With this result we can compute $\delta A_{u,d}$ and $\delta m_{H_{u,d}}^2$ using expressions (2.2). We obtain

$$\delta A_{u,d} = |\lambda_{u,d}|^2 \sum_i \hat{c}_{A_{u,d}}^i \langle Q^2 O_{0;i}^{u,d} \rangle, \quad \hat{c}_{A_{u,d}}^i = -\frac{i}{8} \check{c}_{u,d}^i \int d^4x e^{-ip \cdot x} x^{\Delta_{\mathcal{O}_{0;i}^{u,d}} - 2\Delta_{\mathcal{O}_{u,d}}} \Big|_{p \rightarrow 0} \quad (3.15)$$

If we evaluate $\hat{c}_{A_{u,d}}^i$ with (3.8), we get

$$\hat{c}_{A_{u,d}}^i = -\frac{1}{2^3} \frac{\gamma_{u,d} + 2}{\gamma_{u,d} + 4} \Gamma^2\left(1 + \frac{\gamma_{u,d}}{2}\right) \sin^2\left(\frac{\gamma_{u,d}\pi}{2}\right) \frac{\lambda_{\mathcal{O}_{u,d} \bar{\mathcal{O}}_{u,d} \mathcal{O}_{0;i}^{u,d}}}{C_{\mathcal{O}_{0;i}^{u,d}}} \frac{1}{M^{\gamma_{u,d}+4}},$$

where

$$\gamma_{u,d} = \Delta_{\mathcal{O}_{0;i}^{u,d}} - 2\Delta_{\mathcal{O}_{u,d}}.$$

The $\delta m_{H_{u,d}}^2$ term is given by

$$\delta m_{H_{u,d}}^2 = |\lambda_{u,d}|^2 \sum_i \hat{c}_{m_{H_{u,d}}^2}^i \langle Q^2 \bar{Q}^2 O_{0;i}^{u,d} \rangle, \quad \hat{c}_{m_{H_{u,d}}^2}^i = -\frac{i}{2^5} \check{c}_{u,d}^i \int d^4x e^{-ip \cdot x} x^{\Delta_{\mathcal{O}_{0;i}^{u,d}} - 2\Delta_{\mathcal{O}_{u,d}}} \Big|_{p \rightarrow 0}. \quad (3.16)$$

Note that, unlike the case of μ and B_μ , the same term in the OPE contributes to both $\delta A_{u,d}$ and $\delta m_{H_{u,d}}^2$. Analogous to (3.13) we here have the relation

$$\hat{c}_{m_{H_{u,d}}^2}^i = \frac{1}{4} \hat{c}_{A_{u,d}}^i. \quad (3.17)$$

This will have implications for the A/m_H^2 problem [19], and will also be discussed in section 6.

3.4.4 $a'_{u,d}$ (“wrong Higgs” trilinears)

It is also possible to compute the couplings in $V_{\text{Higgs}}^{(\text{other})}$ (see (2.3)) using the same methods. As mentioned earlier, and explained in [18], $\delta \mathcal{Z}_{u,d}$ arises from supersymmetric wave-function renormalization and does not affect any physical observables at leading order in $\lambda_{u,d}$. Next, we study the couplings $a'_{u,d}$. As seen from (2.1), (2.2) and (2.3), $a'_{u,d}$ arises from effects which cannot be captured by the μ parameter alone. They can be thought of as providing “wrong Higgs” trilinear parameters after one replaces $F_{\mathcal{H}_u}$ and $F_{\mathcal{H}_d}$ by their equation of motion (see (2.1)).⁶

From (2.3), $a'_{u,d}$ is determined by $\langle Q^\alpha(O_{u,d}(x) Q_\alpha(O_{d,u}(0))) \rangle$. Since it can be written as $\langle Q(\cdot) \rangle$, it can only get contributions starting at order F/M .⁷ In order to compute $a'_{u,d}$, we first find the OPE $O_{u,d}(x) \times Q_\alpha(O_{d,u}(0))$ and then act with Q^α . Using the same procedure as for the other parameters, we find

$$\begin{aligned} Q^\alpha(O_{u,d}(x) Q_\alpha O_{d,u}(0)) &= c_{a'_{u,d;1}} Q^2 O_{1,0}(0) + c_{a'_{u,d;2}} Q^\alpha O_{2,1\alpha}(0) \\ &+ \sum_i (c_{a'_{u,d;3}}^i [Q^2 \bar{Q}^2 O_{3,0;i}]_p(0) + c_{a'_{u,d;4}}^i [Q \sigma_\mu \bar{Q} O_{3,1;i}^\mu]_p(0)) + \dots \end{aligned} \quad (3.18)$$

with

$$\begin{aligned} c_{a'_{u,d;1}} &= \frac{\Delta_{\mathcal{O}_{d,u}}}{\Delta_{\mathcal{O}_u} + \Delta_{\mathcal{O}_d}} \frac{\lambda_{\mathcal{O}_{u,d} \mathcal{O}_{d,u} \mathcal{O}_{1,0}}}{C_{\mathcal{O}_{1,0}}}, & c_{a'_{u,d;2}} &= \frac{\lambda_{\mathcal{O}_{u,d} \mathcal{O}_{d,u} \mathcal{O}_{2,1}}}{C_{\mathcal{O}_{2,1}}}, \\ c_{a'_{u,d;3}}^i &= \tilde{c}_{a'_{u,d;3}}^i x^{\Delta_{\mathcal{O}_{3,0;i}} - \Delta_{\mathcal{O}_u} - \Delta_{\mathcal{O}_d} + 1}, & c_{a'_{u,d;4}}^i &= \tilde{c}_{a'_{u,d;4}}^i x^{\Delta_{\mathcal{O}_{3,1;i}}^\mu - \Delta_{\mathcal{O}_u} - \Delta_{\mathcal{O}_d}}, \end{aligned}$$

where

$$\begin{aligned} \tilde{c}_{a'_{u,d;3}}^i &= \frac{1}{8} \frac{\lambda_{\mathcal{O}_{u,d} \mathcal{O}_{d,u} \mathcal{O}_{3,0;i}}}{C_{\mathcal{O}_{3,0;i}}} \times \\ &\times \frac{\Delta_{\mathcal{O}_{u,d}} - \Delta_{\mathcal{O}_{d,u}} - \Delta_{\mathcal{O}_{3,0;i}} - 1}{(\Delta_{\mathcal{O}_{3,0;i}} + 1)(\Delta_{\mathcal{O}_u} + \Delta_{\mathcal{O}_d} - \Delta_{\mathcal{O}_{3,0;i}} - 1)(\Delta_{\mathcal{O}_u} + \Delta_{\mathcal{O}_d} - \Delta_{\mathcal{O}_{3,0;i}} - 3)}, \\ \tilde{c}_{a'_{u,d;4}}^i &= \frac{i}{2} \frac{\lambda_{\mathcal{O}_{u,d} \mathcal{O}_{d,u} \mathcal{O}_{3,1;i}^\mu}}{C_{\mathcal{O}_{3,1;i}^\mu}} \frac{1}{\Delta_{\mathcal{O}_u} + \Delta_{\mathcal{O}_d} - \Delta_{\mathcal{O}_{3,1;i}^\mu}}. \end{aligned}$$

⁶The $a'_{u,d}$ terms are called “maybe soft” in [28], but if there are no SM gauge singlets in the theory (as in the MSSM), they are soft and do not introduce quadratic divergences (see discussion in [28]).

⁷Here we assume that F/M^2 is smaller than unity, as mentioned earlier.

Just like in the case of μ , the first two terms in the right-hand side of (3.18) do not end up contributing to $a'_{u,d}$. Therefore, we can write

$$a'_{u,d} = \sum_i (\hat{c}_{a'_{u,d};3}^i \langle Q^2 \bar{Q}^2 O_{3,0;i} \rangle + \hat{c}_{a'_{u,d};4}^i \langle Q \sigma_\mu \bar{Q} O_{3,1;i}^\mu \rangle), \quad (3.19)$$

with

$$\begin{aligned} \hat{c}_{a'_{u,d};3}^i &= \frac{i}{8} \tilde{c}_{a'_{u,d};3}^i \int d^4x e^{-ip \cdot x} x^{\Delta_{\mathcal{O}_{3,0;i}} - \Delta_{\mathcal{O}_u} - \Delta_{\mathcal{O}_d} + 1} \Big|_{p \rightarrow 0}, \\ \hat{c}_{a'_{u,d};4}^i &= \frac{i}{8} \tilde{c}_{a'_{u,d};4}^i \int d^4x e^{-ip \cdot x} x^{\Delta_{\mathcal{O}_{3,1;i}^\mu} - \Delta_{\mathcal{O}_u} - \Delta_{\mathcal{O}_d}} \Big|_{p \rightarrow 0}. \end{aligned}$$

Using (3.8) we get

$$\begin{aligned} \hat{c}_{a'_{u,d};3}^i &= \frac{1}{28} \frac{\lambda_{\mathcal{O}_{u,d} \mathcal{O}_{d,u} \mathcal{O}_{3,0;i}}}{C_{\mathcal{O}_{3,0;i}}} \frac{\Delta_{\mathcal{O}_{u,d}} - \Delta_{\mathcal{O}_{d,u}} - \Delta_{\mathcal{O}_{3,0;i}} - 1}{\Delta_{\mathcal{O}_{3,0;i}} + 1} \\ &\quad \times \frac{\eta_0 + 1}{\eta_0 + 5} \Gamma^2\left(\frac{\eta_0 + 1}{2}\right) \cos^2\left(\frac{\eta_0 \pi}{2}\right) \frac{1}{M^{\eta_0 + 5}}, \\ \hat{c}_{a'_{u,d};4}^i &= -\frac{i}{24} \frac{\lambda_{\mathcal{O}_{u,d} \mathcal{O}_{d,u} \mathcal{O}_{3,1;i}}}{C_{\mathcal{O}_{3,1;i}}} \frac{\eta_1 + 2}{\eta_1(\eta_1 + 4)} \Gamma^2\left(\frac{\eta_1}{2} + 1\right) \sin^2\left(\frac{\eta_1 \pi}{2}\right) \frac{1}{M^{\eta_1 + 4}}, \end{aligned} \quad (3.20)$$

with $\eta_{0,1}$ given in (3.10).

By comparing (3.6) and (3.19), we see that $Q^2 \bar{Q}^2 O_{3,0}$ and $Q \sigma_\mu \bar{Q} O_{3,1}^\mu$ contribute to both μ and $a'_{u,d}$, with comparable Wilson coefficients. This implies that unless $O_{3,0}$ is the dominant operator contributing to μ , $a'_{u,d}$ may be generated at the same order as μ in general. In simple (spurion-based) models of supersymmetry breaking, however, it can be shown that $O_{3,0}$ gives the dominant contribution to μ . Hence, in such cases, $a'_{u,d}$ is suppressed compared to μ .

Note that although $a'_{u,d}$ may be generated at the same order as μ in general, it can still only be generated at order F^2 . This is because both operators $Q^2 \bar{Q}^2 O_{3,0}$ and $Q \sigma_\mu \bar{Q} O_{3,1}^\mu$, which contribute to $a'_{u,d}$ in general, are generated at order F^2 . This can be easily seen for the former. For the latter, this is true because $\langle Q \sigma_\mu \bar{Q} O_{3,1}^\mu \rangle$ is of the form $\langle \{Q, [\bar{Q}, (\cdot)]\} \rangle$, and all such correlation functions of this form must start at order F^2 [18]. Thus, the wrong-Higgs trilinears will generically be suppressed to other soft parameters which start at order F . We will make more comments about this result in section 6.

3.4.5 γ

Finally, we discuss the remaining parameter in $V_{\text{Higgs}}^{(\text{other})}$, namely γ , which gives rise to a dimensionless coupling between four MSSM sfermions for example (in addition to other terms), after using the equation of motion (see (2.1)). Now, γ is determined by $\langle O_u(x) O_d(0) \rangle$ from (2.3). It can be shown that this dimensionless parameter must be suppressed at least by order F/M^2 in phenomenologically viable models.

Indeed, suppose $\langle O_u(x) O_d(0) \rangle$ is nonvanishing in the supersymmetric limit. Then, since it is a correlation function of scalar chiral primary operators, it does not depend

on the separation $|x|$ of the two operators [29]. This implies that one can take $|x|$ to be very large, and apply the cluster-decomposition principle. Thus, $\lim_{|x| \rightarrow \infty} \langle O_u(x) O_d(0) \rangle = \langle O_u \rangle \langle O_d \rangle \neq 0$. However, since O_u and O_d are charged under the SM gauge symmetry (see (1.1)), this explicitly breaks the SM gauge symmetry. Hence, $\langle O_u(x) O_d(0) \rangle$ can only receive contributions from supersymmetry breaking, and must be at least of order F/M^2 .

4 A weakly-coupled example

In this section we use OPE methods to compute the Higgs parameters for a simple model that was considered in [4]. This model illustrates the μ/B_μ problem in gauge-mediated supersymmetry breaking. The model contains messengers $\Phi_{1,2}$ and $\tilde{\Phi}_{1,2}$, with $\mathcal{O}_u = \Phi_1 \Phi_2$ and $\mathcal{O}_d = \tilde{\Phi}_1 \tilde{\Phi}_2$. Here Φ_1 is a $\mathbf{5}$, $\tilde{\Phi}_1$ a $\bar{\mathbf{5}}$, and Φ_2 and $\tilde{\Phi}_2$ singlets of $SU(5)$. The superpotential that couples the messenger to the Higgs sector is

$$W = \lambda_u \mathcal{H}_u \Phi_1 \Phi_2 + \lambda_d \mathcal{H}_d \tilde{\Phi}_1 \tilde{\Phi}_2 = \lambda_u \epsilon^{ij} (H_u)_i (\Phi_1)_j \Phi_2 + \lambda_d \epsilon_{ij} (H_d)^i (\tilde{\Phi}_1)^j \tilde{\Phi}_2,$$

where i, j are $SU(2)$ indices, while the messenger sector is coupled to the hidden sector via

$$W = \lambda X (\Phi_1 \tilde{\Phi}_1 + \Phi_2 \tilde{\Phi}_2),$$

where X is a spurion that gets a vev in both the first and the last component, $\langle X \rangle = X + \theta^2 F$. The computations below are done at one loop and to leading order in F/X^2 .

To start, let us compute the leading contribution to μ . From (2.2) we see that we have to consider the OPE of the fermionic components of the composite operators $\Phi_1 \Phi_2$ and $\tilde{\Phi}_1 \tilde{\Phi}_2$. We use the results of section 3 to compute the right-hand side of the OPE, and concentrate on the operator of the lowest scaling dimension that has a nonzero supersymmetry-breaking vev consistent with Poincaré and gauge invariance:

$$i \int d^4x e^{-ip \cdot x} Q^\alpha(\Phi_1 \Phi_2(x)) Q_\alpha(\tilde{\Phi}_1 \tilde{\Phi}_2(0)) = \tilde{c}_{X^\dagger F^\dagger}(s) X^\dagger F^\dagger(0) + \dots, \quad s = -p^2.$$

Note that $Q^\alpha(\Phi_1 \Phi_2) = -i\sqrt{2}(\Phi_1 \psi_{\Phi_2}^\alpha + \Phi_2 \psi_{\Phi_1}^\alpha)$ and similarly for $\tilde{\Phi}_{1,2}$. The operator $X^\dagger F^\dagger$, which is an example of an $\mathcal{O}_{3,0}$ in (3.4), turns out to be the leading operator with the correct R -charge. Of course, there are higher dimension operators of the form $X^\dagger F^\dagger (X^\dagger X)^n (F^\dagger F)^m$, with $n, m \geq 0$ but not both zero, which also contribute to the OPE. However, as advocated in [30] we expect a good approximation to the full answer with this truncation. The Wilson coefficient at one loop is given by (messengers run in the loop)⁸

$\xrightarrow{p}$

$= \frac{i(\lambda^*)^2}{\pi^2} \frac{1}{Q^2} \int_0^1 dx x \frac{x(1-x) + 2\xi}{(x(1-x) + \xi)^2},$

$\xrightarrow{p}$

(4.1)

⁸The factor of -2^3 comes about as follows: a 2 because we have two equal diagrams due of the presence of both $\Phi_{1,2}$ and $\tilde{\Phi}_{1,2}$, a $-2 = (-i\sqrt{2})^2$ from the insertions of the operators at the crosses, and a 2 from the fact that our messengers Φ_1 and $\tilde{\Phi}_1$ are in the $\mathbf{5}$ and $\bar{\mathbf{5}}$ of $SU(5)$ respectively, with the branching rule $\mathbf{5} = (\mathbf{1}, \mathbf{2})_{-\frac{1}{2}} + (\mathbf{1}, \mathbf{3})_{\frac{1}{3}}$ into the standard model $SU(3) \times SU(2) \times U(1)$, so that doublets run in the loop.

where $\xi = \mu^2/Q^2$ with μ an arbitrary normalization point necessary in the OPE [31], and $Q^2 = p_E^2 = p^2 = -s$ ($s > 0$ in the physical region). Note that for convergence of the loop-integral we go to the Euclidean region $Q^2 > 0$. The integral over the Feynman parameter x can be performed analytically, and since we will rely on (2.4) we can expand the answer and keep only the $\ln \xi$ -term. From (4.1) we thus find

$$\tilde{c}_{X^\dagger F^\dagger}(s) = \frac{(\lambda^*)^2}{2\pi^2} \frac{1}{s} \ln \frac{-s}{\mu^2} + \dots, \quad (4.2)$$

and since $\text{Im} \ln(-(s \pm i\epsilon)) = \mp\pi$, $s > 0$, we can use (2.4) with $s_0 = 4|\lambda X|^2$ to obtain

$$\tilde{c}_{X^\dagger F^\dagger}(0) = -\frac{\lambda^*}{4\pi^2 \lambda |X|^2}.$$

With this result it is straightforward to find the leading contribution to μ :

$$\mu \approx -\frac{1}{2} \frac{\lambda_u \lambda_d \lambda^*}{16\pi^2 \lambda} \frac{F^\dagger}{X}. \quad (4.3)$$

This is 50% of the result obtained in [4]. The full result can be obtained by computing the contributions of all operators of the form $X^\dagger F^\dagger (X^\dagger X)^n (F^\dagger F)^m$, $m, n \geq 0$, and resumming them. For simplicity, we will not do this resummation for μ or B_μ , but we will do part of it for $A_{u,d}$ and $m_{H_{u,d}}^2$.

The calculation of B_μ proceeds along similar lines. As seen in (2.2) we consider here the OPE $Q^2(\Phi_1 \Phi_2(x)) \times Q^2(\tilde{\Phi}_1 \tilde{\Phi}_2(0))$. Clearly, the operator $(X^\dagger)^2$ is the leading contribution to this OPE. Nevertheless, the vev of this operator is supersymmetry-preserving, which means that its Wilson coefficient must be zero, for the only contributions to B_μ are of the form $Q^2(O_{3,0})$ (see (3.11)). Indeed, at one loop,

$$-i\tilde{c}_{(X^\dagger)^2}(s) = 2^5 \times \text{[Solid Loop Diagram]} + 2^6 \times \text{[Dashed Loop Diagram]} \xrightarrow{p} = 0. \quad (4.4)$$

This is a consequence of supersymmetry. Similar cancellations persist order-by-order in perturbation theory, and also for the operators $(X^\dagger)^2 (X^\dagger X)^n$ for any $n \geq 1$.

We saw before that the leading operator of type $O_{3,0}$ is $X^\dagger F^\dagger$. However, $Q^2(X^\dagger F^\dagger)$ vanishes in the zero-momentum limit. The leading operator of type $Q^2(O_{3,0})$ which contributes to B_μ is $Q^2(X^\dagger F^\dagger (X^\dagger X)) = 4(X^\dagger)^2 F^\dagger F$. The Wilson coefficient of $(X^\dagger)^2 F^\dagger F$ at

one loop is given by

$$-\frac{i}{26}\tilde{c}_{(X^\dagger)^2 F^\dagger F}(s) = \xrightarrow{p} \text{Diagram 1} \xrightarrow{p} + \xrightarrow{p} \text{Diagram 2} \xrightarrow{p} + \xrightarrow{p} \text{Diagram 3} \xrightarrow{p},$$

from which we obtain

$$\tilde{c}_{(X^\dagger)^2 F^\dagger F}(0) = -\frac{\lambda^*}{4\pi^2 \lambda |X|^4},$$

again using the dispersion method of section 2.1. Thus, the leading contribution to B_μ is

$$B_\mu \approx -\frac{1}{8} \frac{\lambda_u \lambda_d \lambda^*}{16\pi^2 \lambda} \frac{|F|^2}{|X|^2}. \quad (4.5)$$

This is 12.5% of the answer in [4]. This underestimation may be attributed to the cancellations that occur for lower-dimension operators like the one in (4.4). Again, a resummation of the contributions of the operators $Q^2(X^\dagger F^\dagger (X^\dagger X)^n (F^\dagger F)^m)$, $m, n \geq 0$, should yield the full one-loop result of [4].

For the calculation of $\delta A_{u,d}$ and $\delta m_{H_{u,d}}^2$, we will first compute contributions to the OPEs $\Phi_1 \Phi_2(x) \times \Phi_1^\dagger \Phi_2^\dagger(0)$ and $\tilde{\Phi}_1 \tilde{\Phi}_2(x) \times \tilde{\Phi}_1^\dagger \tilde{\Phi}_2^\dagger(0)$, and then act with Q^2 and $Q^2 \bar{Q}^2$ according to (2.2). The leading operator of the type O_0 (as in (3.14)) is $X^\dagger X$. Its Wilson coefficient at one loop is given by

$$-\frac{i}{2}\tilde{c}_{X^\dagger X}(s) = \xrightarrow{p} \text{Diagram 1} \xrightarrow{p} + \xrightarrow{p} \text{Diagram 2} \xrightarrow{p},$$

from which we can compute

$$\tilde{c}_{X^\dagger X}(0) = -\frac{1}{16\pi^2 |X|^2}$$

by the same dispersion methods as before. With this result we obtain

$$\delta A_{u,d} \approx \frac{1}{2} \frac{|\lambda_{u,d}|^2}{16\pi^2} \frac{F^\dagger}{X^\dagger}, \quad \delta m_{H_{u,d}}^2 \approx \frac{1}{2} \frac{|\lambda_{u,d}|^2}{16\pi^2} \frac{|F|^2}{|X|^2}.$$

For $\delta A_{u,d}$ this approximation gives 50% of the full one-loop result [32], but for $\delta m_{H_{u,d}}^2$ we get an answer that at first seems incompatible with the one-loop result. Indeed, due to an accidental cancellation the one-loop result for $\delta m_{H_{u,d}}^2$ vanishes at leading order in

F/M^2 [4]. One can also understand this result from arguments using analytic continuation in superspace [8, 32]. Our result for $\delta m_{H_{u,d}}^2$ shows that the OPE, although very useful, can be misleading in such approximate computations. Of course, if the full computation within the OPE is performed, we should recover the full result $\delta m_{H_{u,d}}^2 = 0$ at leading order. We will now show this explicitly.

In order to carry out the analysis, we need to compute the Wilson coefficient of the operator $(X^\dagger X)^n$, $n \geq 1$, in the OPEs $\Phi_1 \Phi_2(x) \times \Phi_1^\dagger \Phi_2^\dagger(0)$ and $\tilde{\Phi}_1 \tilde{\Phi}_2(x) \times \tilde{\Phi}_1^\dagger \tilde{\Phi}_2^\dagger(0)$. We find

$$\tilde{c}_{(X^\dagger X)^n}(s) = \frac{1}{4\pi^2} \frac{\Gamma(2n-1)}{\Gamma(n)\Gamma(n+1)} \left(\frac{|\lambda|^2}{s} \right)^n \ln \frac{-s}{\mu^2} + \dots,$$

and we can now use (2.4) and (2.2) to obtain the full one-loop result for both $\delta A_{u,d}$ and $\delta m_{H_{u,d}}^2$ at leading order in F/M^2 . We find

$$\delta A_{u,d} = \frac{|\lambda_{u,d}|^2}{8\pi^2} \left(\sum_{n=1}^{\infty} \frac{n\Gamma(2n-1)}{2^{2n}\Gamma^2(n+1)} \right) \frac{F^\dagger}{X^\dagger} = \frac{|\lambda_{u,d}|^2}{16\pi^2} \frac{F^\dagger}{X^\dagger} \quad (4.6)$$

and

$$\delta m_{H_{u,d}}^2 = \frac{|\lambda_{u,d}|^2}{8\pi^2} \left(\sum_{n=1}^{\infty} \frac{n^2\Gamma(2n-1)}{2^{2n}\Gamma^2(n+1)} \right) \frac{|F|^2}{|X|^2}. \quad (4.7)$$

The sum in (4.6) is convergent, and (4.6) agrees precisely with the full result (at one-loop and leading order in F/M^2) from Feynman diagram computations as well as analytic continuation methods.

On the other hand, the sum in (4.7) is divergent, but it can be regulated. The most divergent part in $\frac{n^2\Gamma(2n-1)}{2^{2n}\Gamma^2(n+1)} = \frac{1}{16\sqrt{\pi}} \frac{\Gamma(n-1/2)}{\Gamma(n)}$ is $\frac{1}{16\sqrt{\pi}} \frac{1}{\sqrt{n}}$, and we know from ζ -function regularization that

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} = \zeta(1/2). \quad (4.8)$$

Furthermore, subtracting the most divergent part from the full sum and regularizing with a z^n we find

$$\sum_{n=1}^{\infty} \left(\frac{\Gamma(n-1/2)}{\Gamma(n)} - \frac{1}{\sqrt{n}} \right) z^n = \frac{\sqrt{\pi}z}{\sqrt{1-z}} - \text{Li}_{1/2}(z), \quad (4.9)$$

where $\text{Li}_n(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^n}$ is the polylogarithm function. But (4.9) has a well defined limit as $z \rightarrow 1$:

$$\lim_{z \rightarrow 1} \left(\frac{\sqrt{\pi}z}{\sqrt{1-z}} - \text{Li}_{1/2}(z) \right) = -\zeta(1/2). \quad (4.10)$$

Adding (4.8) and (4.9) and using (4.10) we thus obtain $\delta m_{H_{u,d}}^2 = 0$ at leading order and one loop. This agrees with [4] and further elucidates the power of the OPE in this context.

Regarding the parameters in $V_{\text{Higgs}}^{(\text{other})}$, it is straightforward to see using our methods that they are suppressed relative to those in $V_{\text{Higgs}}^{(\text{soft})}$. We will not show this explicitly here.

4.1 A comment on the OPE with unrenormalized operators

Before we conclude this section, let us elaborate on a feature that may be of interest. Consider, as we have so far, the OPE in momentum space,

$$i \int d^4x e^{-ip \cdot x} \mathcal{O}_i(x) \mathcal{O}_j(0) = \sum_k \tilde{c}_{ij}^k(p^2) \mathcal{O}_k(0). \quad (4.11)$$

In computations of the coefficients $\tilde{c}_{ij}^k$, we found that the introduction of an arbitrary normalization point μ (not to be confused with the μ parameter) was necessary in order to avoid IR divergences, as in $\tilde{c}_{X^\dagger F^\dagger}$ in (4.2) for example. In other words, we found that $\tilde{c}_{ij}^k$ were actually functions of p^2 and μ^2 . This is a known consequence of the splitting of UV and IR physics inherent in the OPE [31]. Of course the μ -dependence is also there in the right-hand side of the OPE when renormalized operators are used.

Now, note that if we use bare operators in the left-hand side of the OPE, then there is no μ -dependence in the corresponding (unrenormalized) expectation value of the operator product. Such dependence, therefore, has to disappear from the right-hand side of the OPE as well. To see this cancellation of the renormalization-point dependence, one needs to consider operator mixing and the μ -dependence of operators in the right-hand side of the OPE.

To illustrate this, let us consider the operators $X^\dagger F^\dagger$ and $X^\dagger \Phi \tilde{\Phi} \equiv X^\dagger (\Phi_1 \tilde{\Phi}_1 + \Phi_2 \tilde{\Phi}_2)$. Clearly, both of them appear in the right-hand side of the OPE $Q^\alpha(\Phi_1 \Phi_2(x)) \times Q_\alpha(\tilde{\Phi}_1 \tilde{\Phi}_2(0))$, and they mix under renormalization since they have the same classical scaling dimension and quantum numbers. As they have appeared up to now, these operators have an implicit μ -dependence. What we have neglected up to now, however, is that these operators mix under renormalization. At one loop, and without taking into account wave-function renormalization (which is unnecessary for the point being made here), we have

$$(X^\dagger \Phi \tilde{\Phi}(0))_{\text{Bare}} = X^\dagger \Phi \tilde{\Phi}(0) + \frac{\lambda^*}{8\pi^2} \ln \frac{\Lambda^2}{\mu^2} X^\dagger F^\dagger(0), \quad (4.12)$$

where Λ is the UV cutoff, as computed from the graph

which is both UV and IR divergent and so the integration over virtual momenta is cut-off at Λ from above and at μ from below. Although the vev of $X^\dagger \Phi \tilde{\Phi}$ is finite, that of $(X^\dagger \Phi \tilde{\Phi})_{\text{Bare}}$ is infinite. Now, since the tree-level Wilson coefficient of $X^\dagger \Phi \tilde{\Phi}$ in the OPE $Q^\alpha(\Phi_1 \Phi_2(x)) \times Q_\alpha(\tilde{\Phi}_1 \tilde{\Phi}_2(0))$ is given by

$$-i\tilde{c}_{X^\dagger \Phi \tilde{\Phi}}(s) = -2 \times \begin{array}{c} \begin{array}{c} \xrightarrow{p} \bigotimes \end{array} \begin{array}{c} \xrightarrow{\lambda^*} \bigotimes \end{array} \xrightarrow{p} \\ \begin{array}{c} \downarrow \\ \Phi_{1,2} \end{array} \quad \begin{array}{c} \downarrow \\ \tilde{\Phi}_{1,2} \end{array} \end{array} = -\frac{4i\lambda^*}{s},$$

we see that the μ in (4.2) is indeed replaced by Λ via operator mixing between $X^\dagger F^\dagger$ and $X^\dagger \Phi \tilde{\Phi}$, at this order in λ , when $X^\dagger \Phi \tilde{\Phi}$ is substituted by $(X^\dagger \Phi \tilde{\Phi})_{\text{Bare}}$ via (4.12). Similar replacements $\mu \rightarrow \Lambda$ in physical logarithms of Wilson coefficients will occur order by order in perturbation theory. All other explicit μ -dependence in the ξ -expansion of (4.1), which is regularization-scheme dependent, will similarly disappear when the right-hand side of the OPE is written in terms of the bare operators.

5 Comments on phenomenology

From the general results obtained in section 3, we can make various comments about possible phenomenological consequences. The generality of the OPE formalism has a great advantage in that it provides a powerful organizing principle. It not only leads to an understanding of known phenomenological problems from a different (possibly deeper) perspective, but also provides a unifying framework for describing the various proposals for these problems.

To see how this works in more detail, let us look at the μ/B_μ [4] and A/m_H^2 [8] problems, which in essence are manifestations of the problem of achieving phenomenologically viable electroweak symmetry breaking (EWSB). These problems arise in many models with Higgs-messenger interactions trying to generate the μ and A terms, as such models also tend to produce B_μ and m_H^2 terms which are too large to be viable for EWSB.⁹ The results in section 3 allow us to understand these problems from a different perspective. For instance, we saw in (3.6) that operators $O_{3,0}$, $Q^2 \bar{Q}^2 O_{3,0}$ and $Q\sigma_\mu \bar{Q} O_{3,1}^\mu$ contribute to μ , in contrast to only $Q^2 O_{3,0}$ contributing to B_μ (see (3.12)). In simple models, as in the weakly coupled example in section 4, only $O_{3,0}$ contributes to μ ,¹⁰ while $Q^2 O_{3,0}$ contributes to B_μ . Now, since it is assumed that μ is forbidden in the supersymmetric limit, it usually arises at order F , i.e. $\langle O_{3,0} \rangle \propto F$. Then, if the supersymmetry breaking dynamics is essentially trivial (as in a spurion model), one obtains parametrically $\langle Q^2 O_{3,0} \rangle \propto F \langle O_{3,0} \rangle \propto F^2$. Finally, in weakly coupled models B_μ is typically generated at the same loop order as that of μ , so that the Wilson coefficients $\hat{c}_{B_\mu} \simeq \hat{c}_\mu \sim 1/16\pi^2$. This gives rise to the well-known problematic relation

$$\frac{B_\mu}{|\mu|^2} = \frac{\hat{c}_{B_\mu} \langle Q^2 O_{3,0} \rangle}{|\hat{c}_\mu|^2 |\langle O_{3,0} \rangle|^2} \sim 16\pi^2. \quad (5.1)$$

Similar statements can be made for the A/m_H^2 problem.

The OPE results suggest possible ways of addressing these issues. We will consider the case when $\langle O_{3,0} \rangle \neq 0$, which is generically at order F (since μ is assumed to vanish in the supersymmetric limit). In this case, if $Q\sigma_\mu \bar{Q} O_{3,1}^\mu$ gets a vev as well, then that vev is of order F^2 as mentioned in section 3. This is because $\langle Q\sigma_\mu \bar{Q} O_{3,1}^\mu \rangle$ is of the form $\langle \{Q, [\bar{Q}, (\cdot)]\} \rangle$, and all such correlation functions must start at order F^2 [18]. Thus, this operator (and also $Q^2 \bar{Q}^2 O_{3,0}$) will only have a subleading effect on μ . In this case, it is also easy to see that the wrong-Higgs trilinears $a'_{u,d}$ will be suppressed compared to μ and other soft terms

⁹The A/m_H^2 problem is absent in “Minimal Gauge Mediation” (MGM) with a single spurion, since in this case m_H^2 vanishes at one-loop at leading order in F/M (this can be seen explicitly in the model of section 4). But even in this case, a “little A/m_H^2 problem” remains if one wants large A -terms [19].

¹⁰In this model, operators of type $Q^2 \bar{Q}^2 (O_{3,0;i})$ can always be written as $O_{3,0;j}$ for some i, j .

expected to be generated at order F . In this situation, one can imagine two possible ways of solving the problem of achieving viable EWSB. Let us now discuss each of these from the perspective of the OPE results obtained.

5.1 EWSB with $B_\mu/\mu^2 \lesssim 1$ and $m_{H_{u,d}}^2/A_{u,d}^2 \lesssim 1$

From the discussion in the previous sections, we know that the μ parameter is given predominantly by $\langle O_{3,0} \rangle$, while the B_μ parameter is given by $\langle Q^2 O_{3,0} \rangle$. Similarly, $A_{u,d}$ is given by $\langle Q^2 O_0^{u,d} \rangle$ while $m_{H_{u,d}}^2$ is given by $\langle Q^2 \bar{Q}^2 O_0^{u,d} \rangle$. An interesting feature of the OPE results is that the operators $O_{3,0}$ and $O_0^{u,d}$ are completely independent on each other in general. Also, for a strongly coupled extra sector, the vevs of operators are determined by strong infrared dynamics and cannot be computed in general. Finally, for a strongly coupled sector with nontrivial supersymmetry breaking dynamics (that is not captured by a single spurion), naïve parametrics such as $\langle Q^2 O_{3,0} \rangle \propto F \langle O_{3,0} \rangle \propto F^2$, giving rise to the problematic relation (5.1), need not hold. Thus, with nontrivial supersymmetry breaking dynamics, it is possible to have viable EWSB with $B_\mu/\mu^2 \lesssim 1$ and $m_{H_{u,d}}^2/A_{u,d}^2 \lesssim 1$, as long as

$$\frac{\langle Q^2 O_{3,0} \rangle}{|\langle O_{3,0} \rangle|^2} \simeq \frac{|\hat{c}_\mu|^2}{\hat{c}_{B_\mu}}, \quad \frac{\langle Q^2 \bar{Q}^2 O_0^{u,d} \rangle}{|\langle Q^2 O_0^{u,d} \rangle|^2} \simeq \frac{|\hat{c}_{A_{u,d}}|^2}{\hat{c}_{m_{H_{u,d}}^2}}, \quad (5.2)$$

respectively. The relations (3.13) and (3.17) can also be used in (5.2). Note that the ratio B_μ/μ^2 and $m_{H_{u,d}}^2/A_{u,d}^2$ will be different in general as they are controlled by the dynamics of different operators $O_{3,0}$ and $O_0^{u,d}$. Finally, it is worth remembering that the parameters in the Higgs Lagrangian computed above correspond to parameters at the scale M . In order for the mechanism described above to give viable EWSB, the RG evolution of Wilson coefficients of operators to scale $\sqrt{F}$,¹¹ where the supersymmetry breaking sector fully decouples, should be negligible. For example, this could happen if all states in the extra sector have mass of order M (with supersymmetry breaking splittings $\sim \sqrt{F} \ll M$), and the renormalization from M to $\sqrt{F}$ from the supersymmetry breaking sector is trivial.¹²

There exists another possibility, however, *viz.* that the relations between the Higgs parameters at scale M are given by the naïve expressions such as (5.1), but there is strong renormalization of Wilson coefficients of operators relevant to μ/B_μ and A/m_H^2 between M and $\sqrt{F}$. This mechanism, which has been known for quite some time, is that of conformal sequestering [33–38]. Within the OPE formalism, this mechanism can be understood as follows. In order to describe the flow from M to $\sqrt{F}$, one has to integrate out states with mass of order M , and write down an effective theory in terms of the light degrees of freedom with some coefficients. These coefficients can be obtained by matching to the full theory at the scale M and must then be RG-evolved to lower scales. If this resulting effective theory is itself an approximately superconformal field theory (*different* from the UV superconformal theory in general), then the coefficients of the operators for the Higgs parameters in the effective theory will receive a power-law running.

¹¹We have assumed $F < M^2$.

¹²This has been effectively assumed in the branch-cut structure in section 2.1. Also, note that renormalization of the parameters from *visible* sector states is still present below the scale M .

In general, the matching on to an effective theory at the scale $\simeq M$ is model-dependent. However, one framework, in which the matching is tractable, is that of the case of GMHM [19], in which it is assumed that the extra sector factorizes into a messenger sector and a supersymmetry breaking (hidden) sector, coupled by a small dimensionless coupling κ of the form

$$W \supset \kappa \Lambda^{-(\Delta_{\mathcal{O}_h} + \Delta_{\mathcal{O}_m} - 3)} \mathcal{O}_h \mathcal{O}_m.$$

In this case it is possible to integrate out operators $\mathcal{O}_m$ and consider the effective theory with $\mathcal{O}_h, \mathcal{O}_h^\dagger$ coupled to the Higgs fields. Then, the usual expressions for conformal sequestering of coefficients of the relevant operators can be derived [19]. To complete this section, it is worth noting that the case of extreme sequestering is disfavored both phenomenologically [39, 40] and theoretically [41]; see detailed discussion in [42]. However, [42] shows that it is possible to obtain viable electroweak symmetry breaking with mild sequestering: $B_\mu/\mu^2 \lesssim 1$ and $m_{H_u}^2/A_u^2 \lesssim 1$, and suitable Wilson coefficients.

5.2 EWSB with “lopsided” gauge mediation

Another mechanism for obtaining viable electroweak symmetry breaking is via what is known as “lopsided” gauge mediation [43]. In this class of models, viable EWSB is possible even with $B_\mu \gg \mu^2$ as long as one also has $m_{H_u}^2 \ll B_\mu \ll m_{H_d}^2$. It was argued in [44] that this can be arranged if the couplings λ_u, λ_d satisfy $\lambda_u \ll \lambda_d$ (for the analysis in this paper to be valid, both couplings should still be perturbative). Furthermore, it was shown in [45] that such a setup can be naturally realized within a SQCD-like supersymmetry breaking sector in which one of the Higgses (H_d) in the MSSM mixes strongly with the supersymmetry-breaking sector and is composite. Therefore, the Higgs sector in this case is “hybrid,” with H_u being an elementary field while H_d being composite. Note that in this case the RG-evolution of the Wilson coefficients of the various Higgs parameters from M to $\sqrt{F}$ is negligible.

The OPE results obtained in this paper suggest more general possibilities for obtaining the pattern $B_\mu \gg \mu^2$ and $m_{H_u}^2 \ll B_\mu \ll m_{H_d}^2$, *in addition* to the $\lambda_u \ll \lambda_d$ possibility above. This is because of the same reason as mentioned earlier: the vevs of operators appearing in the OPE for the various Higgs parameters are determined by infrared dynamics in general, and the naïve parametric relations between different Higgs parameters may not hold. For example, even if λ_u and λ_d are comparable to each other, it is possible to obtain $m_{H_u}^2 \ll B_\mu \ll m_{H_d}^2$ if

$$\hat{c}_{m_{H_u}^2} \langle Q^2 \bar{Q}^2 O_0^u \rangle \ll \hat{c}_{B_\mu} \langle Q^2 O_{3,0} \rangle \ll \hat{c}_{m_{H_d}^2} \langle Q^2 \bar{Q}^2 O_0^d \rangle. \quad (5.3)$$

In addition, it is also possible to have $|A_u|^2 > m_{H_u}^2$ if $\frac{\langle Q^2 \bar{Q}^2 O_0^u \rangle}{| \langle Q^2 O_0^u \rangle |^2} < \frac{|\hat{c}_{A_u}|^2}{\hat{c}_{m_{H_u}^2}^2}$ so that the Higgs quartic coupling gets nontrivial radiative contributions from visible sector superpartners such as stops. Note that the coupling (1.1) will also provide a source of extra contributions to the Higgs quartic couplings, and it would be very interesting to compute the general form of these corrections.

6 Summary and future directions

In this work we have developed a systematic and general formalism to compute the parameters in the effective Higgs Lagrangian to quadratic order, in a broad class of supersymmetric frameworks in which the Higgs fields in the visible sector couple to another sector via the superpotential (1.1). It is assumed that the additional sector is superconformal in the UV but develops a mass gap $\sim M$ and supersymmetry breaking splitting $\sim \sqrt{F}$ with F/M not far from the TeV scale. The primary technique used to compute the Higgs parameters is that of the operator product expansion (OPE). The results obtained within this formalism are completely general within the class of frameworks described above, and can be applied even when the additional sector is strongly coupled. The formalism provides a deeper insight into problems affecting simple models of supersymmetry breaking and mediation, and provides new possibilities for solutions. The underlying reason for the existence of these new possibilities which have not been considered before, is the fact that OPE methods imply that different types of operators contribute to different Higgs parameters in general. Furthermore, since the vevs of these operators are determined by infrared dynamics, simple parametric relations between different Higgs parameters, which hold in weakly coupled or spurion based models, may not hold in general.

There are a few interesting directions that are worth exploring in the future. One interesting direction would be to construct realistic models of dynamical supersymmetry breaking and mediation, and apply OPE techniques to explicitly compute the Wilson coefficients and the relevant Higgs parameters. It would also be worth exploring the computation of the quartic terms in the effective Higgs Lagrangian, due to the presence of the superpotential couplings (1.1). This would be crucial for computing the physical Higgs boson masses and mixing angles, and as such is directly relevant for phenomenology. In particular, it is straightforward to see that the contribution to the Higgs quartic couplings, for example the coefficient λ_{H_u} in $\lambda_{H_u}|H_u|^4$, due to the couplings (1.1), will be determined by a four-point function of the form

$$|\lambda_u|^4 \left\langle \int d^4y d^4z d^4w Q^2 O_u(0) \bar{Q}^2 O_u^\dagger(y) Q^2 O_u(z) \bar{Q}^2 O_u^\dagger(w) \right\rangle.$$

Within the OPE formalism described above, a possible way to compute such quartic couplings is by expanding these four-point functions in conformal blocks [46], or their supersymmetric extensions, the superconformal blocks for $\mathcal{N} = 1$ supersymmetry [26]. It would be interesting to develop this approach to the quartic couplings in greater detail.

Another interesting direction to explore would be to apply similar OPE methods to compute terms in the effective Higgs Lagrangian for the case when the Higgs fields couple to a SM gauge-singlet $\mathcal{S}$, via $\mathcal{S}H_u H_d$ in the superpotential. Although such operators have been studied extensively in the literature (mostly in weakly-coupled settings), OPE results may shed an interesting light on the physics in more general cases that would be hard to see otherwise.

Finally, it would be interesting to extend our analysis of Higgs parameters to include the possible contributions of poles appearing in the two-point functions of the additional

sector, similar to what was done for current correlators in [47–51]. In this case one could derive sum-rules relating the contributions of these poles to the Higgs parameters, obtaining results that are applicable to an even more general class of models. We hope to study these and related directions in the near future.

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A Two-point functions of superconformal and conformal primaries

As we already saw in (3.1), in an $\mathcal{N} = 1$ superconformal theory the two-point function of an operator $\mathcal{O}^I$ with its conjugate $\bar{\mathcal{O}}^{\bar{I}}$ is given by

$$\langle \mathcal{O}^I(z_1) \bar{\mathcal{O}}^{\bar{I}}(z_2) \rangle = C_{\mathcal{O}} \frac{\mathcal{I}^{I\bar{I}}(x_{1\bar{2}}, x_{\bar{1}2})}{x_{1\bar{2}}^{2\bar{q}} x_{\bar{1}2}^{2q}}.$$

Now, one can carry out the θ -expansion of both sides of (3.1) and match the various powers of $\theta_{1,2}, \bar{\theta}_{1,2}$ between the two sides to read off two-point functions of the various components of $\mathcal{O}^I$. This achieves a projection of the superconformal two-point function to the conformal subgroup. Nevertheless, this projection is contaminated by the presence of conformal descendants in the θ -expansion of $\mathcal{O}^I$. This contamination has to be removed in order to obtain a projection to conformal primaries.

To illustrate these points, let us work out explicitly the case of a general scalar $\mathcal{N} = 1$ superfield operator $\mathcal{O}$. In this case (3.1) becomes

$$\langle \mathcal{O}(z_1) \bar{\mathcal{O}}(z_2) \rangle = \frac{C_{\mathcal{O}}}{x_{1\bar{2}}^{2\bar{q}} x_{\bar{1}2}^{2q}}. \quad (\text{A.1})$$

Now, the Baker-Campbell-Hausdorff formula and the supersymmetry algebra imply that

$$e^{i\theta Q + i\bar{\theta} \bar{Q}} = e^{i\theta Q} e^{i\bar{\theta} \bar{Q}} e^{\theta P \cdot \sigma \bar{\theta}},$$

and expanding the exponentials it is straightforward to evaluate

$$\begin{aligned} e^{i\theta Q + i\bar{\theta} \bar{Q}} &= 1 + i\theta Q + i\bar{\theta} \bar{Q} + \frac{1}{2} \theta \sigma^\mu \bar{\theta} (Q \sigma_\mu \bar{Q} + 2P_\mu) + \frac{1}{4} \theta^2 Q^2 + \frac{1}{4} \bar{\theta}^2 \bar{Q}^2 \\ &\quad - \frac{i}{4} \theta^2 \bar{\theta}^{\dot{\alpha}} (Q^2 \bar{Q}_{\dot{\alpha}} - 2Q^\alpha \sigma_{\alpha\dot{\alpha}}^\mu P_\mu) + \frac{i}{4} \bar{\theta}^2 \theta^\alpha (\bar{Q}^2 Q_\alpha + 2\sigma_{\alpha\dot{\alpha}}^\mu \bar{Q}^{\dot{\alpha}} P_\mu) \\ &\quad + \frac{1}{16} \theta^2 \bar{\theta}^2 (Q^2 \bar{Q}^2 - 4P^2 - 4Q \sigma^\mu \bar{Q} P_\mu). \end{aligned} \quad (\text{A.2})$$

This implies that $\mathcal{O}(z) \equiv e^{i\theta Q + i\bar{\theta}\bar{Q}}O(x)$ can be expanded as

$$\begin{aligned}\mathcal{O}(z) = & O + i\theta QO + i\bar{\theta}\bar{Q}O + \frac{1}{2}\theta\sigma^\mu\bar{\theta}([Q\sigma_\mu\bar{Q}O]_p + c_1 P_\mu O) + \frac{1}{4}\theta^2 Q^2 O + \frac{1}{4}\bar{\theta}^2 \bar{Q}^2 O \\ & - \frac{i}{4}\theta^2\bar{\theta}^{\dot{\alpha}}([Q^2\bar{Q}_{\dot{\alpha}}O]_p - c_2\sigma_{\alpha\dot{\alpha}}^\mu P_\mu Q^\alpha O) + \frac{i}{4}\bar{\theta}^2\theta^\alpha([\bar{Q}^2 Q_\alpha O]_p - c_3\sigma_{\alpha\dot{\alpha}}^\mu P_\mu \bar{Q}^{\dot{\alpha}} O) \\ & + \frac{1}{16}\theta^2\bar{\theta}^2([Q^2\bar{Q}^2 O]_p - c_4 P^2 O - c_5 P_\mu[Q\sigma^\mu\bar{Q}O]_p),\end{aligned}\quad (\text{A.3})$$

where $[\cdot]_p$ denotes a conformal primary operator, and $c_{1,\dots,5}$ are coefficients we need to evaluate, and that will allow us to see which combinations of $Q\sigma_\mu\bar{Q}O$ and $P_\mu O$, $\bar{Q}^2 Q_\alpha O$ and $\sigma_{\alpha\dot{\alpha}}^\mu P_\mu \bar{Q}^{\dot{\alpha}} O$, $Q^2\bar{Q}_{\dot{\alpha}}O$ and $\sigma_{\alpha\dot{\alpha}}^\mu P_\mu Q^\alpha O$, and $Q^2\bar{Q}^2 O$, $P^2 O$ and $P_\mu[Q\sigma^\mu\bar{Q}]_p$ are conformal primaries. Note that some components in the expansion are already conformal primaries; for example $[Q^2 O]_p = Q^2 O$.

The coefficients $c_{1,\dots,5}$ can be evaluated by expanding both sides of (A.1) using (A.3) on the left-hand side. For example, from the $\theta_1\bar{\theta}_1$ component of $\langle\mathcal{O}(z_1)\bar{\mathcal{O}}(z_2)\rangle$ we can find

$$c_1 = 2\frac{q - \bar{q}}{q + \bar{q}},$$

and comparing (A.3) with (A.2) we find that

$$[Q\sigma_\mu\bar{Q}O]_p = Q\sigma_\mu\bar{Q}O + 4\frac{\bar{q}}{q + \bar{q}}P_\mu O,$$

which, as expected, is zero for $q = 0$ or $\bar{q} = 0$. With our result for c_1 we can use the $\theta_1\bar{\theta}_1\theta_2\bar{\theta}_2$ part of (A.1) to compute

$$\langle[Q\sigma^\mu\bar{Q}O(x)]_p[Q\sigma^\nu\bar{Q}O(0)]_p^\dagger\rangle = 2^5 C_O \frac{q\bar{q}(q + \bar{q} + 1)}{q + \bar{q}} \frac{I^{\mu\nu}(x)}{x^{2(q+\bar{q}+1)}}, \quad I^{\mu\nu}(x) = \eta^{\mu\nu} - 2\frac{x^\mu x^\nu}{x^2}, \quad (\text{A.4})$$

which has the required form, $\sim I^{\mu\nu}$, dictated by conformal invariance, and the correct behavior for chiral ($\bar{D}_{\dot{\alpha}}\Phi = 0$, $\bar{q} = 0$) and anti-chiral ($D_\alpha\Phi = 0$, $q = 0$) superfields. For a linear superfield ($D^2\mathcal{J} = \bar{D}^2\mathcal{J} = 0$, $q = \bar{q} = 1$), whose $\theta\bar{\theta}$ component is a conserved vector current, we also get a consistency check by the fact that $\partial_\mu\langle[Q\sigma^\mu\bar{Q}O(x)]_p[Q\sigma^\nu\bar{Q}O(0)]_p^\dagger\rangle$ correctly vanishes for $q = \bar{q} = 1$.

We can also evaluate

$$c_2 = 2\frac{q - \bar{q} - 1}{q + \bar{q} - 1}, \quad c_3 = 2\frac{q - \bar{q} + 1}{q + \bar{q} - 1},$$

and thus obtain

$$\langle[Q^2\bar{Q}_{\dot{\alpha}}O(x)]_p[\bar{Q}^2 Q_\alpha O^\dagger(0)]_p\rangle = -2^8 i C_O \frac{q\bar{q}(q-1)(q+\bar{q}+1)}{q+\bar{q}-1} \frac{x_{\alpha\dot{\alpha}}}{x^{2(q+\bar{q}+2)}}, \quad (\text{A.5a})$$

$$\langle[\bar{Q}^2 Q_\alpha O(x)]_p[Q^2\bar{Q}_{\dot{\alpha}}O^\dagger(0)]_p\rangle = -2^8 i C_O \frac{q\bar{q}(\bar{q}-1)(q+\bar{q}+1)}{q+\bar{q}-1} \frac{x_{\alpha\dot{\alpha}}}{x^{2(q+\bar{q}+2)}}, \quad (\text{A.5b})$$

where

$$[Q^2\bar{Q}_{\dot{\alpha}}O]_p = Q^2\bar{Q}_{\dot{\alpha}}O - 4\frac{\bar{q}}{q+\bar{q}-1}\sigma_{\alpha\dot{\alpha}}^\mu P_\mu Q^\alpha O, \quad (\text{A.6a})$$

$$[\bar{Q}^2 Q_\alpha O]_p = \bar{Q}^2 Q_\alpha O + 4\frac{q}{q+\bar{q}-1}\sigma_{\alpha\dot{\alpha}}^\mu P_\mu \bar{Q}^{\dot{\alpha}} O. \quad (\text{A.6b})$$

It is easy to see that (A.6a) becomes zero for $q = 0$, $\bar{q} = 0$ or $q = 1$, and correspondingly (A.6b) becomes zero when $\bar{q} = 0$, $q = 0$ or $\bar{q} = 1$. Expressions (A.5) correctly become zero if $\mathcal{O}$ is a chiral, antichiral, or linear superfield. Furthermore, as expected, (A.5a) and (A.5b) become zero for a scalar superfield $\mathcal{S}$ satisfying $D^2\mathcal{S} = 0$ or $\bar{D}^2\mathcal{S} = 0$ respectively.¹³

With a little bit more work we can compute

$$c_4 = 4 \frac{(q - \bar{q})^2 - q - \bar{q}}{(q + \bar{q})(q + \bar{q} - 1)}, \quad c_5 = 4 \frac{q - \bar{q}}{q + \bar{q} - 2},$$

which allow us to find

$$\langle [Q^2 \bar{Q}^2 O(x)]_p [Q^2 \bar{Q}^2 O(0)]_p^\dagger \rangle = 2^{12} C_O \frac{q\bar{q}(q-1)(\bar{q}-1)(q+\bar{q})(q+\bar{q}+1)}{(q+\bar{q}-1)(q+\bar{q}-2)} \frac{1}{x^{2(q+\bar{q}+2)}}, \quad (\text{A.7})$$

where

$$[Q^2 \bar{Q}^2 O]_p = Q^2 \bar{Q}^2 O - 2^4 \frac{\bar{q}(\bar{q}-1)}{(q+\bar{q}-1)(q+\bar{q}-2)} P^2 O - 8 \frac{\bar{q}-1}{q+\bar{q}-2} P_\mu Q^{\sigma\mu} \bar{Q} O,$$

which correctly goes to zero for $\bar{q} = 0$ or $\bar{q} = 1$, as well as $q = 0$ or $q = 1$. As expected, (A.7) becomes zero for a chiral and an antichiral superfield, as well as a superfield $\mathcal{S}$ satisfying $D^2\mathcal{S} = 0$ or $\bar{D}^2\mathcal{S} = 0$. For a linear superfield it also becomes zero, despite the $q + \bar{q} - 2$ in the denominator, due to the “double” zero from $(q-1)(\bar{q}-1)$ in the numerator. In the (Δ, R) -representation the $R \rightarrow 0$ limit produces a $(\Delta-2)^2$ in the numerator which cancels the $\Delta-2$ in the denominator and thus leads to zero when $\Delta \rightarrow 2$.

For a spin-one superconformal operator $\mathcal{O}^\mu$ a similar treatment, starting from

$$\langle \mathcal{O}^\mu(z_1) \bar{\mathcal{O}}^\nu(z_2) \rangle = \frac{1}{2} C_{\mathcal{O}^\mu} \frac{\text{Tr}(\bar{\sigma}^\mu x_{1\bar{2}} \bar{\sigma}^\nu x_{2\bar{1}})}{x_{1\bar{2}}^{2\bar{q}+1} x_{2\bar{1}}^{2q+1}},$$

shows that

$$\langle [Q\sigma_\mu \bar{Q} O^\mu]_p(x) [Q\sigma_\nu \bar{Q} O^\nu]_p^\dagger(0) \rangle = 2^5 C_{\mathcal{O}^\mu} \frac{(2q-3)(2\bar{q}-3)(q+\bar{q}-2)}{(q+\bar{q}-3)} \frac{1}{x^{2(q+\bar{q}+1)}}, \quad (\text{A.8})$$

where

$$[Q\sigma_\mu \bar{Q} O^\mu]_p = Q\sigma_\mu \bar{Q} O^\mu + 4 \frac{\bar{q} - \frac{3}{2}}{q + \bar{q} - 3} P_\mu O^\mu. \quad (\text{A.9})$$

The two-point function coefficient in (A.8) correctly vanishes if $\mathcal{O}^\mu$ is the supercurrent ($D^\alpha \mathcal{J}_{\alpha\dot{\alpha}} = \bar{D}^{\dot{\alpha}} \mathcal{J}_{\alpha\dot{\alpha}} = 0$, $q = \bar{q} = \frac{3}{2}$), in which case in (A.8) we see the vanishing of the trace of the stress-energy tensor. Of course, the condition $D^\alpha \mathcal{O}_{\alpha\dot{\alpha}} = 0$ (respectively, $\bar{D}^{\dot{\alpha}} \mathcal{O}_{\alpha\dot{\alpha}} = 0$) is enough to shorten a multiplet, and in that case (A.8) is also zero since superconformal symmetry requires $q = \frac{3}{2}$ (respectively, $\bar{q} = \frac{3}{2}$).

Finally, let us mention that in the $R \rightarrow 0$ limit our expressions (A.4), (A.7), and (A.8) agree with the zero-spin limit of expressions (3.34), (3.39) and (3.35) of [26] respectively.

¹³A superfield $\mathcal{S}$ satisfying $\bar{D}^2\mathcal{S} = 0$ has twelve fermionic and twelve bosonic degrees of freedom and is called a nonminimal (or complex) scalar superfield. Such a superfield is a superconformal quasi-primary, and has $\bar{q} = 1$ and $q > 1$. Correspondingly, a superfield $\mathcal{S}$ satisfying $D^2\mathcal{S} = 0$ has $q = 1$ and $\bar{q} > 1$.

B From three-point functions to OPEs

It is well known that the three-point function of three operators contains the same information as the OPE of two of them with the third one. Indeed, a three-point function of the form $\langle \mathcal{O}_1 \mathcal{O}_2 \bar{\mathcal{O}}_3 \rangle$ has to reproduce the two-point function $\langle \mathcal{O}_3 \bar{\mathcal{O}}_3 \rangle$ if the OPE $\mathcal{O}_1 \times \mathcal{O}_2$ is used. In this appendix we will work out this correspondence explicitly for the $[Q^2 \bar{Q}^2 O_{3,0}]_p$ contribution to (3.4).

We will thus concentrate on the OPE $Q^\alpha O_u(x) \times Q_\alpha O_d(0)$, which we will recover from the three-point function (3.2). There are multiple structures for the $\bar{t}^{\bar{t}}$ in (3.2), but here we will only work out the term $[Q^2 \bar{Q}^2 O_{3,0}]_p$ in (3.4), which arises from Solution (III) for $\ell = 0$. The coefficient we will compute is $c_{\mu;4}^i$ in (3.4). Other terms in this and other OPEs in section 3 can be obtained similarly.

To start, we have to perform the $\theta_{1,2,3}$ and $\bar{\theta}_{1,2,3}$ expansion of the superconformal three-point function $\langle \mathcal{O}_u(z_1) \mathcal{O}_d(z_2) \mathcal{O}_{3,0}(z_3) \rangle$ and go to order $\theta_1 \theta_2 \theta_3^2 \bar{\theta}_3^2$. In practice this is a very lengthy computation, but straightforward enough to be coded in *Mathematica*. As we can see from (A.3), the result of this computation is the combination

$$\begin{aligned} & \frac{1}{2^5} \langle Q^\alpha O_u(x_1) Q_\alpha O_d(x_2) [Q^2 \bar{Q}^2 O_{3,0}]_p^\dagger(x_3) \rangle \\ & - \frac{c_4}{2^5} \langle Q^\alpha O_u(x_1) Q_\alpha O_d(x_2) (P^2 O_{3,0})^\dagger(x_3) \rangle \\ & + \frac{c_5}{2^5} \langle Q^\alpha O_u(x_1) Q_\alpha O_d(x_2) (P_\mu [Q \sigma^\mu \bar{Q} O_{3,0}]_p)^\dagger(x_3) \rangle \end{aligned} \quad (\text{B.1})$$

of three-point functions involving both primaries and descendants.

Now, in order to compute the Wilson coefficient of $[Q^2 \bar{Q}^2 O_{3,0}]_p$ in the OPE $Q^\alpha O_u \times Q_\alpha O_d$, we need to substitute

$$Q^\alpha O_u(x_1) Q_\alpha O_d(x_2) \sim w(x_{12}^2) [Q^2 \bar{Q}^2 O_{3,0}]_p(x_2) \quad (\text{B.2})$$

in (B.1), and compute w using the known result (A.7) for the resulting two-point function. Obviously, the last two terms in (B.1) complicate the computation. Indeed, there are other contributions to the OPE $Q^\alpha O_u \times Q_\alpha O_d$, besides the one in (1.2), which also need to be evaluated since they result in nonzero two-point functions with the conformal descendants $(P^2 O_{3,0})^\dagger$ and $(P_\mu [Q \sigma^\mu \bar{Q} O_{3,0}]_p)^\dagger$. Since the θ -expansion of the superconformal three-point function yields (B.1), the Wilson coefficients of these extra contributions are necessary in order to obtain $w(x_{12}^2)$.

Therefore, we need to compute the Wilson coefficients $w_{1,\dots,4}$ in

$$\begin{aligned} Q^\alpha O_u(x_1) Q_\alpha O_d(x_2) \sim & w_1(x_{12}^2) \frac{x_{12}^\mu x_{12}^\nu}{x_{12}^2} i^2 \partial_\mu \partial_\nu O_{3,0} + w_2(x_{12}^2) i^2 \partial^2 O_{3,0} \\ & + w_3(x_{12}^2) \frac{x_{12}^\mu x_{12}^\nu}{x_{12}^2} i \partial_\mu [Q \sigma_\nu \bar{Q} O_{3,0}]_p + w_4(x_{12}^2) i \partial_\mu [Q \sigma^\mu \bar{Q} O_{3,0}]_p, \end{aligned}$$

which in turn require orders $\theta_1 \theta_2$ and $\theta_1 \theta_2 \theta_3 \bar{\theta}_3$ of the three-point function (3.2) for their evaluation. This is a tedious computation, but in the end we find that $w(x_{12}^2) =$

$\tilde{w}x_{12}^{\Delta_{\mathcal{O}_{3,0}}-\Delta_{\mathcal{O}_u}-\Delta_{\mathcal{O}_d}+1}$ with

$$\tilde{w} = \frac{1}{2^4} \frac{\lambda_{\mathcal{O}_u \mathcal{O}_d \mathcal{O}_{3,0}}}{C_{\mathcal{O}_{3,0}}} \frac{(\Delta_{\mathcal{O}_u} - \Delta_{\mathcal{O}_d} - \Delta_{\mathcal{O}_{3,0}} - 1)(\Delta_{\mathcal{O}_u} - \Delta_{\mathcal{O}_d} + \Delta_{\mathcal{O}_{3,0}} + 1)}{\Delta_{\mathcal{O}_{3,0}}(\Delta_{\mathcal{O}_{3,0}} + 1)(\Delta_{\mathcal{O}_u} + \Delta_{\mathcal{O}_d} - \Delta_{\mathcal{O}_{3,0}} - 1)(\Delta_{\mathcal{O}_u} + \Delta_{\mathcal{O}_d} - \Delta_{\mathcal{O}_{3,0}} - 3)},$$

where $C_{\mathcal{O}_{3,0}}$ is the coefficient in the superconformal two-point function $\langle \mathcal{O}_{3,0}(z_1) \bar{\mathcal{O}}_{3,0}(z_2) \rangle$, and $\lambda_{\mathcal{O}_u \mathcal{O}_d \mathcal{O}_{3,0}}$ the coefficient in the superconformal three-point function $\langle \mathcal{O}_u(z_1) \mathcal{O}_d(z_2) \bar{\mathcal{O}}_{3,0}(z_3) \rangle$. As expected, our answer for w is symmetric under $\mathcal{O}_u \leftrightarrow \mathcal{O}_d$.

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